Accelerated Simulation of Cardiac Arrhythmias by Projective Integration on Unstructured Meshes

SINADINOVIC Marko

Optional Thesis approved by **Prof. Thomas REY**

Acknowledgements

Abstract

Résumé FR ?

Contents

Introduction

Chapter 1

Mathematical Modeling of Excitable Media

1.1 From Cell to Tissue : Principles of Reaction-Diffusion Systems

1.2 Phenomenological Derivation

1.2.1 Biological Interpretation of Parameters

1.2.2 Dimensional Analysis : Scaling from Canine to Human Physiology

1.3 Zero-Dimensional Analysis : Local Dynamics

1.3.1 Phase Plane Analysis and Nullclines

1.3.2 Fixed Points and Linear Stability Analysis

1.3.3 The Stiffness Ratio : Eigenvalue Disparity

1.4 One-Dimensional Analysis : Wave Propagation

1.4.1 Traveling Wave Ansatz and Coordinate Transformation

1.4.2 Analytical Derivation of Conduction Velocity

1.4.3 The Curvature Effect : Eikonal-Curvature Relation in 2D

Chapter 2

Spatial Discretization : The Finite Element Method

2.1 Weak Formulation of the Reaction-Diffusion Problem

2.1.1 Derivation using Weighted Residuals

2.1.2 Application of Green's First Identity

2.2 Galerkin Discretization on Unstructured Meshes

Delaunay Triangulation

2.2.1 Linear Lagrange Basis Functions

2.2.2 Assembly of Mass (M) and Stiffness (K) Matrices

2.3 Numerical Techniques

Mass Lumping : Diagonalization for Explicit Time-Stepping

2.3.1 Spectral Analysis : Eigenmodes of the Laplacian on Square Domains

2.3.2 Verification of the Discrete Laplacian Operator

Chapter 3

Temporal Integration: Addressing Stiffness - The Projective Method

3.1 The Time-Stepping Dilemma

3.1.1 Explicit Euler : The CFL Condition

3.1.2 The Stiffness Constraint : Stability vs. Accuracy

3.2 Projective Integration Schemes (PFE)

3.2.1 Separation of Fast and Slow Manifolds

3.2.2 The Algorithm

3.2.3 Error Estimation and Step-Size Adaptivity

3.3 Stability Analysis

3.3.1 Linear Stability Domain of the Projective Forward Euler Method

3.3.2 Theoretical Speedup Analysis

Chapter 4

High-Performance Implementation

4.1 Software Architecture

Chapter 5

Numerical Experiments and Results

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