

GrumpyIR Dynamic Semantics

April 13, 2021

The GrumpyIR dynamic (small-step operational) semantics is given as a six-place relation between a function environment δ , a variable environment ρ , heap stores μ and μ' , and expressions e and e' , written $\delta \mid \rho \vdash e \mid \mu \longrightarrow e' \mid \mu'$, pronounced “under δ and ρ , in memory state μ , e steps to e' resulting in new state μ' ”. Formally, the dynamic semantics is taken to be the smallest relation satisfying the following transition rules, where metavariables i and n range over numbers, e over expressions, v over values, l over locations, p over function pointers, x over variables, and b over booleans or binary operators depending on context:

Variables

$$\frac{\text{E-VAR} \quad \rho(x) = v}{\delta \mid \rho \vdash x \mid \mu \longrightarrow v \mid \mu}$$

A variable x steps to a value v whenever ρ maps x to v .

Unary and binary operators

$$\begin{array}{c} \text{E-NEG} \\ \frac{b \in \{\text{true}, \text{false}\}}{\delta \mid \rho \vdash (\text{neg } b) \mid \mu \longrightarrow \neg b \mid \mu} \end{array} \qquad \begin{array}{c} \text{E-NEG1} \\ \frac{\delta \mid \rho \vdash e \mid \mu \longrightarrow e' \mid \mu'}{\delta \mid \rho \vdash (\text{neg } e) \mid \mu \longrightarrow (\text{neg } e') \mid \mu'} \end{array}$$

$$\begin{array}{c} \text{E-BINOP} \\ \frac{n_1 \ b \ n_2 = n \quad b \in \{+, *, -, /\}}{\delta \mid \rho \vdash (b \ n_1 \ n_2) \mid \mu \longrightarrow n \mid \mu} \end{array} \qquad \begin{array}{c} \text{E-BINOP1} \\ \frac{\delta \mid \rho \vdash e_1 \mid \mu \longrightarrow e'_1 \mid \mu'}{\delta \mid \rho \vdash (b \ e_1 \ n_2) \mid \mu \longrightarrow (b \ e'_1 \ n_2) \mid \mu'} \end{array}$$

$$\begin{array}{c} \text{E-BINOP2} \\ \frac{\delta \mid \rho \vdash e_2 \mid \mu \longrightarrow e'_2 \mid \mu'}{\delta \mid \rho \vdash (b \ e_1 \ e_2) \mid \mu \longrightarrow (b \ e_1 \ e'_2) \mid \mu'} \end{array}$$

Let-expressions and sequencing

E-LET

$$\frac{}{\delta \mid \rho \vdash (\text{let } x \ v_1 \ v_2) \mid \mu \longrightarrow v_2 \mid \mu}$$

E-LET1

$$\frac{\delta \mid \rho \vdash e_1 \mid \mu \longrightarrow e'_1 \mid \mu'}{\delta \mid \rho \vdash (\text{let } x \ e_1 \ e_2) \mid \mu \longrightarrow (\text{let } x \ e'_1 \ e_2) \mid \mu'}$$

E-LET2

$$\frac{\delta \mid [x \mapsto v_1] \rho \vdash e_2 \mid \mu \longrightarrow e'_2 \mid \mu'}{\delta \mid \rho \vdash (\text{let } x \ v_1 \ e_2) \mid \mu \longrightarrow (\text{let } x \ v_1 \ e'_2) \mid \mu'}$$

E-SEQ

$$\frac{}{\delta \mid \rho \vdash (\text{seq } v \ e_2) \mid \mu \longrightarrow e_2 \mid \mu}$$

E-SEQ1

$$\frac{\delta \mid \rho \vdash e_1 \mid \mu \longrightarrow e'_1 \mid \mu'}{\delta \mid \rho \vdash (\text{seq } e_1 \ e_2) \mid \mu \longrightarrow (\text{seq } e'_1 \ e_2) \mid \mu'}$$

Arrays

$$\frac{\text{E-ALLOC} \quad l \text{ is a freshly allocated store location} \quad 0 \leq n \quad \forall i, v_i = v}{\delta \mid \rho \vdash (\text{alloc } n \ v) \mid \mu \longrightarrow l \mid [l \mapsto (v_1, v_2, \dots, v_n)]\mu}$$

$$\frac{\text{E-ALLOC1} \quad \delta \mid \rho \vdash e_{size} \mid \mu \longrightarrow e'_{size} \mid \mu'}{\delta \mid \rho \vdash (\text{alloc } e_{size} \ e_{init}) \mid \mu \longrightarrow (\text{alloc } e'_{size} \ e_{init}) \mid \mu'}$$

$$\frac{\text{E-ALLOC2} \quad \delta \mid \rho \vdash e_{init} \mid \mu \longrightarrow e'_{init} \mid \mu'}{\delta \mid \rho \vdash (\text{alloc } n \ e_{init}) \mid \mu \longrightarrow (\text{alloc } n \ e'_{init}) \mid \mu'}$$

$$\frac{\text{E-SET} \quad 0 \leq i \quad \forall j \neq i, v_j = \mu(l)_j}{\delta \mid \rho \vdash (\text{set } l \ i \ v) \mid \mu \longrightarrow \text{tt} \mid [l \mapsto (v_1, \dots, v_{i-1}, v, v_{i+1}, \dots, v_n)]\mu}$$

$$\frac{\text{E-SET1} \quad \delta \mid \rho \vdash e_{arr} \mid \mu \longrightarrow e'_{arr} \mid \mu'}{\delta \mid \rho \vdash (\text{set } e_{arr} \ e_{ix} \ e) \mid \mu \longrightarrow (\text{set } e'_{arr} \ e_{ix} \ e) \mid \mu'}$$

$$\frac{\text{E-SET2} \quad \delta \mid \rho \vdash e_{ix} \mid \mu \longrightarrow e'_{ix} \mid \mu'}{\delta \mid \rho \vdash (\text{set } l \ e_{ix} \ e) \mid \mu \longrightarrow (\text{set } l \ e'_{ix} \ e) \mid \mu'}$$

$$\frac{\text{E-SET3} \quad \delta \mid \rho \vdash e \mid \mu \longrightarrow e' \mid \mu'}{\delta \mid \rho \vdash (\text{set } l \ i \ e) \mid \mu \longrightarrow (\text{set } l \ i \ e') \mid \mu'} \quad \frac{\text{E-GET} \quad 0 \leq i \quad \mu(l)_i = v}{\delta \mid \rho \vdash (\text{get } l \ i) \mid \mu \longrightarrow v \mid \mu}$$

$$\frac{\text{E-GET1} \quad \delta \mid \rho \vdash e_{arr} \mid \mu \longrightarrow e'_{arr} \mid \mu'}{\delta \mid \rho \vdash (\text{get } e_{arr} \ e_{ix}) \mid \mu \longrightarrow (\text{get } e'_{arr} \ e_{ix}) \mid \mu'}$$

$$\frac{\text{E-GET2} \quad \delta \mid \rho \vdash e_{ix} \mid \mu \longrightarrow e'_{ix} \mid \mu'}{\delta \mid \rho \vdash (\text{get } l \ e_{ix}) \mid \mu \longrightarrow (\text{get } l \ e'_{ix}) \mid \mu'}$$

Conditionals and function calls

E-COND-TRUE

$$\frac{}{\delta \mid \rho \vdash (\text{cond true } e_1 \ e_2) \mid \mu \longrightarrow e_1 \mid \mu}$$

E-COND-FALSE

$$\frac{}{\delta \mid \rho \vdash (\text{cond false } e_1 \ e_2) \mid \mu \longrightarrow e_2 \mid \mu}$$

E-COND

$$\frac{\delta \mid \rho \vdash e_{\text{cond}} \mid \mu \longrightarrow e'_{\text{cond}} \mid \mu'}{\delta \mid \rho \vdash (\text{cond } e_{\text{cond}} \ e_1 \ e_2) \mid \mu \longrightarrow (\text{cond } e'_{\text{cond}} \ e_1 \ e_2) \mid \mu'}$$

E-CALL

$$\frac{\delta(p) = ((x_1, x_2, \dots, x_n), e) \quad \delta \mid [x_i \mapsto v_i] \rho_0 \vdash e \mid \mu \longrightarrow^* v \mid \mu'}{\delta \mid \rho \vdash (\text{call } p \ v_1 \ v_2 \ \dots \ v_n) \mid \mu \longrightarrow v \mid \mu'}$$

E-CALL1

$$\frac{\delta \mid \rho \vdash e_i \mid \mu \longrightarrow e'_i \mid \mu'}{\delta \mid \rho \vdash (\text{call } e_f \ v_1 \ \dots \ e_i \ \dots \ e_n) \mid \mu \longrightarrow (\text{call } e_f \ e_1 \ \dots \ e'_i \ \dots \ e_n) \mid \mu'}$$

E-CALL2

$$\frac{\delta \mid \rho \vdash e_f \mid \mu \longrightarrow e'_f \mid \mu'}{\delta \mid \rho \vdash (\text{call } e_f \ v_1 \ v_2 \ \dots \ v_n) \mid \mu \longrightarrow (\text{call } e'_f \ v_1 \ v_2 \ \dots \ v_n) \mid \mu'}$$

where ρ_0 is the empty variable environment and \longrightarrow^* is the reflexive transitive closure of \longrightarrow .