# Compression Strain Measurement by Digital Speckle Correlation

by D. Zhang, X. Zhang and G. Cheng

ABSTRACT—In this paper, an optimized digital speckle correlation algorithm, named big-window correlation, is proposed to iterate strain directly. Verified by some experiments, the sensitivity and accuracy of the displacement gradient measurement with this method can be improved greatly. Finally, this method was applied to the measurement of the compression strain for polyurethane foam plastics materials. Then the material properties, such as the module of elasticity and the Poisson ratio, with different mass densities were obtained.

KEY WORDS—Digital speckle correlation, PFP, deformation measurement

#### Introduction

Digital speckle correlation methodology has been widely used to measure the deformation of an object surface in optical mechanics with the development of the digital imageprocessing technique. One basic assumption in this technique is that during the deformation, the gray level distribution before and after loading is almost the same if the surface is illuminated by a constant light source. Although the subpixel technique had been introduced, this method was often applied to the measurement of displacement, which can reach a precision of 0.1 pixels. The displacement gradients were usually obtained by displacement difference with numerical methods. Recent research has indicated that the strain could be derived directly in the correlation iteration, but the sensitivity was about 1 percent. 1,2 The method proposed in this paper will use a big subset, in which the pixel members are still less, to iterate the displacement gradients directly with high accuracy.

Polyurethane foam plastics (PFP) has a wide application in engineering. The mechanical behavior of compression is one of the most important properties. Some electric testing methods have been employed to measure the compression strain. Since the PFP has a large range of compression strain, the electric testing method is not suitable for the measurement of PFP with a large deformation. In this paper, the digital speckle correlation was implemented to measure the compression strain of these materials, and some important results were obtained.

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## **Big-window Digital Speckle Correlation**

The experimental scheme used in our digital speckle correlation is shown in Fig. 1. The rough surface is illuminated by a uniform incoherent light, and a CCD camera captures the speckle light field in the normal direction of the surface. For planar deformation analysis, the motion of the object surface is parallel to the camera image plane. Digitized by an image board, the image is sampled into a size of  $512 \times 512$  pixels with 256 gray levels. When two images are digitized before and after deformation, respectively, the digital speckle correlation becomes a job of comparing subsets of numbers between these images. The correlation formula in our study, taken in this paper, is expressed as follows:

$$C = \frac{\langle F_1 \cdot F_2 \rangle - \langle F_1 \rangle \cdot \langle F_2 \rangle}{\left[\langle (F_1 - \langle F_1 \rangle)^2 \rangle \cdot \langle (F_2 - \langle F_2 \rangle)^2 \rangle\right]^2}$$
 (1)

where  $F_1$  is the gray level matrix of the subset at coordinates (x, y) for one image, and  $F_2$  is the gray value matrix of the subset at coordinates  $(x^*, y^*)$  for the second image.  $\langle \bullet \rangle$  means to get the mean value of the elements in a matrix.

For planar analysis, a small segment line PQ moved to  $P^*Q^*$  after deformation, as shown in Fig. 2. So the displacement of point Q can be written as

$$u_{Q} = u_{P} + \left(\frac{\partial u}{\partial x}\right)_{P} dx + \left(\frac{\partial u}{\partial y}\right)_{P} dy$$

$$v_{Q} = v_{P} + \left(\frac{\partial v}{\partial x}\right)_{P} dx + \left(\frac{\partial v}{\partial y}\right)_{P} dy$$
(2)

where  $u_P$  and  $v_P$  are the displacements of point P in the x- and y-direction, and  $u_Q$  and  $v_Q$  are the displacements of point Q in the x, y-direction.  $\frac{\partial u}{\partial x}$ ,  $\frac{\partial u}{\partial y}$ ,  $\frac{\partial v}{\partial x}$  and  $\frac{\partial v}{\partial y}$  are its displacement gradients. The terms dx, dy are the distances from the subset center to point Q. So the new coordinate of  $Q^*$  after deformation is expressed as

$$x_Q^* = x_Q + u_Q$$
  
$$y_Q^* = y_Q + v_Q$$
 (3)

According to the basic assumption of digital speckle correlation, the gray level distribution around Q and  $Q^*$  should be same, so the following expression is derived:

