```
In[1]:= Remove["Global`*"]
(* Y.Jinyun,Symmetric gaussian quadrature formulae for tetrahedronal regions,
Comp.Meth.Appl.Mech.Eng.43 (1984),349--353.*)
(* http://nines.cs.kuleuven.be/ecf/mtables.html *)

(* Here we export the integration point coordinates and the weights for *)
(* the reference tetrahedron (0,0,0|1,0,0|0,1,0|0,0,1) *)

(* These are used for the cyclic permutation *)
F1 = {{0, 0, 1}, {0, 1, 0}, {-1, -1, -1}};
F2 = {{0, 0, 1}, {-1, -1, -1}, {1, 0, 0}};
F3 = {{-1, -1, -1}, {0, 0, 1}, {0, 1, 0}};

(* Create empty tables for weights and coordinates *)
NGP = 29;
weights = Table[ToExpression[StringJoin["w", ToString[i]]], {i, 1, NGP}];
coords = Table[{
    ToExpression[StringJoin["x", ToString[i]]],
    ToExpression[StringJoin["y", ToString[i]]],
    ToExpression[StringJoin["z", ToString[i]]]},
   {i, 1, NGP}];

(* Start filling the weight and coordinate tables *)

(* 1 Center point *)
coords[[1]] = {1 / 4, 1 / 4, 1 / 4};
weights[[1]] = 0.0150668817433579497383277309990912;

(* 2..5: 4 first non-centers point near 0,0,0 and permutations *)
coords[[2]] = {0.0574269173173568195799787251408230,
   0.0574269173173568195799787251408230, 0.0574269173173568195799787251408230};
coords[[3]] = F1.coords[[2]] + {0, 0, 1};
coords[[4]] = F2.coords[[2]] + {0, 1, 0};
coords[[5]] = F3.coords[[2]] + {1, 0, 0};
weights[[2]] = 3.18663904649853147632014415654494 * 10 ^ (-3);
weights[[3]] = 3.18663904649853147632014415654494 * 10 ^ (-3);
weights[[4]] = 3.18663904649853147632014415654494 * 10 ^ (-3);
weights[[5]] = 3.18663904649853147632014415654494 * 10 ^ (-3);

(* 6..17: 12 points: first set of non-centers points near axis 0,
0,0-0.25,0.25,0.25 and permutations *)
coords[[6]] = {0.231298543651914663423853440991853,
   0.231298543651914663423853440991853, 0.486051028570607278709198710768507};
coords[[7]] = F1.coords[[6]] + {0, 0, 1};
coords[[8]] = F2.coords[[6]] + {0, 1, 0};
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coords[[9]] = F3.coords[[6]] + {1, 0, 0};
coords[[10]] = {0.4860510285706072787091987107685070,
   0.2312985436519146634238534409918 53, 0.2312985436519146634238534409918 53};
coords[[11]] = F1.coords[[10]] + {0, 0, 1};
coords[[12]] = F2.coords[[10]] + {0, 1, 0};
coords[[13]] = F3.coords[[10]] + {1, 0, 0};
coords[[14]] = {0.2312985436519146634238534409918 53,
   0.4860510285706072787091987107685070, 0.2312985436519146634238534409918 53};
coords[[15]] = F1.coords[[14]] + {0, 0, 1};
coords[[16]] = F2.coords[[14]] + {0, 1, 0};
coords[[17]] = F3.coords[[14]] + {1, 0, 0};
weights[[6]] = 7.2691564011109382427152201950077 7 * 10 ^ (-3);
weights[[7]] = 7.2691564011109382427152201950077 7 * 10 ^ (-3);
weights[[8]] = 7.2691564011109382427152201950077 7 * 10 ^ (-3);
weights[[9]] = 7.2691564011109382427152201950077 7 * 10 ^ (-3);
weights[[10]] = 7.2691564011109382427152201950077 7 * 10 ^ (-3);
