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Uniqueness and Observability of Conceptual Rainfall-Runoff Model Parameters: The Percolation Process Examined

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Many researchers have expressed concerns regarding the uniqueness of parameter estimates for conceptual rainfall-runoff (R-R) models obtained through calibration. Recent studies (Sorooshian et al., this issue; Sorooshian and Gupta, this issue) have revealed that even though stochastic parameter estimation techniques can help, the problems are not all due to inefficiencies in the calibration techniques used but are caused by the manner in which the model is structurally formulated. Thus even when calibrated under ideal conditions (simulation studies), it is often impossible to obtain unique estimates for the parameters. It is possible to resolve this problem, at least in part, by appropriate reparameterizations of the pertinent model equations. In this paper the percolation equation of the soil moisture accounting model of the National Weather Service River Forecast System (SMA-NWSRFS) will be discussed. It is shown that a logical reparameterization of this equation can result in conditions that improve the chances of obtaining unique parameter estimates. It is believed that these results have implications for other conceptual R-R models in which similar approaches are used in the representation of the percolation/infiltration process.

INTRODUCTION

The most important problem in the automatic calibration of conceptual rainfall-runoff (R-R) models is the inability to obtain unique and conceptually realistic parameter estimates [see, for example, *Ibbitt*, 1970; *Johnston and Pilgrim*, 1976; *Pickup*, 1977; *Mein and Brown*, 1978; *Brazil and Hudlow*, 1981]. This paper is the third in a three-paper sequence in which the main causes of this problem are discussed. In the first paper, *Sorooshian et al.* [this issue] demonstrated that part of this problem is caused by the use of an objective function which does not properly account for the nature of the uncertainties present in both the data and the model. However, as discussed by *Sorooshian and Gupta* [this issue] in the second paper, there are often factors inherent to the structure of the model that make the determination of a unique parameter set impossible.

One of the most often-reported problems has been the inability to identify the parameters associated with the infiltration or percolation process [*Johnston and Pilgrim*, 1976; *Ibbitt*, 1970]. *Sorooshian and Gupta* [this issue], working with the soil moisture accounting model of the U.S. National Weather Service river forecast system (SMA-NWSRFS), reported the existence of an extended valley in the response surface of the parameters of the percolation equation. They demonstrated the existence of a structural problem that prevents accurate calibration of the model. As shown by *Gupta* [1982], the percolation equation of the SMA-NWSRFS is similar in structure to the Horton infiltration equation, variations of which are used in many models.

In this paper the properties of the percolation process equation of the SMA-NWSRFS are examined in detail and the reasons for the high degree of interaction between its parameters are exposed. It is shown that a suitable reparameterization of the equation can result in improved identifiability of the model. The effectiveness of the proposed approach is demonstrated using simulation studies.

THE PERCOLATION PROCESS

The SMA-NWSRFS model has been discussed extensively in the literature. A full description of the model may be found in the works of *Peck* [1976] and/or *Brazil and Hudlow* [1981]. The model recognizes the presence of different vertically stratified zones of soil in the ground (as do most conceptual R-R models). In brief, the stratification is represented by an upper zone which extends from the surface to the depth of short rooted plants and a lower zone representing ground water storage. Each zone has both tension and free water storage representations. The percolation process connects the upper and lower zones, simulating the effects of gravity and downward suction. To quote *Brazil and Hudlow* [1981],

> The percolation function of the SMA-NWSRFS (developed by Burnash et al., 1973) is considered to be the key element in the transfer of water within the model, as it affects water movement throughout the soil profile and is itself dependent on the current state of the storage system.

It is a complex nonlinear function relating the capacities and contents of both the upper and lower zones and their free water depletion coefficients. The percolation mechanism has been designed to coincide with observed characteristics of the motion of moisture through the soil mantle, including the formation and transmission characteristics of the wetting front, as reported by *Hanks et al.* [1969] and *Green et al.* [1970]. It is generally considered to be a reasonably good representation of the true process. Because the percolation process helps decide on how much of the water in the upper zone contributes to the

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quickly reacting components of the storm hydrograph and how much is transfered to relatively longer term storage in the lower zone, it is critical that the exact nature of the nonlinear relationship between the factors affecting percolation be adequately identified.

The equation used in the SMA-NWSRFS to compute PD_t, the percolation demand (sometimes referred to as potential percolation) during the tth computational time interval is given below; the notation has been adopted from *Sorooshian and Gupta* [this issue].

$$PD_t = \beta[1 + Z(L_t)^X]U_t \tag{1}$$

where

$$\beta = (T_4\, K_2) + (T_5\, K_3) \tag{2}$$

is the minimum percolation demand when both the upper and lower zones are full,

$$L_t = \frac{(T_3 + T_4 + T_5) - (C_{3t} + C_{4t} + C_{5t})}{T_3 + T_4 + T_5} \tag{3}$$

is the lower zone deficiency ratio at the beginning of tth time interval,

$$U_t = \frac{C_{2t}}{T_2} \tag{4}$$

is the upper zone free water contents ratio at the beginning of the tth time interval, and

- Z parameter limiting maximum percolation rate, unitless;
- X parameter controlling nonlinearity of the percolation equation, unitless;
- T_2 threshold parameter limiting upper zone free water contents, mm;
- T_3 threshold parameter limiting lower zone tension water contents, mm;
- T_4 threshold parameter limiting lower zone free primary water contents, mm;
- T_5 threshold parameter limiting lower zone free secondary water contents, mm;
- K_2 lower zone primary storage depletion constant, unitless;
- K_3 lower zone secondary storage depletion constant, unitless;
- C_{it} content at beginning of tth time interval of the reservior with associated threshold T_i.

