MCMTpy: A Python Package for Source Parameters Inversion Based on Cut-And-Paste Algorithm and Markov Chain Monte Carlo

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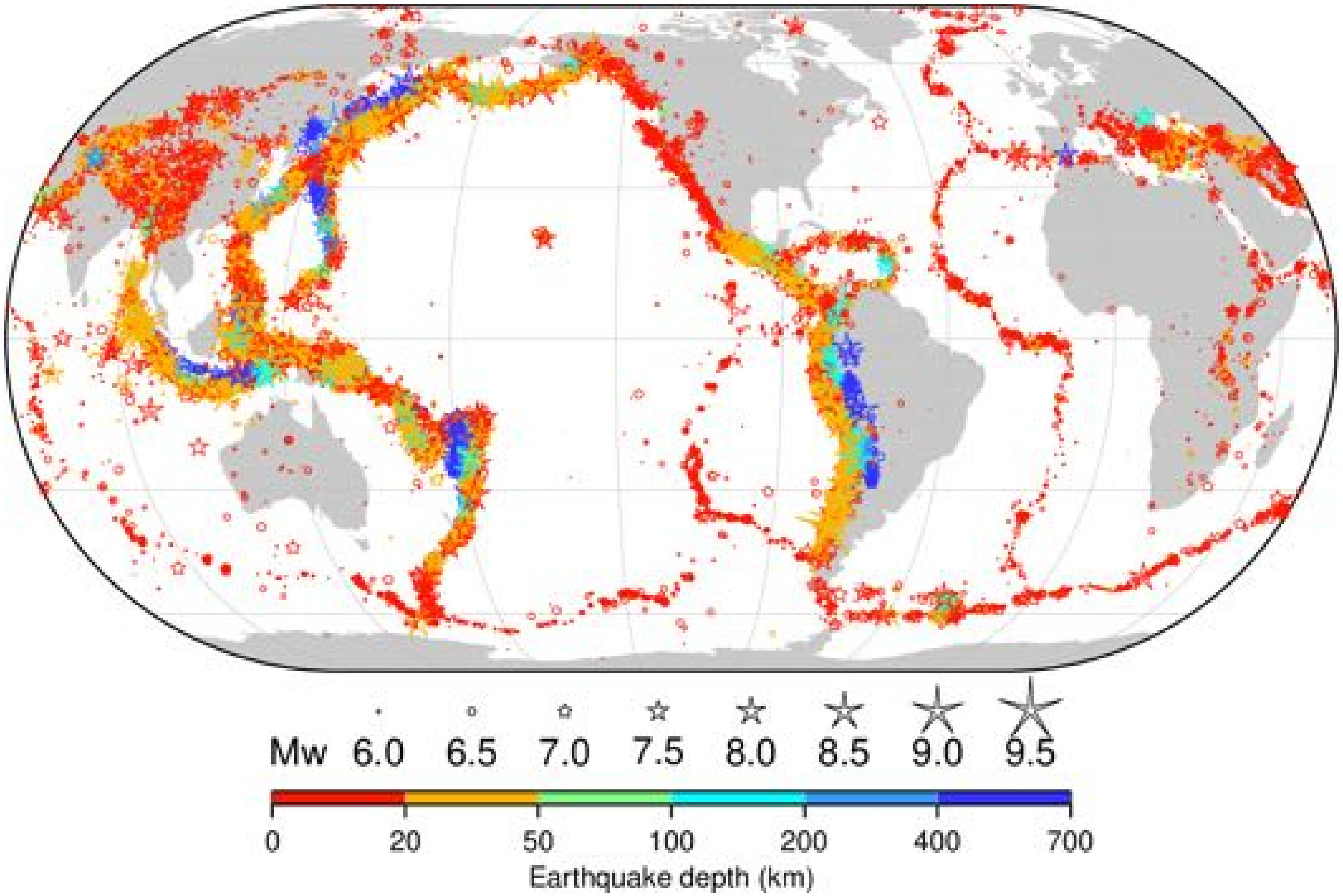
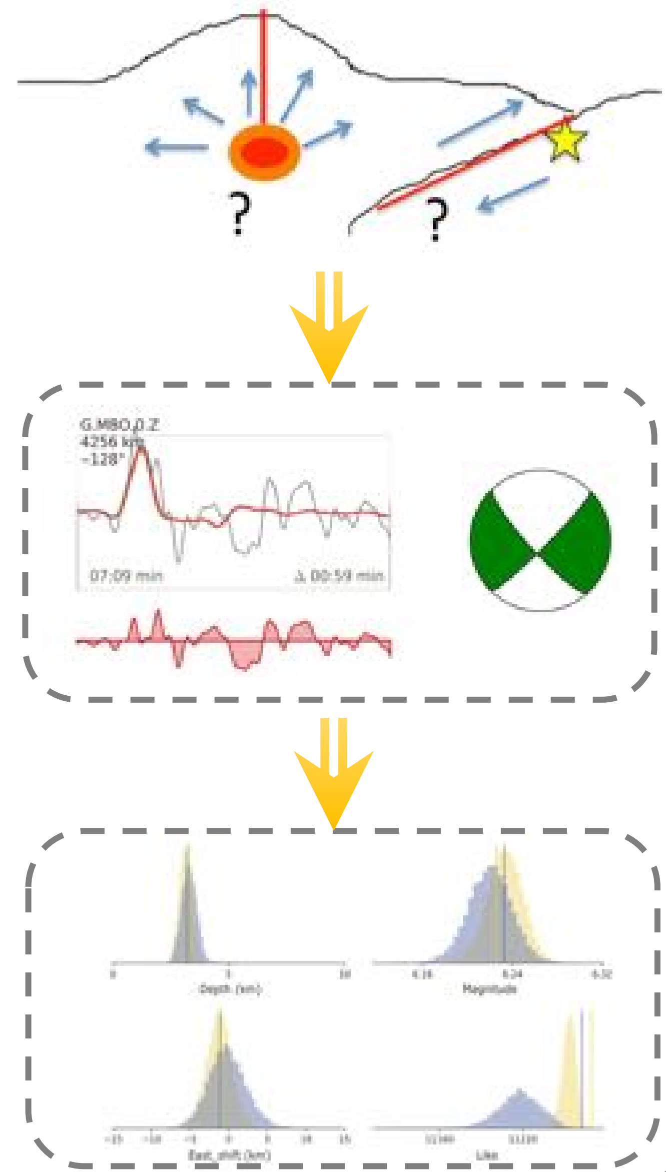
2021.11.01

