Binary phase mask for extending depth-of-field

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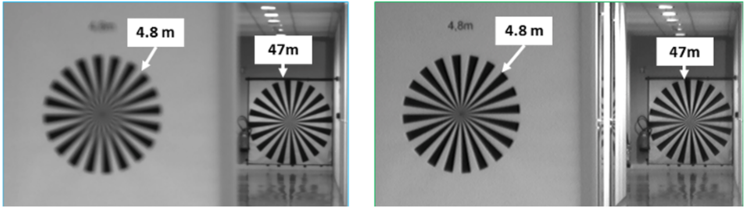
Introduction

Introduction

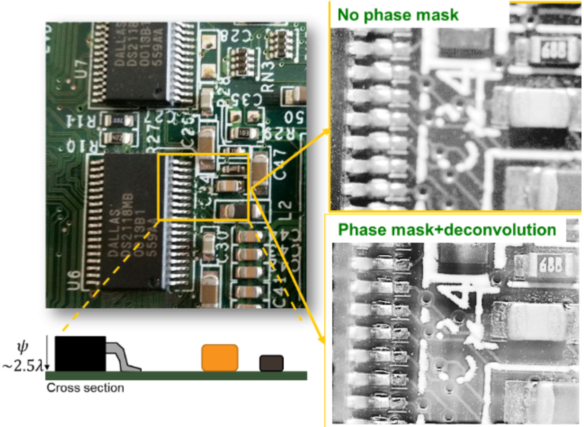
- **Objective:** Extend the depth of field.
- **First Solution:** Reduce the aperture of the system $\rightarrow$ This limits the amount of light reaching the sensor, affecting the image quality and brightness.
- **Second Solution:** Use a binary phase mask.
- **Wiener filter:** Enhance the quality of the final image.



Expected result of the binary phase mask [1]

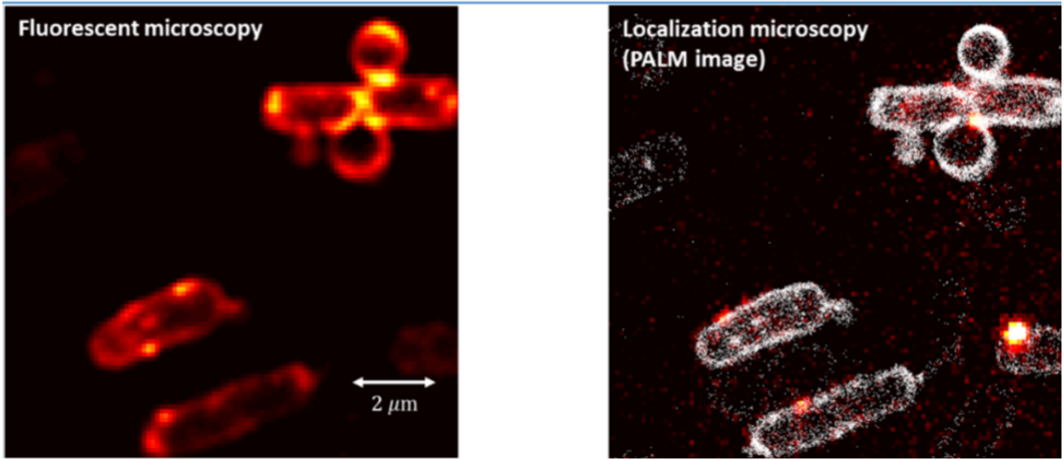
[1] *https://www.lcf.institutoptique.fr/groupes-de-recherche/imagerie-et-information/equipe-co-conception/co-design-optique/profondeur-de-champ.*

Use cases



[2] *https://www.institutoptique.fr/offres-emploi/these-conception-conjointe-de-masques-de-phase-et-dalgorithmes-pour-la-microscopie-de?.*

Use cases



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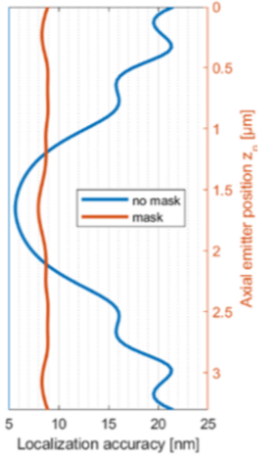
Use cases