If during the tth time interval the percolation demand PD_t is greater than the free water C_{2t} available for percolation in the upper zone, the entire available upper zone free water is transferred to the lower zone as actual percolation (i.e., the percolation demand is not completely satisfied). The above equations contain a total of eight parameters that must be estimated. The two parameters that are important to our discussion are Z and X. Note that they are the only two parameters that are not explicitly connected with the operational mechanics of any other part of the model. Parameter X essentially determines the degree of nonlinearity in the equation and Z primarily determines the magnitude of the percolation demand. *Sorooshian and Gupta* [this issue] discussed the identifiability of these two parameters. Their response surface studies of the Z versus X subspace uncovered the presence of an extended valley which explained the inability to obtain unique values for these parameters. Through the use of synthetic data studies it was demonstrated that this problem was largely related to the structural representation of the percolation subprocess. (Note that this implies that even a manual estimation procedure would encounter similar difficulties.) These findings prompted the authors to address the following questions.

1. How does the presence of the extended valley influence the results of the parameter estimation procedure, and how is its existence related to the nature of the data used for calibration?
2. Does the existence of the valley have any bearing on the reliability of the calibrated model?
3. If the answer to 2 is in the affirmative, what can be done to alleviate the problem?

In order to address these questions, the SMA-NWSRFS was modified as described below. Simulation studies were conducted with this model under controlled conditions.

DESCRIPTION OF THE MODEL USED

In order to eliminate unneccessary complications caused by high dimensionality, the SMA-NWSRFS model was modified. The model is depicted in Figure 1. The two tension storages have been removed, and the two lower zone free water storages have been lumped together. Evapotranspiration was assumed to be neglegible, and flow measurements was assumed to be made at the point of channel inflow. The percolation equation (the primary focus of this sutdy) remains essentially unchanged and is presented below:

$$\tilde{PD}_t = \tilde{\beta}[1 + Z(\tilde{L}_t)^X]\tilde{U}_t \tag{5}$$

where

$$\tilde{\beta} = (LM)(LK) \qquad \tilde{L}_t = (LM - LC_t)/LM$$

(which is the lower zone deficiency ratio at the beginning of tth time interval),

$$\tilde{U}_t = \frac{UC_t}{UM}$$

which is the upper zone contents ratio at the beginning of tth time interval), and where

- UM upper zone maximum storage capacity, mm;
- UK upper zone recession constant, unitless;
- LM lower zone maximum storage capacity, mm;
- LK lower zone recession constant, unitless;
- Z, X constants to fit the percolation equation, unitless;
- UC_t upper zone contents at the beginning of tth time interval, mm;

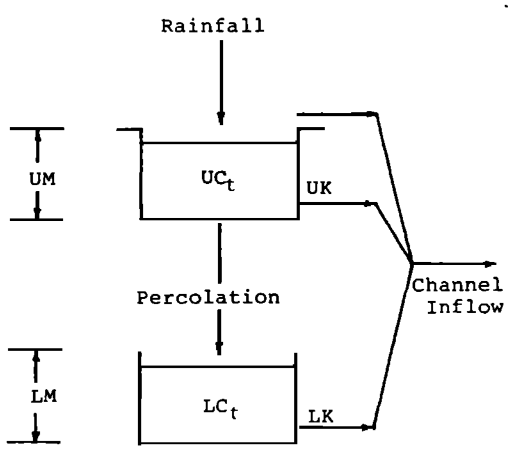

Fig. 1. The modified rainfall-runoff model.

LC_t lower zone contents at the beginning of tth time interval, mm.

STUDY OF THE PERCOLATION PROCESS

We first address the question of how the presence of the extended valley affects the results of the parameter estimation procedure and how its existence is related to the nature of the data used for calibration. It is clear that the ability to identify values for the parameters of a model through calibration is dependent on both the structure of the model, and the type of the data used. Poorly 'activating' data may render a group of model parameters nonobservable or highly interactive, thereby causing poor response surface characteristics. As discussed earlier, the percolation process of the SMA-NWSRFS model (and many others) operates based on a 'demand-supply' principle. In each time interval the percolation equation is used to compute the demand for water transfer to the lower zone. The actual amount transfered (supply) depends on the magnitude of available water in the upper zone. If this available moisture is such that the demand is never met, then the quantity of the water transfered to the lower zone will be insensitive to moderate changes in the values of the parameters of the percolation equation. On the other hand, if the demand is always met, the output should be quite sensitive to variations in the parameters. This would imply that if the process is adequately activated by the data, then the percolation equation parameters would be properly observable. However, if even in this case the parameters prove difficult to observe, then it is fair to conclude that an identifiability problem exists which is inherent to the structure of the model.

