A Numerical Experiment for Locating Small Variation in Multiple Scattering Media Based on MCMC Method

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Abstract

The structural (scatters) change usually exhibits the characteristics of localized variations, such as the crack propagation and the emergence of underground cavities. The scatter imaging is an important method to detect localized variations which has been applied to various fields including nondestructive testing and environmental monitoring. In this study, we consult a numerical simulation experiment, using Markov Chain Monte Carlo (MCMC) method to locate small variation in multiple scattering media. We are trying to extend this method to environmental monitoring with ambient noise.

Model

Table 1. Wave field simulation parameters

L	$15\,mm * 15\,mm$	D	$3.6\,mm^2/us$
C_0	$1500\,m/s$	f_0	$2\,MHz$
N	$5000\,points$	wl	$0.75\,mm$

L is the length of model, D is diffusion coefficient, C_0 is the velocity, f_0 is the main frequency, N is the number of scatters, and wl is wave length.

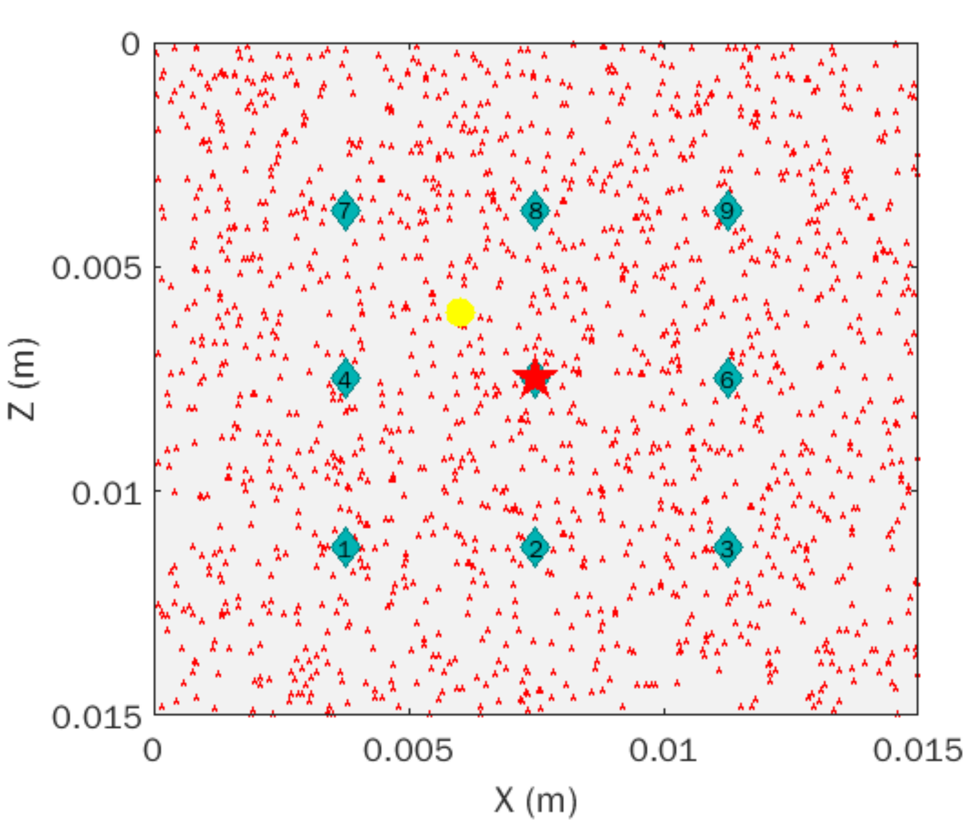


Figure 1. Scatter model. Red dots represent random scatters, total 5000 points. Blue crosses represent receivers position, and red star indicates the source position. The yellow circle is the reflector.

Diffusion Theory

$$I_\infty(S,R,t) = \frac{1}{4\pi Dt}\exp\left(-\frac{\|S-R\|^2}{4Dt}\right) \qquad (1)$$