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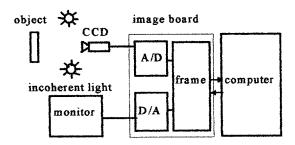


Fig. 1—Experimental setup for digital speckle correlation

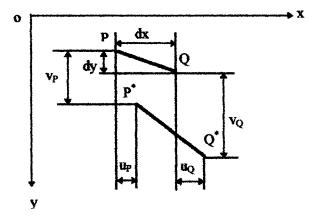


Fig. 2—Deformation of subset

$$f(x_{Q}, y_{Q}) = f^{*}(x_{Q}^{*}, y_{Q}^{*})$$

$$= f^{*} \left[ x_{Q} + u_{P} + \left( \frac{\partial u}{\partial x} \right)_{P} dx + \left( \frac{\partial u}{\partial y} \right)_{P} dy, \right.$$

$$\cdot y_{Q} + v_{P} + \left( \frac{\partial v}{\partial x} \right)_{P} dx + \left( \frac{\partial v}{\partial y} \right)_{P} dy \right]$$
(4)

where  $f(x_Q, y_Q)$  and  $f^*(x_Q^*, y_Q^*)$  are the gray level value distribution around Q and  $Q^*$ , respectively. Equation 4 indicates that the coordinate of  $Q^*$  can be determined if the displacement and the displacement gradients in point P are known. Digital speckle correlation is performed by determined values u, v,  $\frac{\partial u}{\partial x}$ ,  $\frac{\partial u}{\partial y}$ ,  $\frac{\partial v}{\partial x}$  and  $\frac{\partial v}{\partial x}$ , which maximize the correlation coefficient in eq (1). In this paper, a coarse-fine search routine is taken in the correlation iteration. Due to the discrete nature of the digital image, there is no gray level value between sampled points. To get a continuous intensity distribution for accurate measurement, a bilinear interpolation was used to obtain the subpixel level value.

Traditional correlation generally uses a subset of about  $30 \times 30$  pixels in neighborhood to search the correlative area. This may lead to a reduction of the precision for displacement gradients. According to eq (2), the displacement of point Q is determined by both the displacement gradients and the distance between P and Q. If the displacement gradients are less than 1 percent and a subset with  $30 \times 30$  pixels is used to search the correlative area, the maximum deformation in the x-direction is about 0.15 pixels. Since the results in Table 1 and those in Ref. 3 show that the accuracy of displacement, derived from digital speckle correlation with  $512 \times 512$  res-

olution by bilinear interpolation, is about 0.1 pixels, it is impossible to get displacement gradients less than 1 percent by traditional correlative iteration. For larger displacement gradients, the error will be great.<sup>3,4</sup>

The subset in big-window correlation extracts every gray level value from each of the 10 pixels (in this paper) in column and row. So the subset covers a large area with fewer pixel numbers. The distance between the center point and the neighbor points in the subset is 10 times the distance in comparison with the traditional correlation subset. Consequently, the displacement gradients could reach about 0.1 percent when deformation in the subset is about 0.1 pixels. So it becomes possible to iterate displacement gradients of 0.1 percent or less through correlation iteration directly. In Fig. 2, the straight-line PQ still yields straight after deformation under the condition of small or uniform strain. Thus, the second assumption of digital speckle correlation still stands. Since dx and dy become larger,  $\frac{\partial u}{\partial x}$ ,  $\frac{\partial u}{\partial y}$ ,  $\frac{\partial v}{\partial x}$  and  $\frac{\partial v}{\partial y}$  can be detected much smaller than those obtained by the general subset. This can improve the precision approximately 10 times according to eq (3) and eq (4). In our experiment, the error of displacement gradients is less than 0.04 percent by the big-window correlation.

In the correlation method, the results are very sensitive to the number of subsets. If the number in the subset is not enough, the square difference of the results should be bigger even in a uniform strain field. On the other hand, if the number is too large, the correlation would use more computation time. To define a suitable window size, an experiment was done. The curve in Fig. 3 shows the relationship between the square difference and the window size. In this paper, the window size was chosen as  $25 \times 25$ .

## **Experiments**

To verify the ability of this big-window correlation method, two experiments were performed to test the accuracy. The experimental setup is shown in Fig. 1.

