NNS in QF

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Introduction to Partial Moments

Q. What are partial moments?

A. The elements of variance.

Univariate:

$$LPM(\color{red}{degree}, \color{steelblue}{target}, variable) = \frac{1}{T}\sum_{t=1}^{T}\left[\max(0, \color{steelblue}{target} - variable_t)\right]^{\color{red}{degree}}$$

$$UPM(\color{red}{degree}, \color{steelblue}{target}, variable) = \frac{1}{T}\sum_{t=1}^{T}\left[\max(0, variable_t - \color{steelblue}{target})\right]^{\color{red}{degree}}$$

- Grounded in probability-weighted integration and rigorous measure theory

$$LPM(n, t, X) = \int_{-\infty}^{t} (t - x)^n \, dF(x)$$

- Time-series or cross-sectional

Partial Moment Equivalences

Mean

```r
library(NNS)
set.seed(123); x = rnorm(100); y = rnorm(100)

mean(x)
```

```
[1] 0.09040591
```

```r
UPM(degree = 1, target = 0, variable = x) - LPM(degree = 1, target
```

```
[1] 0.09040591
```

Variance

```
# Sample Variance (base R):
var(x)
```

```
[1] 0.8332328
```

```
# Sample Variance:
mu_x = mean(x)
(UPM(2, mu_x, x) + LPM(2, mu_x, x)) * (length(x) / (length(x) - 1))
```

```
[1] 0.8332328
```

```
# Population Adjustment of Sample Variance (base R):
var(x) * ((length(x) - 1) / length(x))
```

```
[1] 0.8249005
```

```
# Population Variance:
UPM(2, mu_x, x) + LPM(2, mu_x, x)
```

```
[1] 0.8249005
```

```
# Variance is also the co-variance of itself:
{(Co.LPM(degree_lpm = 1, x = x, y = x, target_x = mu_x, target_y =
  + Co.UPM(degree_upm = 1, x, x, mu_x, mu_x)
  - D.LPM(degree_lpm = 1, degree_upm = 1, x, x, mu_x, mu_x)
  - D.UPM(1, 1, x, x, mu_x, mu_x))}
```

```
[1] 0.8249005
```

Skewness

```
PerformanceAnalytics::skewness(x)
```

```
[1] 0.06049948
```

```
((UPM(3, mu_x, x) - LPM(3, mu_x, x)) / (UPM(2, mu_x, x) + LPM(2, mu
```

```
[1] 0.06049948
```

Kurtosis

```
PerformanceAnalytics::kurtosis(x)
```

```
[1] -0.161053
```

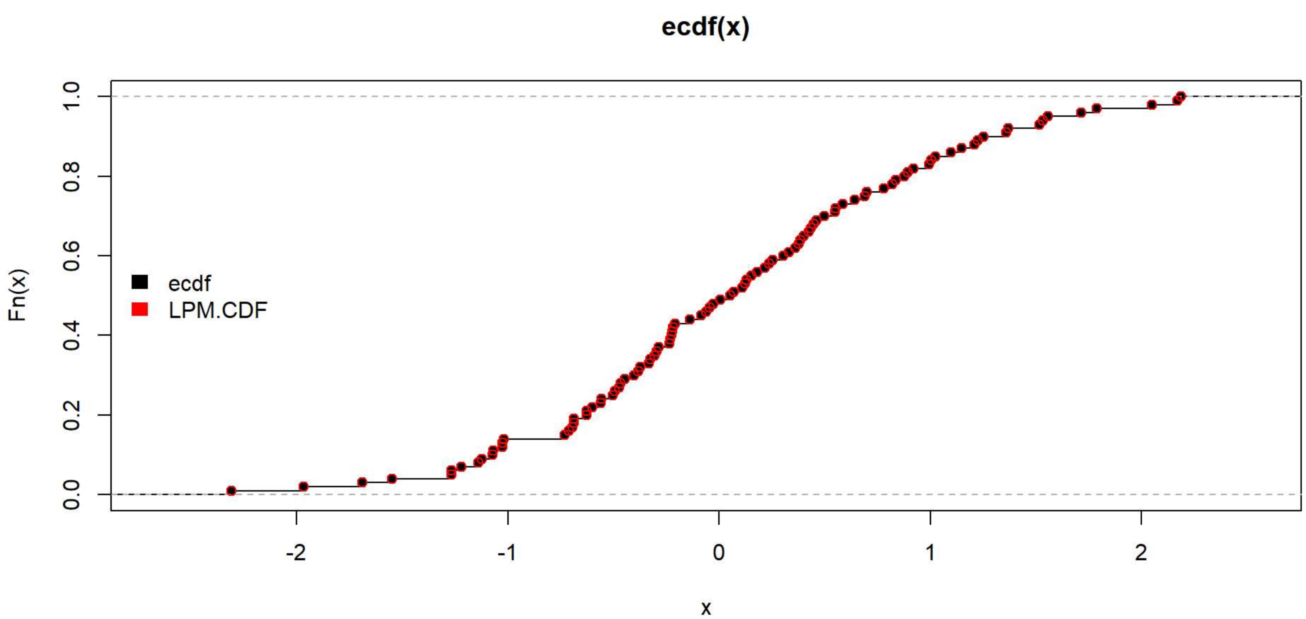
```
((UPM(4, mu_x, x) + LPM(4, mu_x, x)) / (UPM(2, mu_x, x) + LPM(2, mu
```

```
[1] -0.161053
```

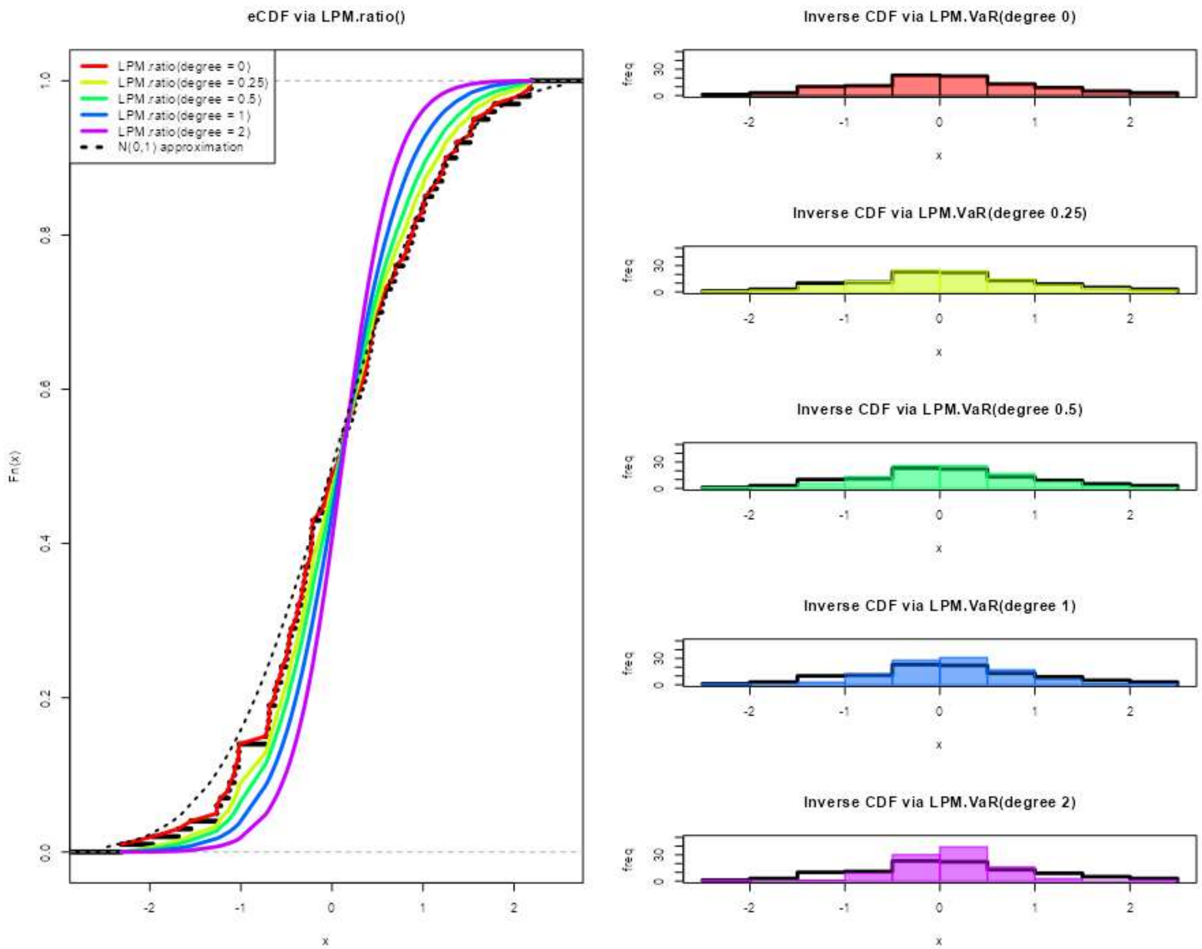
More Equivalences

LPM degree 0 is the eCDF for any distribution

```
LPM.CDF = LPM(degree = 0, target = sort(x), variable = x)

plot(ecdf(x))
points(sort(x), LPM.CDF, col='red')
legend('left', legend = c('ecdf','LPM.CDF'), fill=c('black','red'),
```