To determine which of these two conditions is present, the following study was conducted. First a parameter set P1 (see Table 1) was specified for the model. Then a synthetic daily rainfall record (input) was used to generated a sequence of 'true' model daily flows (output). Figure 2*a* is a graph of both percolation demand and actual percolation for this case. It was found that the percolation demand exceeded the actual percolation during a substantial number (34%) of the time intervals, thereby reducing the sensitivity of the model output to the parameters of the percolation equation (the critical parameters being Z and X). In order to examine this sensitivity, the following procedure was employed. The simple least squares (SLS) criterion

$$\mathrm{SLS} = \sum_{t=1}^{n} \varepsilon_t^2$$

where ε_t is the difference between the simulated and observed flows for time interval t) was selected as a suitable measure of the fit of the model to the true data when different values are selected for the parameters. (The SLS has been the most commonly used criterion for parameter estimation of dynamic system models, and its advantages and disadvantages have been extensively discussed in the literature.) A contour plot was created by evaluating the SLS criterion at various points of a grid in the $Z - X$ parameter subspace while maintaining the remaining parameters at their true values. (See Figure 3*a*). The figure displays the existence of an extended valley similar to that reported by *Sorooshian and Gupta* [this issue] when using the SMA-NWSRFS model. Note that the valley is oriented in a direction demonstrating lower sensitivity to Z than to X. This result seems to imply that if the response could somehow be made more sensitive to the parameter Z, then the extent of the valley could perhaps be reduced to the point that reasonably elliptical contours which display a unique optimum might be obtained.

TABLE 1. Parameter Sets Used in the Studies With the Modified Model.

Parameter	*P*1	*P*2
UM	10.0	10.0
LM	20.0	20.0
UK	0.5	0.5
LK	0.2	0.2
Z	50.0	5.0
X	3.0	3.0

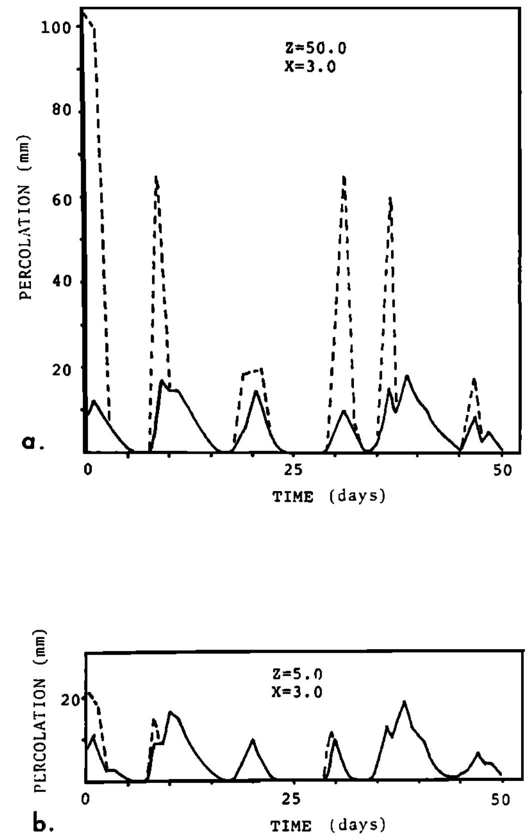

Fig. 2. Percolation demand-and actual percolation-values for the two cases (Z = 50) and (Z = 5), respectively.

One condition under which this might happen is when the data is such that the percolation demand is always satisfied. This condition can be simulated effectively by reducing the true value of Z (to reduce the magnitude of the percolation demand) and repeating the entire experiment described above. Figures 2*b* and 3*b* represent the results when parameter set P2 was used (true value of Z = 5). In this case the percolation demand exceeds supply less than 10% of the time. However, instead of the extent of the valley being reduced, the angle of its orientation has changed to make Z relatively more sensitive and X less so.