#### Rigid Body Translation

A steel plate, whose surface was smeared with aluminum powder, was mounted on a precise movable platform. The

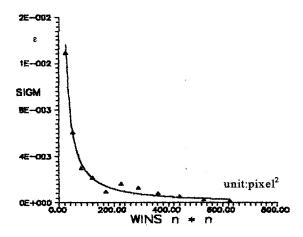


Fig. 3—Relationship between subset size and square difference

TABLE 1—RESULTS OF DISPLACEMENT OBTAINED BY DIGITAL SPECKLE CORRELATION

Displacement (mm)	Pixel	mm	Error (mm)
0.100	$2.80 \pm 0.03$	0.101 ± 0.001	0.001
0.610	$17.12 \pm 0.05$	$0.616 \pm 0.002$	0.002
1.020	$28.58 \pm 0.06$	$1.029 \pm 0.002$	0.002

displacement of the platform can be read directly by a vernier micrometer. The speckle images of the steel plate during the motion were recorded by the CCD camera. In the speckle image, five subsets were selected. The big-window correlation was implemented to iterate the displacement in the five subsets. Results and those read from the micrometer are listed in Table 1. Surprisingly, the precision of the big-window correlation for displacement measurement is a little higher than 0.1 pixels.

### Uniform Strain

A tension sample, whose sizes are  $254 \times 20 \times 0.8 \text{ mm}^3$ , was fixed on a WKW-100B testing machine. In the middle of the sample, the strain can be considered to be uniform if the deformation is limited in the elastic range. The strains were measured both by an axial extensometer and the bigwindow correlation. Also, five subsets were selected in the speckle image. The strains obtained by both methods are given in Table 2. The results show good agreement with those obtained by the extensometer. The error estimation of local strain measured by big-window correlation is limited in a range of about 0.04 percent.

#### **Compression Strain Analysis for PFP**

PFP has a rough surface with a lot of foam. The strain cannot be measured by a strain gauge. Because of the viscoelasticity and the low module of elasticity, some optical methods are not suitable for the measurement either. In this paper, the compression strains of PFP were measured by bigwindow correlation. The PFP testing samples with two mass densities (0.3g/cm<sup>3</sup> and 0.5g/cm<sup>3</sup>) were made in a cube with the same sizes of 40 mm long, 25 mm wide and 25 mm high. Three testing samples were chosen from each sort of mass densities. The testing surface was smeared with aluminum powder to increase reflection brightness. The loading speed was controlled to be 0.05 mm/min as a quasi-static loading. Since the digital image-processing system captures 25 frames per second, we can assume that no oversampling occurs. In the experiments, the out-of-plane deformation might affect planar displacement while using digital correlation. To decrease this effect, a sequence of speckle images with different load states was frozen, and the correlation was done between each two neighborhood images. For the testing samples whose mass density is bigger, speckle images were captured when the loading step reached 500N. With the increase of the loading, the loading step gradually decreased until the total loading reached 8200N. The captured speckle images were digitized and stored in the computer, then analyzed by big-window correlation one by one. The results are shown in Table 3 and Fig. 4. The same procedure was taken for the other samples with smaller mass densities. The results are shown in Table 4 and Fig. 5. In Fig. 4 and Fig. 5, when the loading was not big, the deformation should be in the elastic range. A least square fitting with powers of 1 was used to derive the elastic modules for the two sorts of PFP:

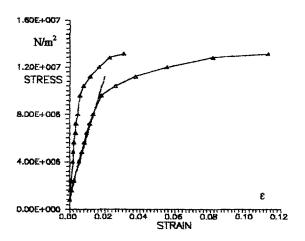


Fig. 4—Strain versus loading for PFP (0.5 g/cm<sup>3</sup>)

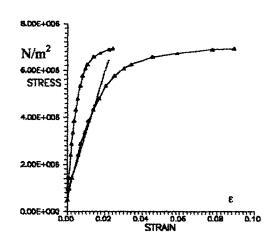


Fig. 5—Strain versus loading for PFP (0.3 g/cm<sup>3</sup>)

$$E_{0.5} = 5.002 \times 10^3 N/m^2$$
$$E_{0.3} = 2.506 \times 10^3 N/m^2$$

In the meantime, the Poisson ratio, which means the ratio of strain in the x- and y-directions here, was also obtained by the strains in the orthogonal directions. Fig. 6 and Fig. 7 show how the Poisson ratio changes with the increasing loading.