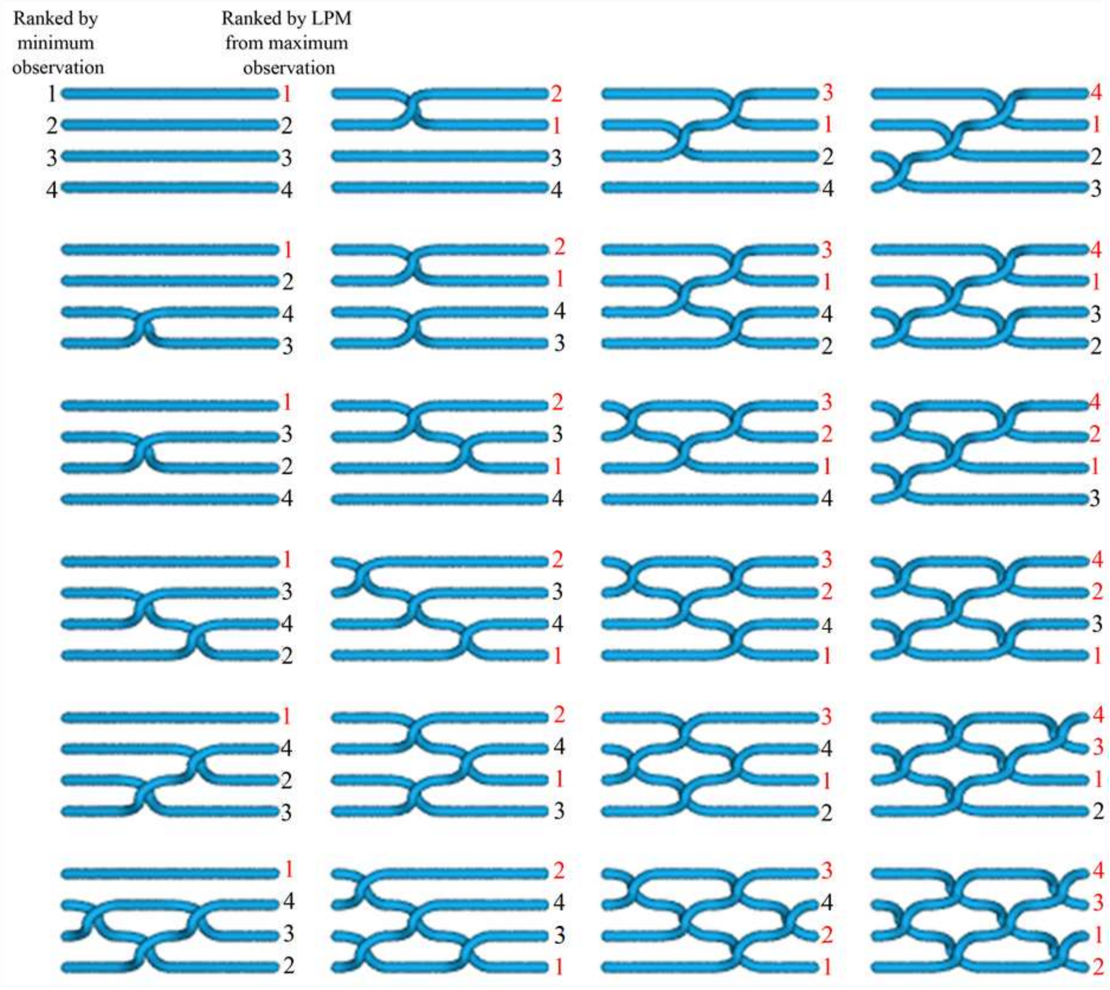


More CDFs and PDFs

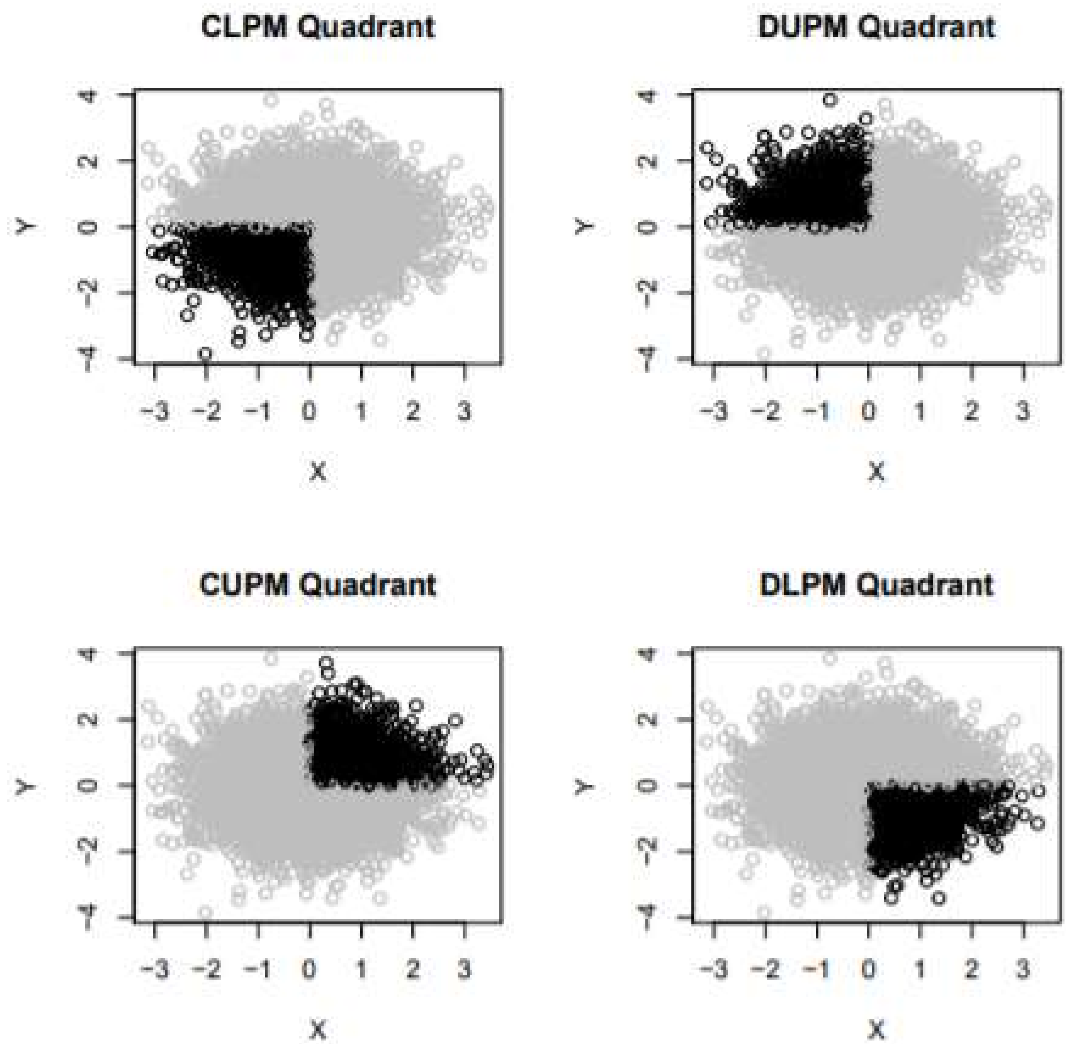


Still empirical CDFs, all we've done is increase the ***degree***

Comparing CDFs Led to Stochastic Dominant Efficient Sets



Multivariate



$$CLPM(degree, target, x, y) = \frac{1}{T} \sum_{t=1}^{T} [\max(0, target - x_t)]^{degree} [\max(0, target - y_t)]^{degree}$$

$$CUPM(degree, target, x, y) = \frac{1}{T} \sum_{t=1}^{T} [\max(0, x_t - target)]^{degree} [\max(0, y_t - target)]^{degree}$$

$$DLPM(degree, target, x, y) = \frac{1}{T} \sum_{t=1}^{T} [\max(0, x_t - target)]^{degree} [\max(0, target - y_t)]^{degree}$$

$$DUPM(degree, target, x, y) = \frac{1}{T} \sum_{t=1}^{T} [\max(0, target - x_t)]^{degree} [\max(0, y_t - target)]^{degree}$$

Covariance Equivalence

Covariance Elements and Covariance Matrix

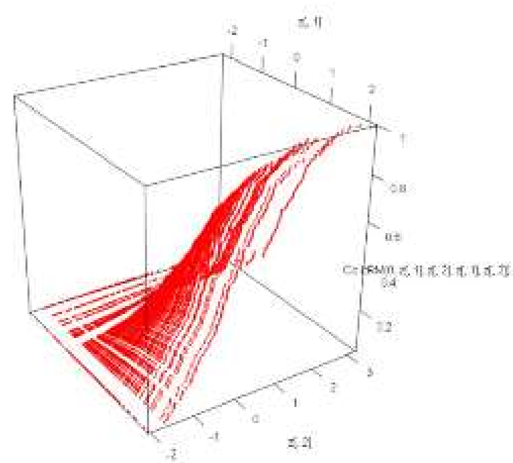
The covariance matrix (Σ) is equal to the sum of the co-partial moments matrices less the divergent partial moments matrices.

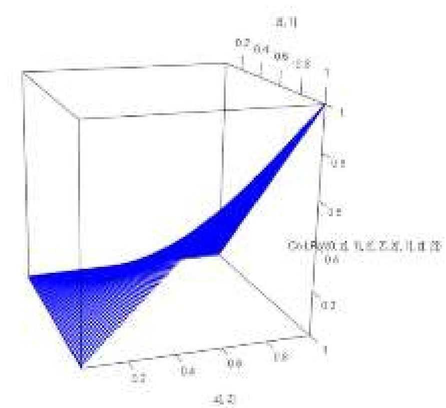
$$\Sigma = CLPM + CUPM - DLPM - DUPM$$

```
> cov.mtx = PM.matrix(LPM_degree = 1, UPM_degree = 1, target = 'mean', variable = cbind(x,y), pop_adj = TRUE)
> cov.mtx
$cupm
          x         y
x 0.4299250 0.1033601
y 0.1033601 0.5411626

$dupm
          x         y
x 0.0000000 0.1469182
y 0.1560924 0.0000000

$dlpm
          x         y
x 0.0000000 0.1560924
y 0.1469182 0.0000000

$clpm
          x         y
x 0.4033078 0.1559295
y 0.1559295 0.3939005

$cov.matrix
            x           y
x  0.83323283 -0.04372107
y -0.04372107  0.93506310


# Reassembled Covariance Matrix
> cov.mtx$cupm + cov.mtx$clpm - cov.mtx$dupm - cov.mtx$dlpm
            x           y
x  0.83323283 -0.04372107
y -0.04372107  0.93506310


# Standard Covariance Matrix
> cov(cbind(x,y))
            x           y
x  0.83323283 -0.04372107
y -0.04372107  0.93506310
```