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Physics Context

Physics Context

Defocus

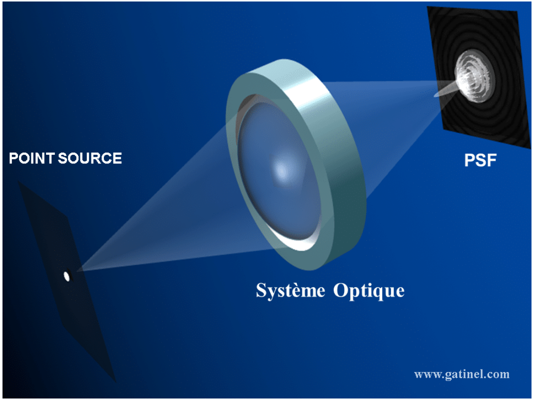
Defocus refers to the optical phenomenon where an image is blurred due to the lack of perfect convergence of light rays from a point source on the sensor plane. This is caused by deviations in focus. The defocus parameter ψ can be expressed as:

$$\psi = \frac{\pi R^2}{\lambda}\left(\frac{1}{f} - \frac{1}{d_o} - \frac{1}{d_i}\right)$$

where:

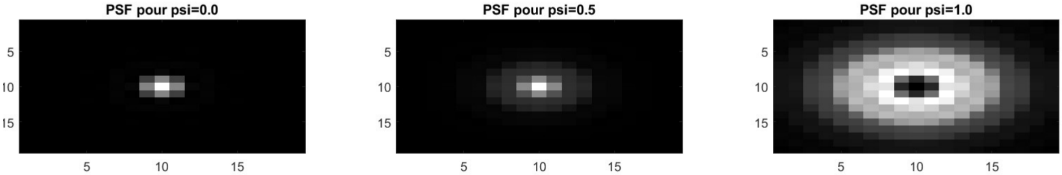
- f: Focal length of the lens.
- d_o: Distance of the object from the lens.
- d_i: Distance of the image plane from the lens.
- R: Aperture radius.
- λ: Wavelength of the light.

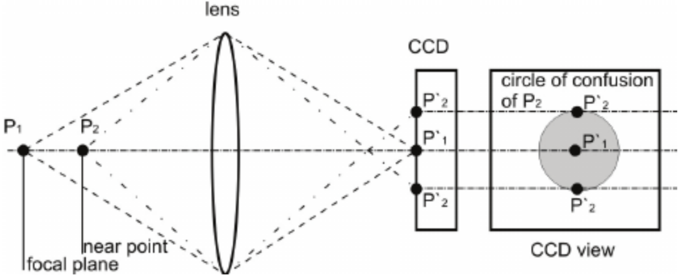
PSF



[3] https://www.gatinel.com/recherche-formation/acuite-visuelle-definition/psf

PSF


PSF pour psi=0.0
PSF pour psi=0.5
PSF pour psi=1.0
5
10
15


lens
CCD
circle of confusion of P2
P1
P2
P'2
P'1
P'2
near point
focal plane
CCD view

PSF

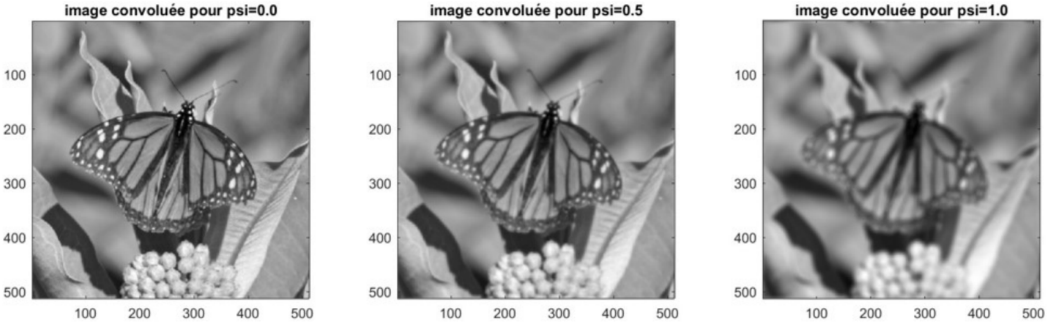
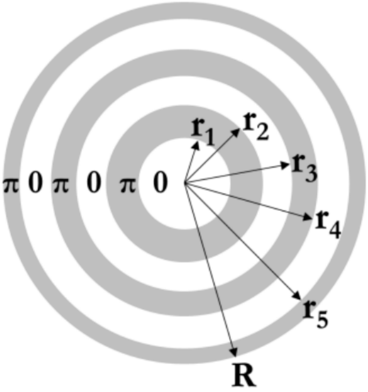