Global seismicity and plate tectonics



Map showing the earthquakes in version 3.0 of the ISC-GEM catalogue (more than 24,000 earthquakes (Storchak, et al., 2012), http://www.isc.ac.uk/iscgem/overview.php.

Inversion Uncertainty Assessment Important



(https://hvasbath.github.io/beat/index.html)

CATALOG

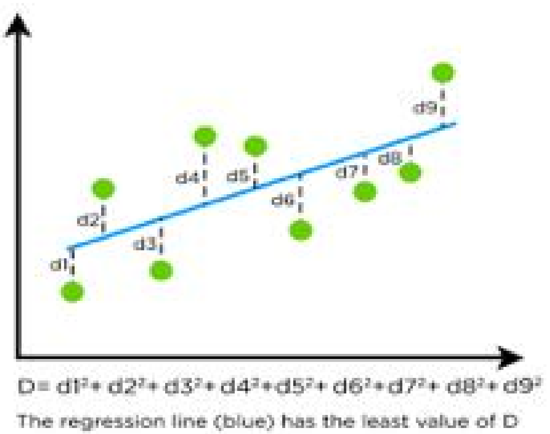
- Introduction
- Method
- Application
- Conclusion

Introduction

Local optimization

- find the minimum of the objective function

$$min \ \|\mathbf{Gm} - \mathbf{d}\|_2^2 + \alpha^2 \|\mathbf{m}\|_2^2$$



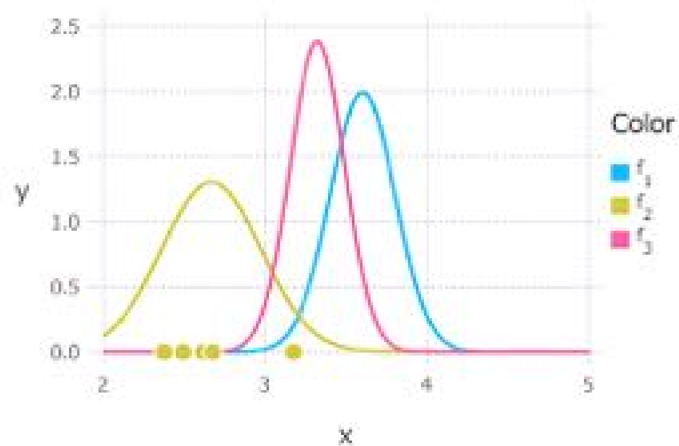
Statistical inverse method (Beyesian)

- find the distribution of model parameters (posterior distribution)

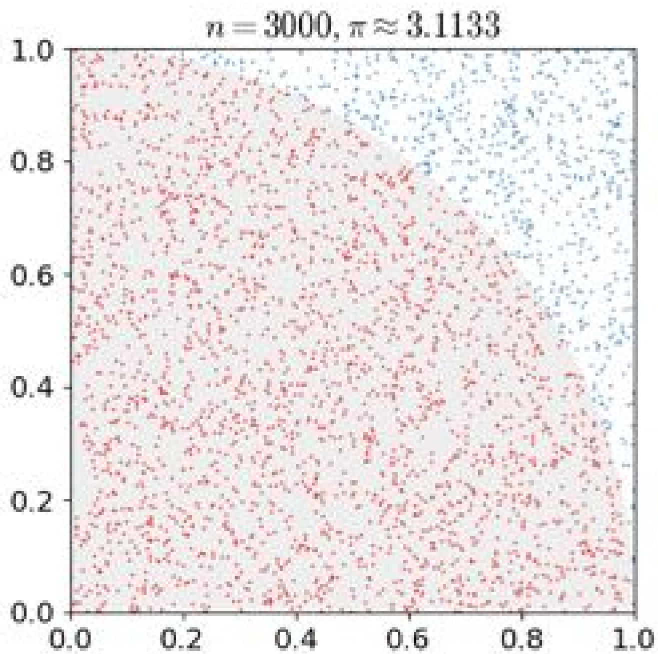
$$\pi_{post}(m|d_{obs}) = k \, \pi_{prior}(m) \, \pi_{like}(d_{obs}|m)$$

$$\pi_{post}(m|d_{obs}) \propto \pi_{like}(d_{obs}|m) = exp\left\{-\frac{1}{2}S(m)\right\}$$

$$S(m) = \left(d_{obs} - f(m)\right)^T C_k^{-1}\left(d_{obs} - f(m)\right)$$



Monte Carlo



Monte Carlo method was first proposed by the United States in the 1940s in the Second World War to develop the atomic bomb **"Manhattan Project"** plan, for secret choice of the gambling city of Monaco Monte Carlo as the code name.

Stanisław Ulam

John von Neumann

Turing

1. Draw a square, then inscribe a quadrant within it
2. Uniformly scatter a given number of points over the square
3. Count the number of points inside the quadrant, i.e. having a distance from the origin of less than 1
4. The ratio of the inside-count and the total-sample-count is an estimate of the ratio of the two areas, $\frac{\pi}{4}$.

