Declarative Programming

Representation and Reasoning with Datalog

Declarative Programming

- Knowledgebases and queries in propositional logic are made up of propositions and connectives.
- Predicate logic adds the notion of *predicates* and *variables*.
- We take a non-theoretical approach to predicate logic by introducing *declarative programming*.
- Declarative programming is the use of mathematical logic to describe the logic of computation without describing its control flow.
- Useful for: expert systems, diagnostic systems, machine learning, parsing text, theorem proving, …

Declarative Programming

- Programmer gives a declarative specification of the problem, using the language of logic.
- The programmer should not have to tell the computer what to do.
- To get information, the programmer simply asks a query.

Datalog

- Prolog is a declarative (logical) programming language and stands for PROgramming in LOGic
- We only look at a subset of the language which is (roughly) equivalent to Datalog.

http://www.learnprolognow.org

Basic idea of Datalog/Prolog

- Describe the situation of interest
- Ask a question
- Prolog logically deduces new facts about the situation we described
- Prolog gives us its deductions back as answers

Consequences

- Think declaratively, not procedurally
  - Challenging
  - Requires a different mindset
  - Has similarities with other programming paradigms such as functional programming.

- High-level language
  - Not as efficient
  - Useful in many AI applications

The interpreter

- Prolog has an interactive interpreter
- After starting the interpreter, it can start reading your Prolog files and answer your queries.
- To exit Prolog simply type the command `halt.` (note the full-stop)
- Prolog program files usually have the extension `.pl` or `.pro`

Where is the program written?

- Facts and Rules are stored in one or more files forming our Knowledge Base

- Files containing KB are loaded into the interpreter

- After changing these files, the files should be loaded again to be effective

- Queries are asked in the interactive mode in front of the question prompt: ?-

Reading Files

- `consult (filename).`
  - Reads and compiles a Prolog source file. Example:
  - `consult ('/home/user/prolog/sample.pl').`

- `reconsult (filename).`
  - Reconsult a changed source files. Example

  `reconsult('/home/user/prolog/sample.pl').`

- `[‘filename’].` (valid but don’t use it; can be confused with lists!)

  `['/home/user/prolog/sample.pl'].`

- `make.`
  - Reconsult all changed source files.

Knowledge Base 1

```
woman(mia).
woman(jody).
woman(yolanda).
playsAirGuitar(jody).
party.
```

Knowledge Base 1

```
woman(mia).
woman(jody).
woman(yolanda).
playsAirGuitar(jody).
party.
```

```
?-
```

Knowledge Base 1

```
woman(mia).
woman(jody).
woman(yolanda).
playsAirGuitar(jody).
party.
```

```
?- woman(mia).
```

Knowledge Base 1

```
woman(mia).
woman(jody).
woman(yolanda).
playsAirGuitar(jody).
party.
```

```
?- woman(mia).
yes
?-
```

Knowledge Base 1

```
woman(mia).
woman(jody).
woman(yolanda).
playsAirGuitar(jody).
party.
```

```
?- woman(mia).
yes
?- playsAirGuitar(jody).
```

Knowledge Base 1

```
woman(mia).
woman(jody).
woman(yolanda).
playsAirGuitar(jody).
party.
```

```
?- woman(mia).
yes
?- playsAirGuitar(jody).
yes
?-
```

Knowledge Base 1

```
woman(mia).
woman(jody).
woman(yolanda).
playsAirGuitar(jody).
party.
```

```
?- woman(mia).
yes
?- playsAirGuitar(jody).
yes
?- playsAirGuitar(mia).
no
```

Knowledge Base 1

```
woman(mia).
woman(jody).
woman(yolanda).
playsAirGuitar(jody).
party.
```

```
?- tattoed(jody).
```

Knowledge Base 1

```
woman(mia).
woman(jody).
woman(yolanda).
playsAirGuitar(jody).
party.
```

```
?- tattoed(jody).
no
?-
```

Knowledge Base 1

```
woman(mia).
woman(jody).
woman(yolanda).
playsAirGuitar(jody).
party.
```

```
?- tattoed(jody).
ERROR: predicate tattoed/1 not defined.
?-
```

Knowledge Base 1

```
woman(mia).
woman(jody).
woman(yolanda).
playsAirGuitar(jody).
party.
```

```
?- party.
```

Knowledge Base 1

```
woman(mia).
woman(jody).
woman(yolanda).
playsAirGuitar(jody).
party.
```

```
?- party.
yes
?-
```

Knowledge Base 1

```
woman(mia).
woman(jody).
woman(yolanda).
playsAirGuitar(jody).
party.
```

```
?- rockConcert.
```

Knowledge Base 1

```
woman(mia).
woman(jody).
woman(yolanda).
playsAirGuitar(jody).
party.
```

```
?- rockConcert.
no
?-
```

Knowledge Base 2

```
happy(yolanda).
listens2music(mia).
listens2music(yolanda):- happy(yolanda).
playsAirGuitar(mia):- listens2music(mia).
playsAirGuitar(yolanda):- listens2music(yolanda).
```

Knowledge Base 2

```
happy(yolanda).
listens2music(mia).
listens2music(yolanda):- happy(yolanda).
playsAirGuitar(mia):- listens2music(mia).
playsAirGuitar(yolanda):- listens2music(yolanda).
```

fact

Knowledge Base 2

```
happy(yolanda).                          fact
listens2music(mia).                      fact
listens2music(yolanda):- happy(yolanda).
playsAirGuitar(mia):- listens2music(mia).
playsAirGuitar(yolanda):- listens2music(yolanda).
```

Knowledge Base 2

```
happy(yolanda).                                    fact
listens2music(mia).                                fact
listens2music(yolanda):- happy(yolanda).           rule
playsAirGuitar(mia):- listens2music(mia).
playsAirGuitar(yolanda):- listens2music(yolanda).
```

Knowledge Base 2

```
happy(yolanda).                                    fact
listens2music(mia).                                fact
listens2music(yolanda):- happy(yolanda).           rule
playsAirGuitar(mia):- listens2music(mia).          rule
playsAirGuitar(yolanda):- listens2music(yolanda).
```

Knowledge Base 2

```
happy(yolanda).
listens2music(mia).
listens2music(yolanda):- happy(yolanda).
playsAirGuitar(mia):- listens2music(mia).
playsAirGuitar(yolanda):- listens2music(yolanda).
```

fact
fact
rule
rule
rule

Knowledge Base 2

```
happy(yolanda).
listens2music(mia).
listens2music(yolanda):- happy(yolanda).
playsAirGuitar(mia):- listens2music(mia).
playsAirGuitar(yolanda):- listens2music(yolanda).
```

head body

Knowledge Base 2

```
happy(yolanda).
listens2music(mia).
listens2music(yolanda):- happy(yolanda).
playsAirGuitar(mia):- listens2music(mia).
playsAirGuitar(yolanda):- listens2music(yolanda).
```

```
?-
```

Knowledge Base 2

```
happy(yolanda).
listens2music(mia).
listens2music(yolanda):- happy(yolanda).
playsAirGuitar(mia):- listens2music(mia).
playsAirGuitar(yolanda):- listens2music(yolanda).
```

```
?- playsAirGuitar(mia).
yes
?-
```

Knowledge Base 2

```
happy(yolanda).
listens2music(mia).
listens2music(yolanda):- happy(yolanda).
playsAirGuitar(mia):- listens2music(mia).
playsAirGuitar(yolanda):- listens2music(yolanda).
```

```
?- playsAirGuitar(mia).
yes
?- playsAirGuitar(yolanda).
yes
```

Clauses

```
happy(yolanda).
listens2music(mia).
listens2music(yolanda):- happy(yolanda).
playsAirGuitar(mia):- listens2music(mia).
playsAirGuitar(yolanda):- listens2music(yolanda).
```

There are five clauses in this knowledge base: two facts, and three rules.

The end of a clause is marked with a full stop.

Predicates

```
happy(yolanda).
listens2music(mia).
listens2music(yolanda):- happy(yolanda).
playsAirGuitar(mia):- listens2music(mia).
playsAirGuitar(yolanda):- listens2music(yolanda).
```

*There are three **predicates***
in this knowledge base:
happy, listens2music, and playsAirGuitar

Knowledge Base 3

```
happy(vincent).
listens2music(butch).
playsAirGuitar(vincent):- listens2music(vincent), happy(vincent).
playsAirGuitar(butch):- happy(butch).
playsAirGuitar(butch):- listens2music(butch).
```

Expressing Conjunction

```
happy(vincent).
listens2music(butch).
playsAirGuitar(vincent):- listens2music(vincent), happy(vincent).
playsAirGuitar(butch):- happy(butch).
playsAirGuitar(butch):- listens2music(butch).
```

The comma "," expresses conjunction in Prolog

Knowledge Base 3

```
happy(vincent).
listens2music(butch).
playsAirGuitar(vincent):- listens2music(vincent), happy(vincent).
playsAirGuitar(butch):- happy(butch).
playsAirGuitar(butch):- listens2music(butch).
```

```
?- playsAirGuitar(vincent).
no
?-
```

Knowledge Base 3

```
happy(vincent).
listens2music(butch).
playsAirGuitar(vincent):- listens2music(vincent), happy(vincent).
playsAirGuitar(butch):- happy(butch).
playsAirGuitar(butch):- listens2music(butch).
```

```
?- playsAirGuitar(butch).
yes
?-
```

Expressing Disjunction

```
happy(vincent).
listens2music(butch).
playsAirGuitar(vincent):- listens2music(vincent), happy(vincent).
playsAirGuitar(butch):- happy(butch).
playsAirGuitar(butch):- listens2music(butch).
```

```
happy(vincent).
listens2music(butch).
playsAirGuitar(vincent):- listens2music(vincent), happy(vincent).
playsAirGuitar(butch):- happy(butch); listens2music(butch).
```

Prolog and Logic

- Clearly Prolog has something to do with logic
- Operators
  - Implication :-
  - Conjunction ,
  - Disjunction ;
- Use of modus ponens

Knowledge Base 4

```
woman(mia).
woman(jody).
woman(yolanda).

loves(vincent, mia).
loves(marsellus, mia).
loves(pumpkin, honey_bunny).
loves(honey_bunny, pumpkin).
```

Prolog Variables

```
woman(mia).
woman(jody).
woman(yolanda).

loves(vincent, mia).
loves(marsellus, mia).
loves(pumpkin, honey_bunny).
loves(honey_bunny, pumpkin).
```

```
?- woman(X).
```

Variable Instantiation

```
woman(mia).
woman(jody).
woman(yolanda).

loves(vincent, mia).
loves(marsellus, mia).
loves(pumpkin, honey_bunny).
loves(honey_bunny, pumpkin).
```

```
?- woman(X).
X=mia
```

Asking Alternatives

```
woman(mia).
woman(jody).
woman(yolanda).

loves(vincent, mia).
loves(marsellus, mia).
loves(pumpkin, honey_bunny).
loves(honey_bunny, pumpkin).
```

```
?- woman(X).
X=mia;
```

Asking Alternatives

```
woman(mia).
woman(jody).
woman(yolanda).

loves(vincent, mia).
loves(marsellus, mia).
loves(pumpkin, honey_bunny).
loves(honey_bunny, pumpkin).
```

```
?- woman(X).
X=mia;
X=jody
```

Asking Alternatives

```
woman(mia).
woman(jody).
woman(yolanda).

loves(vincent, mia).
loves(marsellus, mia).
loves(pumpkin, honey_bunny).
loves(honey_bunny, pumpkin).
```

```
?- woman(X).
X=mia;
X=jody;
X=yolanda
```

Asking Alternatives

```
woman(mia).
woman(jody).
woman(yolanda).

loves(vincent, mia).
loves(marsellus, mia).
loves(pumpkin, honey_bunny).
loves(honey_bunny, pumpkin).
```

```
?- woman(X).
X=mia;
X=jody;
X=yolanda;
no
```

Knowledge Base 4

```
woman(mia).
woman(jody).
woman(yolanda).

loves(vincent, mia).
loves(marsellus, mia).
loves(pumpkin, honey_bunny).
loves(honey_bunny, pumpkin).
```

```
?- loves(marsellus,X), woman(X).
```

Knowledge Base 4

```
woman(mia).
woman(jody).
woman(yolanda).

loves(vincent, mia).
loves(marsellus, mia).
loves(pumpkin, honey_bunny).
loves(honey_bunny, pumpkin).
```

```
?- loves(marsellus,X), woman(X).
X=mia
?-
```

Knowledge Base 4

```
woman(mia).
woman(jody).
woman(yolanda).

loves(vincent, mia).
loves(marsellus, mia).
loves(pumpkin, honey_bunny).
loves(honey_bunny, pumpkin).
```

```
?- loves(pumpkin,X), woman(X).
```

Knowledge Base 4

```
woman(mia).
woman(jody).
woman(yolanda).

loves(vincent, mia).
loves(marsellus, mia).
loves(pumpkin, honey_bunny).
loves(honey_bunny, pumpkin).
```

```
?- loves(pumpkin,X), woman(X).
no
?-
```

Knowledge Base 5

```
loves(vincent,mia).
loves(marsellus,mia).
loves(pumpkin, honey_bunny).
loves(honey_bunny, pumpkin).

jealous(X,Y):- loves(X,Z), loves(Y,Z).
```

Knowledge Base 5

```
loves(vincent,mia).
loves(marsellus,mia).
loves(pumpkin, honey_bunny).
loves(honey_bunny, pumpkin).

jealous(X,Y):- loves(X,Z), loves(Y,Z).
```

```
?- jealous(marsellus,W).
```

Knowledge Base 5

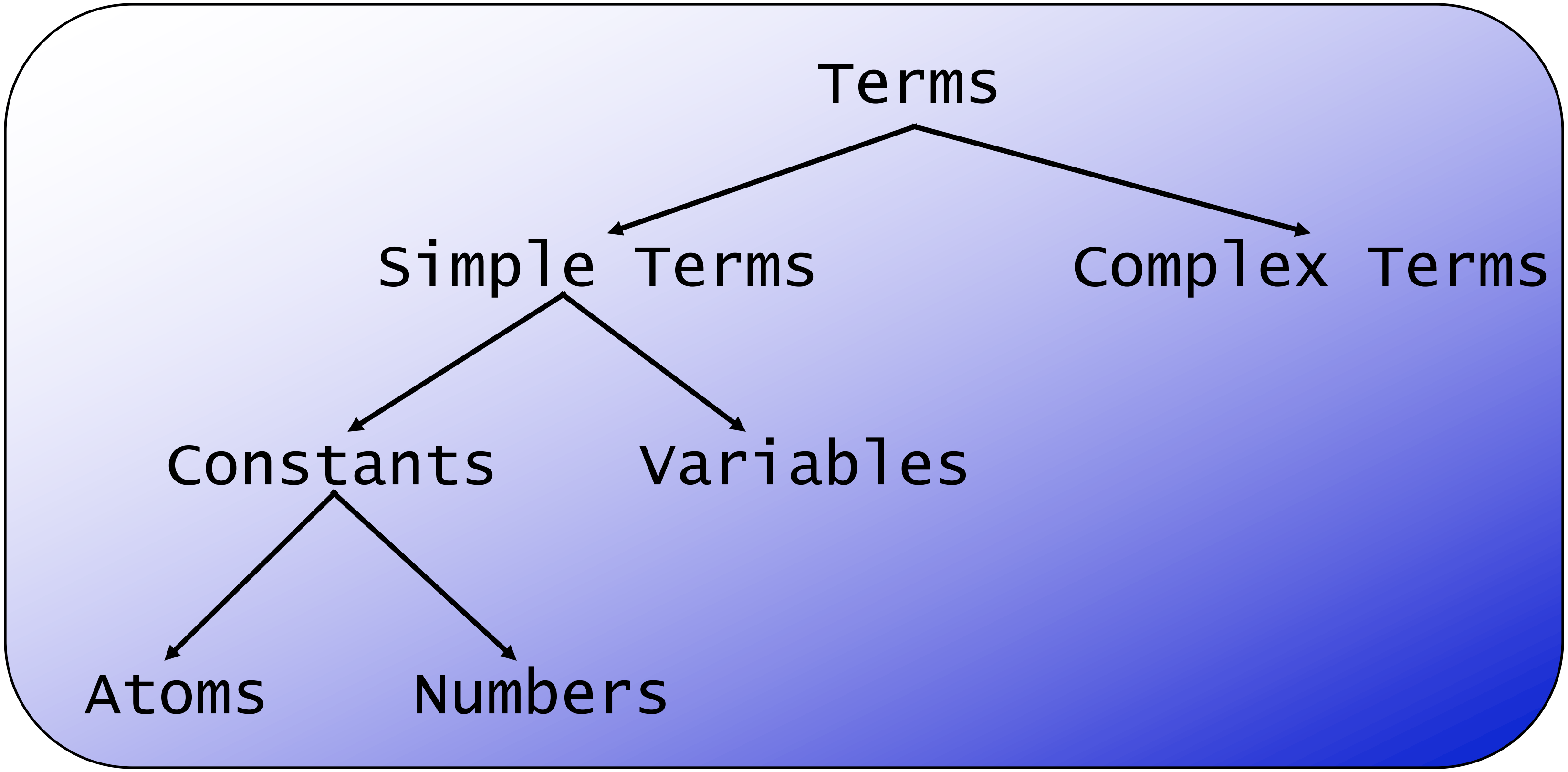
```
loves(vincent,mia).
loves(marsellus,mia).
loves(pumpkin, honey_bunny).
loves(honey_bunny, pumpkin).

jealous(X,Y):- loves(X,Z), loves(Y,Z).
```

```
?- jealous(marsellus,W).
W=vincent
?-
```

Prolog Syntax

- What exactly are facts, rules and queries built out of?



Atoms

- A sequence of characters of upper-case letters, lower-case letters, digits, or underscore, starting with a lowercase letter
  - *Examples*: **butch**, **big_kahuna_burger**, **playGuitar**

- An arbitrary sequence of characters enclosed in single quotes
  - *Examples*: **'Vincent'**, **'Five dollar shake'**, **'@$%'**

- A sequence of special characters
  - *Examples*: **:** **,** **;** **.** **:-**

Numbers

- Integers: 12, -34, 22342
- Floats: 34573.3234

Variables

- A sequence of characters of upper-case letters, lower-case letters, digits, or underscore, starting with either an uppercase letter or an underscore

- Examples:

  X, Y, Variable, Vincent, _tag

Complex Terms

- Atoms, numbers and variables are building blocks for complex terms
- Complex terms are built out of a functor directly followed by a sequence of arguments
- Arguments are put in round brackets, separated by commas
- The functor must be an atom

Examples of complex terms

- Examples we have seen before:
  - playsAirGuitar(jody)
  - loves(vincent, mia)
  - jealous(marsellus, W)

- Complex terms inside complex terms:
  - hide(X,father(father(father(butch))))

Arity

- The number of arguments a complex term has is called its arity

- Examples:

woman(mia) is a term with arity 1
loves(vincent,mia) has arity 2
father(father(butch)) arity 1

Arity is important

- In Prolog you can define two predicates with the same functor but with different arity
- Prolog would treat this as two different predicates
- In Prolog documentation arity of a predicate is usually indicated with the suffix "/" followed by a number to indicate the arity

Example of Arity

```
happy(yolanda).
listens2music(mia).
listens2music(yolanda):- happy(yolanda).
playsAirGuitar(mia):- listens2music(mia).
playsAirGuitar(yolanda):- listens2music(yolanda).
```

- This knowledge base defines
  - happy/1
  - listens2music/1
  - playsAirGuitar/1

Unification

- Working definition:
  - Two terms unify if they are the same term or if they contain variables that can be uniformly instantiated with terms in such a way that the resulting terms are equal.

Unification

- This means that:
  - **mia** and **mia** unify
  - **42** and **42** unify
  - **woman(mia)** and **woman(mia)** unify

- This also means that:
  - **vincent** and **mia** do not unify
  - **woman(mia)** and **woman(jody)** do not unify

Unification

- What about the terms:
  - **mia** and **X**

Unification

- What about the terms:
  - **mia** and **X**
  - **woman(Z)** and **woman(mia)**

Unification

- What about the terms:
  - **mia** and **X**
  - **woman(Z)** and **woman(mia)**
  - **loves(mia,X)** and **loves(X,vincent)**

Instantiations

- When Prolog unifies two terms it performs all the necessary instantiations, so that the terms are equal afterwards
- This makes unification a powerful programming mechanism

Revised Definition 1/3

1. If T_1 and T_2 are constants, then T_1 and T_2 unify if they are the same atom, or the same number.

Revised Definition 2/3

1. If T_1 and T_2 are constants, then T_1 and T_2 unify if they are the same atom, or the same number.
2. If T_1 is a variable and T_2 is any type of term, then T_1 and T_2 unify, and T_1 is instantiated to T_2. (and vice versa)

Revised Definition 3/3

1. If T_1 and T_2 are constants, then T_1 and T_2 unify if they are the same atom, or the same number.
2. If T_1 is a variable and T_2 is any type of term, then T_1 and T_2 unify, and T_1 is instantiated to T_2. (and vice versa)
3. If T_1 and T_2 are complex terms then they unify if:
    1. They have the same functor and arity, and
    2. all their corresponding arguments unify, and
    3. the variable instantiations are compatible.

Prolog unification: =/2

```
?- mia = mia.
yes
?-
```

Prolog unification: =/2

```
?- mia = mia.
yes
?- mia = vincent.
no
?-
```

Prolog unification: =/2

```
?- mia = X.
X=mia
yes
?-
```

How will Prolog respond?

```
?- X=mia, X=vincent.
```

How will Prolog respond?

?- X=mia, X=vincent.

no

?-

Why? After working through the first goal, Prolog has instantiated X with **mia**, so that it cannot unify it with **vincent** anymore. Hence the second goal fails.

Example with complex terms

```
?- k(s(g),Y) = k(X,t(k)).
```

Example with complex terms

```
?- k(s(g),Y) = k(X,t(k)).
X=s(g)
Y=t(k)
yes
?-
```

Example with complex terms

```
?- k(s(g),t(k)) = k(X,t(Y)).
```

Example with complex terms

```
?- k(s(g),t(k)) = k(X,t(Y)).
X=s(g)
Y=k
yes
?-
```

One last example

```
?- loves(X,X) = loves(marsellus,mia).
```

Programming with Unification

```
vertical( line(point(X,Y),
                 point(X,Z))).

horizontal( line(point(X,Y),
                   point(Z,Y))).
```

Programming with Unification

```
vertical( line(point(X,Y),
                point(X,Z))).

horizontal( line(point(X,Y),
                  point(Z,Y))).
```

```
?-
```

Programming with Unification

```
vertical( line(point(X,Y),
                point(X,Z))).

horizontal( line(point(X,Y),
                  point(Z,Y))).
```

```
?- vertical(line(point(1,1),point(1,3))).
yes
?-
```

Programming with Unification

```
vertical( line(point(X,Y),
                point(X,Z))).

horizontal( line(point(X,Y),
                  point(Z,Y))).
```

```
?- vertical(line(point(1,1),point(1,3))).
yes
?- vertical(line(point(1,1),point(3,2))).
no
?-
```

Programming with Unification

```
vertical( line(point(X,Y),
                point(X,Z))).

horizontal( line(point(X,Y),
                  point(Z,Y))).
```

```
?- horizontal(line(point(1,1),point(1,Y))).
Y = 1;
no
?-
```

Programming with Unification

```
vertical( line(point(X,Y),
                point(X,Z))).

horizontal( line(point(X,Y),
                  point(Z,Y))).
```

```
?- horizontal(line(point(2,3),Point)).
Point = point(_554,3);
no
?-
```

Proof Search

- Now that we know about unification, we are in a position to learn how Prolog searches a knowledge base to see if a query is satisfied.
- In other words: we are ready to learn about <u>proof search</u>

- Prolog has a specific way of answering queries:
  - ➢ Search knowledge base from top to bottom
  - ➢ Processes clauses from left to right
  - ➢ Backtracking to recover from bad choices

Example

```
f(a).
f(b).
g(a).
g(b).
h(b).
k(X):- f(X), g(X), h(X).
```

```
?- k(Y).
```

Example: search tree


```
f(a).
f(b).
g(a).
g(b).
h(b).
k(X):- f(X), g(X), h(X).
```

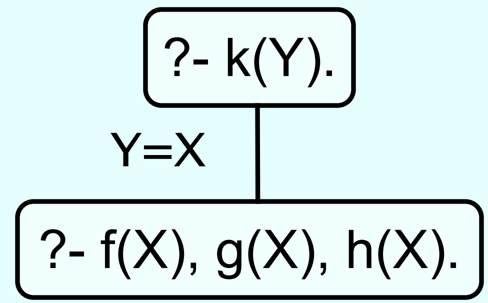
```
?- k(Y).
```

```
?- k(Y).
```

Example: search tree

```
f(a).
f(b).
g(a).
g(b).
h(b).
k(X):- f(X), g(X), h(X).
```

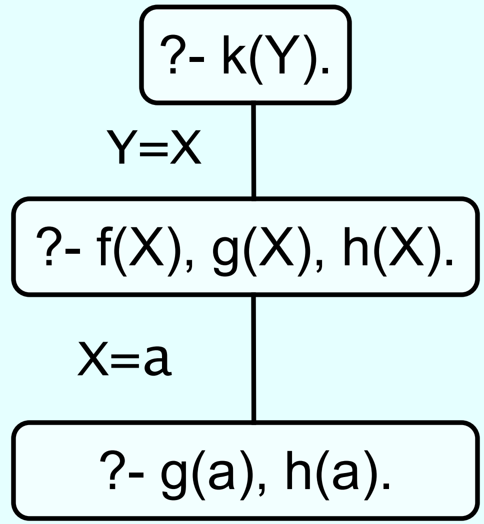
```
?- k(Y).
```



Example: search tree

```
f(a).
f(b).
g(a).
g(b).
h(b).
k(X):- f(X), g(X), h(X).
```

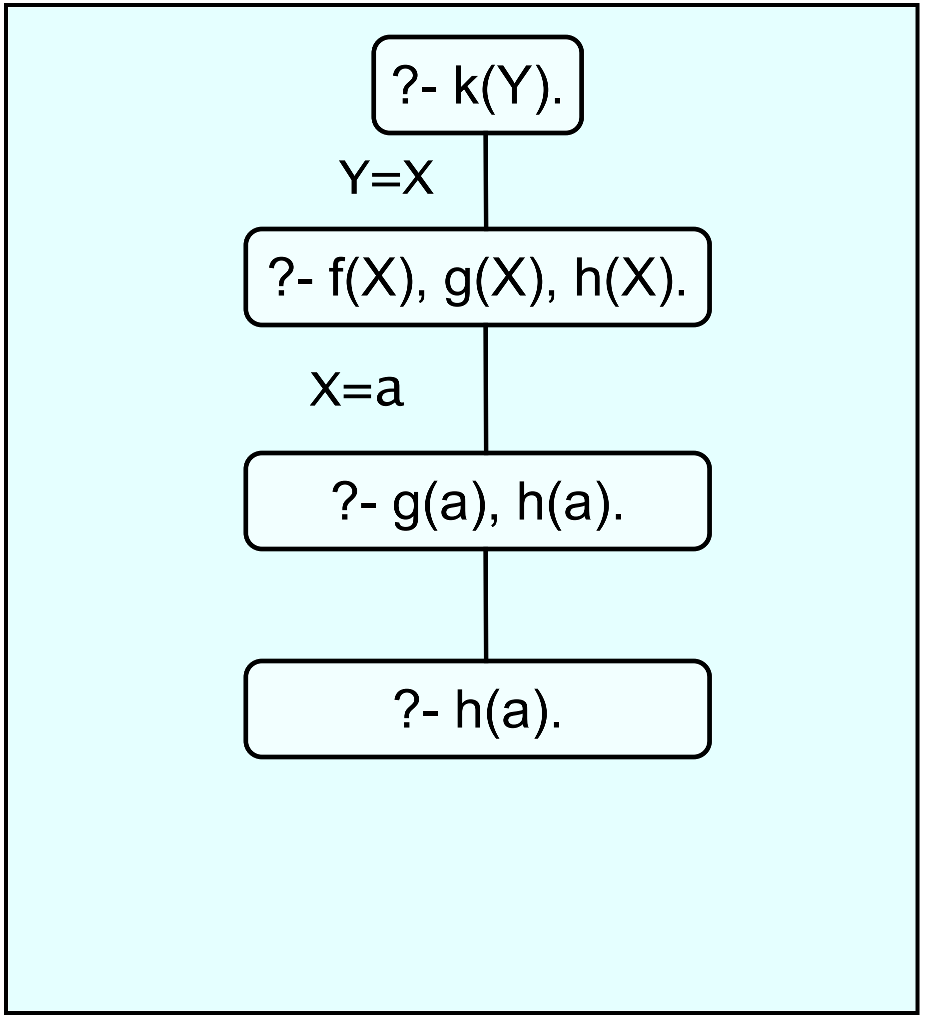
```
?- k(Y).
```



Example: search tree

```
f(a).
f(b).
g(a).
g(b).
h(b).
k(X):- f(X), g(X), h(X).
```

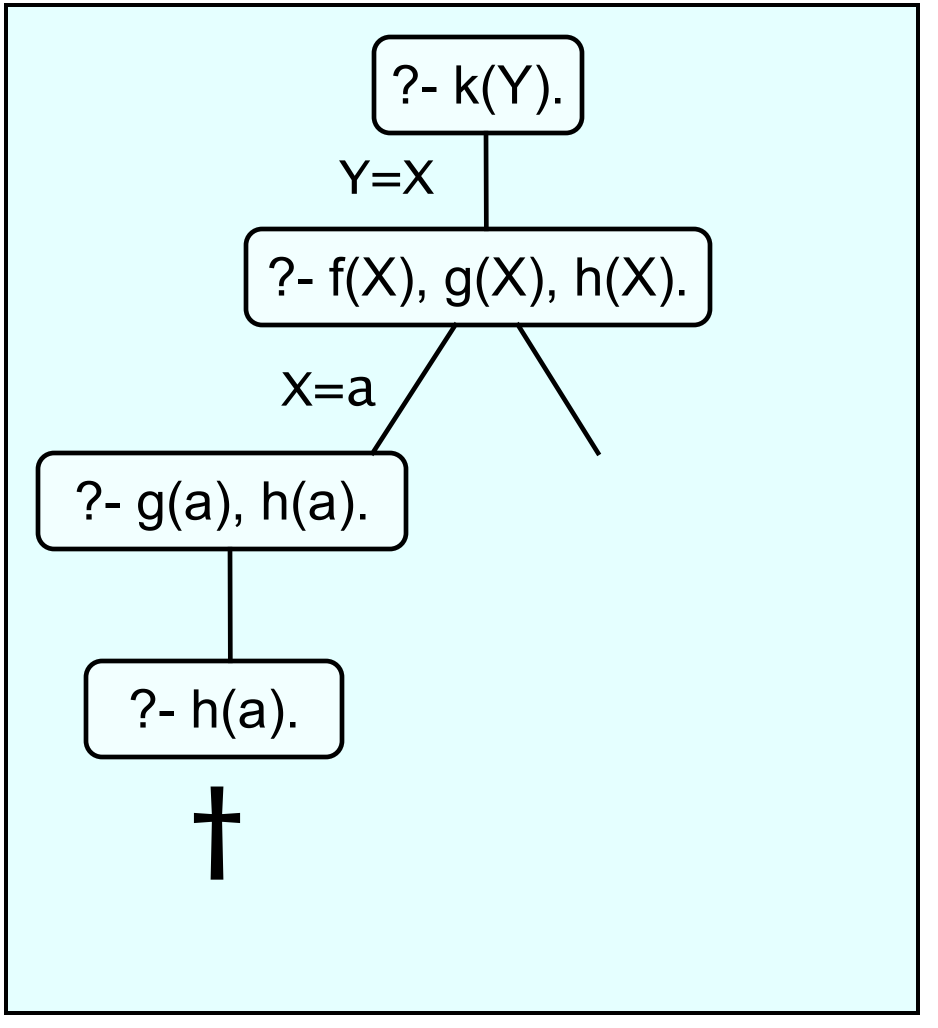
```
?- k(Y).
```



Example: search tree

```
f(a).
f(b).
g(a).
g(b).
h(b).
k(X):- f(X), g(X), h(X).
```

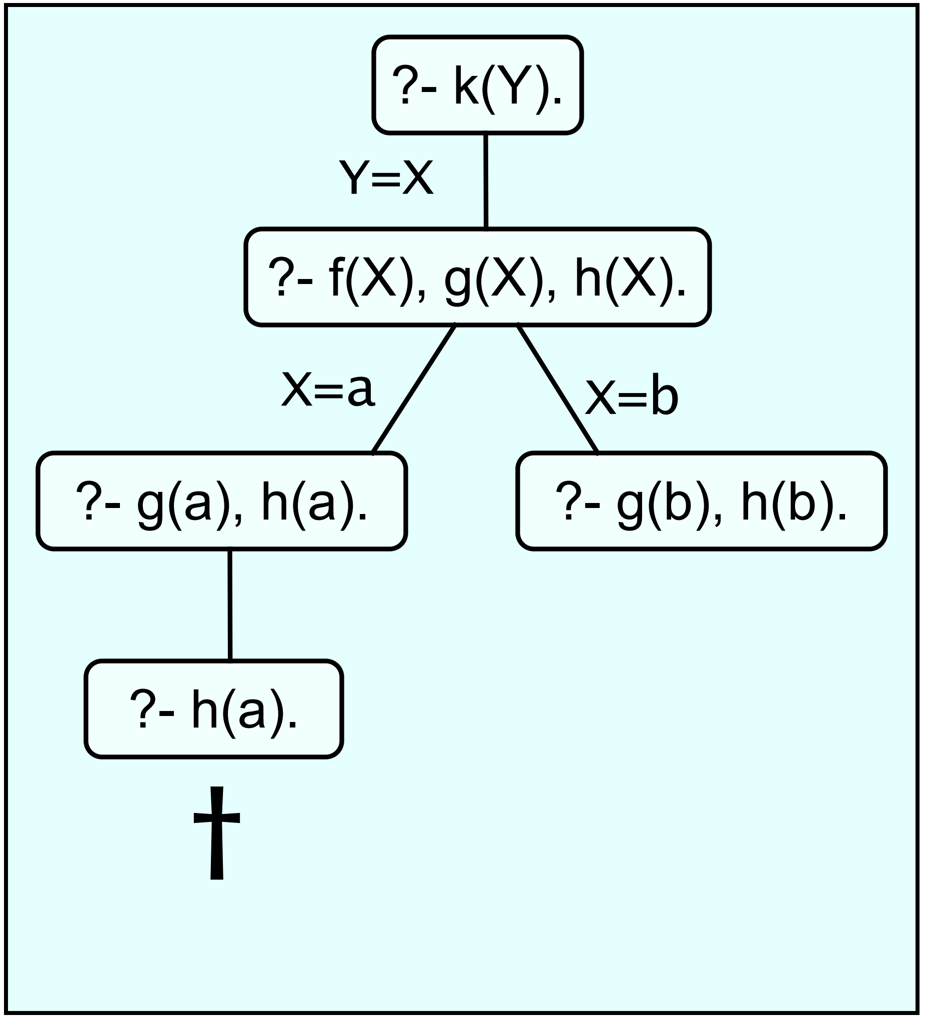
```
?- k(Y).
```



Example: search tree

```
f(a).
f(b).
g(a).
g(b).
h(b).
k(X):- f(X), g(X), h(X).
```

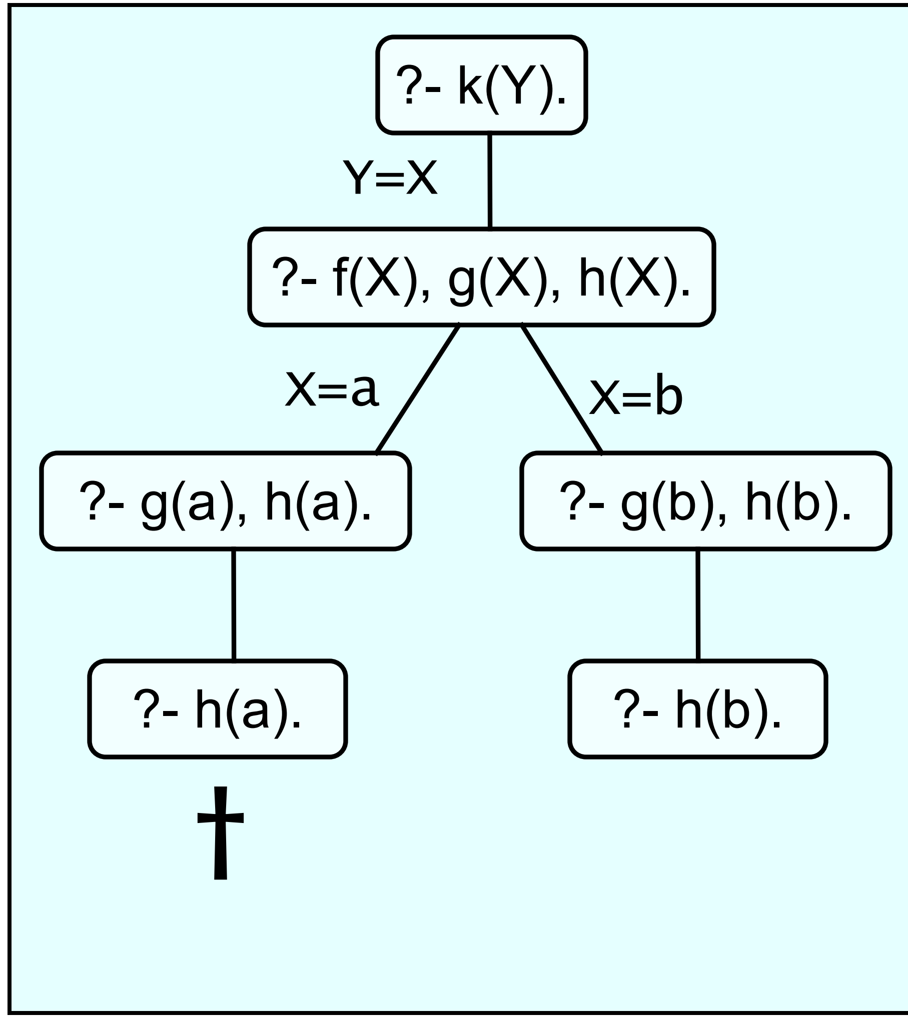
```
?- k(Y).
```



Example: search tree

```
f(a).
f(b).
g(a).
g(b).
h(b).
k(X):- f(X), g(X), h(X).
```

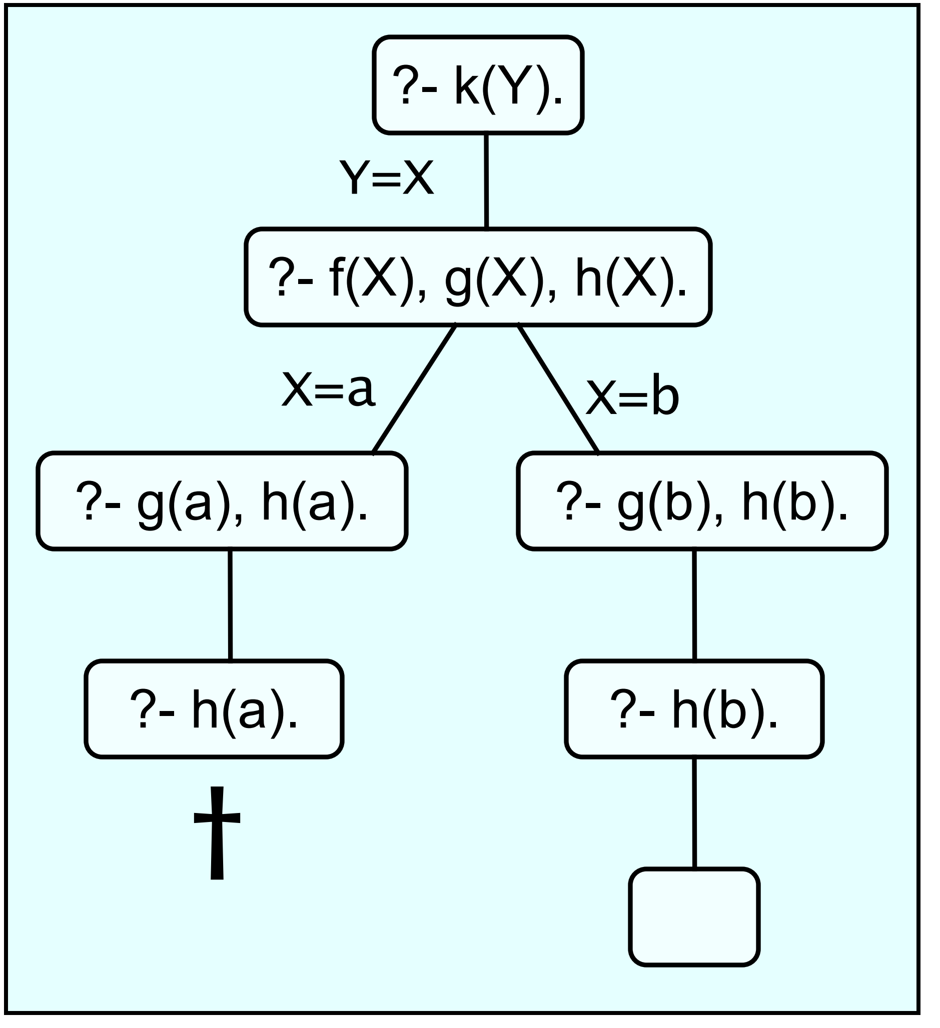
```
?- k(Y).
```



Example: search tree

```
f(a).
f(b).
g(a).
g(b).
h(b).
k(X):- f(X), g(X), h(X).
```

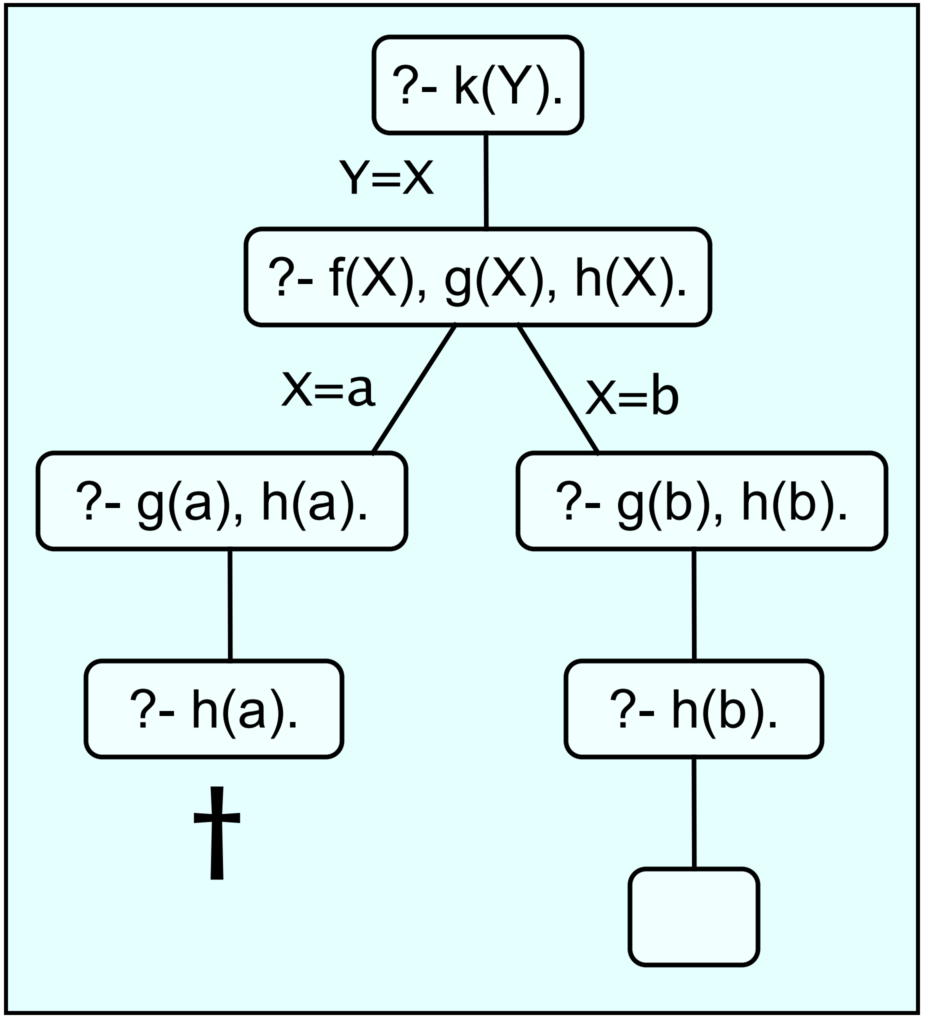
```
?- k(Y).
Y=b
```



Example: search tree

```
f(a).
f(b).
g(a).
g(b).
h(b).
k(X):- f(X), g(X), h(X).
```

```
?- k(Y).
Y=b;
no
?-
```



Another example

```
loves(vincent,mia).
loves(marsellus,mia).

jealous(A,B):-
    loves(A,C),
    loves(B,C).
```

```
?- jealous(X,Y).
```

Another example

```
loves(vincent,mia).
loves(marsellus,mia).

jealous(A,B):-
    loves(A,C),
    loves(B,C).
```

```
?- jealous(X,Y).
```

```
?- jealous(X,Y).
```

Another example

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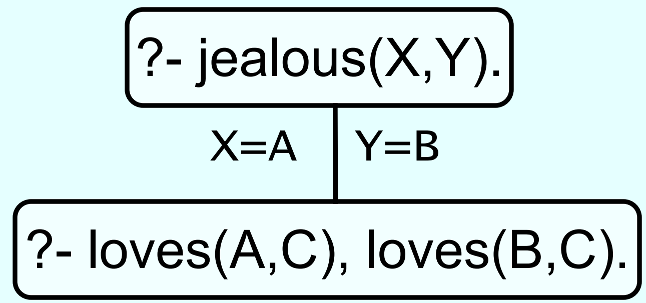
loves(vincent,mia).
loves(marsellus,mia).

jealous(A,B):-
    loves(A,C),
    loves(B,C).
```

```
?- jealous(X,Y).
```



Another example

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loves(vincent,mia).
loves(marsellus,mia).

jealous(A,B):-
    loves(A,C),
    loves(B,C).
```
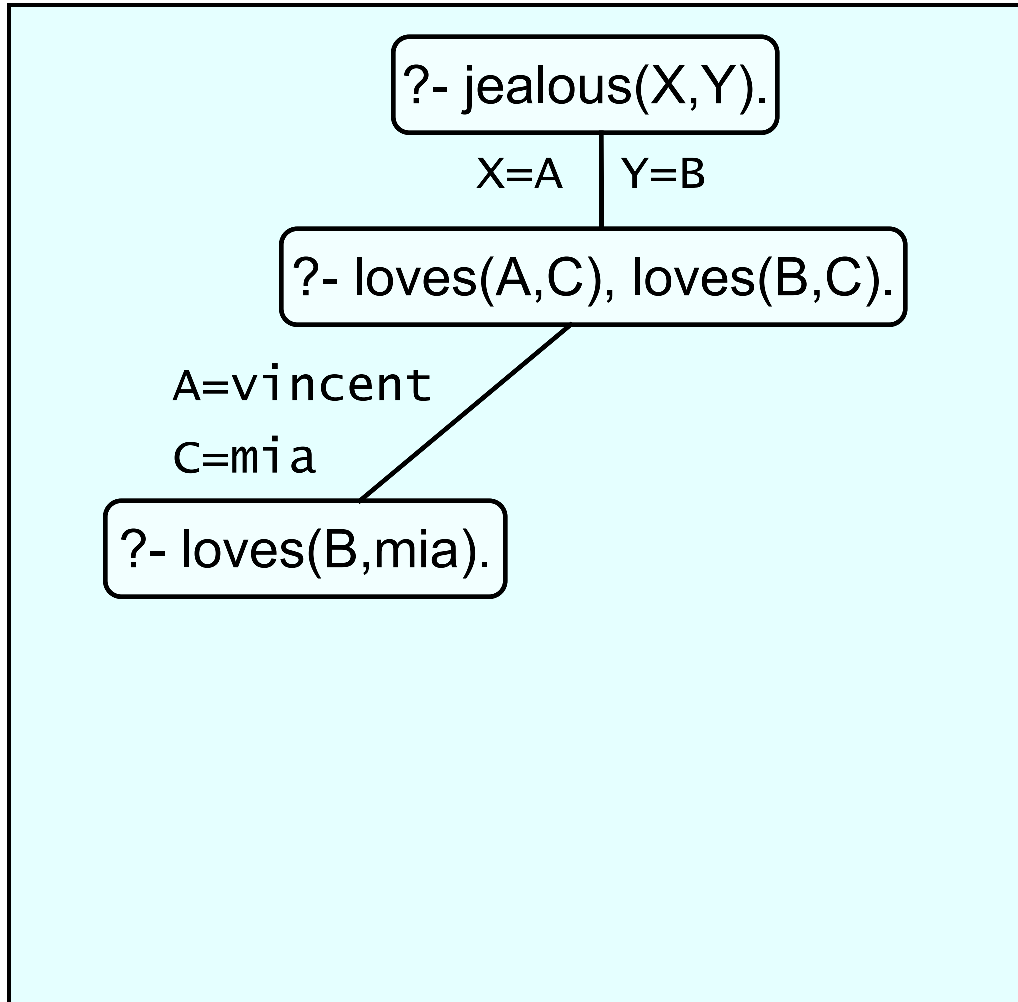
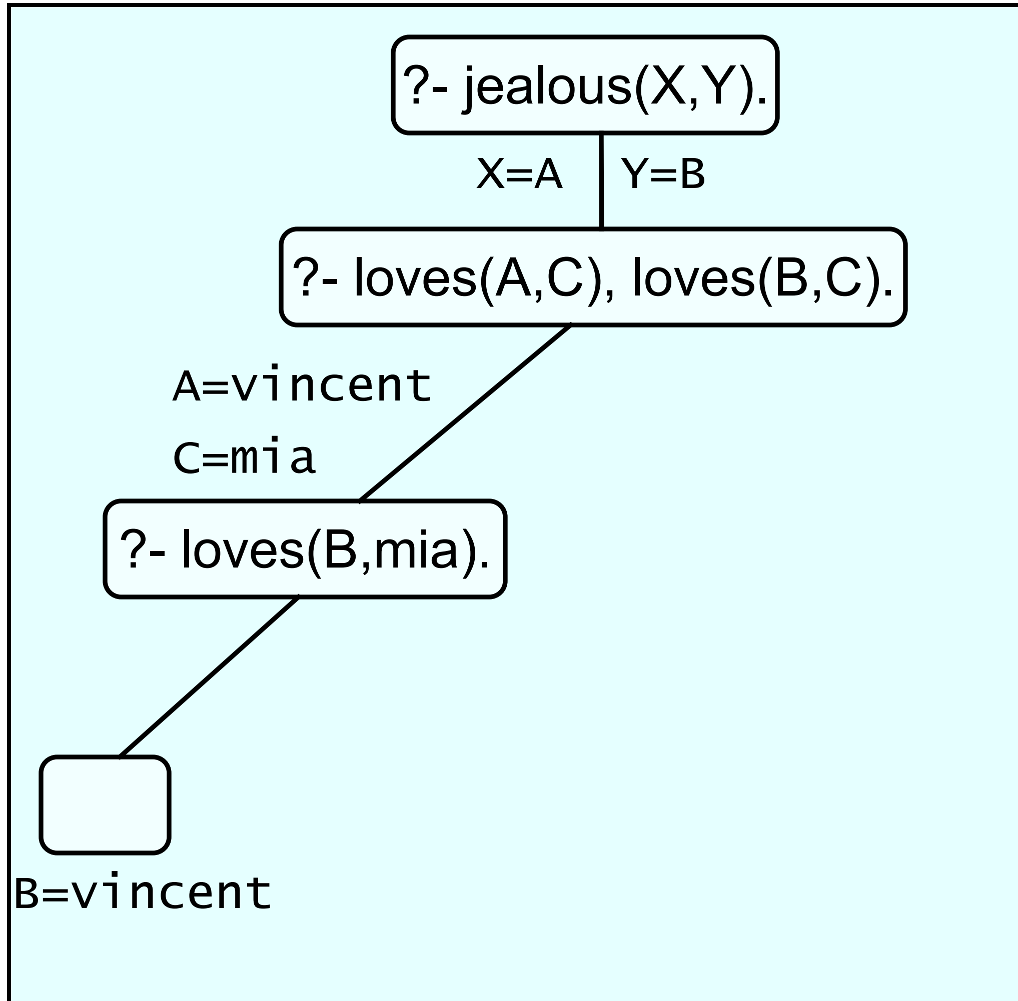

```
?- jealous(X,Y).
```



Another example

```
loves(vincent,mia).
loves(marsellus,mia).

jealous(A,B):-
    loves(A,C),
    loves(B,C).
```

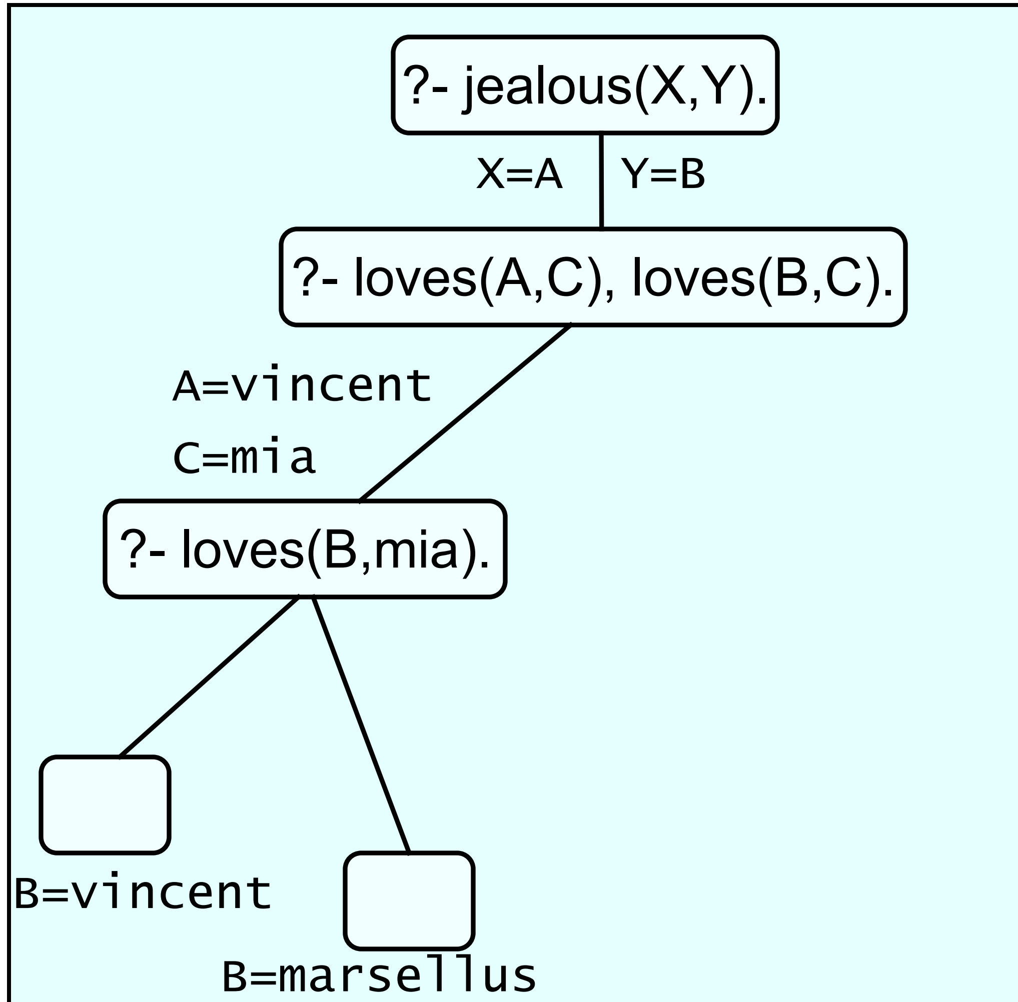
```
?- jealous(X,Y).
X=vincent
Y=vincent
```



Another example

```
loves(vincent,mia).
loves(marsellus,mia).

jealous(A,B):-
    loves(A,C),
    loves(B,C).
```

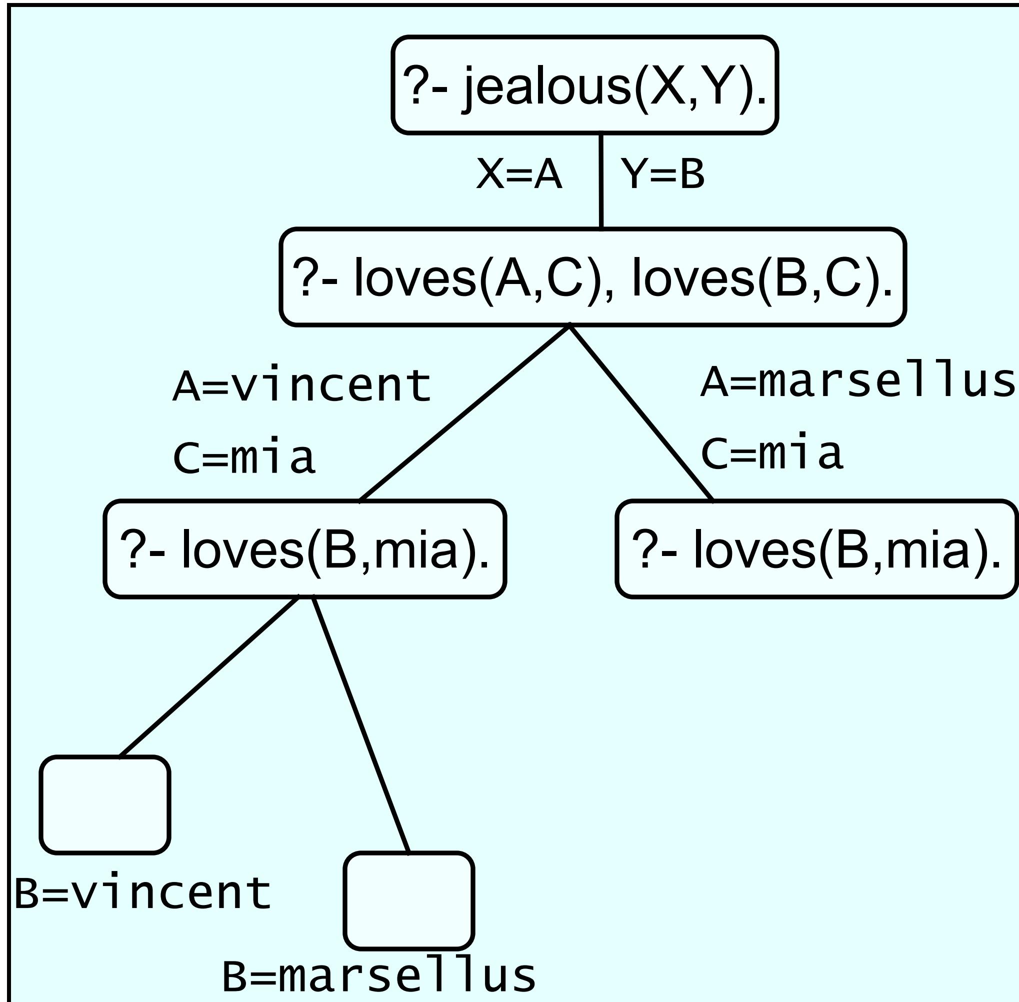
```
?- jealous(X,Y).
X=vincent
Y=vincent;
X=vincent
Y=marsellus
```



Another example

```
loves(vincent,mia).
loves(marsellus,mia).

jealous(A,B):-
    loves(A,C),
    loves(B,C).
```

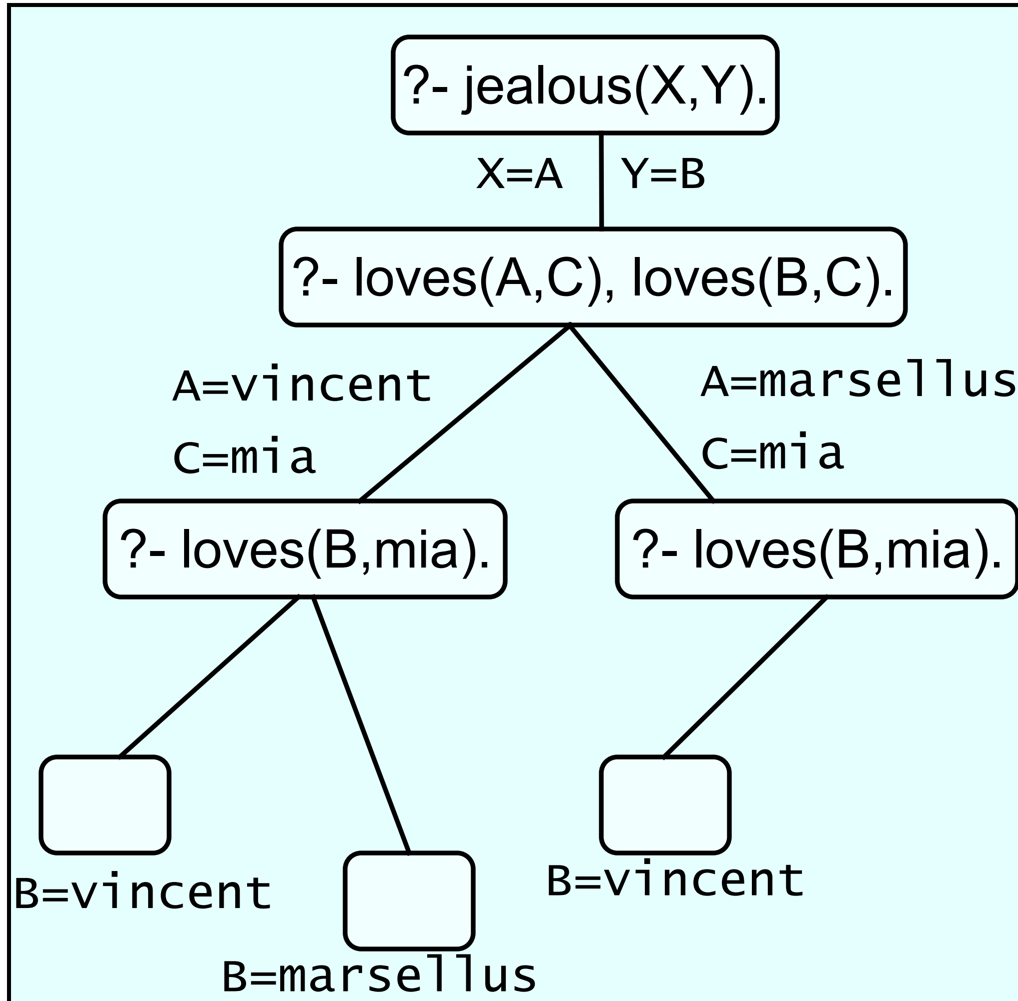
```
?- jealous(X,Y).
X=vincent
Y=vincent;
X=vincent
Y=marsellus;
```



