Scheme2 Core Typing Relation

November 21, 2021

The Scheme2 Core static semantics is given as a three-place relation between a variable typing context Γ, expression e, and type T, written $\Gamma \vdash e : T$, pronounced "under Γ, e has type T". Formally, the static semantics is taken to be the smallest relation closed under the following rules:

Variables and values

$$\frac{}{\Gamma \vdash n : \text{num}}\ \text{T-NUM} \qquad \frac{}{\Gamma \vdash \mathit{false} : \text{bool}}\ \text{T-FALSE} \qquad \frac{}{\Gamma \vdash \mathit{true} : \text{bool}}\ \text{T-TRUE} \qquad \frac{(x : \text{T}) \in \Gamma}{\Gamma \vdash x : \text{T}}\ \text{T-VAR}$$

Unary operators

$$\frac{\Gamma \vdash e : \text{bool}}{\Gamma \vdash (\text{not } e) : \text{bool}}\ \text{T-NOT}$$

Binary operators

$$\frac{\Gamma \vdash e_1 : \text{num} \qquad \Gamma \vdash e_2 : \text{num} \qquad b \in \{+, *, -, /\}}{\Gamma \vdash (b\, e_1\, e_2) : \text{num}}\ \text{T-BINOP-ARITH}$$

$$\frac{\Gamma \vdash e_1 : \text{num} \qquad \Gamma \vdash e_2 : \text{num} \qquad b \in \{=, <\}}{\Gamma \vdash (b\, e_1\, e_2) : \text{bool}}\ \text{T-BINOP-COMP}$$

Conditionals

$$\frac{\Gamma \vdash e_{cond} : \text{bool} \qquad \Gamma \vdash e_1 : T \qquad \Gamma \vdash e_2 : T}{\Gamma \vdash (\text{if } e_{cond}\, e_1\, e_2) : T}\ \text{T-IF}$$

Functions

$$\textsc{T-fun}\ \frac{\Gamma, x : T_1 \vdash e : T_2}{\Gamma \vdash (\text{fun}\ x\ T_1\ e) : T_1 \to T_2} \qquad \textsc{T-app}\ \frac{\Gamma \vdash e_1 : T_1 \to T_2 \qquad \Gamma \vdash e_2 : T_1}{\Gamma \vdash (e_1\ e_2) : T_2}$$

Recursion

$$\textsc{T-rec}\ \frac{\Gamma, x : T \vdash e : T}{\Gamma \vdash (\text{rec}\ x\ T\ e) : T}$$

Products

$$\textsc{T-pair}\ \frac{\Gamma \vdash e_1 : T_1 \qquad \Gamma \vdash e_2 : T_2}{\Gamma \vdash (\text{pair}\ e_1\ e_2) : T_1 * T_2} \qquad \textsc{T-fst}\ \frac{\Gamma \vdash e : T_1 * T_2}{\Gamma \vdash (\text{fst}\ e) : T_1} \qquad \textsc{T-snd}\ \frac{\Gamma \vdash e : T_1 * T_2}{\Gamma \vdash (\text{snd}\ e) : T_2}$$

Sums

$$\textsc{T-inl}\ \frac{\Gamma \vdash e : T_1}{\Gamma \vdash (\text{inl}\ T_2\ e) : T_1 + T_2} \qquad \textsc{T-inr}\ \frac{\Gamma \vdash e : T_2}{\Gamma \vdash (\text{inr}\ T_1\ e) : T_1 + T_2}$$

$$\textsc{T-case}\ \frac{\Gamma \vdash e_1 : T_1 + T_2 \qquad \Gamma \vdash e_2 : T_1 \to T \qquad \Gamma \vdash e_3 : T_2 \to T}{\Gamma \vdash (\text{case}\ e_1\ e_2\ e_3) : T}$$

Lists

$$\textsc{T-nil}\ \frac{}{\Gamma \vdash (\text{nil}\ T) : \text{list}\ T} \qquad \textsc{T-cons}\ \frac{\Gamma \vdash e_1 : T \qquad \Gamma \vdash e_2 : \text{list}\ T}{\Gamma \vdash (\text{cons}\ e_1\ e_2) : \text{list}\ T}$$

$$\textsc{T-fold}\ \frac{\Gamma \vdash e_1 : \text{list}\ T_1 \qquad \Gamma \vdash e_2 : T_1 \to T_2 \to T_2 \qquad \Gamma \vdash e_3 : T_2}{\Gamma \vdash (\text{fold}\ e_1\ e_2\ e_3) : T_2}$$