Bayesian Framwork ⇒ Inversion Uncertainty Assessment

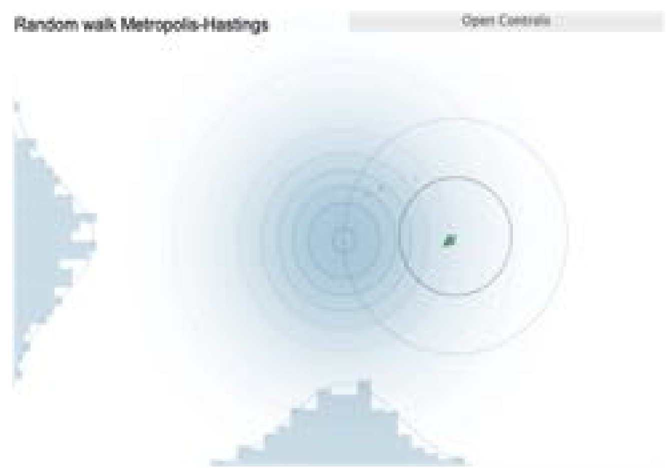
Detailed Balance Condition:

$$\pi(i)P(i,j) = \pi(j)P(j,i), \quad for\ all\ i,j$$

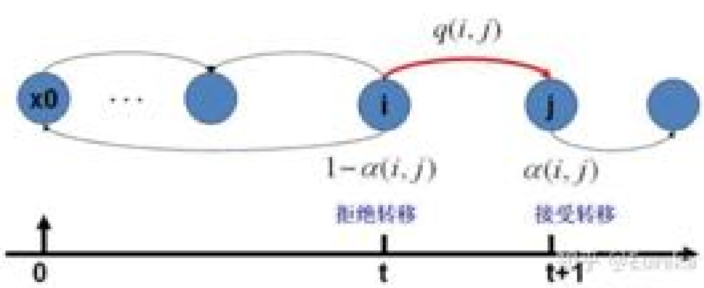
For each transition matrix

$$\pi(i)Q(i,j)\alpha(i,j) = \pi(j)Q(j,i)\alpha(j,i)$$

$\alpha(i,j)$ is acceptance rate



https://chi-feng.github.io/mcmcdemo/app.html?algorithm=RandomWalkMH&target=banana



Method

= **CAP + MCMC**

$$S(m) = \begin{cases} S_{time}(m), & k \leq N_k \\ S_{wave}(m), & k > N_k \end{cases}$$

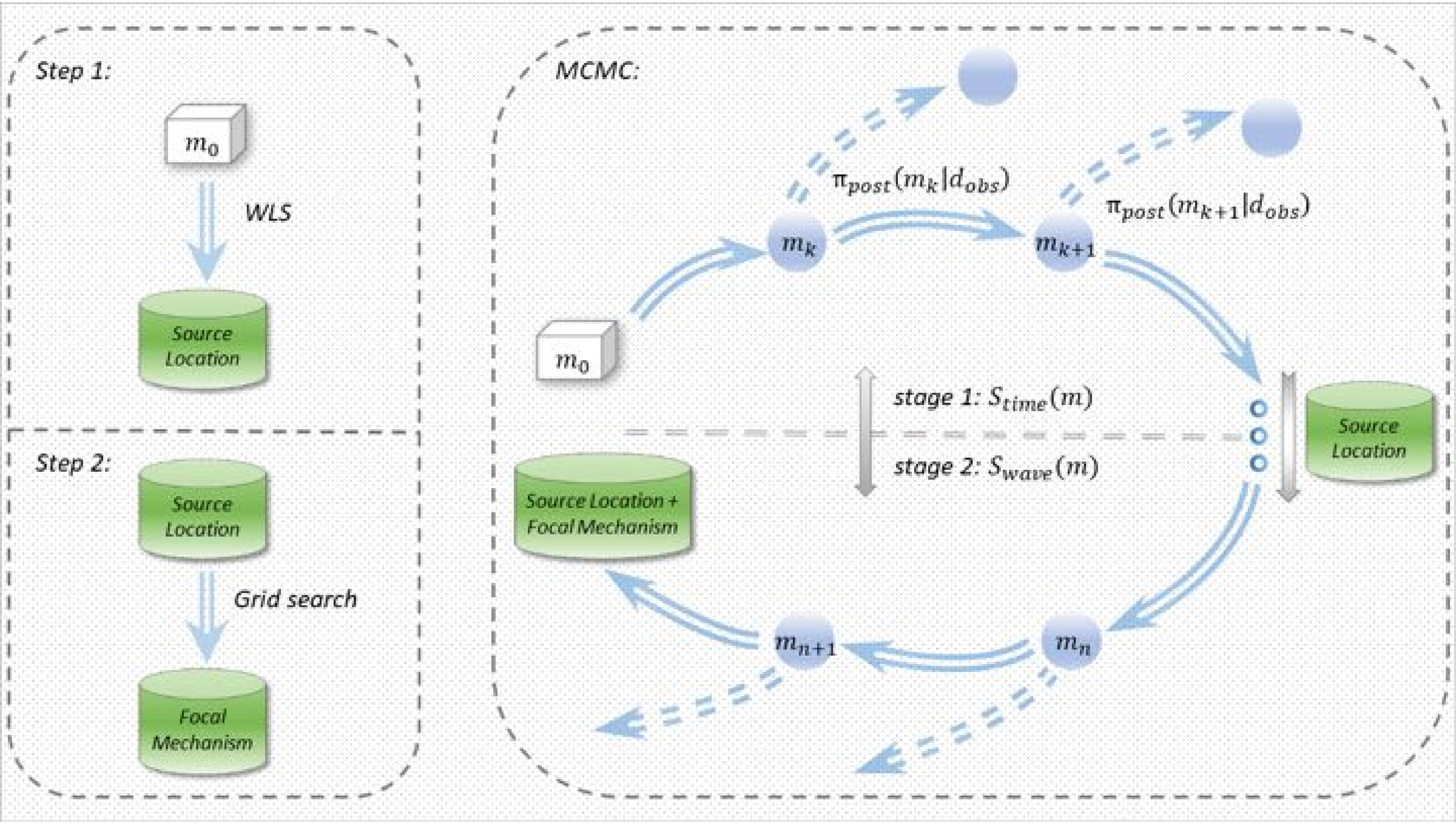
$$S_{time}(m) = \frac{1}{N} \sum_{i=1}^{N} \frac{\left[T_i(m) - T_i^{obs} - T_0\right]^2}{\sigma_i^2},$$