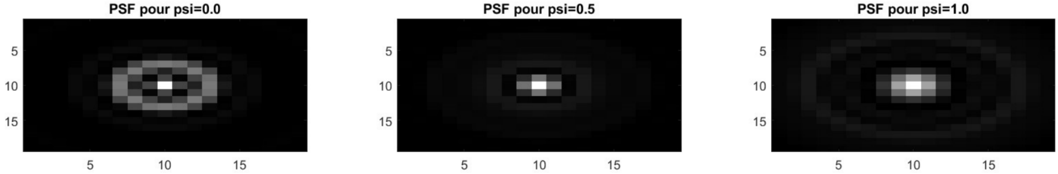
image convoluée pour psi=0.0
image convoluée pour psi=0.5
image convoluée pour psi=1.0
100
200
300
400
500

Binary phase mask



[1] https://www.lcf.institutoptique.fr/groupes-de-recherche/imagerie-et-information/equipe-co-conception/co-design-optique/profondeur-de-champ

Binary phase mask



Mathematical formulation

Mathematical formulation

Imaging model

Given a perfect scene $O(r)$, the PSF of the system $h_\psi(r)$ and measurement noise $n(r)$ the obtained scene is:

$$I_\psi(r) = h_\psi(r) * O(r) + n(r)$$

$$h_\psi(r) = \left| F\{P(r)\exp[i(\phi(r) + \psi r^2)]\} \right|^2$$

Defocus: $\psi = \frac{\pi R^2}{\lambda}\left(\frac{1}{f} - \frac{1}{d_o} - \frac{1}{d_i}\right)$

Phase mask function: $\phi(r)$

Mathematical formulation

Deconvolution

Our goal is to get an accurate estimator of the original scene:

$$\hat{O}_\psi(r) = d(r) * I_\psi(r)$$

where $d(r)$ is a linear deconvolution filter.

Mathematical formulation

How do we choose this filter?

Criteria

For a specific defocus ψ we define the mean squared error:

$$MSE_\psi = \langle |\hat{O}_\psi(r) - O(r)|^2 \rangle$$

We want to design $d(r)$ to minimize the MSE averaged over n_{MSE} defocus values:

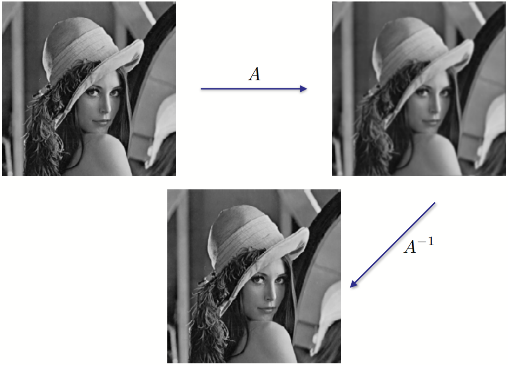
$$MSE_{\text{mean}} = \frac{1}{n_{\text{MSE}}} \sum_{i=1}^{n_{\text{MSE}}} MSE_{\psi_i}$$

where $\psi_i \in [0, \psi_{\text{defoc max}}]$ are n_{MSE} uniformely distributed defocus values.