Copulas

```
# Data
set.seed(123); x = rnorm(100); y = rnorm(100); z = expand.grid(x, y

# Plot
rgl::plot3d(z[,1], z[,2], Co.LPM(0, z[,1], z[,2], z[,1], z[,2]), co

# Uniform values
u_x = LPM.ratio(0, x, x); u_y = LPM.ratio(0, y, y); z = expand.gric

# Plot
rgl::plot3d(z[,1], z[,2], Co.LPM(0, z[,1], z[,2], z[,1], z[,2]), co
```





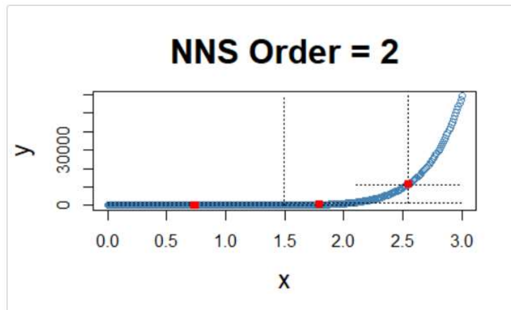
Partitioning Led to:

Nonlinear Correlation & Dependence

Nonlinear Relationship

Note the fact that all observations occupy the co-partial moment quadrants.

```
x = seq(0, 3, .01) ; y = x ^ 10
```




```
cor(x, y)
```

```
## [1] 0.6610183
```

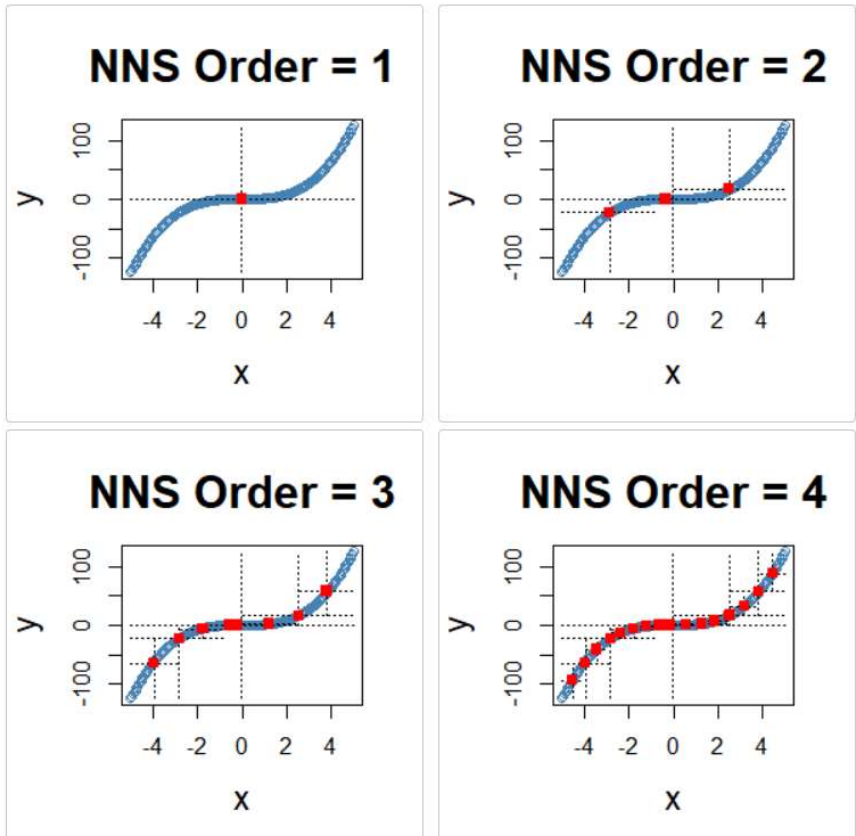
```
NNS.dep(x, y)
```

```
## $Correlation
## [1] 0.9325653
##
## $Dependence
## [1] 0.9325653
```

Compared to Mutual Information, Distance Correlation, Chatterjee’s Xi…

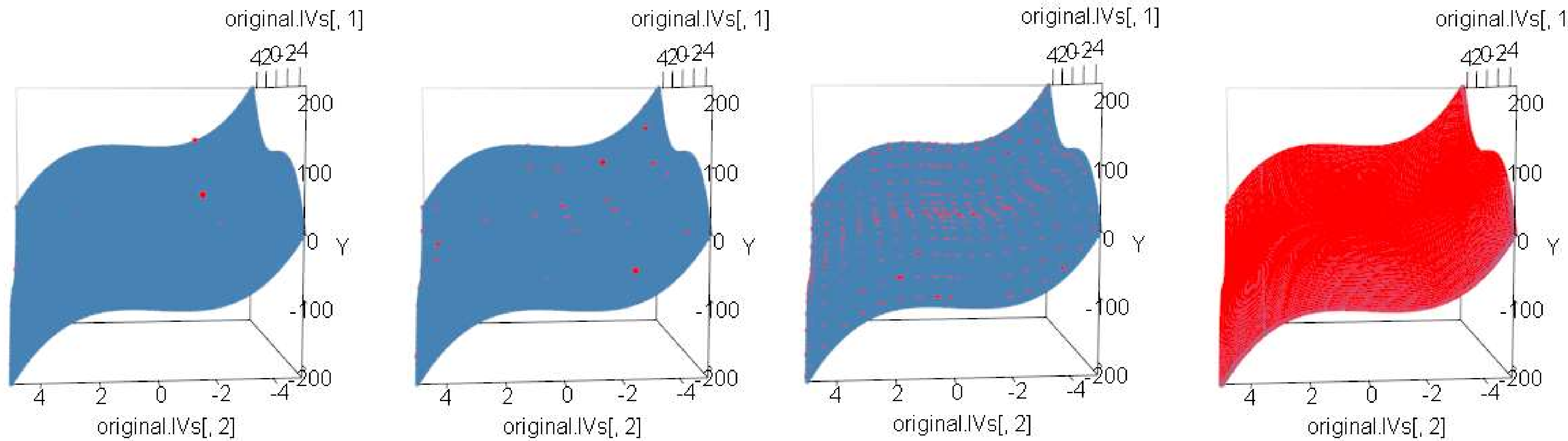
Partitioning Also Led to:

Nonlinear Regression



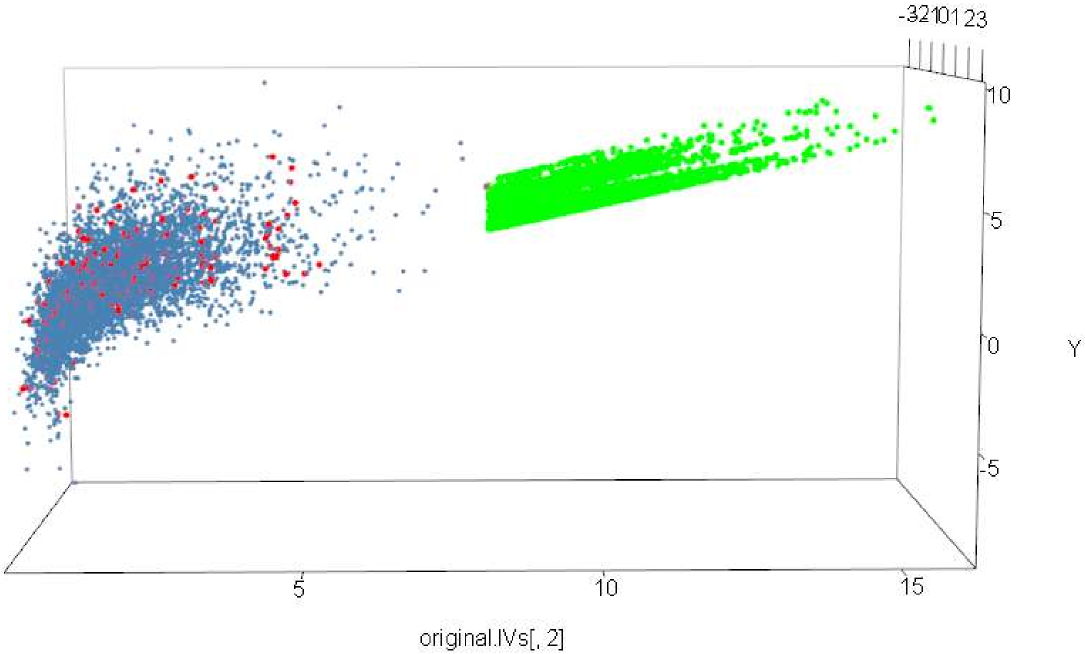
Offers dynamic bandwidth solution

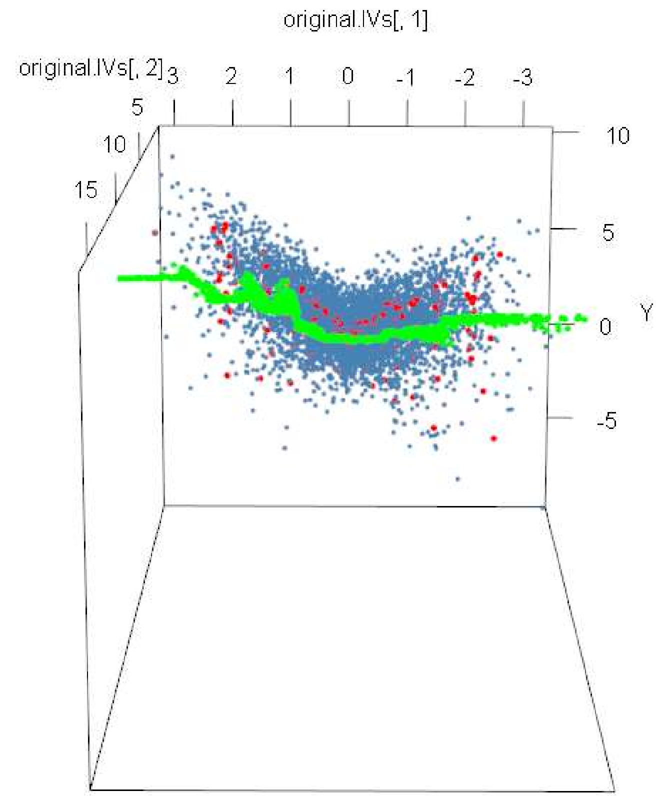
Multivariate Regression



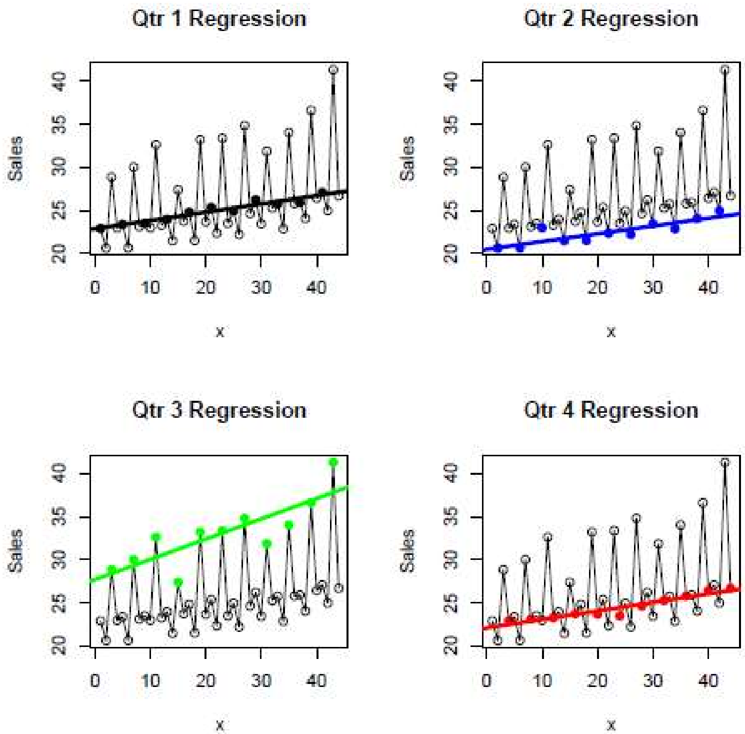
- Similar to kNN regression… cluster centroids instead of observations
- k-fold cross-validation for number of centroids

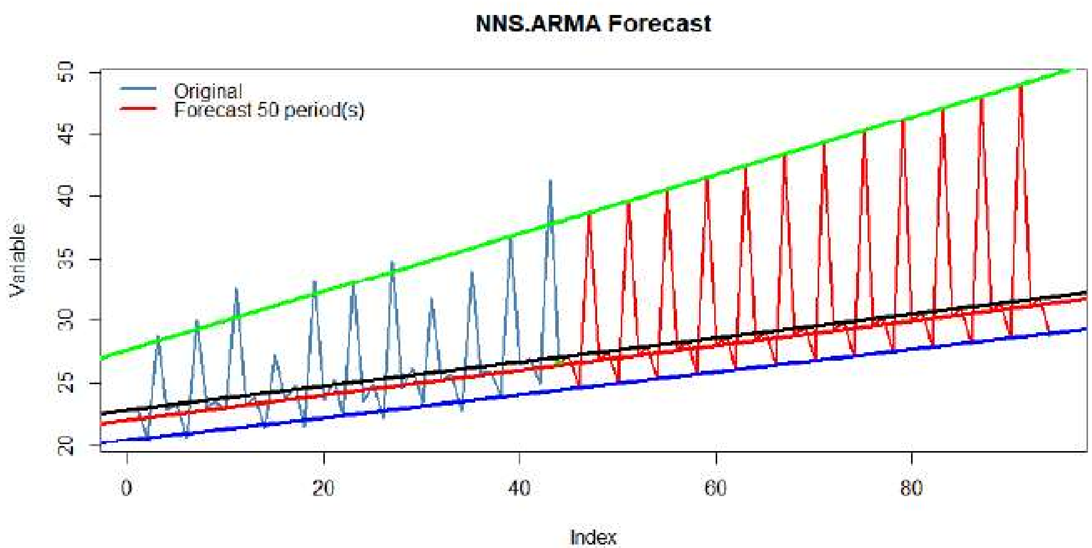
Very good at extrapolation





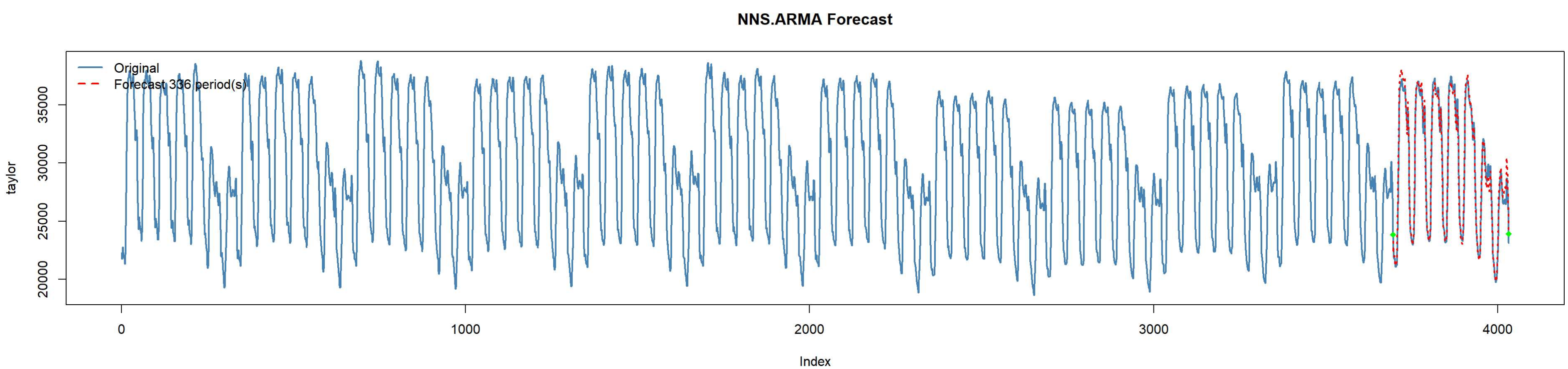
Nonlinear Regression Led to Time-Series Forecasting:





Time-Series Forecasting with Nonlinear Regression

```r
require(forecast); data(taylor)

NNS.ARMA(taylor, h = 336, training.set = length(taylor)-336,
           seasonal.factor = 336, plot = TRUE, method = 'nonlin')
```



and a multivariate extension via a nonparametric vector autoregression!

Generative and Synthetic Data

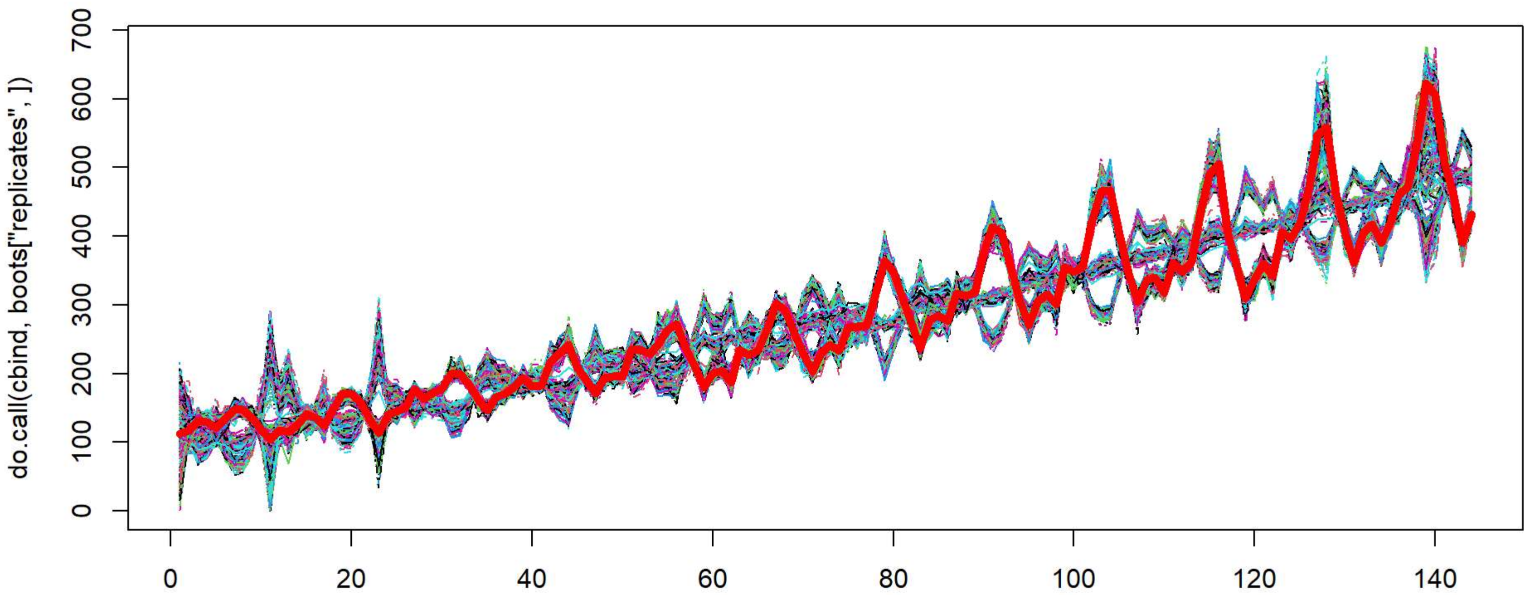
Maximum Entropy Bootstrap for Time-Series

- Specify a `rho` or `drift` for any replicate… ensures $\rho \in [-1, 1]$ correlation spectrum is covered.

- Standard IID MC or block bootstrap cannot accomplish this.