$$S_{waveform} = \frac{1}{N} \sum_{i=1}^{N} \frac{w_i^{Station}}{\tau_i^2} \left(w_i^p \left(\frac{r_i}{r_0}\right)^{ps_p} \left(e_i^{p_z} + e_i^{p_r} + e_i^{p_t}\right) + w_i^s \left(\frac{r_i}{r_0}\right)^{ps_s} \left(e_i^{s_z} + e_i^{s_r} + e_i^{s_t}\right) + w_i^{surf} \left(\frac{r_i}{r_0}\right)^{ps_{surf}} \left(e_i^{surf_z} + e_i^{surf_r} + e_i^{surf_t}\right) \right),$$

Algorithm 1. MCMTpy algorithm to sample proposal distribution $\pi_{post}(m|d_{obs})$

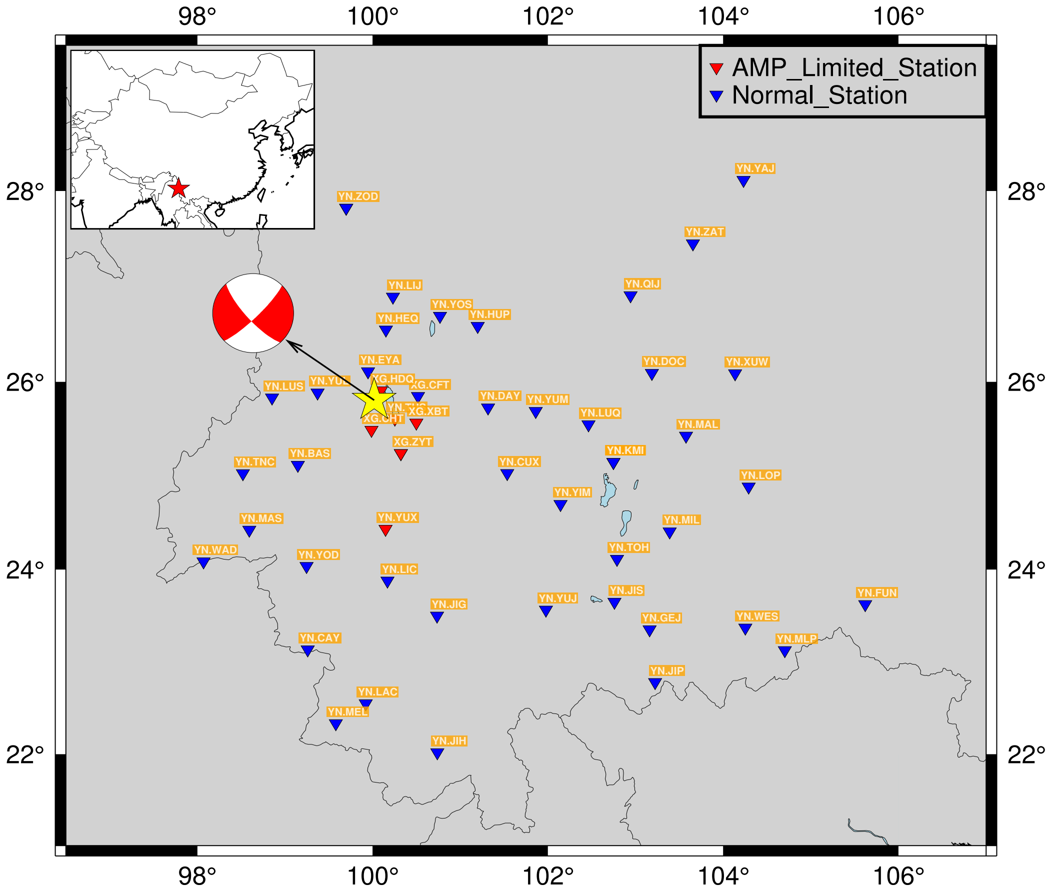
```
Choose initial m_0, S(m_0)
Compute π_post(m_0|d_obs)
for k = 0, ..., N − 1 do
    If k < N_k then
        Define S(m) = S_time(m)
    else
        If k < N_k + N_mag then
            Estimate M_0 with formula 15
        end if
        Define S(m) = S_wave(m)
    end if
    Draw sample y with random walk with formula 18
    Compute π_post(y|d_obs)
    Compute β(m_k, y) = min{ π_post(y|d_obs) / π_post(m_k|d_obs), 1 }
    Draw random number u~μ([0,1])
    If μ<β(m_k, y) then
        Accept: set m_{k+1} = y
    else
        Reject: set m_{k+1} = m_k
    end if
end for
```

Two step workflow **VS** MCMTpy workflow



Application

Application to the 21 May 2021 Ms 6.4 Yangbi Earthquake



According to CENC catalog:

Latitude 25.70°

Longitude 99.88°

Depth 10 km

GMT 13:48:35

Magnitude Ms 6.4

Misfit curve

$$S(m) = \begin{cases} S_{time}(m), & k \leq N_k \\ S_{wave}(m), & k > N_k \end{cases}$$

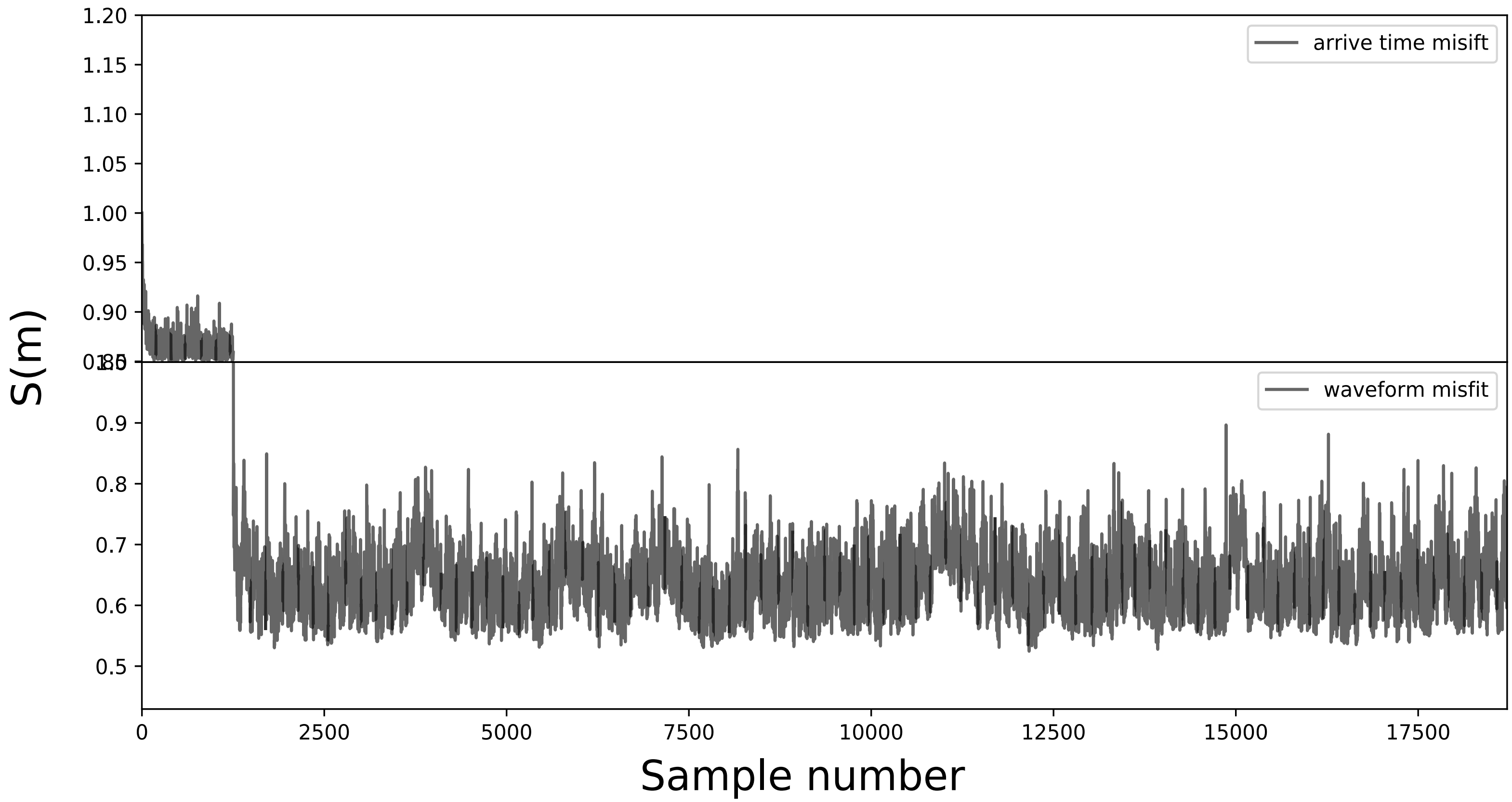
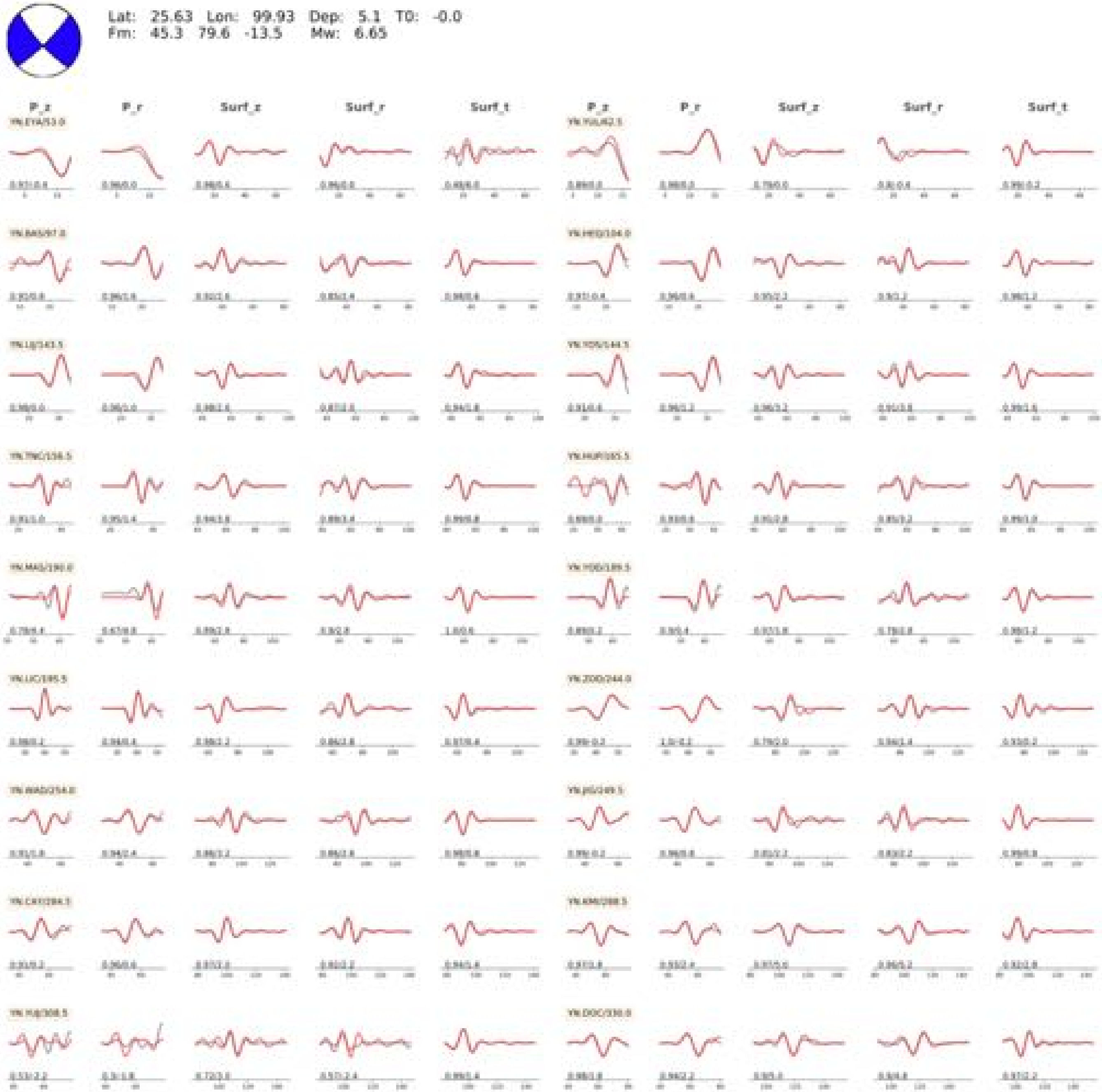


