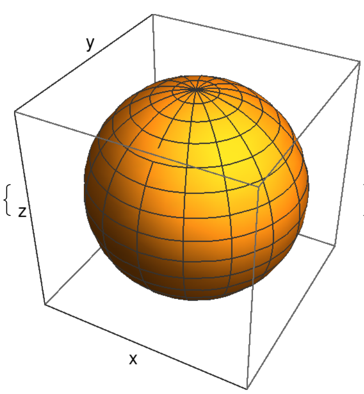
# Global parameters

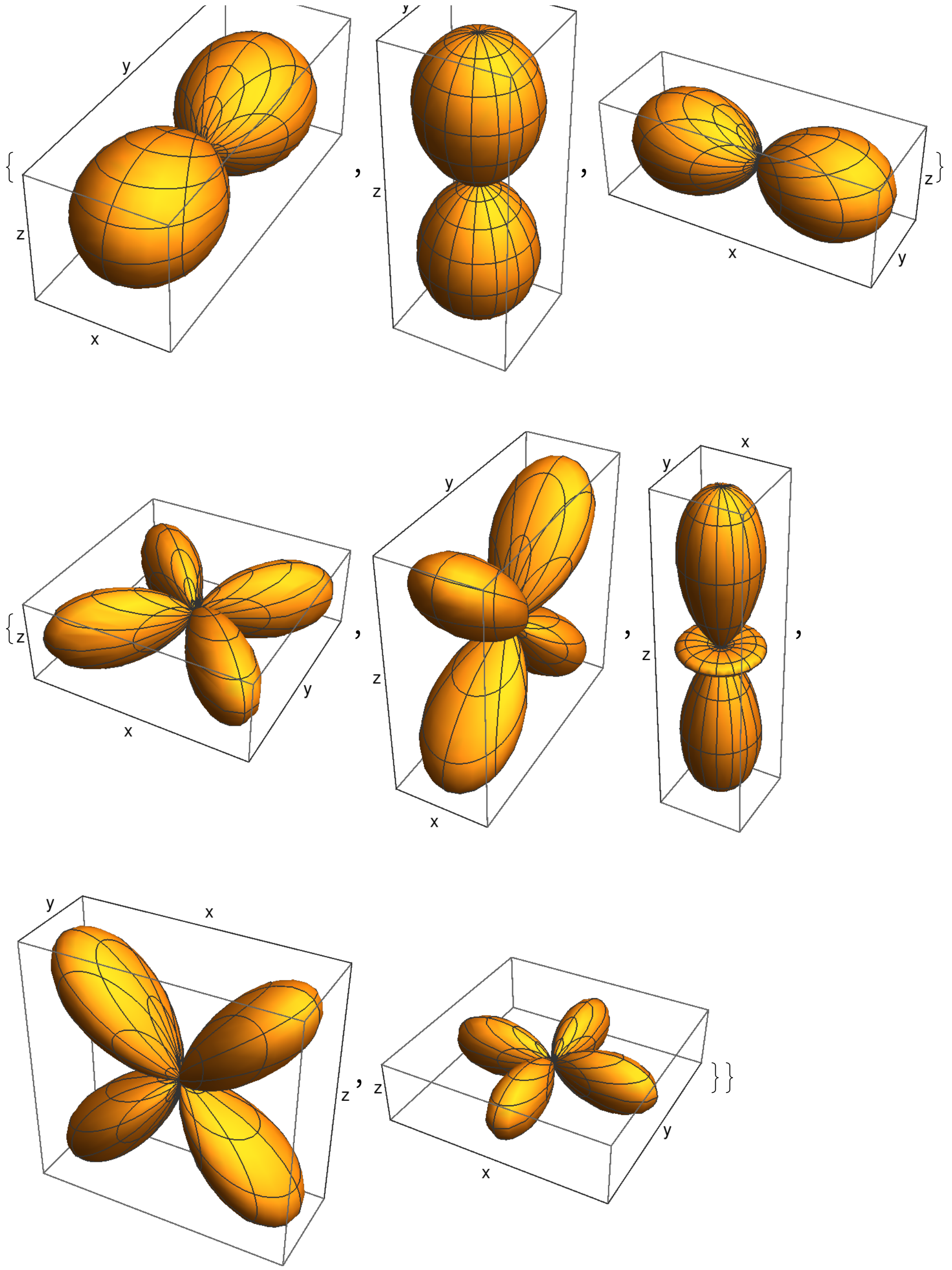
```
In[•]:= r = Sqrt[x^2 + y^2 + z^2];
rbm = {x, y, z};
tospherical = {x → r * Sin[θ] Cos[ϕ],
   y → r * Sin[θ] Sin[ϕ], z → r * Cos[θ], x^2 + y^2 + z^2 → r^2, Sqrt[x^2 + y^2 + z^2] → r};
tocartesian = {Sin[θ] → Sqrt[x^2 + y^2]/r, Cos[θ] → z/r,
   Sin[ϕ] → y/Sqrt[x^2 + y^2], Cos[ϕ] → x/Sqrt[x^2 + y^2]};
L[f_] := -I * {x, y, z} × Grad[f, {x, y, z}]
```

```
In[•]:= Y[l_, m_, θ_: θ, ϕ_: ϕ] :=
 If[m == 0, SphericalHarmonicY[l, m, θ, ϕ],
   If[m < 0, I * 1/Sqrt[2]
     (SphericalHarmonicY[l, -m, θ, ϕ] + (-1)^(Abs[m]+1) * SphericalHarmonicY[l, m, θ, ϕ]),
    1/Sqrt[2] (SphericalHarmonicY[l, -m, θ, ϕ] +
      (-1)^Abs[m] * SphericalHarmonicY[l, m, θ, ϕ])]] // FullSimplify
Yxyz[l_, m_, θ_: θ, ϕ_: ϕ] := ReplaceAll[
   TrigExpand[Y[l, m, θ, ϕ]], tocartesian
  ] // Simplify
Olm[l_, m_, θ_: θ, ϕ_: ϕ] := Sqrt[(4 π)/(2 l + 1)] * r^l * Yxyz[l, m, θ, ϕ]
```

```
In[•]:= Table[SphericalPlot3D[Evaluate[Y[l, m, θ, ϕ]^2],
   {θ, 0, Pi}, {ϕ, 0, 2 π}, AxesLabel → {"x", "y", "z"}, Ticks → None,
   DisplayFunction → Identity, PlotRange → All], {l, 0, 2}, {m, -l, l}] // Short
```

Out[•]//Short= {{ 

},




# Basis

In[∘]:=
$$A = \sqrt{\frac{3}{4 * \pi}};$$
$$phix = A * \frac{x}{r};$$
$$phiy = A * \frac{y}{r};$$
$$phiz = A * \frac{z}{r};$$
```
phip = {phix, phiy, phiz}
```
Out[∘]=
$$\left\{\frac{\sqrt{\frac{3}{\pi}}\ x}{2\sqrt{x^2+y^2+z^2}}, \frac{\sqrt{\frac{3}{\pi}}\ y}{2\sqrt{x^2+y^2+z^2}}, \frac{\sqrt{\frac{3}{\pi}}\ z}{2\sqrt{x^2+y^2+z^2}}\right\}$$

# Electric Multipole

In[∘]:=
```
Q[l_, m_, basis_, basis2_, θ_: θ, ϕ_: ϕ] :=
 Module[{e = 1},
  electricpole = -e * Olm[l, m];
  Qmat = ReplaceAll[KroneckerProduct[basis, electricpole * basis2], tospherical];
  Qmat = Integrate[Qmat * Sin[θ], {θ, 0, π}, {ϕ, 0, 2 π}];
  Qmat
 ]

Q[0, 0, phip, phip] // MatrixForm
Q[2, 2, phip, phip] // MatrixForm
```
Out[∘]//MatrixForm=
$$\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

Out[∘]//MatrixForm=
$$\begin{pmatrix} -\frac{1}{5}\sqrt{3}\ (x^2+y^2+z^2) & 0 & 0 \\ 0 & \frac{1}{5}\sqrt{3}\ (x^2+y^2+z^2) & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

# Magnetic Multipole

In[•]:=
```
Clear[M, magneticpole]
M[l_, m_, basis_, basis2_, θ_:θ, ϕ_:ϕ] :=
 Module[{μB = 1},
  magneticpole[b_] := -μB * Grad[Olm[l, m], {x, y, z}].(2 * L[b])/(l + 1);
  ket := Map[magneticpole, basis2];
  Mmat = ReplaceAll[KroneckerProduct[basis*, ket], tospherical] // FullSimplify;
  Mmat = Integrate[Mmat * Sin[θ], {θ, 0, π}, {ϕ, 0, 2 π}];
  Mmat
 ]
M[1, 1, phip, phip] // MatrixForm
M[1, 0, phip, phip] // MatrixForm
```

Out[•]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & i \\ 0 & -i & 0 \end{pmatrix}$$

Out[•]//MatrixForm=

$$\begin{pmatrix} 0 & i & 0 \\ -i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

# Toroidal Multipole

トロイダルはl=2の項が活性となるが，M(l=1)と完全に比例関係にあるため問題無い.
※手計算で確認→活性でした.

In[•]:=
```
Clear[T]
Clear[troidalpole]

T[l_, m_, basis_, basis2_, θ_: θ, ϕ_: ϕ] :=
 Module[{μB = 1},
  t[b_] := {x, y, z}/(l + 1) × ((2 * L[b])/(l + 2));
  troidalpole[b_] := -μB * Grad[Olm[l, m], {x, y, z}].t[b];
  ket = Map[troidalpole, basis2];
  Tmat = ReplaceAll[KroneckerProduct[basis*, ket], tospherical] // FullSimplify;
  Tmat = Integrate[Tmat * Sin[θ], {θ, 0, π}, {ϕ, 0, 2 π}];
  Tmat
 ]
T[0, 0, phip, phip] // MatrixForm // FullSimplify
T[2, 0, phip, phip] // MatrixForm // FullSimplify
```

Out[•]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Out[•]//MatrixForm=

$$\begin{pmatrix} \frac{1}{10} i \left(x^2 + y^2 + z^2\right) & 0 & 0 \\ 0 & \frac{1}{10} i \left(x^2 + y^2 + z^2\right) & 0 \\ 0 & 0 & -\frac{1}{5} i \left(x^2 + y^2 + z^2\right) \end{pmatrix}$$

# Electric Toroidal Multipole

In[ ]:=
```
Clear[G, electrictroidal]

G[l_, m_, basis_, basis2_, θ_: θ, ϕ_: ϕ] :=
 Module[{e = 1},
  mα[b_] := (2 * L[b])/(l + 1);
  tβ[b_] := ({x, y, z})/(l + 1) × ((2 * L[b])/(l + 2));
  g[b_] := Sum[-e * mα[tβ[b][[i]]][[j]] *
     D[Olm[l, m], rbm[[i]], rbm[[j]]], {i, 1, 3}, {j, 1, 3}];
  electrictroid = Map[g, basis2];
  Gmat = ReplaceAll[
     KroneckerProduct[basis, electrictroid], tospherical] // FullSimplify;
  Gmat = Integrate[Gmat * Sin[θ], {θ, 0, π}, {ϕ, 0, 2 π}];
  Gmat
 ]
G[0, 0, phip, phip] // MatrixForm // FullSimplify
G[3, 1, phip, phip] // MatrixForm // FullSimplify
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Out[ ]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$