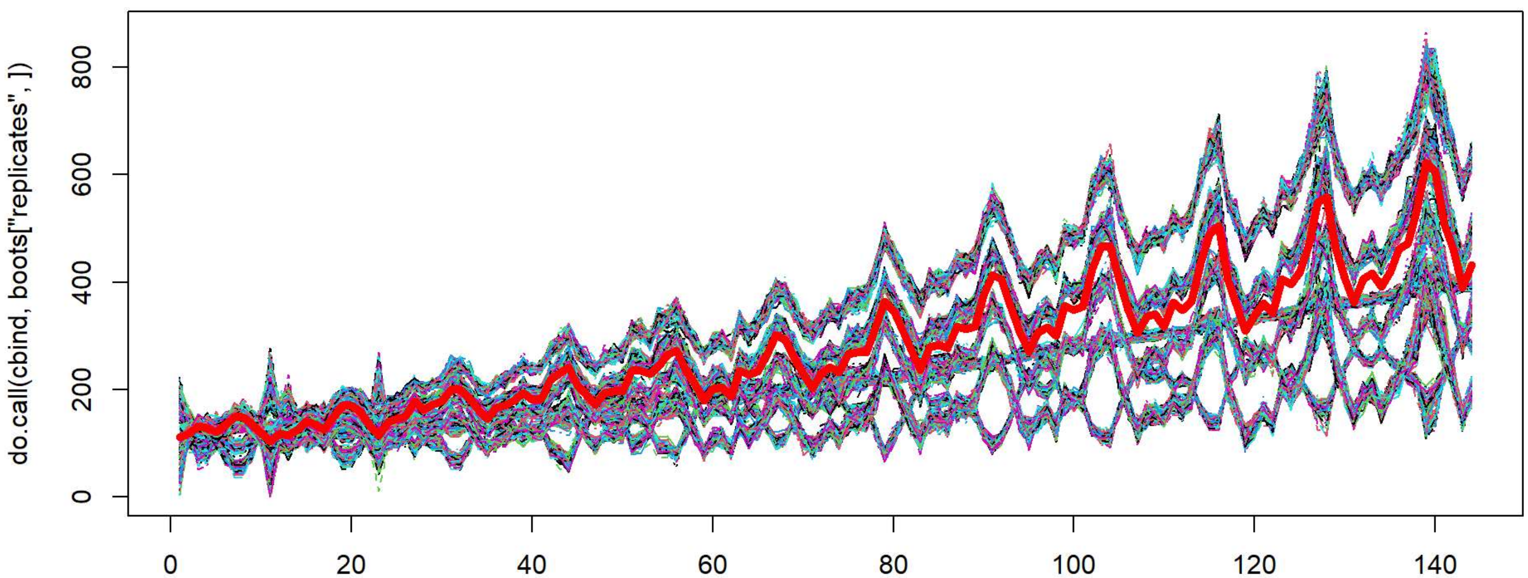
Static Drift

```
boots = NNS.meboot(AirPassengers, reps = 100, rho = seq(-1, 1, .25),
                   xmin = 0, target_drift_scale = 1)

matplot(do.call(cbind, boots["replicates", ]), type = "l")
lines(1:length(AirPassengers), AirPassengers, lwd = 5, col = "red")
```



Specify a Drift for Replicates (RfR perhaps)

```
boots = NNS.meboot(AirPassengers, reps = 100, rho = seq(-1, 1, .25),
                   xmin = 0, target_drift = 1:4)

matplot(do.call(cbind, boots["replicates", ]), type = "l")
lines(1:length(AirPassengers), AirPassengers, lwd = 5, col = "red")
```



Ultimately Led to This General Statisical Toolkit:

NNS

NONLINEAR NONPARAMETRIC STATISTICS

NNS version 11.1 licence GPL-3

NNS offers:

- Numerical Integration & Numerical Differentiation
- Partitional & Hierarchial Clustering
- Nonlinear Correlation & Dependence
- Causal Analysis
- Nonlinear Regression & Classification
- ANOVA
- Seasonality & Autoregressive Modeling
- Normalization
- Stochastic Dominance
- Advanced Monte Carlo Sampling

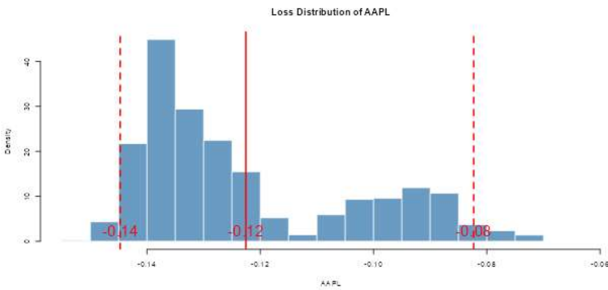
Finance Applications?
You bet!

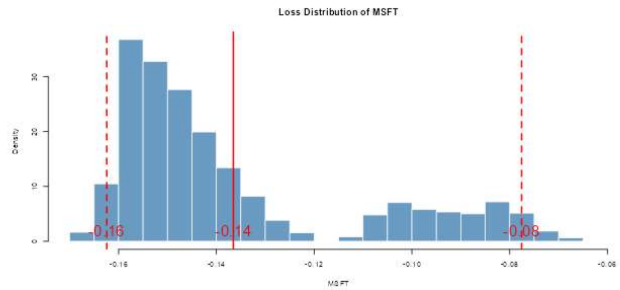
Stress Testing

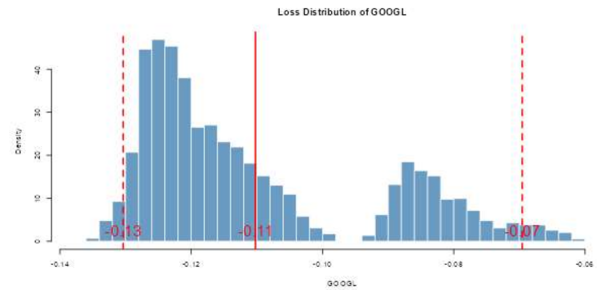
- Restrict observations to $CLPM$ and use `NNS.reg()`, maintains dependence structure
- Reverse the procedure… instead of what happens to portfolio if S&P 500 drops 10%, what portfolio returns would lead to a 10% S&P 500 drop? Distribution of scenarios.

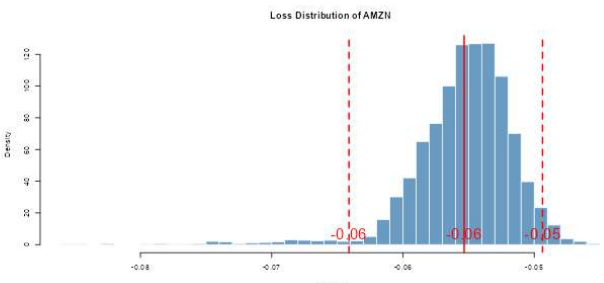
https://github.com/OVVO-Financial/Finance/blob/main/stress_test.md

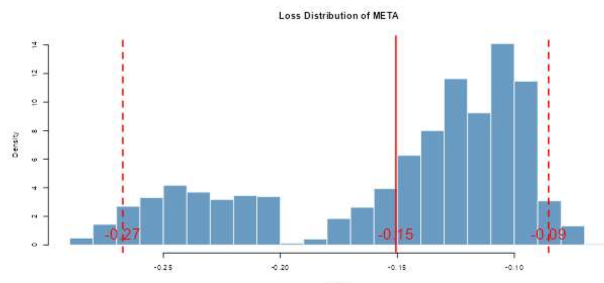
Distribution of Losses

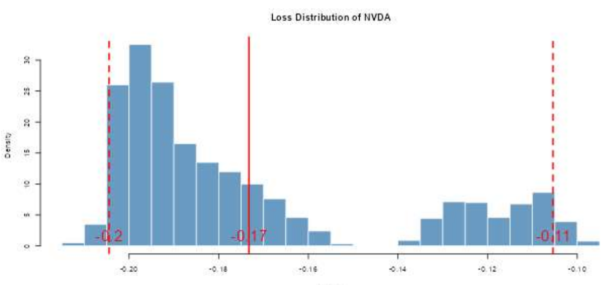












Utility Theory

- Markowitz dedicated a quarter of his 1959 book to utility theory. Was it a waste of time? Absolutely not!
- Each of us has unique risk profiles, costs of capital, and preferences. Parsing variance to reflect these subjective interpretations is critical for representing individual preferences.
- Partial moments are the ***perfect*** method to achieve this. Summary statistics just don’t cut it!

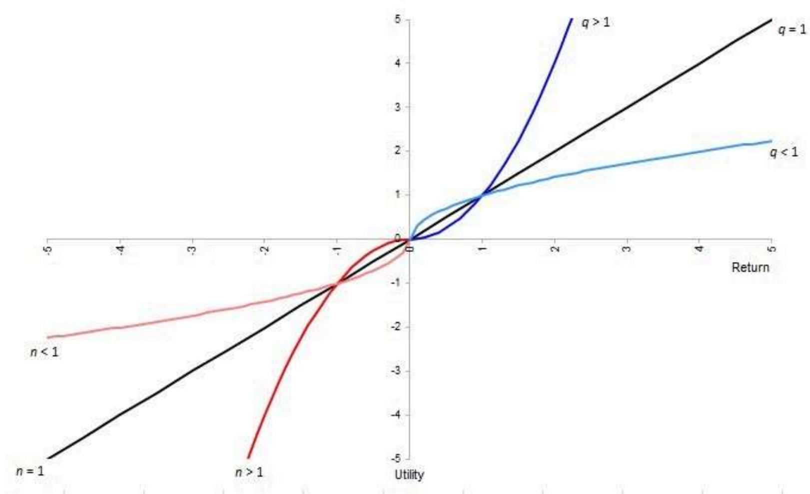
Let's Revisit Degrees

$$LPM(n, target, variable) = \frac{1}{T}\sum_{t=1}^{T}[\max(0, target - variable_t)]^n$$