The results of this study indicate that the existence of the extended valley is inherent to the model structure but that its orientation depends on the degree to which the parameters of the process are activated by the data. Thus the parameters Z and X are nonidentifiable (rather than nonobservable). (Note that we conducted similar experiments using values of Z be-

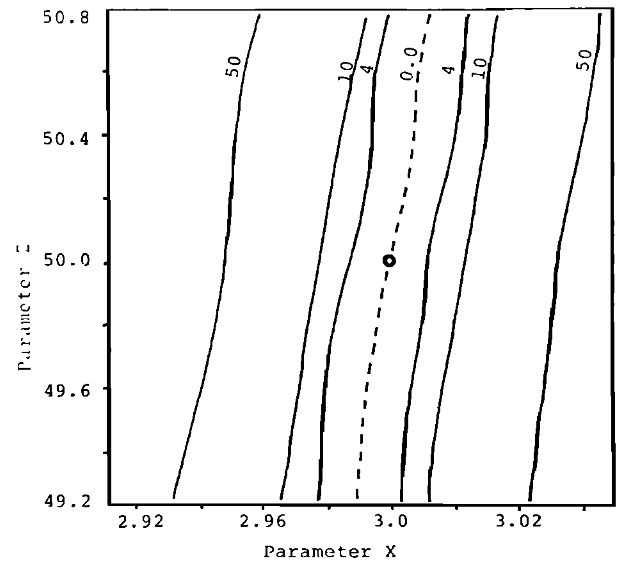

Fig. 3a. SLS contour plot for case ($Z = 50$) showing nature and orientation of the valley in Z versus X parameter space.

tween 50 and 5. The results indicated a progressive change in the angle of orientation of the valley. An extended examination of the response surface confirmed the existence of the valley well outside the regions presented in this paper).

The bearing that the above results have on the reliability of the calibrated model can now be discussed. The existence of an extended valley clearly implies an inability to obtain a unique parameter set for the model using either the manual or automatic calibration procedures. In the case of the automatic procedure the search will terminate at the first point at the bottom of the valley that it encounters. Thus the results of the calibration are highly dependent on the choice of parameter values used to initiate the search. Recognizing that each pair of Z and X along the valley results in the same value of the objective function, one might presume that it does not matter which pair is chosen. This presumption would be false for two basic reasons.

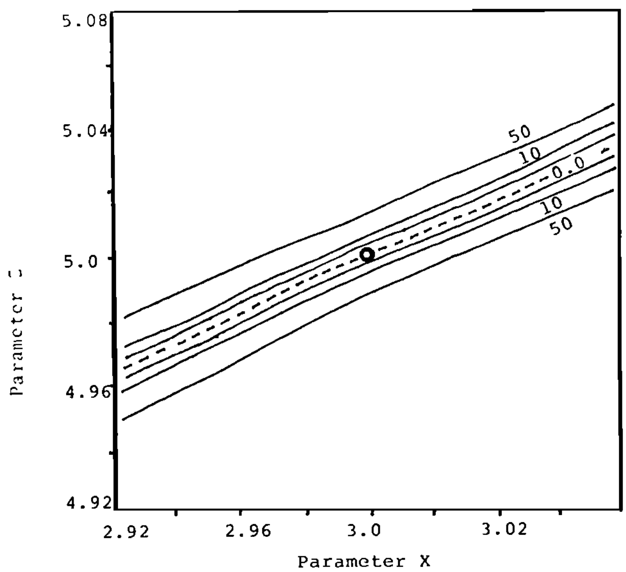

Fig. 3b. SLS contour plot for case ($Z = 5$) showing nature and orientation of the valley in Z versus X parameter space.

To understand the first reason, let us pretend that the true values of the parameters are known. Due to the nature of the calibration data being used, there will exist a valley (say, VI) on the $Z - X$ response surface oriented in some direction, this direction being dependent on the data (see Figure 4). Let us assume that the search technique selects a point (shown by the crossed circle) in the valley at some distance from the true optimum (shown by the open circle). Now a second data set is selected to be used for forecasting. If the degree of activation provided by this data is different from that of the calibration record, the bottom of the valley will still pass through the true parameter set but will have a different orientation (say, V2). Hence the selected parameter set will no longer be contained in the indifference set for the new data and will give nonoptimal forecasting results, due to poor reproduction of the percolation process. It is important therefore to recognize that any pair of Z and X obtained through calibration would be satisfactory only if the orientation of the valley was insensitive to the data used for calibration.

The second reason concerns parameter interaction. Among other things, parameter interaction implies that if there is an error in the estimation of one parameter, there will be errors in the estimated values of the other interacting parameters (as a result of the compensating changes required to minimize the sum of the squares of the residuals). The magnitude of these errors depends on the extent and importance of the interaction. A look at the percolation equation (5) reveals the presence of five parameters interacting directly through this process. We have shown that the identification of the true values of Z and X is close to impossible. Hence it is reasonable to expect that this will affect the identification of the other parameters of the model. Here again, whether a manual or an automatic technique is used, parameters identified are likely to be unsatisfactory and certainly nonoptimal, as was also demonstrated by *Sorooshian and Gupta* [this issue] on the SMA-NWSRFS model using real data.

