Applied Mathematics for Computer Science

Sheet 1

This series of exercises is intended for first-year high school students. It is designed to train students in mathematics and computer science by practicing on a variety of problems. Each problem consists of a theoretical, practical, and application section.

The main goal of this sheet is to understand vectors, their "informatic" representation, to understand the usefulness of these mathematical objects in computer science, how to deal with them using a programming language, apply and use mathematical formulas in a computer science problem.

The reader can use any programming language. You should not use any existing libraries. The answers will be written by hand on the sheet, except for the codes, you can do it only when I indicate it in the question. These will be put on Github repositories accessible only by the creator and me. Execute all your tasks from a text editor and your command terminal.

Preambular

A) Github

1 Create a Github account.

2 Create a folder named AMCS1 using only the command terminal of your operating system.

3 Build a tree of 3 folders: Theoretical, Practical, Application.

4 Push the folder to Github with a *gitignore* folder appropriate to the programming language and a $ReadMe$ explaining what the AMCS1 folder consists of.

Recall : Each programme must be written in the appropriate folder.

B) Pen and paper

1 Write in black

2 Put an appropriate title to your paper.

Recall : Particular attention will be paid to cleanliness, quality of writing, and clarity of words.

Theorical part

A) Course questions

We place ourselves in an orthonormal plane (O, x, y) with the scalar product $\langle \vec{\cdot}, \vec{\cdot} \rangle$.We consider u, v two vectors of dimension 3 such as $u = (u_1, u_2, u_3)$ and $v = (v_1, v_2, v_3)$.

1 Give the definition of *vector*, *scalar product* $\langle \vec{\cdot}, \vec{\cdot} \rangle$ and *plane*.

2 Explain the meaning of : vectors of dimension 2 with real values. What does this imply ?

3 Regarding vectors, what operations are allowed on them ? Why ?

4 Add the vectors u and v. What is the dimension of the vector obtained ? Why ?

5 What does collinearity mean in geometry ? Show that what you are saying is true by giving a example (geometric, algebric...)

6 What is a control terminal, and what components do it communicate with? Explain how a computer works.

7 What is an intrasec command ? Explain the word. *Indication* : *what does intra mean*

8 Are machines with a CPU capable of handling exact numbers ? Like 3 for example ? If so, why ? If not, why ?

9 How are colors defined on a computer ? Give a standard.

10 According to your knowledge, are there areas in hacking that require a mix of vectors, probability, and geometry ? If so, how do you think it works ? If not, why is it not possible ?

Now that you are used to the subject, having all the definitions and notions in mind, let us move on to the practical case. This section still has theory, but with a non-numerical application.

Practice

A) Mathematics

1 We consider u a vector of dimension 3 such as $u = (u_1, u_2, u_3)$. Demonstrate the following result: $\forall n \in \mathbb{N}^*$, $\forall$ i $\in \{1, 2, 3\}, \forall (u_i) \in \mathbb{R}$, $\in \frac{1}{n} \cdot (u) = \left(\frac{u_1}{n}, \frac{u_2}{n}, \frac{u_3}{n}\right)$

Is the result valid for $n = 0$? $n \to +\infty$? Explain. What happens geometrically when we divide values of a vector by a very large number?

2 We consider u a vector of dimension 3 such as $u = (u_1, u_2, u_3)$ Prove that:

$\forall\ (a, b) \in \mathbb{R}$, $\forall$ i $\in \{1, 2, 3\}, \forall (u_i) \in \mathbb{R}$:

$(a + b) \cdot u = a \cdot u + b \cdot u = (a \cdot u_1, a \cdot u_2, a \cdot u_3) + (\text{b} \cdot u_1, b \cdot u_2, b \cdot u_3)$

B) Computer Science

1 Back to question 4, create a list with 3 items, and a list with 2 items, add them up, what happens? Is there a link between mathematics and computer science ?

2 Regarding question 3, try the different operations on 2 lists. What do you see ?

3 What are these lists used for in computer science? What are their most common uses? Imagine yourself a common case. Comment on your code

4 Write a code that asks a user to enter 2 vectors (of some size) and choose an operation. Apply what you have learned from the previous questions.

5 What is a library in computer science ? Search for image processing, graphics and math libraries of your programming language. Quote me your sources.

6 Optional : Do this on a local web page using only: CSS, $HTML$, JS.

7 Optional: What security actions can you take to secure your site before deploying it on Github? On a public website on the internet ? Explain it.

Now you have acquired some skills in IT and mathematics, you know some definitions, some rules and some methods. However, this is not enough for the world of work, you must know apply what you have learned and understood in a specific case, or to solve a problem or to response to a request.

Problem

A) Contextualisation

You are a computer science student and you own an academic discord server. You notice that your comrades on this server often use very dark images. However some users have problems with views and cannot see the images properly. To help students clear images, you must find a solution.

You need to write a function that takes an image as input and lightens each pixel. This function must be implemented in a discord bot that is present on the server. It has the usual permissions and students will be allowed to use some of its commands. The student writes in the discord channel the command /light and pastes the image link and the bot responds by sending the lightened image.

B) Hint

RGB ?

RGB is a color model used to represent colors in digital images. The term "RGB" stands for Red, Green, and Blue, which are the three primary colors of light. By combining these three colors at different intensities, a wide range of colors can be created.
Each color component (R, G, B) is represented by an integer value ranging from 0 to 255. For example:

- (255, 0, 0) represents pure red.
- (0, 255, 0) represents pure green.
- (0, 0, 255) represents pure blue.
- (255, 255, 255) represents white.
- (0, 0, 0) represents black.

In a digital image, each pixel is represented by a triplet of RGB values. By adjusting the values of each component, the color of the pixel can be changed. For example, increasing the value of the red component will make the pixel redder.

Libraries

Depending on the language you will be using (GO, Python, Java, C++ or other), you will find resources to read and use in order to solve this kind of problem. Here is a non-exhaustive list of libraries that you can use depending on your programming language:

- Python :
  CV2, image processing library
  MatPlotLib, High quality 2D graphics
  Python library for discord bots
- GO :
  Go Images
- Java :
  Working with Images
- JavaScript :
  JS library for discord bots
  Official API Discord documentation
- C++ :
  CV2, image processing library
  C++ library for discord bots

Program

You need to create a discord bot that works only on *one server*, that has only *one command / fonction* that can be used by all users. Don't forget to give it the right permissions. This bot can be hosted locally, no need to put it on a server.

In order for the bot to read the image, remember to provide the image link and analyze its colors. Each image consists of a large number of pixels of different colors. You need to write a function that increases the brightness of each pixel, and then apply this transformation to each pixel of the desired image. Finally, you need to use this transformed image to respond to the command call.