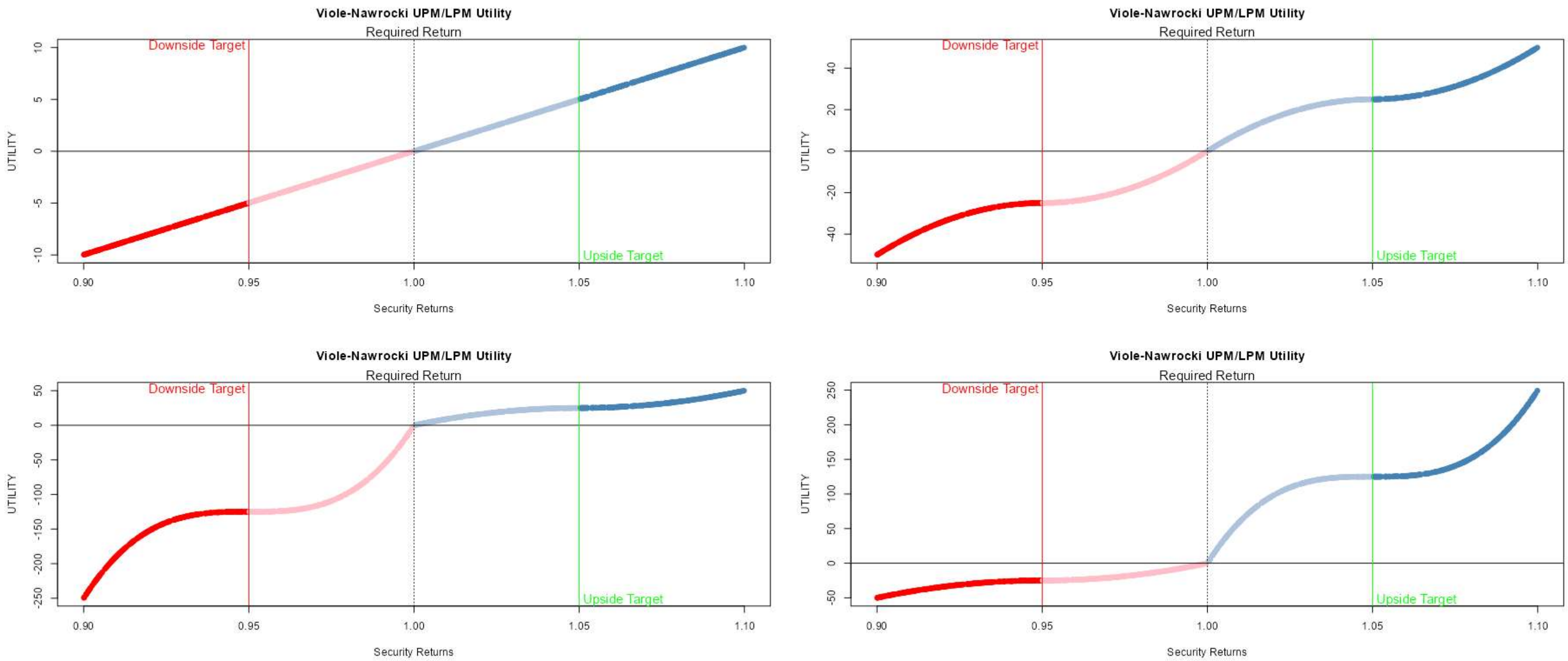
$$UPM(q, target, variable) = \frac{1}{T}\sum_{t=1}^{T}[\max(0, variable_t - target)]^q$$

loss-aversion: n

gain-seeking: q



Utility Functions



NOBODY is linear $(n = 1)$ wrt losses!

Utility Embedded Into Covariance Matrix

Covariance Elements and Covariance Matrix

The covariance matrix (Σ) is equal to the sum of the co-partial moments matrices less the divergent partial moments matrices.

$$\Sigma = CLPM + CUPM - DLPM - DUPM$$

```
> cov.mtx = PM.matrix(LPM_degree = 1, UPM_degree = 1, target = 'mean', variable = cbind(x,y), pop_adj = TRUE)
> cov.mtx
$cupm
          x         y
x 0.4299250 0.1033601
y 0.1033601 0.5411626

$dupm
          x         y
x 0.0000000 0.1469182
y 0.1560924 0.0000000

$dlpm
          x         y
x 0.0000000 0.1560924
y 0.1469182 0.0000000

$clpm
          x         y
x 0.4033078 0.1559295
y 0.1559295 0.3939005

$cov.matrix
            x           y
x  0.83323283 -0.04372107
y -0.04372107  0.93506310


# Reassembled Covariance Matrix
> cov.mtx$cupm + cov.mtx$clpm - cov.mtx$dupm - cov.mtx$dlpm
            x           y
x  0.83323283 -0.04372107
y -0.04372107  0.93506310


# Standard Covariance Matrix
> cov(cbind(x,y))
            x           y
x  0.83323283 -0.04372107
y -0.04372107  0.93506310
```

Matrix Properties

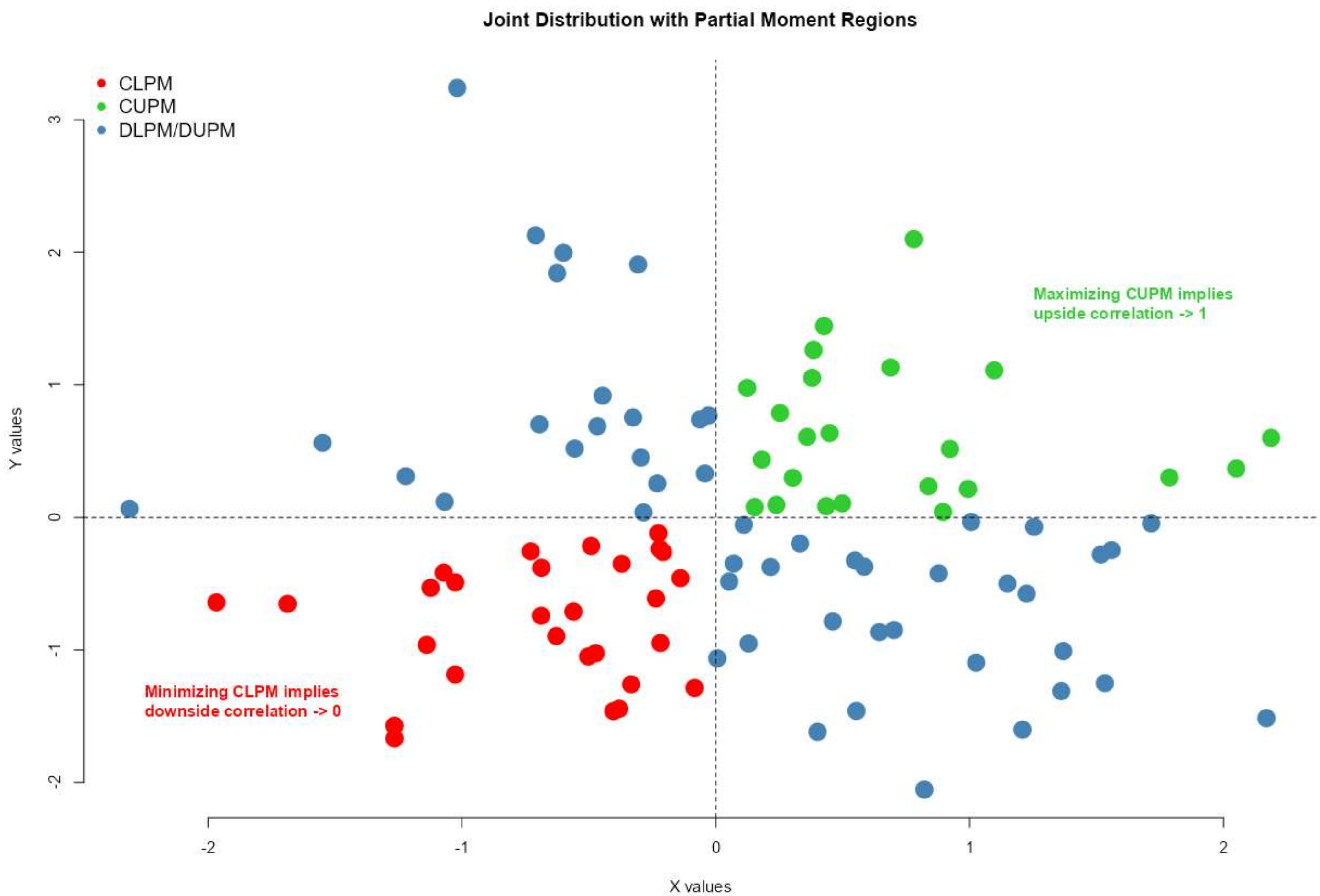
Positive Semi-Definite

```
> dim(Returns)
[1] 1429  501
> pm_matrices = NNS::PM.matrix(1, 1, "mean", Returns, pop_adj = T)
> matrixcalc::is.positive.semi.definite(pm_matrices$clpm)
[1] TRUE
> matrixcalc::is.positive.semi.definite(pm_matrices$cupm)
[1] TRUE
```