REPARAMETERIZATION OF THE PERCOLATION FUNCTION

Having clearly demonstrated that the extended valley is a problem inherent to the structure of the model, we now examine what can be done to alleviate the problem. The objective is to see if (5) can be modified so as to obtain elliptical response surface contours in the resulting parameter space (circular contours being the ideal) without seriously affecting the conceptual

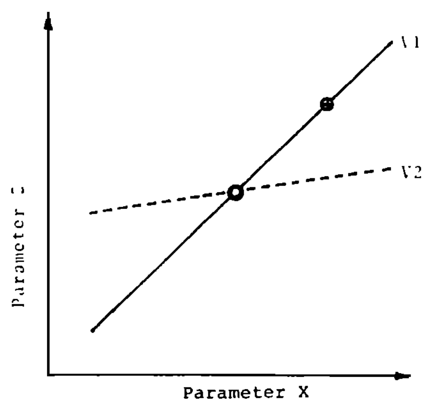

Fig. 4. Illustration of effect of choosing incorrect parameter set from calibration data valley. Valley for calibration data is indicated by the solid line; valley for forecast data is shown as a dashed line. The 'true' parameter set B shown by the open circle, while the parameter set selected during calibration is shown by the crossed circles.

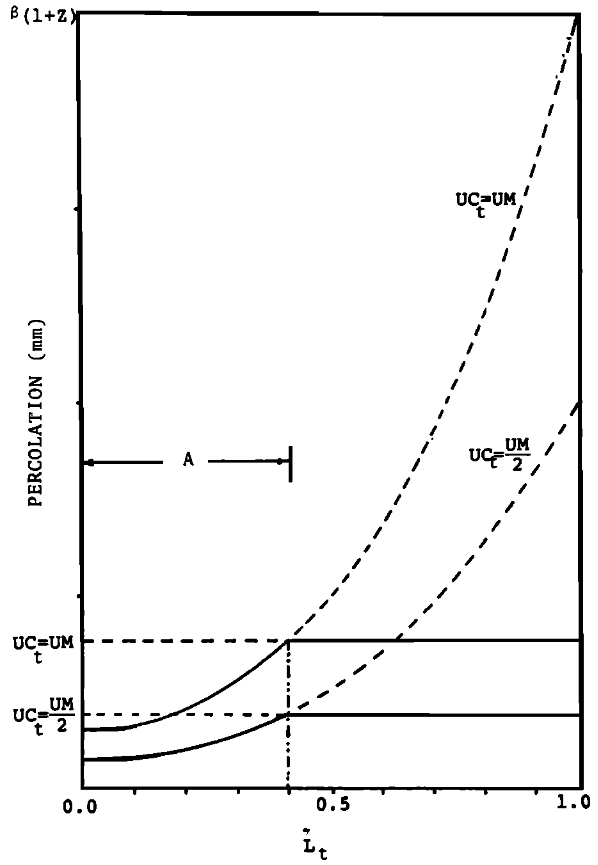

Fig. 5. Representation of percolation demand as a function of lower zone deficiency ratio at the tth time interval.

representation of the process intended by the developers of the SMA-NWSRFS model.

To illustrate the problem, a graphical representation of the percolation demand and actual percolation (for the tth time interval) versus the change in the lower zone deficiency ratio ($\tilde{L}_t = 0.0$ indicates lower zone full, $\tilde{L}_t = 1.0$ indicates lower zone empty) for the cases of upper zone full (UC_t = UM) and upper zone half full (UC_t = UM/2) is shown in Figure 5. The curved lines represent the percolation demand for each of the cases, while the horizontal lines represent the amount of water available in the upper zone. The maximum percolation that can occur is equal to UM, which is the maximum capacity of the upper zone. Hence percolation demand will always be met for $\tilde{L}_t$ less than or equal to some value A ($0 \leq A \leq 1$) and will never be met for $\tilde{L}_t$ greater than A. (Note that an important assumption here is that the maximum possible percolation demand [$\bar{\beta}(1 + Z)$] exceeds the size of the upper zone storage UM. This is considered to be realistic for most watersheds.) A is therefore a threshold. The implication of this is that irrespective of the data selected for the estimation procedure, the output of the model is only sensitive to the parameters of the percolation equation (Z and X) during those time periods for which $\tilde{L}_t$ is less than or equal to A, ($\tilde{L}_t \leq A$). Also, from Figure 5 it can be seen that the value A is significantly influenced by the magnitude of the upper zone UM. If UM is small, then A shifts to the left, making the parameters even less observable.

In Figure 6 the manner in which this gives rise to the existence of the valley is demonstrated. Curve 1 indicates the demand curve corresponding to the true parameter set (say $\hat{Z}$, $\hat{X}$). Curve 2 corresponds to the case when Z has been reduced substantially to $Z^* < \hat{Z}$, while X is maintained equal to its true value $\hat{X}$. It is easy to see that fairly large changes in Z do not cause correspondingly large deviations in the percolation curve for $\tilde{L}_t \leq A$. These deviations can be minimized by an appropriate compensating change in X (e.g., see curve 3, where $Z = Z^*$ and X has been reduced to $X^* < \hat{X}$), thus making the errors in computed percolation almost nelegible. This will evidently give rise to the existence of a valley of virtually identical objective function values in the $Z - X$ response surface.

