Measure and Integration I (MAA5616), Fall 2020
Homework 1, due Thursday, Sep. 3

1. Prove that bijections on a nonempty set form a group.

2. Prove deMorgan's laws:

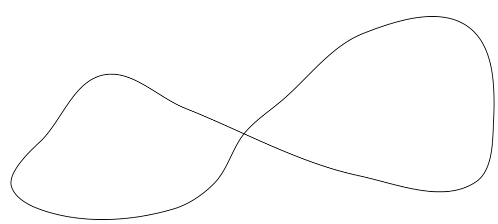
$$\left(\bigcup_{\alpha\in A} E_\alpha\right)^c = \bigcap_{\alpha\in A} E_\alpha^c, \qquad \left(\bigcap_{\alpha\in A} E_\alpha\right)^c = \bigcup_{\alpha\in A} E_\alpha^c.$$

3. Prove that $\mathbb{N}\times\mathbb{N}$ has the cardinality of $\mathbb{N}$.

4. Prove that $\mathbb{R}^{\mathbb{N}}$ (the set of all sequences of real numbers) has the cardinality $\mathfrak{c}$ (the cardinality of $\mathbb{R}$).
Hint: use expansions with base $b \geq 2$ and the previous question.

5. Show that the cardinality of open sets in $\mathbb{R}$ is $\mathfrak{c}$.
Hint: use that open subsets of $\mathbb{R}$ are countable disjoint unions of intervals, and intervals are countable unions of intervals with rational endpoints.

6. A figure 8 in the plane looks like so:



What is the largest possible cardinality of a set of such continuous curves in the plane, assuming the curves are pairwise disjoint? Would the answer be different if we considered figure 0?