Transposes of One Another

```
> pm_matrices = NNS::PM.matrix(1, 1, "mean", mag_7_returns, pop_adj = T)
> pm_matrices
$cupm
              AAPL         AMZN        GOOGL         META         MSFT         NVDA         TSLA
AAPL  0.0001845879 0.0001257108 0.0001172715 0.0001499652 0.0001290853 0.0001904245 0.0001842340
AMZN  0.0001257108 0.0002194009 0.0001282581 0.0001768822 0.0001294801 0.0001975487 0.0001893952
GOOGL 0.0001172715 0.0001282581 0.0001814092 0.0001648536 0.0001303299 0.0001885374 0.0001629186
META  0.0001499652 0.0001768822 0.0001648536 0.0003509361 0.0001520305 0.0002415405 0.0002034251
MSFT  0.0001290853 0.0001294801 0.0001303299 0.0001520305 0.0001633589 0.0002029688 0.0001696033
NVDA  0.0001904245 0.0001975487 0.0001885374 0.0002415405 0.0002029688 0.0005310263 0.0003034602
TSLA  0.0001842340 0.0001893952 0.0001629186 0.0002034251 0.0001696033 0.0003034602 0.0007570225

$dupm
              AAPL         AMZN        GOOGL         META         MSFT         NVDA         TSLA
AAPL  0.000000e+00 1.584127e-05 1.027990e-05 1.529635e-05 7.045037e-06 1.889708e-05 2.855015e-05
AMZN  1.241220e-05 0.000000e+00 1.186534e-05 1.222438e-05 9.328401e-06 2.026998e-05 3.713813e-05
GOOGL 1.049113e-05 8.067339e-06 0.000000e+00 1.055072e-05 7.663992e-06 1.751569e-05 3.895092e-05
META  1.366089e-05 1.565359e-05 1.123151e-05 0.000000e+00 1.383689e-05 2.654724e-05 5.462058e-05
MSFT  6.947570e-06 7.269397e-06 6.539639e-06 1.220053e-05 0.000000e+00 1.139308e-05 3.088400e-05
NVDA  1.607725e-05 2.083073e-05 1.837323e-05 2.652328e-05 1.397346e-05 0.000000e+00 4.768053e-05
TSLA  3.725337e-05 4.413379e-05 4.755441e-05 6.066748e-05 4.622815e-05 6.768202e-05 0.000000e+00

$dlpm
              AAPL         AMZN        GOOGL         META         MSFT         NVDA         TSLA
AAPL  0.000000e+00 1.241220e-05 1.049113e-05 1.366089e-05 6.947570e-06 1.607725e-05 3.725337e-05
AMZN  1.584127e-05 0.000000e+00 8.067339e-06 1.565359e-05 7.269397e-06 2.083073e-05 4.413379e-05
GOOGL 1.027990e-05 1.186534e-05 0.000000e+00 1.123151e-05 6.539639e-06 1.837323e-05 4.755441e-05
META  1.529635e-05 1.222438e-05 1.055072e-05 0.000000e+00 1.220053e-05 2.652328e-05 6.066748e-05
MSFT  7.045037e-06 9.328401e-06 7.663992e-06 1.383689e-05 0.000000e+00 1.397346e-05 4.622815e-05
NVDA  1.889708e-05 2.026998e-05 1.751569e-05 2.654724e-05 1.139308e-05 0.000000e+00 6.768202e-05
TSLA  2.855015e-05 3.713813e-05 3.895092e-05 5.462058e-05 3.088400e-05 4.768053e-05 0.000000e+00

$clpm
              AAPL         AMZN        GOOGL         META         MSFT         NVDA         TSLA
AAPL  0.0001761168 0.0001402162 0.0001400667 0.0001632621 0.0001391880 0.0002161162 0.0002228611
AMZN  0.0001402162 0.0002132808 0.0001500598 0.0001929205 0.0001455512 0.0002330055 0.0002182102
GOOGL 0.0001400667 0.0001500598 0.0001857633 0.0001826160 0.0001418376 0.0002097453 0.0001989730
META  0.0001632621 0.0001929205 0.0001826160 0.0003562793 0.0001681899 0.0002635573 0.0002452277
MSFT  0.0001391880 0.0001455512 0.0001418376 0.0001681899 0.0001637322 0.0002146819 0.0002047791
NVDA  0.0002161162 0.0002330055 0.0002097453 0.0002635573 0.0002146819 0.0004638918 0.0003588282
TSLA  0.0002228611 0.0002182102 0.0001989730 0.0002452277 0.0002047791 0.0003588282 0.0006856359

$cov.matrix
              AAPL         AMZN        GOOGL         META         MSFT         NVDA         TSLA
AAPL  0.0003607047 0.0002376736 0.0002365672 0.0002842700 0.0002542807 0.0003715663 0.0003412916
AMZN  0.0002376736 0.0004326817 0.0002583852 0.0003419247 0.0002584335 0.0003894535 0.0003263335
GOOGL 0.0002365672 0.0002583852 0.0003671725 0.0003256874 0.0002579639 0.0003623938 0.0002753862
META  0.0002842700 0.0003419247 0.0003256874 0.0007072154 0.0002941830 0.0004520273 0.0003333648
MSFT  0.0002542807 0.0002584335 0.0002579639 0.0002941830 0.0003270911 0.0003922841 0.0002972703
NVDA  0.0003715663 0.0003894535 0.0003623938 0.0004520273 0.0003922841 0.0009949181 0.0005469258
TSLA  0.0003412916 0.0003263335 0.0002753862 0.0003333648 0.0002972703 0.0005469258 0.0014426584

> identical(pm_matrices$dlpm, t(pm_matrices$dupm))
[1] TRUE
```

Logical Objective Functions



Consistent objective for all risk preferences!

Expected Partial Moments

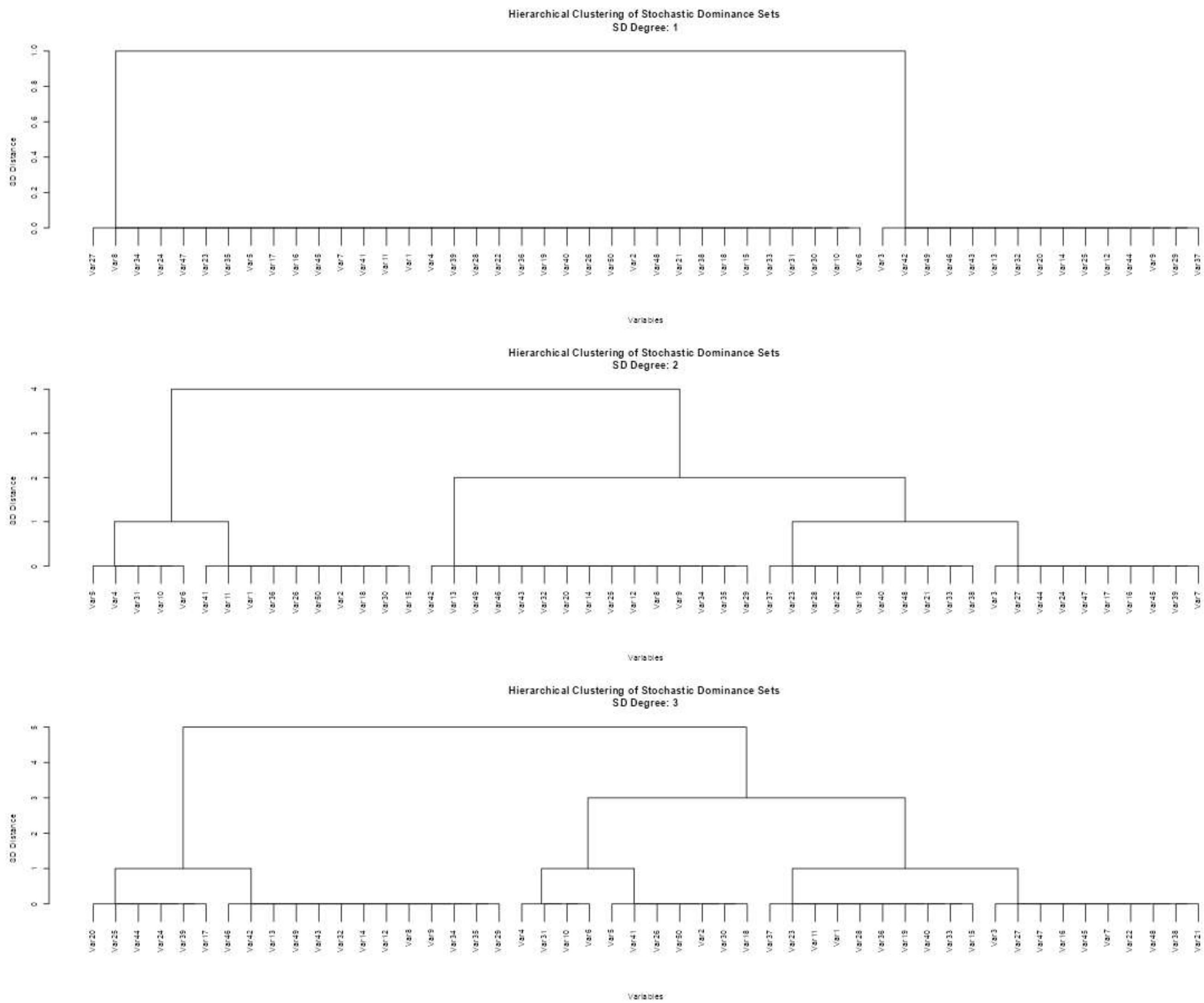
Longer post and paper link on conditioning returns via entropy proxies

https://www.linkedin.com/pulse/expected-partial-moments-fred-viole/

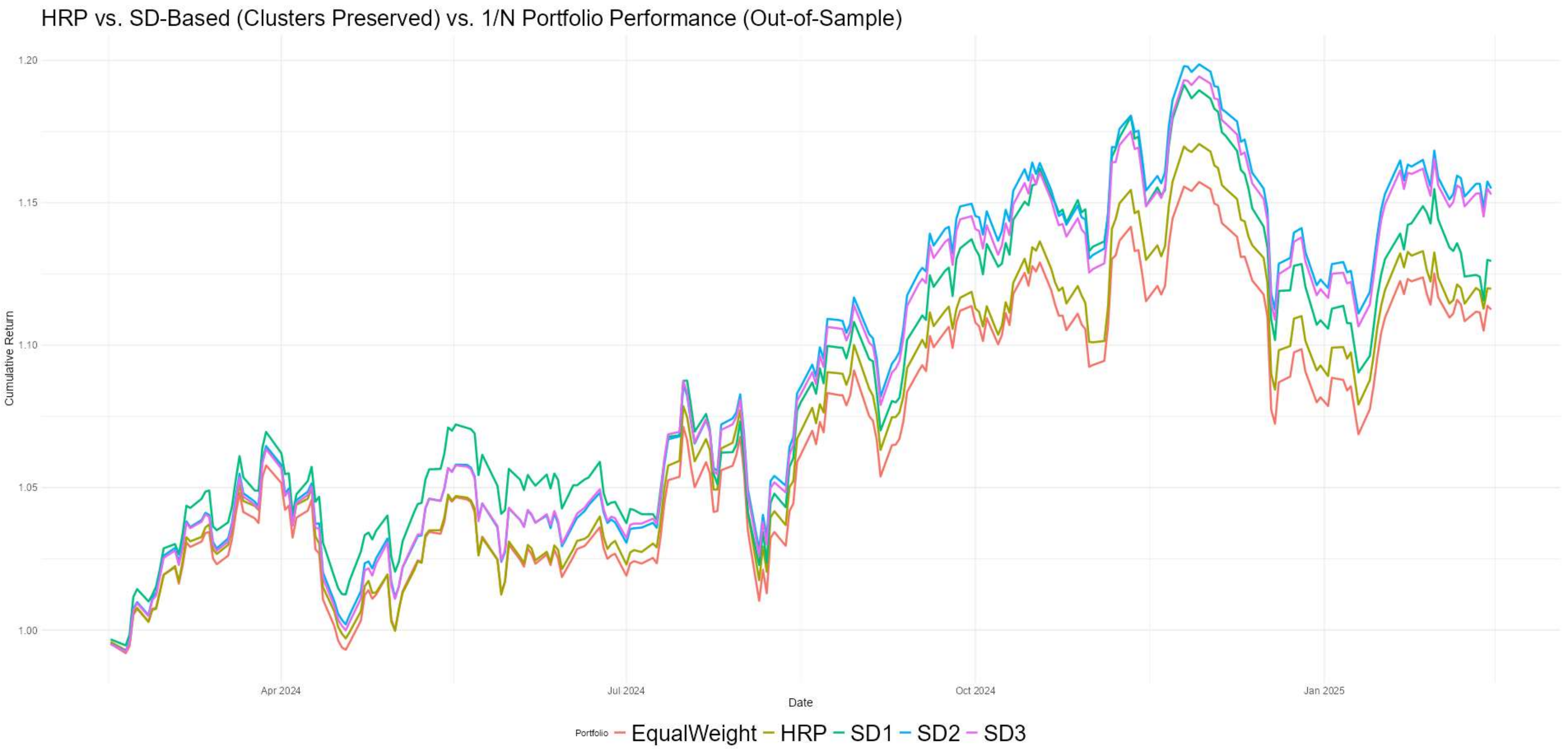
Other Portfolio “Theories”

- Diversification for its own sake, with no identified utility function or preference being satisfied.
- Hierarchical Risk Parity (HRP) aims to create portfolios that are more robust to input variations, avoiding extreme mean-variance allocations.
- Stochastic Portfolio Theory (SPT) values growth rates but ignores investor preferences — ***how*** you achieve final wealth matters!

Stochastic Dominant Clusters

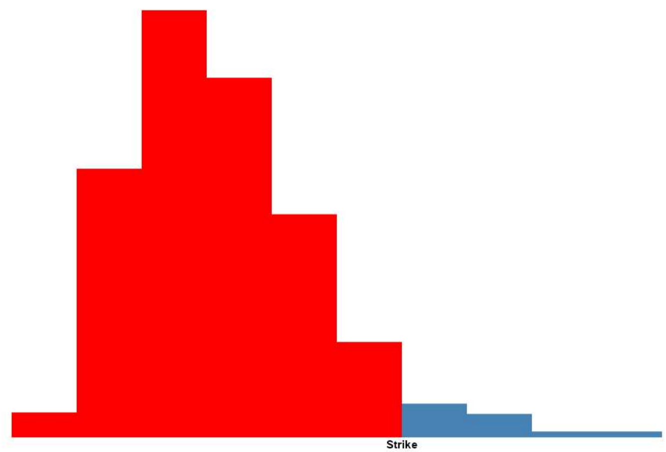


SD Clusters vs. HRP OOS



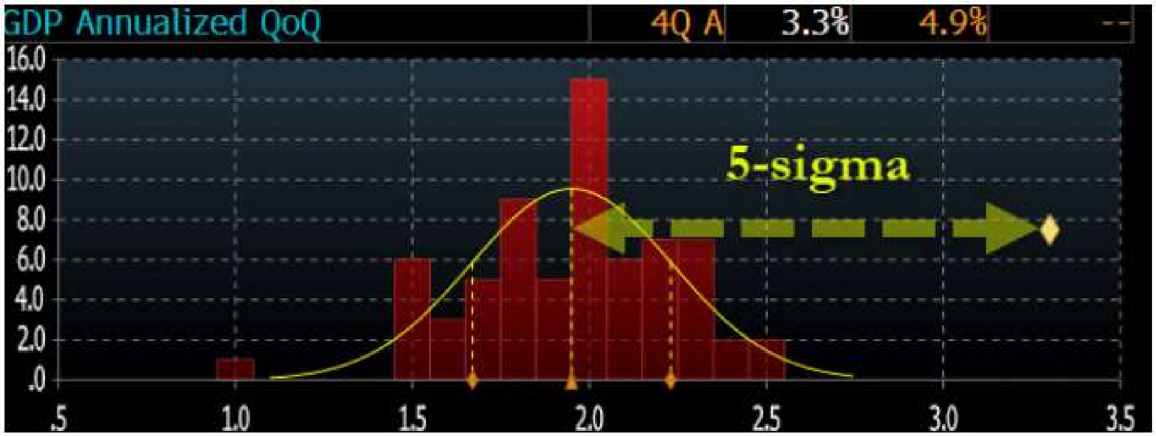
Options Pricing

An obvious extension is to use UPM for calls and LPM for puts



- No more $N(d1)$ and $N(d2)$!
- Preserves the empirical structure of real-world probabilities ($\mathbb{P}$) while directly adjusting for risk-neutral valuation ($\mathbb{Q}$)

Macro Forecasts



REPORTED THURSDAY, JANUARY 25 AT 8:30 AM

0.1	+37	1 / 21
YOUR DELTA TO ACTUAL	POINTS	YOUR RANK

	Value (%)
Estimize Consensus 21 estimates	1.8
› Your Estimate	3.4
› Reported Values	3.3

Partial Moments in the Wild

To showcase the effectiveness of NNS in quantitative finance, I’ve designed the following applications:

- MacroNow nowcasting
  https://ovvo.shinyapps.io/macronow_intro
- Options
  https://ovvo.shinyapps.io/options_intro
- Portfolio
  https://ovvo.shinyapps.io/portfolio_intro

More Informed Dispersion Measures

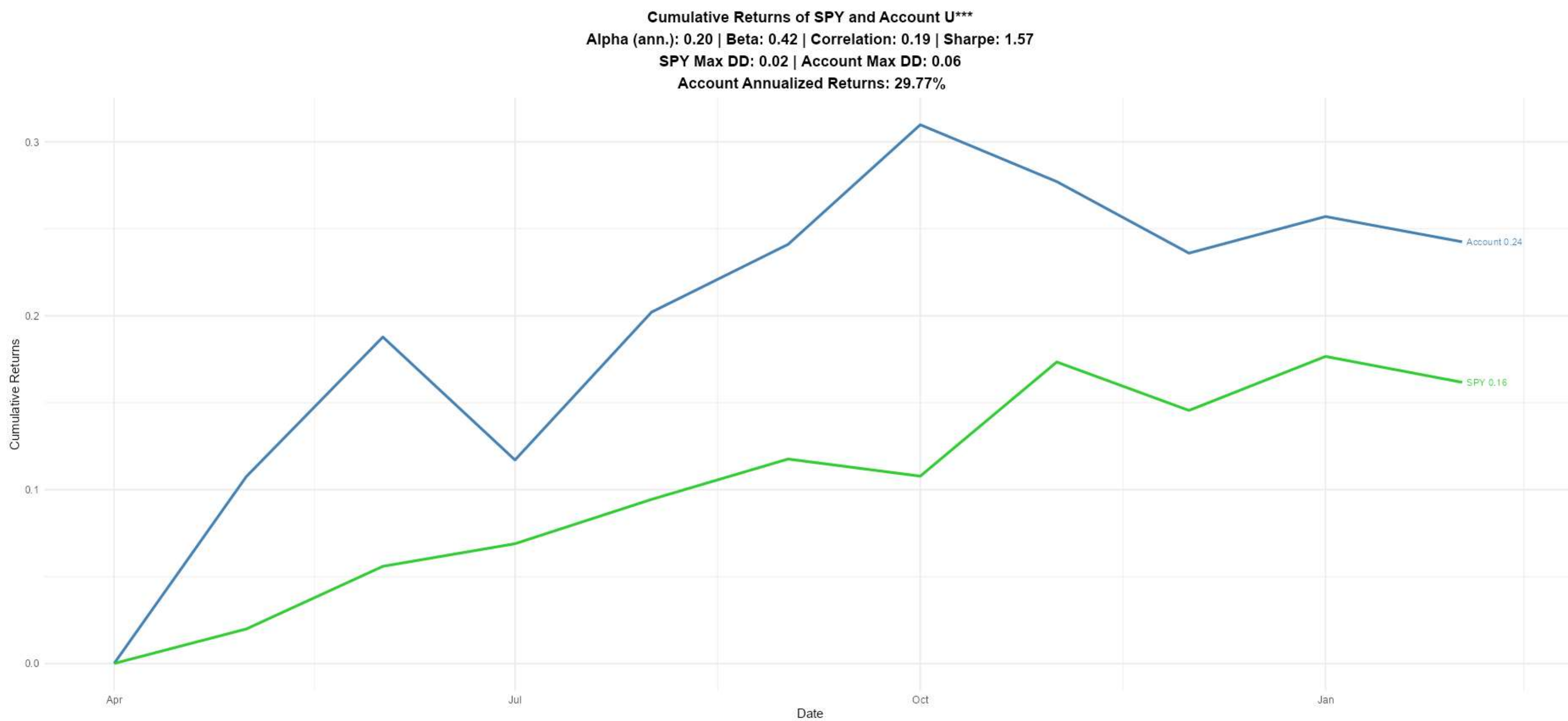
$$RD_t = \sqrt{\sum_{i=1}^{N} w_i (R_{i,t} - R_{I,t})^2}$$

where $R_{I,t} = \sum_{i=1}^{N} w_i R_{i,t}$ for N index members

$$RD_t^{LPM} = \sqrt{\sum_{i=1}^{N} w_i [\max(0, R_{I,t} - R_{i,t})]^2}$$

$$RD_t^{UPM} = \sqrt{\sum_{i=1}^{N} w_i [\max(0, R_{i,t} - R_{I,t})]^2}$$

Better Stats to Arb



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