Mathematical formulation

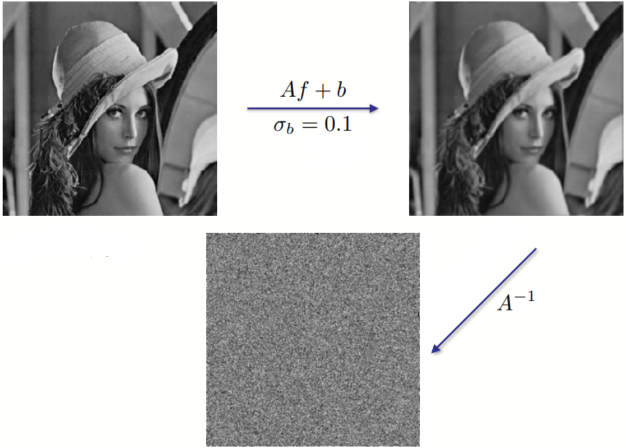
Context: Wiener filters

Why don't we just filter with the inverse of the PSF?



Naïve inverse filtering without noise

Mathematical Formulation



Naïve inverse filtering with noise

Mathematical Formulation

Wiener filter

Minimizing the MSE for a particular defocus value ψ:

$$\tilde{d}(\nu) = \frac{\tilde{h}^*_\psi(\nu)}{|\tilde{h}_\psi(\nu)|^2 + \frac{S_{nn}(\nu)}{S_{OO}(\nu)}}.$$

where $S_{nn}(\nu) = |\tilde{n}(\nu)|^2, \quad S_{OO}(\nu) = |O(\nu)|^2$ (Power Spectral Densities)

Mathematical Formulation

Averaged Wiener filter

Ideal filter that minimizes the averaged MSE [4]:

$$\tilde{d}(\nu)_{\text{mean}} = \frac{\frac{1}{n_d}\sum_{i=1}^{n_d} \tilde{h}^*_{\psi_i}(\nu)}{\frac{1}{n_d}\sum_{i=1}^{n_d} |\tilde{h}_{\psi_i}(\nu)|^2 + \frac{S_{nn}(\nu)}{S_{OO}(\nu)}}.$$

where $\psi_i \in [0, \psi_{\text{defoc max}}]$, $\quad S_{nn}(\nu) = |\tilde{n}(\nu)|^2, \quad S_{OO}(\nu) = |O(\nu)|^2,$

[4] *F. Diaz, F. Goudail, B. Loiseaux, and J.-P. Huignard, “In- crease in depth of field taking into account deconvolution by optimization of pupil mask,” Opt. Lett. 34, 2970–2972 (2009).*

Designing the phase mask

Designing the phase mask

How to assess the reconstructions?

Restored image quality (RIQ)

$$RIQ_{\psi}(\mathrm{dB}) = 10\log_{10}\left[\frac{\int S_{OO}(\nu)\,\mathrm{d}\nu}{MSE_{\psi}}\right]$$

$$RIQ_{\mathrm{mean}}(\mathrm{dB}) = 10\log_{10}\left[\frac{\int S_{OO}(\nu)\,\mathrm{d}\nu}{MSE_{\mathrm{mean}}}\right]$$

Designing the phase mask

1. Choose an image which we consider the perfect undisturbed scene $O(r)$
2. Convolve this image by a simulated PSF $h_\psi(r)$ (defocus ψ + phase mask $\phi(r)$).
3. Add noise: $I_\psi(r) = h_\psi(r) * O(r) + n(r)$
4. Restore the original image with the Wiener Filter: $\hat{O}_\psi(r) = d(r) * I_\psi(r)$
5. Compute Mean Squared Error: $MSE_\psi = \langle |\hat{O}_\psi(r) - O(r)|^2 \rangle$
6. Compute spectral power of the image: $S_{OO}(\nu) = |O(\nu)|^2$
7. Compute Restored Image Quality criterion: $RIQ_\psi(\mathrm{dB}) = 10 \log_{10} \left[\frac{\int S_{OO}(\nu)\, \mathrm{d}\nu}{MSE_\psi} \right]$