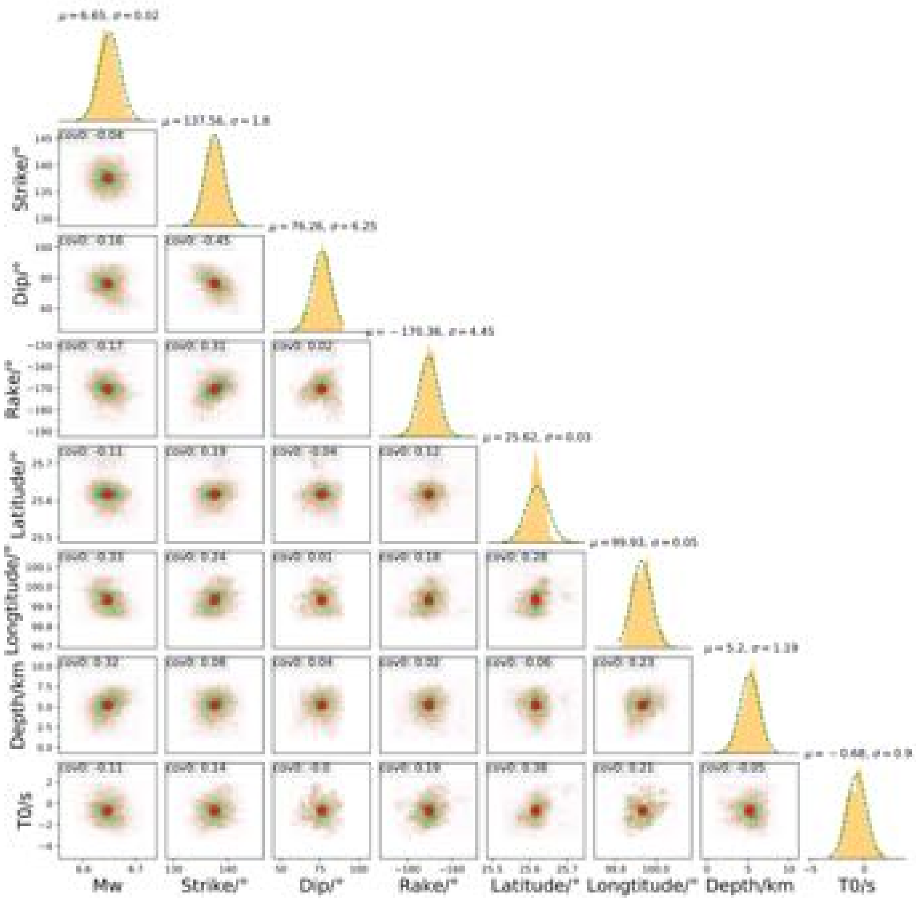
Table 1. Source Parameters for 2021 Yangbi Earthquake of Different Agencies

Method	Mag	Lat/Lon	Depth	Plane 1 and 2 (Strike/Dip/rake)	Percent DC (%)	Beachball
Global CMT	6.1 M_{wc}	25.76/100.01	15.0	315/86/168 46/78/4	62	
USGS W-phase	6.1 M_{ww}	25.74/100.02	17.5	135/82/-165 43/75/-9	93	
P Wave First Motion	NO	25.67/99.87	NO	141/68/-153 40/65/-24	NO	
CAP	6.4 M_s	25.67/99.87	5.0	138/82/-161 45/71/-8	NO	
MCMTpy	6.65 M_w	25.62/99.93	5.2	138/76/-170 46/80/-14	91	

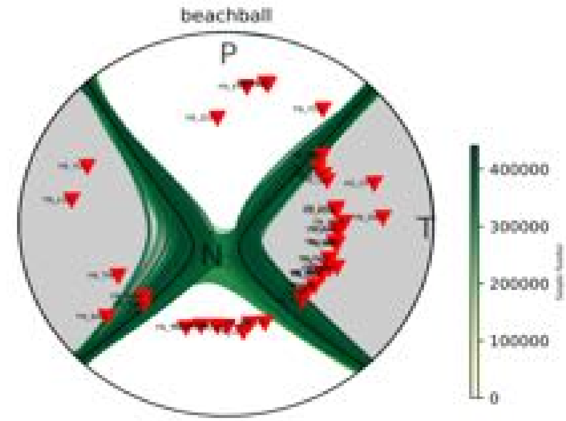
P Wave First Motion and CAP results are from the Institute of Geophysics, China Earthquake Administration, using global network data and local stations, respectively.

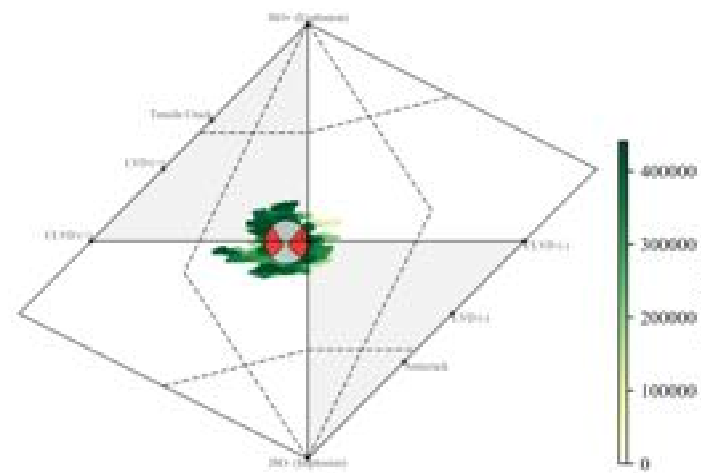

Lat: 25.63 Lon: 99.93 Dep: 5.1 T0: -0.0
Fm: 45.3 79.6 -13.5 Mw: 6.65
P_z
P_r
Surf_z
Surf_r
Surf_t
P_z
P_r
Surf_z
Surf_r
Surf_t