## Conclusion

The big-window correlation proposed in this paper increases the precision and sensitivity for strain measurement. The experiments, which measure the displacement of the rigid body and the local strain, have verified that this method

TABLE 2—RESULTS OF STRAIN OBTAINED BY ELECTRIC METHOD AND DIGITAL SPECKLE CORRELATION (UNIT: 0.1 PERCENT)

Result by Electric Method	Result by Correlation Method	Error	
0.6	$0.6 \pm 0.2$	0.2	
1.2	1.1 ± 0.1	0.1	
1.6	$1.7 \pm 0.3$	0.3	
2.2	$2.2 \pm 0.2$	0.2	
3.2	$3.2 \pm 0.3$	0.3	
4.4	$4.3 \pm 0.3$	0.3	
5.4	$5.2 \pm 0.4$	0.4	

TABLE 3-DATA OF COMPRESSION STRAIN AND POISSON RATIO FOR PFP (0.5 a/cm3)

Stress				Stress			
(10 <sup>6</sup> N/m <sup>2</sup> )	$\varepsilon_x(10^{-3})$	$\varepsilon_y(10^{-3})$	$\mu(10^{-1})$	(10 <sup>6</sup> N/m <sup>2</sup> )	$\varepsilon_x(10^{-3})$	$\varepsilon_{\rm y}(10^{-3})$	$\mu(10^{-1})$
1.6	0.31	-1.27	2.45	8.0	4.74	-13.3	3.55
2.4	0.78	-2.63	2.95	9.6	5.87	-18.2	3.23
4.0	1.80	-5.33	3.39	10.4	8.12	-26.0	3.11
4.8	2.20	-7.03	3.12	11.2	11.5	-37.5	3.07
5.6	2.63	-8.36	3.15	12.0	16.8	-55.2	3.03
6.4	3.31	-9.89	3.34	12.8	22.7	-81.4	2.78
7.2	3.69	-11.5	3.19	13.12	30.7	-112.8	2.72

TABLE 4-DATA OF COMPRESSION STRAIN AND POISSON RATIO FOR PFP (0.3 g/cm3)

Stress				Stress			
(10 <sup>6</sup> N/m <sup>2</sup> )	$\varepsilon_x(10^{-3})$	$\varepsilon_y(10^{-3})$	$\mu(10^{-1})$	(10 <sup>6</sup> N/m <sup>2</sup> )	$\varepsilon_x(10^{-3})$	$\varepsilon_{\gamma}(10^{-3})$	$\mu(10^{-1})$
0.96	0.27	-1.03	2.58	5.33	6.99	-20.7	3.37
1.44	0.76	-2.50	3.01	5.76	8.09	-25.3	3.19
2.40	1.82	-5.70	3.19	6.08	9.53	-30.2	3.15
2.88	2.40	-7.29	3.29	6.24	10.5	-34.2	3.07
3.36	3.02	-9.24	3.26	6.56	14.0	-45.7	3.07
3.84	3.76	-11.3	3.31	6.72	17.5	-58.8	2.97
4.32	4.49	-13.9	3.22	6.88	22.2	-77.6	2.85
4.80	5.64	<b>-17.1</b>	3.30	6.91	24.3	-89.2	2.73

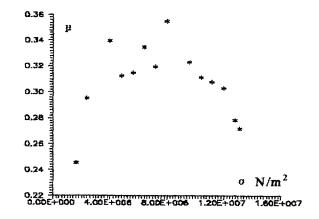


Fig. 6—Poisson ratio versus loading for PFP (0.5 g/cm<sup>3</sup>)

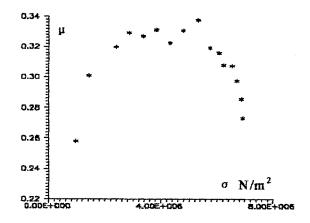


Fig. 7—Poisson ratio versus loading for PFP (0.3 g/cm<sup>3</sup>)

has good stability and is suitable in engineering. This method was also applied to the measurement of the compression strain for the PFP with two sorts of mass densities. Some interesting results and conclusions have been given.

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