USEFUL TRIGONOMETRIC FORMULAS

1. **Pythagorean identity:**

$$\sin^2 A + \cos^2 A = 1$$
$$\tan^2 A + 1 = \sec^2 A$$
$$1 + \cot^2 A = \csc^2 A.$$

2. **Trigonometric functions in a triangle (draw the picture!):**

$$\sin A = \frac{\text{opp}}{\text{hyp}} \quad \cos A = \frac{\text{adj}}{\text{hyp}} \quad \tan A = \frac{\text{opp}}{\text{adj}}$$
$$\csc A = \frac{\text{hyp}}{\text{opp}} \quad \sec A = \frac{\text{hyp}}{\text{adj}} \quad \cot A = \frac{\text{adj}}{\text{opp}}.$$

3. **Relations between** $\sin, \cos, \tan$, **etc:**

$$\tan A = \frac{\sin A}{\cos A} \quad \cot A = \frac{1}{\tan A}$$
$$\sec A = \frac{1}{\cos A} \quad \csc A = \frac{1}{\sin A}.$$

4. **Addition formulas:**

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$
$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$
$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$
$$\cos(A - B) = \cos A \cos B + \sin A \sin B.$$

   Notice, the second and fourth can be obtained from the first and third, respectively, by replacing B with $-B$ and using that sin is an odd function, cos even.

5. **Double angle formulas:**
   (plug in $A = B$ above)

$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A.$$

6. **Half-angle formulas:**
   (use the second double angle formula and the Pythagorean identity)

$$\cos^2 A = \frac{1}{2}(1 + \cos 2A)$$
$$\sin^2 A = \frac{1}{2}(1 - \cos 2A).$$

7. **Product formulas:**
   (solve addition formulas as a linear system for $\sin A \cos B$, etc)

$$\sin A \cos B = \frac{1}{2}\left(\sin(A-B) + \sin(A+B)\right)$$

$$\sin A \sin B = \frac{1}{2}\left(\cos(A-B) - \cos(A+B)\right)$$

$$\cos A \cos B = \frac{1}{2}\left(\cos(A-B) + \cos(A+B)\right).$$

8. Other (**less relevant**):

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\sin 3A = -4\sin^3 A + 3\sin A$$

$$\cos 3A = 4\cos^3 A - 3\cos A.$$

9. The unit circle: