Twin and Quadruple Primes

By,

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PAIRS OF PRIMES

Since all primes end in 1, 3, 7 or 9 (after the seed row of 2, 3, 5, 7) we can visualize four number lines simultaneously rather than a single aggregated number line. The four number lines consist of the following sequences S:

S_1: 11,21,31,…
S_3: 13,23,33,…
S_7: 17,27,37,…
S_9: 19,29,39,…

Given the sequences of possible primes after the division of 2, 3, and 5 in Figure 1 below, we can see how the how the prime quadruples are aligned at every 3rd row (highlighted in red below) and by extension the regularity of twin primes (highlighted in blue).[1]

x	S_1	S_3	S_7	S_9
1	**11**	**13**	**17**	**19**
2		23		**29**
3	**31**		37	
4	**41**	**43**	**47**	**49**
5		53		**59**
6	**61**		67	
7	**71**	**73**	**77**	**79**
8		83		**89**
9	**91**		97	
10	**101**	**103**	**107**	**109**
11		113		**119**
12	**121**		127	
13	**131**	**133**	**137**	**139**
14		143		**149**
15	**151**		157	
16	**161**	**163**	**167**	**169**
17		173		**179**
18	**181**		187	
19	**191**	**193**	**197**	**199**
20		203		209

Figure 1. Sequences after division by 2, 3, and 5.

[1] One can illustrate the pattern of all even number differences between possible primes from these sequences.

It has recently been discovered that the distance between prime numbers is indeed bound.

> "Now Zhang has broken through this barrier. His paper shows that there is some number *N* smaller than 70 million such that there are infinitely many pairs of primes that differ by *N*. No matter how far you go into the deserts of the truly gargantuan prime numbers — no matter how sparse the primes become — you will keep finding prime pairs that differ by less than 70 million."[2,3]

The question to really ask is, ***"What is the probability a number is prime?"*** If we become more certain after more possible factors have been tried (think Bayesian evidence accumulation from a zero prior probability), then a test of one factor will increase the probability, ***to at a minimum, greater than 0***. Thus, if the probability a number is prime is greater than 0 if one divisibility test (by 2 or 3) has been performed, ***then the joint probability of finding twin primes is greater than 0*** (however remote it becomes due to remaining possible factors).

x	S_1	S_3	S_7	S_9
10	**101**	**103**	**107**	**109**

$Pr(101 \in primes) > 0$

$Pr(103 \in primes) > 0$

Where,

$$Pr(101 \in primes) = Pr(103 \in primes) > 0$$

The probability of a number being prime does not drop simply as the number increases; **numbers that share the whole number of its square root have the same probability of being prime,**[4] contrary to the probability suggested by $\frac{1}{ln(x)}$ via the Prime Number Theorem.

Thus the joint probability,

$$\boldsymbol{Pr(101 \in primes) * Pr(103 \in primes) > 0}$$

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[2] http://www.wired.com/wiredscience/2013/05/twin-primes/all/

[3] At the time of this paper, Zhang's *N* has been reduced to 5,414.
http://michaelnielsen.org/polymath1/index.php?title=Bounded_gaps_between_primes

[4] See the uniform probability over the range between squares explained here:
https://github.com/OVVO-Financial/Number-Theory/blob/master/Number%20Theory%20Papers/On%20the%20Distribution%20of%20Prime%20Numbers.pdf

Furthermore all reduced sequences are of alternating remainders when divided by 3.

x	S_1	S_3	S_7	S_9
1	**2**	**1**	**2**	**1**
2		2		**2**
3	**1**		1	
4	**2**	**1**	**2**	**1**
5		2		**2**
6	**1**		1	
7	**2**	**1**	**2**	**1**
8		2		**2**
9	**1**		1	
10	**2**	**1**	**2**	**1**

Given that half of the primes have a remainder of 1 when divided by 3 and the other half have a remainder of 2 (see link[5] for remainder law characterization), each set of twin primes and quadruples must contain an even amount of remainder 1's and remainder 2's.

Since these twin and quadruple possible prime sets exist at every 3rd row, and these sets have a probability greater than 0 (and can never equal 0) of being prime while satisfying the remainder law; we can state that there are indeed an infinite number of twin, quadruple, and any other even numbered differenced prime sets.

[5] https://www.simonsfoundation.org/quanta/20131119-together-and-alone-closing-the-prime-gap/