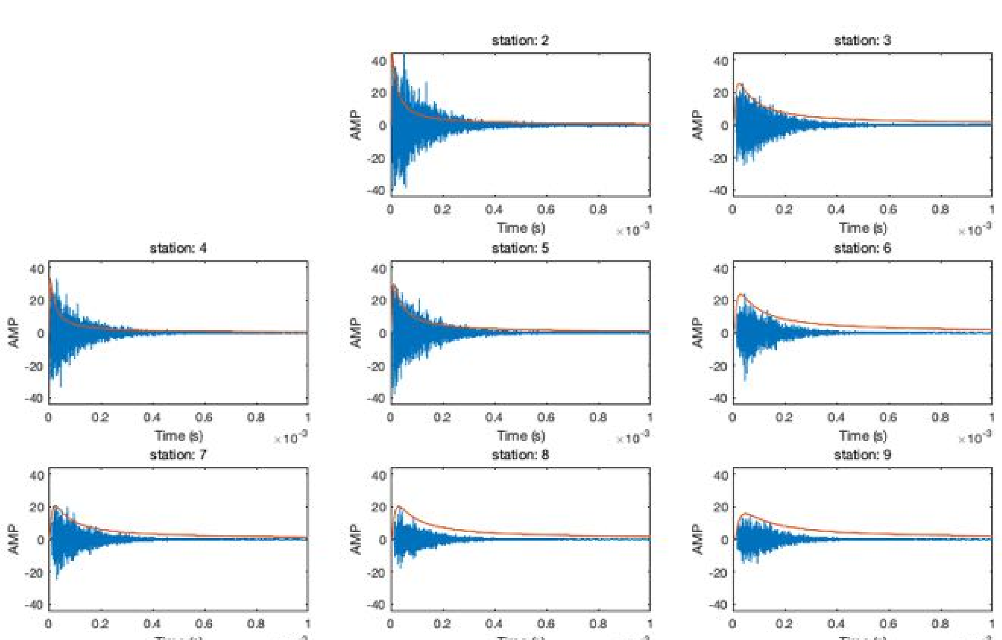


Figure 2. Use the diffusion equation (1) to describe the probability of scattering wave propagation. S represents source and R represents receiver. Coda waveforms are shown in blue lines. Orange line indicates the intensity of the wave field for each receiver.

