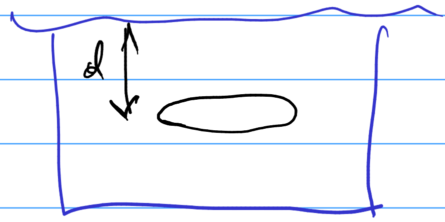
Section 8.3: Applications to physics

Pressure at depth d in liquid of density ρ:

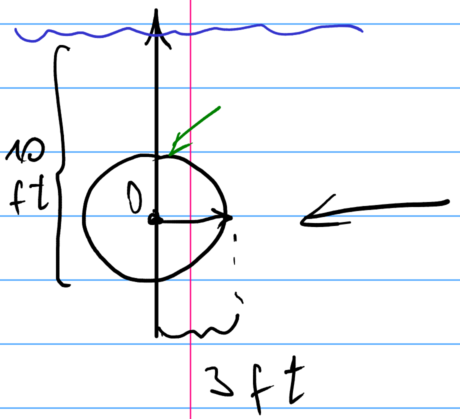
$$p = \rho \cdot g \cdot d$$

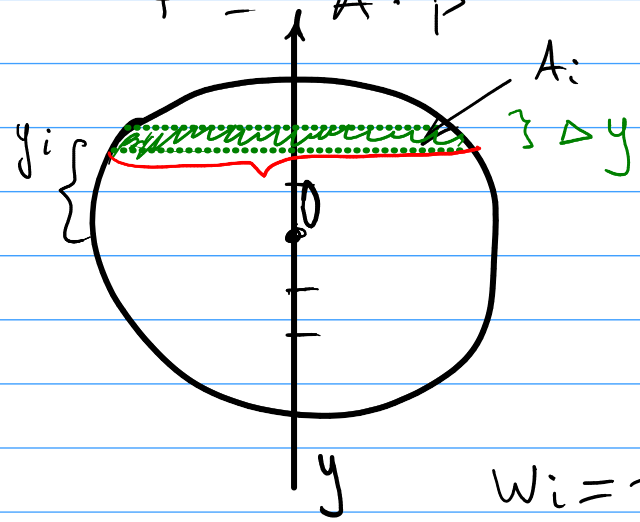
$g \to 9.8\,m/s^2$

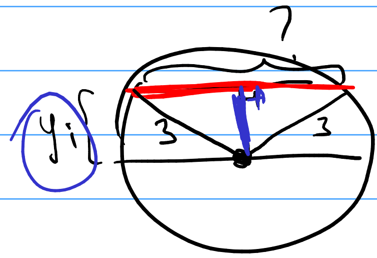


Ex. Find the force acting on a cylindrical drum (see figure)

$$F = A \cdot p$$







$$W_i = 2\sqrt{3^2 - y_i^2}$$

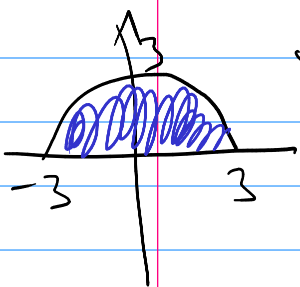
$$F = \lim_{n\to\infty} \sum_{i=1}^{n} F_i = \qquad F_i = A_i \cdot p_i$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} A_i \cdot p_i = \lim_{n\to\infty} \sum_{i=1}^{n} A_i \cdot \underbrace{(7-y_i)}_{d_i}$$

$$= \rho g \cdot \lim_{n\to\infty} \sum_{i=1}^{n} w_i \cdot (7-y_i)\,\Delta y$$

$$= \rho g \lim_{n\to\infty} \sum_{i=1}^{n} 2\sqrt{9-y_i^2}\,(7-y_i)\,\Delta y$$

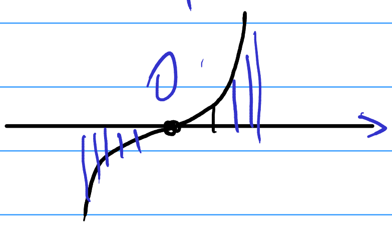
$$= \rho g \underbrace{\int_{-3}^{3} 2(7-y)\sqrt{9-y^2}\,dy}_{\text{hydrostatic force}}$$



$$= \underbrace{\rho g\, 14 \int_{-3}^{3}\sqrt{9-y^2}\,dy}_{14\rho g\cdot\frac{\pi 3^2}{2}} - \underbrace{\rho g\cdot 2\int_{-3}^{3} \underbrace{y\sqrt{9-y^2}}_{\text{odd function}}\,dy}_{=0}$$

$$= 63\pi\rho g$$

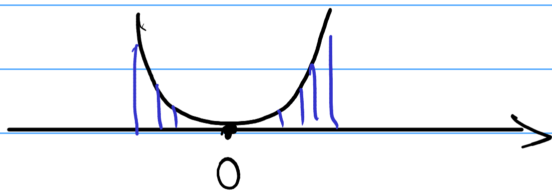
odd function:



$$f(-x) = -f(x)$$

$$\int_{-a}^{a} f(x)\,dx = 0$$

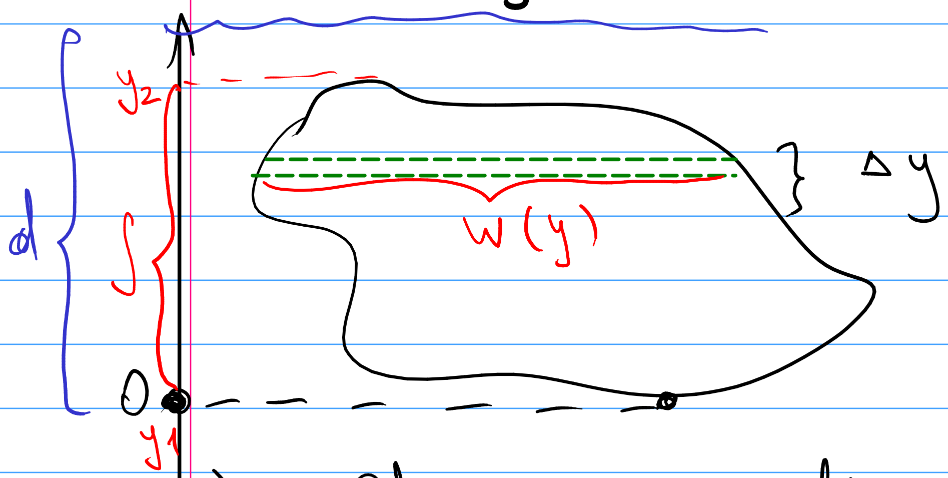
even function



$$f(x) = f(-x)$$

$$\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$$

Summary:



1). Choose coordinates

2). Find depth depending on y:

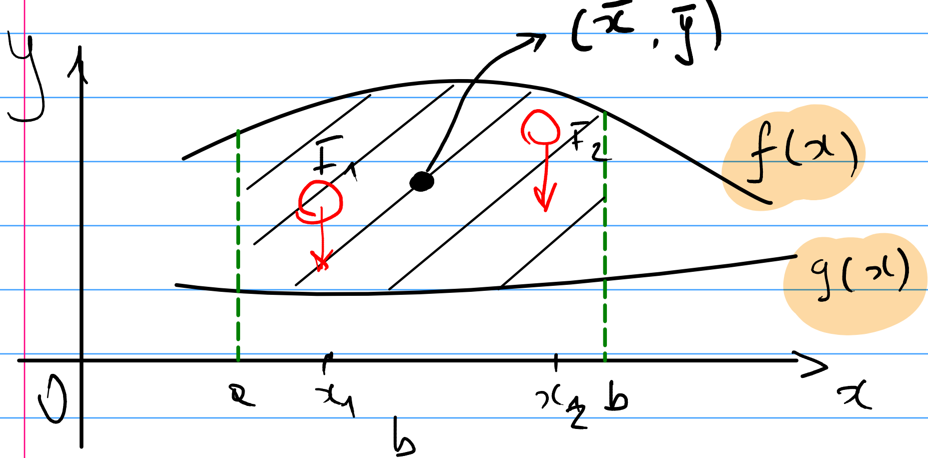
$$d_i = d - y_i$$

3). Find $w_i = f(y_i)$

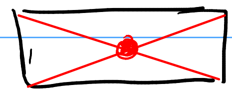
4). $$F = \lim_{n \to \infty} \sum F_i = \lim_{n \to \infty} \sum \rho \cdot g \cdot (d - y_i) \overbrace{w_i \cdot \Delta y}^{A_i}$$

$$F = \int_{y_1}^{y_2} \rho g (d-y) f(y)\, dy.$$

Center of mass



$f(x) \geq g(x)$

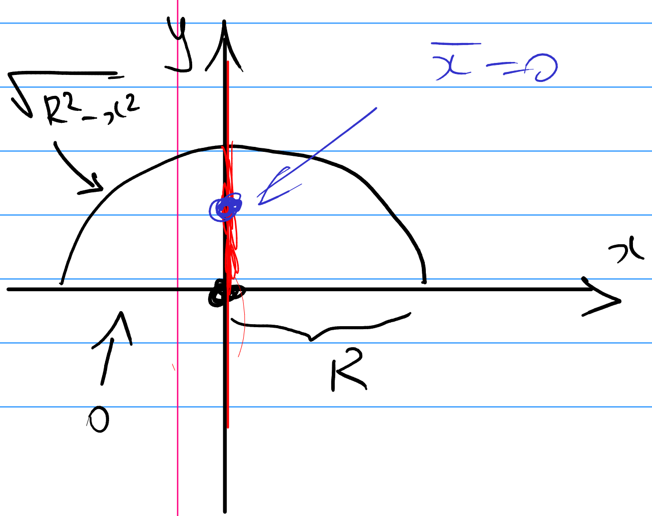


$$\bar{x} = \frac{1}{A} \int_a^b x \left(f(x) - g(x)\right) dx$$

$$\bar{y} = \frac{1}{A} \int_a^b \frac{1}{2} \left(f(x)^2 - g(x)^2\right) dx$$

where $A = \int_a^b \left(f(x) - g(x)\right)$ (area of the region)

Ex. Find the center of mass of a semicircular plate of radius R.



$\bar{x} = 0$

by symmetry: $(\bar{x}, \bar{y})$ is on every axis of symmetry

$$A = \pi \frac{R^2}{2}$$

$$\bar{y} = \frac{1}{A} \int_{-R}^{R} \frac{1}{2} \left(\sqrt{R^2 - x^2}\right)^2 dx$$

$$= \frac{1}{2A} \int_{-R}^{R} (R^2 - x^2) dx$$

$$= \frac{1}{2A} \cdot 2 \int_{0}^{R} R^2 - x^2 \, dx$$

$$= \frac{1}{\pi R^2} \cdot 2 \cdot \left[R^2 x - \frac{x^3}{3}\right]_0^R$$

$$= \frac{2}{\pi R^2} \cdot \left(R^3 - \frac{R^3}{3}\right) = \frac{4}{3\pi R^2} \cdot R^3$$

$$= \frac{4R}{3\pi}.$$

Center of mass: $\left(0, \underset{<R}{\frac{4R}{3\pi}}\right)$