Binghamton University, State University of New York
Department of Electrical and Computer Engineering

EECE 629 – MACHINE PATTERN RECOGNITION (Fall 2017)

Assignment #3

Due Date: 10/31/2017 (Tuesday)

Note: This is an individual assignment, no group submission is accepted.

1. Suppose in a *c*-category supervised learning environment we sample the full distribution $p(\mathbf{x})$ and subsequently train a PNN classifier according to Algorithm 1.

Algorithm 1 (PNN training)

<u>begin</u> <u>initialize</u> $j = 0, n = \#\text{patterns}$
    <u>do</u> $j \leftarrow j + 1$
        normalize : $x_{jk} \leftarrow x_{jk} / \left(\sum_{i}^{d} x_{ji}^2 \right)^{1/2}$
        train : $w_{jk} \leftarrow x_{jk}$
        <u>if</u> $\mathbf{x} \in \omega_i$ <u>then</u> $a_{ic} \leftarrow 1$
    <u>until</u> $j = n$

   a. Show that even if there are unequal category priors and hence unequal numbers of points in each category, the recognition method properly accounts for such priors.
   b. Suppose we have trained a PNN with the assumption of equal category priors, but later wish to use it for a problem having the cost matrix λ_{ij}, representing the cost of choosing category ω_i when in fact the pattern came from ω_j. How should we do this?
   c. Suppose instead we know a cost matrix λ_{ij} *before* training. How shall we train a PNN for minimum risk?

2. Consider the following set of two-dimensional vectors from three categories:

ω_1		ω_2		ω_3	
x_1	x_2	x_1	x_2	x_1	x_2
10	0	5	10	2	8
0	-10	0	5	-5	2
5	-2	5	5	10	-4

a. Plot the decision boundary resulting from the nearest-neighbor rule just for categorizing ω_1 and ω_2. Find the sample means $\mathbf{m}_1$ and $\mathbf{m}_2$ and on the same figure sketch the decision boundary corresponding to classifying $\mathbf{x}$ by assigning it to the category of the nearest sample mean.
b. Repeat (a) for categorizing only ω_1 and ω_3.
c. Repeat (a) for categorizing only ω_2 and ω_3.
d. Repeat (a) for a three-category classifier, classifying ω_1, ω_2, and ω_3.

3. Computer Problem.

	ω_1			ω_2			ω_3		
sample	x_1	x_2	x_3	x_1	x_2	x_3	x_1	x_2	x_3
1	0.28	1.31	-6.2	0.011	1.03	-0.21	1.36	2.17	0.14
2	0.07	0.58	-0.78	1.27	1.28	0.08	1.41	1.45	-0.38
3	1.54	2.01	-1.63	0.13	3.12	0.16	1.22	0.99	0.69
4	-0.44	1.18	-4.32	-0.21	1.23	-0.11	2.46	2.19	1.31
5	-0.81	0.21	5.73	-2.18	1.39	-0.19	0.68	0.79	0.87
6	1.52	3.16	2.77	0.34	1.96	-0.16	2.51	3.22	1.35
7	2.20	2.42	-0.19	-1.38	0.94	0.45	0.60	2.44	0.92
8	0.91	1.94	6.21	-0.12	0.82	0.17	0.64	0.13	0.97
9	0.65	1.93	4.38	-1.44	2.31	0.14	0.85	0.58	0.99
10	-0.26	0.82	-0.96	0.26	1.94	0.08	0.66	0.51	0.88

Consider *k*-nearest-neighbor density estimations in different numbers of dimensions.

a. Write a program to find the *k*-nearest-neighbor density for *n* (unordered) points in one dimension. Use your program to plot such a density estimate for the x_1 values in category ω_3 in the table above for $k = 1, 3,$ and 5.
b. Write a program to find the *k*-nearest-neighbor density estimate for *n* points in two dimensions. Use your program to plot such a density estimate for the $(x_1, x_2)^t$ values in ω_2 for $k = 1, 3,$ and 5.
c. Write a program to find the *k*-nearest-neighbor density estimate for the three-dimensional data from the three categories in the table above. Use your program with $k = 1, 3,$ and 5 to estimate the relative densities at the following points: $(-0.41, 0.82, 0.88)^t$, $(0.14, 0.72, 4.1)^t$, and $(-0.81, 0.61, -0.38)^t$.