Designing the phase mask

Optimal value for the radius r_1 of a single ring mask: **Minimax approach**

Algorithm 1 Optimization of Radius with RIQs

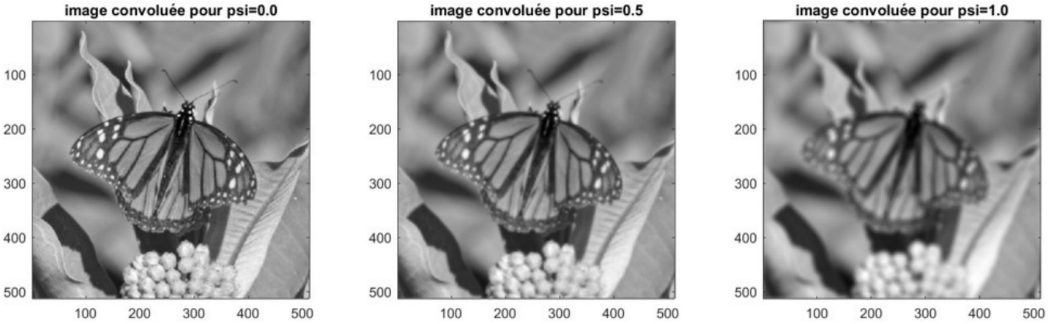
Initialization:
Define range of radius candidates `rs`
Define a range of defocus values `psis`
Initialize an empty array `minRIQs`
for each radius r in `rs` **do**
    Initialize temporary array for RIQ values `RIQs`
    for each defocus ψ in `psis` **do**
        Simulate imaging model from a noiseless image
        Compute reconstruction
        Compute RIQ and store in `RIQs`
    end for
    For the current radius, store the minimum value of `RIQs` in `minRIQs`
end for
Output the radius corresponding to the highest RIQ in `minRIQs`

Designing the phase mask

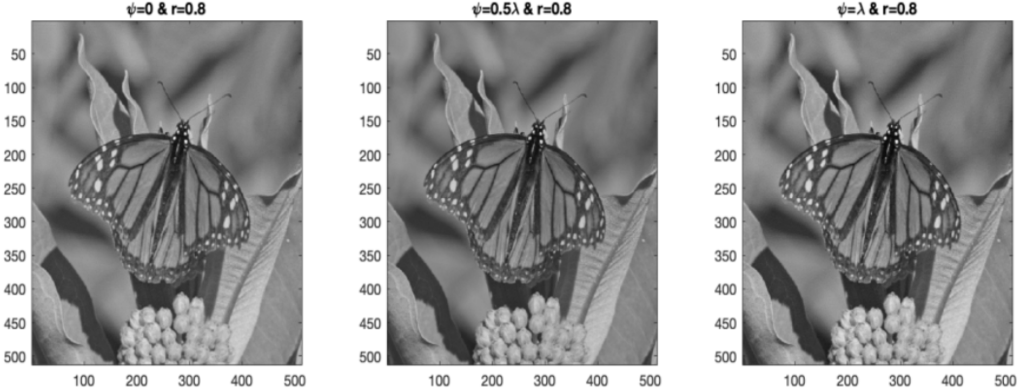
Designing a 3-ring mask: We do the same but with a (r_1, r_2) pair of values!

Obtained results

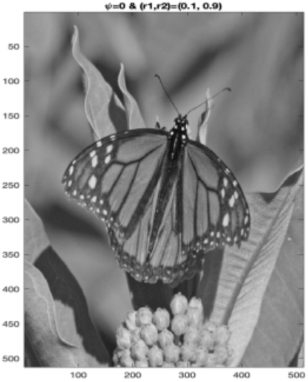
Original effect of defocus

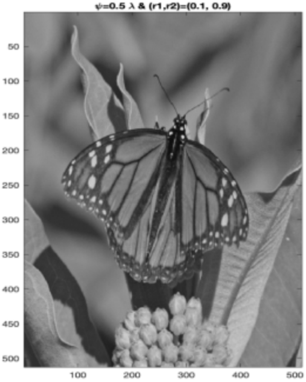


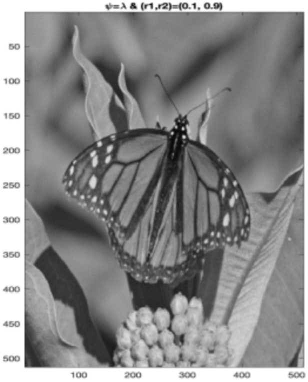
Using 1-ring mask



Using 3-ring mask







Questions?