Inversion Result



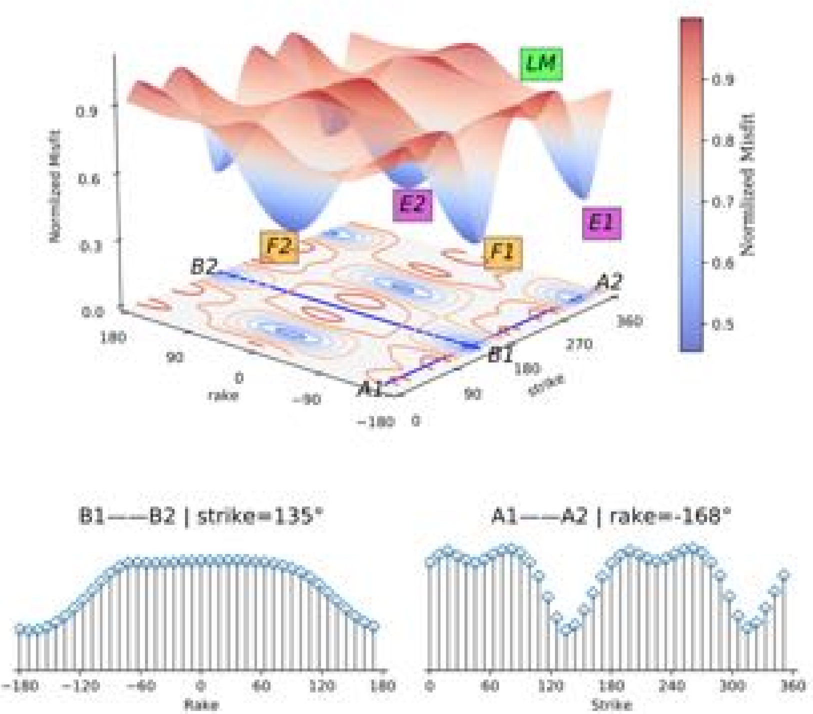
Double Couple Inversion

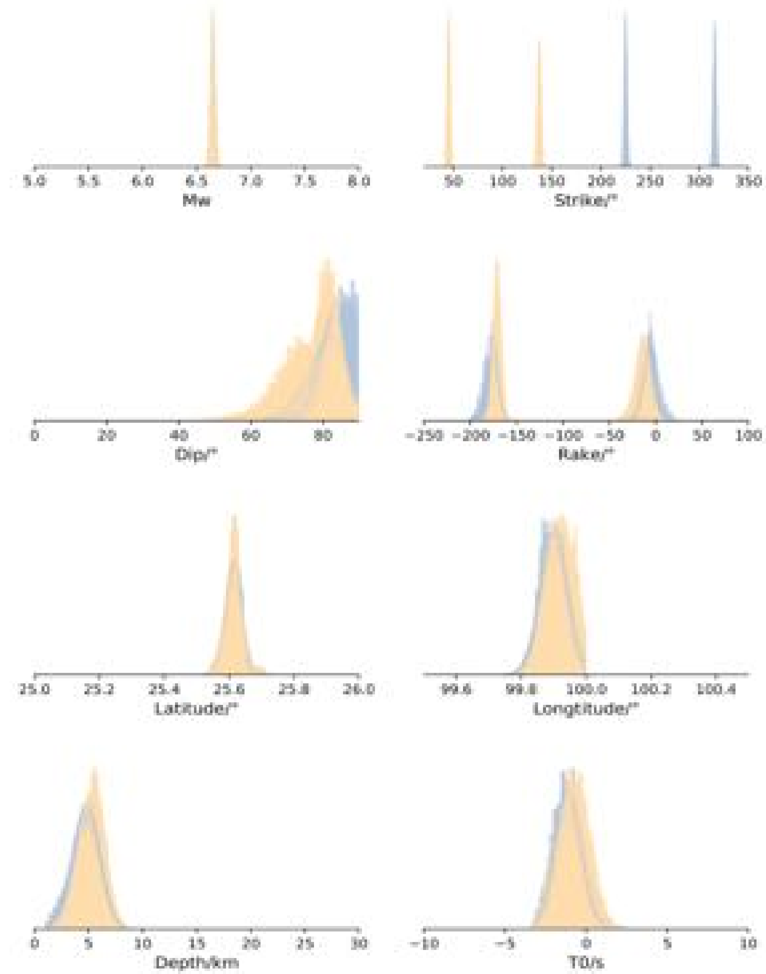




Moment Tensor Inversion

Reduce the Dependence of **Prior Model** and Limitations





two-dimensional normalized misfit space of strike-rake

30 random initial models

- Source parameters inversion under **double-couple** assumption with Markov Chain Monte Carlo method.

- Source parameters inversion under double-couple assumption with **grid search method**.

- Source parameters inversion for full **moment-tensor** solution with Markov Chain Monte Carlo method.

- Jupyter notebooks for source **parameter calculation and conversion**, such as moment tensor decomposition and Hudson plot.

More Information

MCMTpy Public https://github.com/OUCyf

MCMTpy is a Python package designed for seismic source study. It provides functionality for focal mechanism inversion and source parameters analysis.

Jupyter Notebook ☆ 11 MIT License Updated 15 days ago

README.rst

MCMTpy

About

The package is still undergoing development.

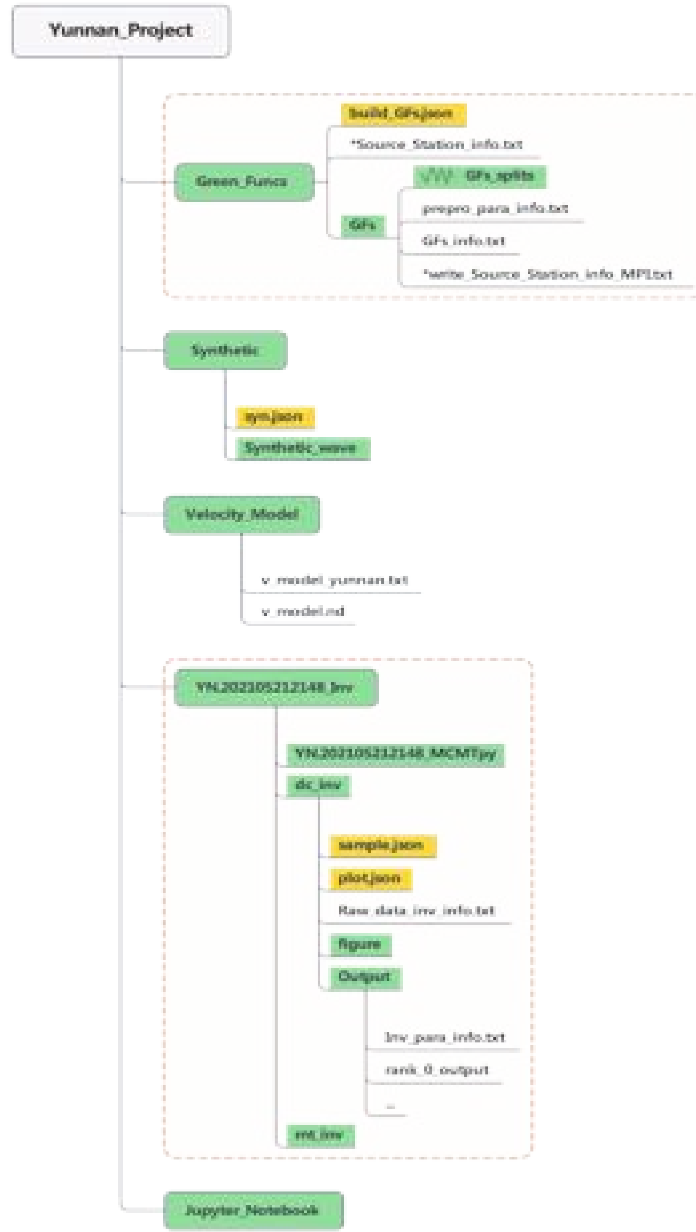
MCMTpy is a Python package designed for seismic source study. It provides functionality for focal mechanism inversion and source parameters analysis.

MCMTpy has mainly provided functions as:

- Focal mechanism inversion of the double couple source and moment tensor using improved **Markov chain Monte Carlo** method.
- A series of scripts is provided for source parameter calculation and conversion.
- Moment tensor decompose, Huston plot and Beachball with station projected.

and it has the features as:

- The package MCMTpy runs on Unix-like systems including Mac and Linux.
- It has a close integration with Obspy, pyfk and pyasdf.
- It can be done in parallel based on mpi4py.
- It's is efficient to do the seismological research based on python.



Conclusion

- We present the MCMTpy package using the MCMC method to combine source location inversion and focal mechanism inversion on one Markov chain.

- The new method considers both **phase arrival time** and **waveform information** to effectively reduces the dependence of the prior model and avoid trapping into local minima.

- This method can mitigate the **cumulative uncertainties** of the source location, provides a way to quantify source parameter's uncertainties by statistical inference.

Thanks！