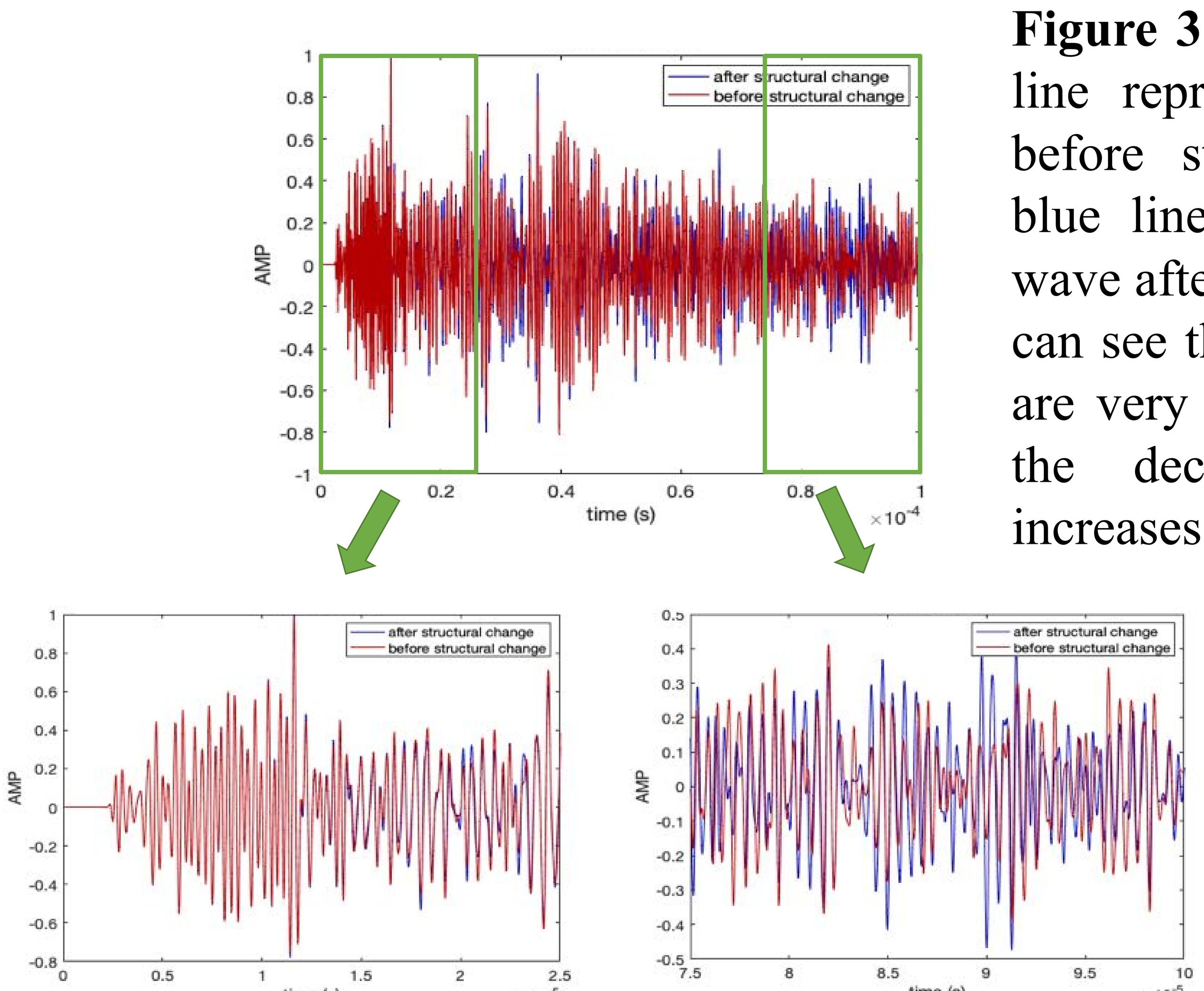


Figure 3. Coda waveform. Red line represents the coda-wave before structural change, and blue line represents the coda-wave after structural change. we can see that the two waveforms are very match 1e-5 s ago, but the decorrelation coefficient increases gradually over time.

Sensitivity Kernel

$$K(S,R,x_0,t) = \frac{\int_0^t I(S,x_0,u)I(x_0,R,t-u)du}{I(S,R,t)} \qquad (2)$$

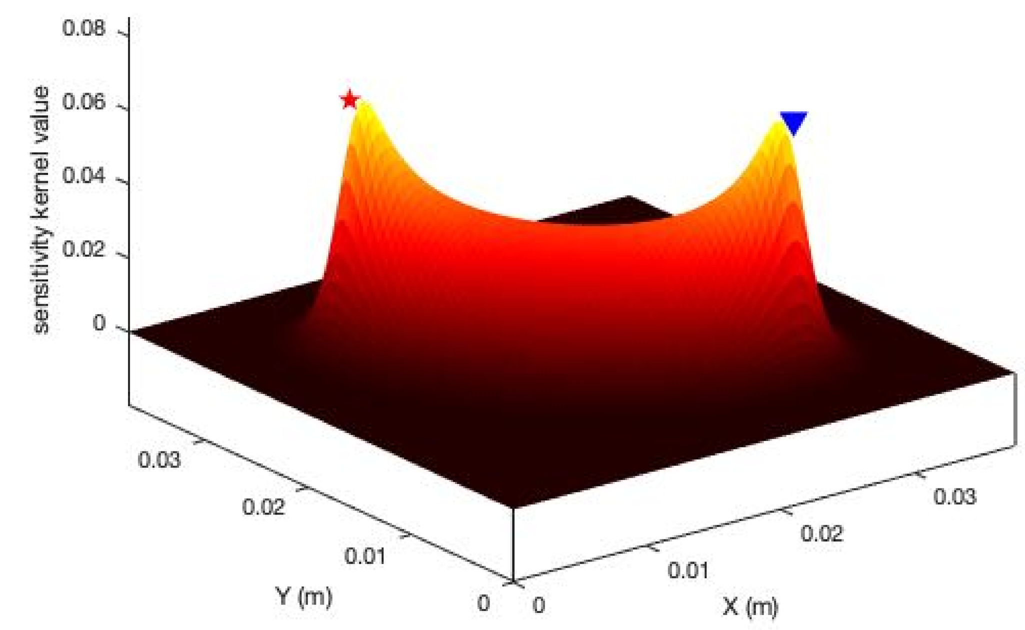


Figure 4. 2-D sensitivity kernel. Red star represents source S and blue triangle represents receiver R. $I(S,R,t)$ represents the intensity of the wave field from S to R within t time by equation (1). The sensitivity kernel indicates the probability the coda wave from S to R passing location x within t time.

Inversion

Theoretical decorrelation coefficient   Observation of decorrelation coefficient

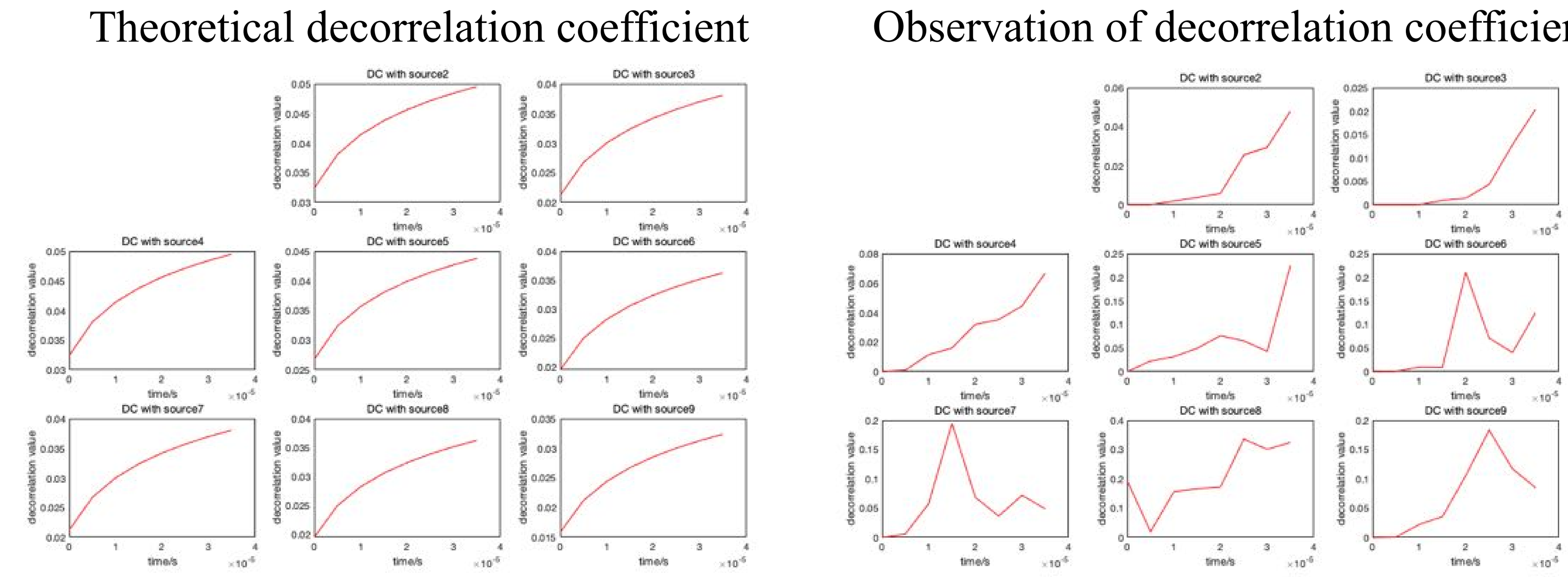
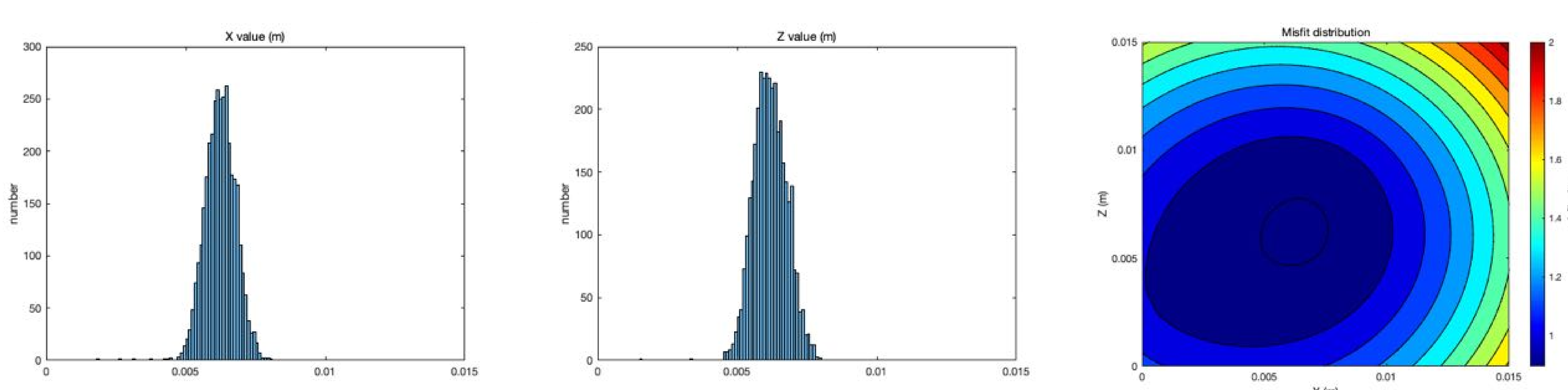


Figure 5. Decorrelation coefficient curves. Left figure is theoretical decorrelation coefficient calculated by diffusion equation and right figure is observation of decorrelation coefficient. We can clearly see the decorrelation coefficient increases gradually over time.



$$DC^{th} = \oint_\Omega G(x)m(x)\,dx, \quad \text{where } G = \frac{c_0 K}{2} \qquad (3)$$

Figure 6. MCMC inverion result. 1-D marginal posterior distributions of the reflector position and 2-D misfit distribution. The misfit function is defined by equation (3) and *m(x)* represents the position of reflector.

Summary

- ✓ We calculate the seismogram of 2-D scattering field with the SPECFEM2D software, and use the diffusion equation to describe the probability of scattering wave propagation.
- ✓ The MCMC inversion results show that the region where the scattering point changes can be accurately located.
- ✓ The inversion results show that the constraint of scattering cross section is poor and needs further improvement

Reference:

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[2] V. Rossetto, L. Margerin, T. Planès, E. Larose, Locating a weak change using diffuse waves: theoretical approach and inversion procedure, J. Appl. Phys. 109 (2011) 034903.

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