weights[[11]] = 7.2691564011109382427152201950077 7 * 10 ^ (-3);
weights[[12]] = 7.2691564011109382427152201950077 7 * 10 ^ (-3);
weights[[13]] = 7.2691564011109382427152201950077 7 * 10 ^ (-3);
weights[[14]] = 7.2691564011109382427152201950077 7 * 10 ^ (-3);
weights[[15]] = 7.2691564011109382427152201950077 7 * 10 ^ (-3);
weights[[16]] = 7.2691564011109382427152201950077 7 * 10 ^ (-3);
weights[[17]] = 7.2691564011109382427152201950077 7 * 10 ^ (-3);

(* 18..29: 12 points: second set of non-centers points near axis 0,
0,0-0.25,0.25,0.25 and permutations *)
coords[[18]] = {0.0475690988147229596460214192031380,
   0.0475690988147229596460214192031380, 0.6081079894015280860743902216333 50};
coords[[19]] = F1.coords[[18]] + {0, 0, 1};
coords[[20]] = F2.coords[[18]] + {0, 1, 0};
coords[[21]] = F3.coords[[18]] + {1, 0, 0};
coords[[22]] = {0.6081079894015280860743902216333 50,
   0.0475690988147229596460214192031380, 0.0475690988147229596460214192031380};
coords[[23]] = F1.coords[[22]] + {0, 0, 1};
coords[[24]] = F2.coords[[22]] + {0, 1, 0};
coords[[25]] = F3.coords[[22]] + {1, 0, 0};
coords[[26]] = {0.0475690988147229596460214192031380,
   0.6081079894015280860743902216333 50, 0.0475690988147229596460214192031380};
coords[[27]] = F1.coords[[26]] + {0, 0, 1};
coords[[28]] = F2.coords[[26]] + {0, 1, 0};
coords[[29]] = F3.coords[[26]] + {1, 0, 0};
weights[[18]] = 4.3019459936652776758729763917751 9 * 10 ^ (-3);
weights[[19]] = 4.3019459936652776758729763917751 9 * 10 ^ (-3);
weights[[20]] = 4.3019459936652776758729763917751 9 * 10 ^ (-3);
weights[[21]] = 4.3019459936652776758729763917751 9 * 10 ^ (-3);
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weights[[22]] = 4.3019459936652776758729763917751 9 * 10 ^ (-3);
weights[[23]] = 4.3019459936652776758729763917751 9 * 10 ^ (-3);
weights[[24]] = 4.3019459936652776758729763917751 9 * 10 ^ (-3);
weights[[25]] = 4.3019459936652776758729763917751 9 * 10 ^ (-3);
weights[[26]] = 4.3019459936652776758729763917751 9 * 10 ^ (-3);
weights[[27]] = 4.3019459936652776758729763917751 9 * 10 ^ (-3);
weights[[28]] = 4.3019459936652776758729763917751 9 * 10 ^ (-3);
weights[[29]] = 4.3019459936652776758729763917751 9 * 10 ^ (-3);

(* Draw reference tetrahedron with integration points *)
Graphics3D[{
  Flatten[{
    Line[{{0, 0, 0}, {0, 0, 1}, {0, 1, 0},
      {1, 0, 0}, {0, 0, 1}, {0, 0, 0}, {0, 1, 0}, {0, 0, 0}, {1, 0, 0}}],
    Table[{Sphere[coords[[i]], (0.01 * Abs[weights[[i]]]) ^ (1 / 3)]}, {i, 1, NGP}]
   }]
 }]

(* Test whether accurate up to 6th order *)
Print["Accurate up to sixth order?"]
$Assumptions = {Element[{x, y, z}, Reals], x > 0, y > 0, z > 0};
Funk[x_, y_, z_] = (x + 0.001) ^ 2 * (y + 1.1) ^ 2 * (z + 0.1) ^ 2;
Print["Numerical integration by MMA minus numerical
    integration by IP-Scheme on 6th order polynomial: ", NumberForm[
   Integrate[Integrate[Integrate[Funk[x, y, z], {x, 0, 1 - y - z}], {y, 0, 1 - z}],
     {z, 0, 1}] - Sum[Funk[coords[[i, 1]], coords[[i, 2]], coords[[i, 3]]] *
      weights[[i]], {i, 1, NGP}], 20]];
Funk[x_, y_, z_] = (x + 0.001) ^ 2 * (y + 1.1) ^ 3 * (z + 0.1) ^ 2;
Print["Numerical integration by MMA minus numerical
    integration by IP-Scheme on 7th order polynomial: ", NumberForm[
   Integrate[Integrate[Integrate[Funk[x, y, z], {x, 0, 1 - y - z}], {y, 0, 1 - z}],
     {z, 0, 1}] - Sum[Funk[coords[[i, 1]], coords[[i, 2]], coords[[i, 3]]] *
      weights[[i]], {i, 1, NGP}], 20]];
Funk[x_, y_, z_] = (x + 0.001) ^ 2 * (y + 1.1) ^ 3 * (z + 0.1) ^ 3;
Print["Numerical integration by MMA minus numerical
    integration by IP-Scheme on 8th order polynomial: ", NumberForm[
   Integrate[Integrate[Integrate[Funk[x, y, z], {x, 0, 1 - y - z}], {y, 0, 1 - z}],
     {z, 0, 1}] - Sum[Funk[coords[[i, 1]], coords[[i, 2]], coords[[i, 3]]] *
      weights[[i]], {i, 1, NGP}], 20]];
Funk[x_, y_, z_] = (x + 0.001) ^ 3 * (y + 1.1) ^ 3 * (z + 0.1) ^ 3;
Print["Numerical integration by MMA minus numerical
    integration by IP-Scheme on 9th order polynomial: ", NumberForm[
   Integrate[Integrate[Integrate[Funk[x, y, z], {x, 0, 1 - y - z}], {y, 0, 1 - z}],
     {z, 0, 1}] - Sum[Funk[coords[[i, 1]], coords[[i, 2]], coords[[i, 3]]] *
      weights[[i]], {i, 1, NGP}], 20]];
```
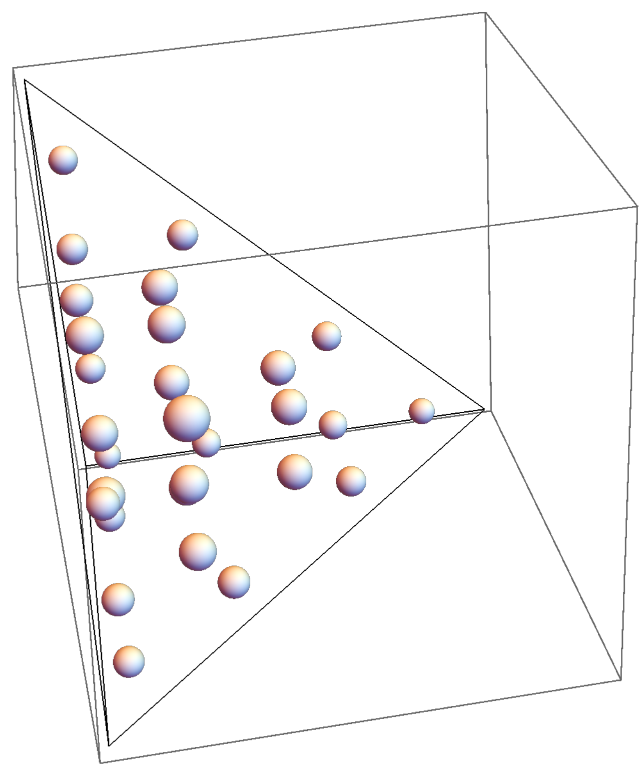

```
(* Export weights and coordinates for use in UEL *)
str = OpenWrite["~/29GP.txt"];
WriteString[str,
  StringJoin["\n! Integration points and weights for reference element
     ((0 0 0)(1 0 0)(0 1 0)(0 0 1))\n"]];
WriteString[str, StringJoin["      double precision GP(29,4)\n"]];
For[i = 1, i ≤ 29, i++,
 WriteString[str, StringJoin["      GP(", ToString[i],
   ",1)=", ToString[FortranForm[coords[[i, 1]]]], "\n"]];
 WriteString[str, StringJoin["      GP(", ToString[i], ",2)=",
   ToString[FortranForm[coords[[i, 2]]]], "\n"]];
 WriteString[str, StringJoin["      GP(", ToString[i], ",3)=",
   ToString[FortranForm[coords[[i, 3]]]], "\n"]];
 WriteString[str, StringJoin["      GP(", ToString[i],
   ",4)=", ToString[FortranForm[weights[[i]]]], "\n"]];
]
Close[str]
```

... Remove: There are no symbols matching "Global`*".

Out[66]=



```
Accurate up to sixth order?
Numerical integration by MMA minus numerical integration
   by IP-Scheme on 6th order polynomial: 2.732189474663471 × 10^-17
Numerical integration by MMA minus numerical integration
   by IP-Scheme on 7th order polynomial: -2.028334428368084 × 10^-9
Numerical integration by MMA minus numerical integration
   by IP-Scheme on 8th order polynomial: -2.279332739609299 × 10^-8
Numerical integration by MMA minus numerical integration
   by IP-Scheme on 9th order polynomial: -2.119993284594086 × 10^-7
Out[81]= /home/gluege/29GP.txt
```