It is obvious that in order to make the parameters identifiable, an appropriate reparameterization is required so as to make the response surface better conditioned. One possible (mathematical) approach is to conduct a linear transformation from the original cartesian coordinate system to the canonical coordinate system using an eigenvalue-eigenvector decomposition in the parameter space. An appropriate rescaling can then be employed. This approach, however, will result in a new (transformed) parameter set which may not possess any physical significance. In addition there is a computational drawback associated with this, because it requires that an accurate estimate of the Hessian matrix be obtained at each iteration of the search. The reader familiar with conceptual R-R models will recognize that this is rather difficult in light of the fact that explicit computation of partial derivatives is often not possible.

As discussed at the beginning of this section it would be far more desirable to obtain a (linear or nonlinear) reparameterization of the equation that preserves the conceptual significance of the parameters. In the context of this problem a parameter is required which is a function of both Z and X (and perhaps other parameters) and which represents the direction

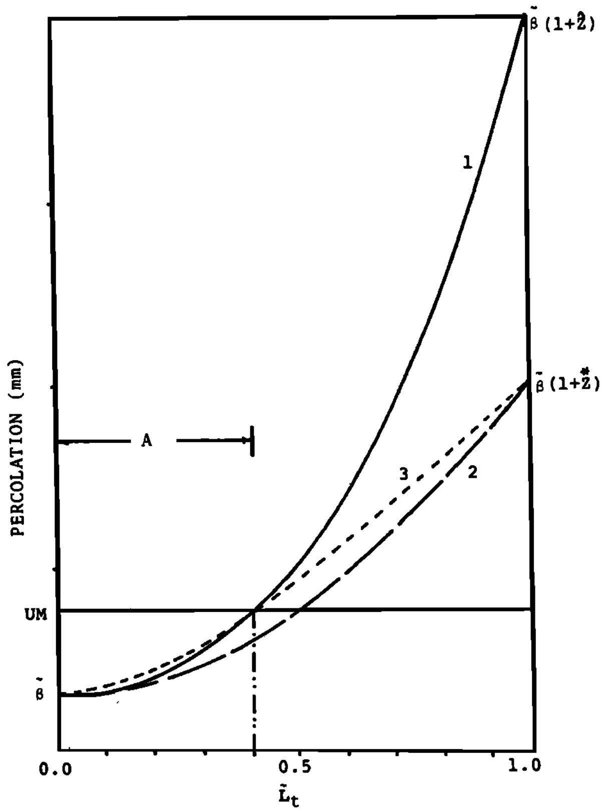

Fig. 6. Exaggerated example showing the effect of Z versus X interaction on the percolation demand curve. $\hat{Z}$, $\hat{X}$ are true values; Z^*, X^* are new values. Curve 1 is for ($Z = \hat{Z}$, $X = \hat{X}$); curve 2, for ($Z = Z^* < \hat{Z}, X = \hat{X}$); Curve 3 for ($Z = Z^*, X = X^* < \hat{X}$).

along the valley. This parameter should replace the relatively insensitive parameter Z. Proper scaling of the new parameter would then allow reasonably elliptical contours of the objective function to be obtained in the neighborhood of the 'best' parameter set.

From Figure 5 it is clear that there is one 'invariant' present which is unaffected by the amount of water available for percolation. This is the value A, which represents the value of $\tilde{L}_t$ at which the percolation demand and the available water curves intersect. This value has the nice property of being scaled between zero and one, which means that a reasonable initial estimate for A can be made because it is bounded. (Z, on the other hand, can theoretically vary between zero and infinity.) This scaled property means that large changes in Z correspond to much smaller changes in A (as can be seen from Figure 6), thus making A a far more sensitive parameter.

The relationship between the parameters A, Z, and X is now derived. Let the contents of the upper zone at the beginning of the tth time interval be equal to UC_t. The percolation demand $\tilde{\mathrm{P}}\mathrm{D}_t$ will be equal to UC_t when $\tilde{L}_t = A$. By substituting these relationships into (5), we get

$$\mathrm{UC}_t = \tilde{\beta}\,[1 + Z(A)^X]\,\frac{\mathrm{UC}_t}{\mathrm{UM}} \qquad (6)$$

By simplifying and rearranging (6), A is obtained as a nonlinear function of Z, X, and other parameters:

$$A = \left[\frac{\mathrm{UM} - \tilde{\beta}}{\tilde{\beta}\,Z}\right]^{1/X} \qquad (7)$$

Similarly,

$$Z = \left[\frac{\mathrm{UM} - \tilde{\beta}}{\tilde{\beta}\,A^X}\right] \qquad (8)$$

By substituting for Z in (5) we obtain a relationship for the actual percolation ($\tilde{\mathrm{P}}\mathrm{A}_t$):

$$\widetilde{\mathrm{PA}}_t = \left[\tilde{\beta} + (\mathrm{UM} - \tilde{\beta})\left(\frac{\tilde{L}_t}{A}\right)^X\right]U_t \qquad \forall\, \tilde{L}_t \le A$$

$$\widetilde{\mathrm{PA}}_t = \mathrm{UC}_t \qquad \forall\, \tilde{L}_t > A \qquad (9)$$

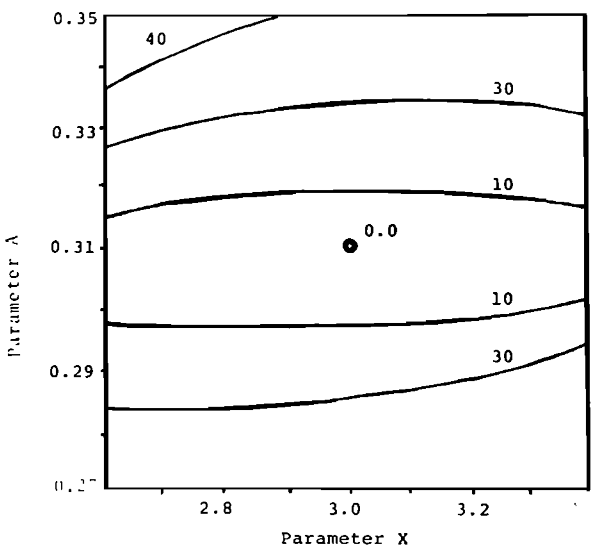

Fig. 7a. SLS contour plot of A versus X for nominal parameter set $A = 0.31, X = 3.0$.

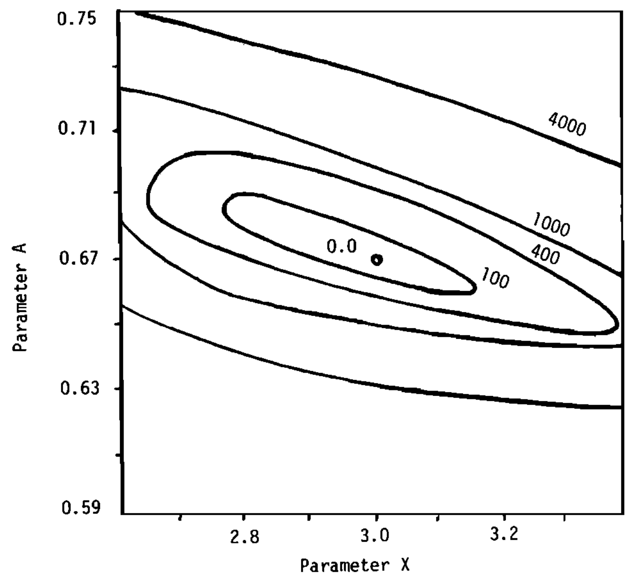

Fig. 7b. SLS contour plot of A versus X for nominal parameter set $A = 0.67, X = 3.0$.

Notice that due to this reparameterization there is no need to calculate a percolation demand value, as the actual value of percolation depends simply on whether the $\tilde{L}_t$ is greater than or less than parameter A.

EXPERIMENTAL TESTING OF THE REPARAMETERIZED PERCOLATION EQUATION

The reparameterized percolation equation (9) was programed into the model replacing the original equation (5). When using parameter set P1 ($Z = 50.0$) and (7), the value of A is 0.31. Similarly, for parameter set P2 ($Z = 5.0$) the value of A is 0.67. The $A - X$ contour plots of the SLS criterion for these two cases are presented in Figures 7a and 7b. These are four important results that we observed.

1. The contours are (in each case) approximately elliptical in shape and display a unique optimum.
2. The degree of interaction between the two parameters A and X is small and, more importantly, is relatively insensitive to different nominal values of the parameter A.
3. The objective function is relatively less sensitive to changes in parameter X than to changes in parameter A.
4. As the nominal value of A gets larger (reflecting a higher degree of activation of the parameters of the percolation equation), the contours are seen to concentrate more tightly around the nominal point, indicating better observability and improved precision of the estimates of both the parameters A and X.

The results of the reparameterization indicate a marked improvement in the identifiability of the parameters of the model. In contrast, when the (5) version of the percolation equation was used, the angle of interaction between the parameters Z and X varied substantially with changes in the degree of activation of the parameters, and it was impossible to identify a unique optimum (see Figures 3a and 3b) even when the degree of activation of the parameters was increased. Equation (9) therefore seems to be a better representation of the percolation equation than (5).

There remains one final aspect of the new version of the percolation equation that needs to be discussed. Figure 7a is the $A - X$ contour plot corresponding to nominal parameter set P1, for which the parameters were activated during only 66% of the computational time intervals (in contrast, for parameter set P2, activation is greater than 90%). As can be seen, the objective function is relatively far more sensitive to changes in parameter A than to changes in parameter X. In such a situation the optimization algorithm employed to search for the optimum will find it difficult to precisely determine the optimal value of the parameter X, unless some kind of rescaling procedure is employed that will result in near-circular contours of the objective function in the region of the optimum. Interestingly enough, the reparameterization has resulted in virtually negligible $A - X$ parameter interaction, thereby permitting the parameters to be individually rescaled. (Note that if the degree of interaction is significant, then the rescaling would have to be applied in the canonical coordinate system to be successful.) In order to demonstrate that the nominal value of X is indeed observable for the case of parameter set P1 and to illustrate one possible approach to rescaling, the following procedure was employed. It was decided to rescale the parameter A. To achieve a proper rescaling, let R be a new parameter which is a function of parameter A. We require that in the neighborhood of the optimum, a percent change ΔR in the nominal value of R will cause a change in the objective function equal to that caused by a percent change ΔX in X. Therefore

$$\frac{\Delta R}{R} = \frac{\Delta A}{A} S \tag{10}$$

where S is the proper scaling factor so that a 2% change in A corresponds to a S percent change ($S > 1$) in R. By integrating both sides we get

$$\ln (R) = S[\ln (A)] \tag{11}$$

hence

$$A = (R)^{1/S} \tag{12}$$

By substituting (12) into (9) we get

$$\begin{aligned} \widetilde{\mathrm{PA}}_t &= [\tilde{\beta} + (UM - \tilde{\beta})(L_t)^X/(R)^{X/S}]\tilde{U}_t \qquad \forall\, \bar{L}_t \le R^{1/S} \\ \widetilde{\mathrm{PA}}_t &= UC_t \qquad \forall\, \bar{L}_t > R^{1/S} \end{aligned} \tag{13}$$

In order to implement (13), a value for the scaling factor S must be chosen. In Figure 8 we present the $R - X$ contour plot of the SLS criterion in the neighborhood of the nominal point using $S = 5$. (Note that $R = A^S = 0.0029$ for nominal value $A = 0.31$). The improvement in contour shape is dramatic, and a clear unique optimum parameter set can be distinguished. The exact value of the scaling factor that will give best results is, of course, unknown to us a priori, and the value of $S = 5$ employed in this example was chosen subjectively by examining the resulting contour plot. Research is currently being conducted to establish a mathematically rigorous approach by which the appropriate scale factor for each parameter might be chosen.

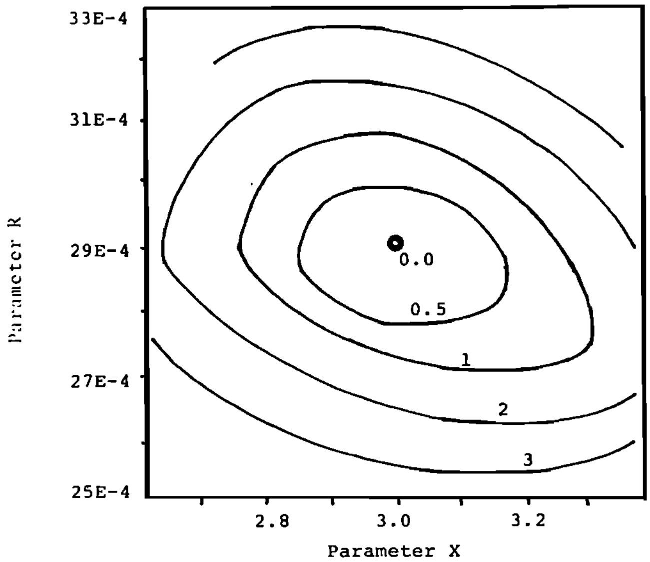

Fig. 8. SLS contour plot of R versus X for nominal parameter set $R = 29E - 4$, $X = 3.0$.

CONCLUSIONS

The principal conclusions of the paper are summarized below.

1. The percolation equation of the SMA-NWSRFS was found to contain parameters that are difficult to estimate using available calibration techniques. A thorough investigation of this equation [see also *Sorooshian and Gupta*, this issue] indicated that the two parameters of the equation (Z and X) related only to the percolation process were at the root of the problem.

2. Response surface studies indicated the presence of an extended valley in the $Z - X$ parameter space. The existence of this valley was shown to be a problem inherent to the structural formulation of the model and not caused by poor activation of the parameters of the percolation equation.

3. The orientation of the valley was found to be significantly affected by the degree of activation of the parameters of the percolation equation, thereby implying that the choice of an arbitrary set of values of Z and X corresponding to a point in the valley other than the 'best' set is likely to result in nonoptimal forecasting performance.

4. The approach used to solve the problem involved a reparameterization of the percolation equation. The criteria used to achieve this were (1) that the original behavior of the model should not be affected, and (2) that the conceptual realism of the parameters should be preserved. Analysis of the reparameterized model demonstrated the observability of a unique optimum parameter set. Clearly, the new form of the equation makes the model better identifiable and is therefore more suitable for parameter estimation using existing calibration methodology.

As discussed in the introduction, we believe that the results of this work are probably relevant to conceptual rainfall-runoff models other than the SMA-NWSRFS. We have examined here one of the more critical aspects of these models, namely the percolation process. There are, however, other aspects of the structures of these models which also cause problems for the calibration scheme (e.g., threshold parameters). Future studies of these problems are essential in order to further improve the reliability of conceptual rainfall-runoff models. We are presently involved in extending this work. We hope that this paper will encourage further research along these lines.

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