Another example

```
loves(vincent,mia).
loves(marsellus,mia).

jealous(A,B):-
    loves(A,C),
    loves(B,C).
```

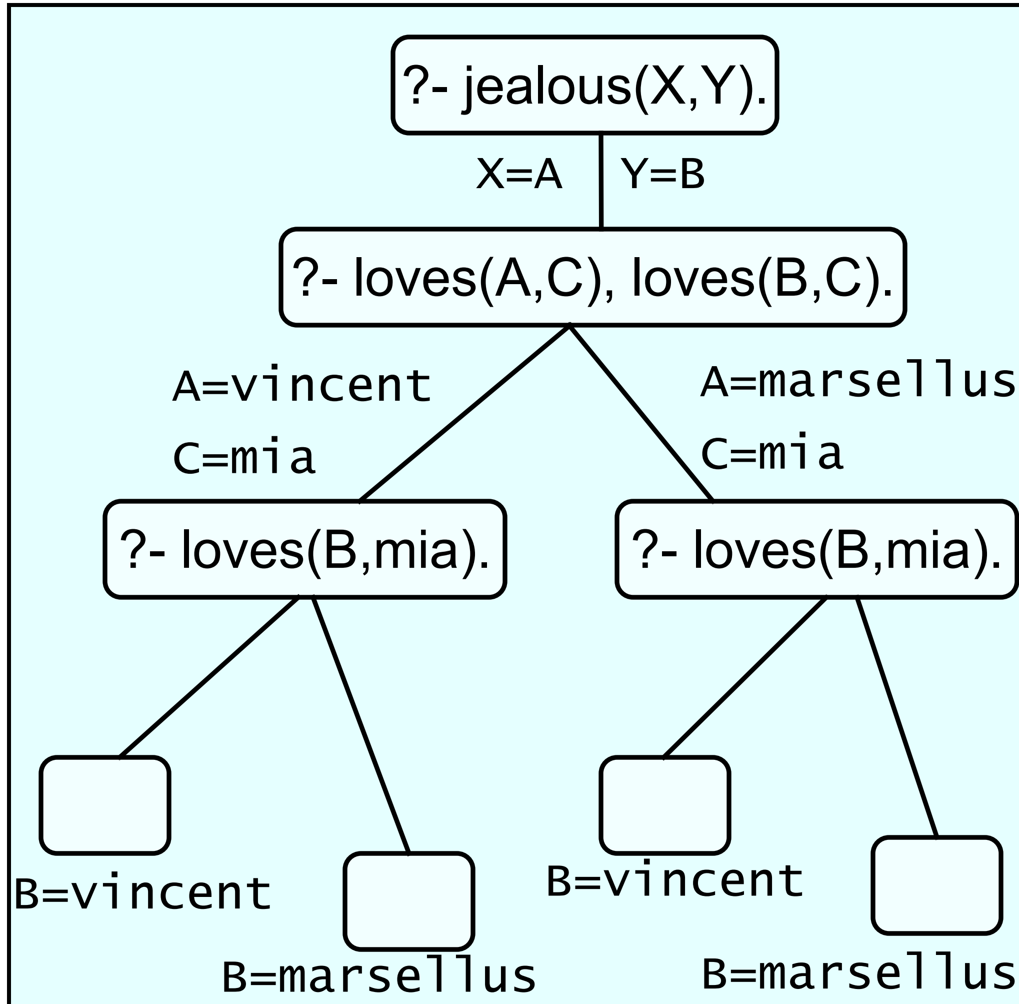
```
….
X=vincent
Y=marsellus;
X=marsellus
Y=vincent
```



Another example

```
loves(vincent,mia).
loves(marsellus,mia).

jealous(A,B):-
    loves(A,C),
    loves(B,C).
```

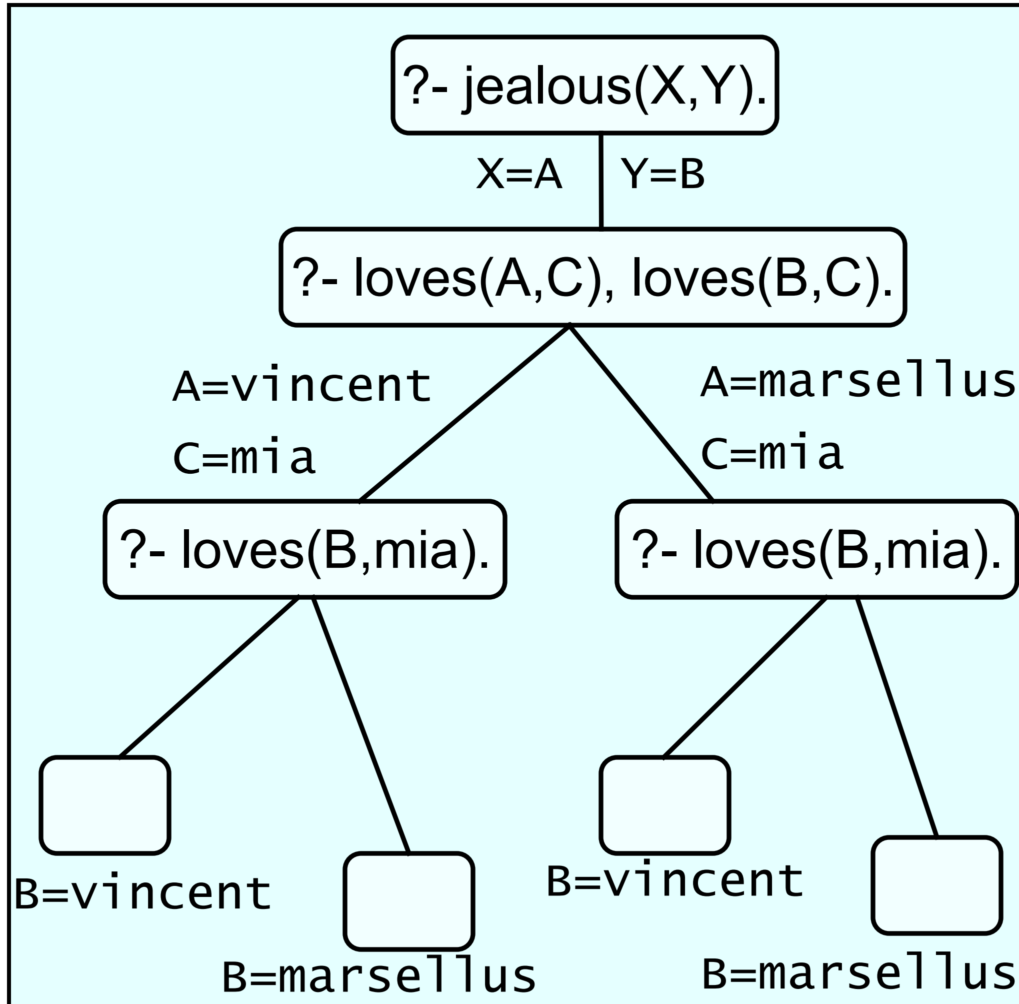
```
….
X=marsellus
Y=vincent;
X=marsellus
Y=marsellus
```



Another example

```
loves(vincent,mia).
loves(marsellus,mia).

jealous(A,B):-
    loves(A,C),
    loves(B,C).
```

```
….
X=marsellus
Y=vincent;
X=marsellus
Y=marsellus;
no
```



Recursive Definitions

- Prolog predicates can be defined recursively
- A predicate is recursively defined if one or more rules in its definition refers to itself

Example: Descendant

```
child(anna,bridget).
child(bridget,caroline).
child(caroline,donna).
child(donna,emily).


descend(X,Y):- child(X,Y).
descend(X,Y):- child(X,Z), descend(Z,Y).
```

```
?-
```

Example: Descendant

```
child(anna,bridget).
child(bridget,caroline).
child(caroline,donna).
child(donna,emily).


descend(X,Y):- child(X,Y).
descend(X,Y):- child(X,Z), descend(Z,Y).
```

```
?- descend(anna,donna).
yes
```

Another recursive definition

```
p:- p.
```

```
?- p.
```

Another recursive definition

```
p:- p.
```

```
?- p.
ERROR: out of memory
```

Example: Successor

- Suppose we use the following way to write numerals:
  1. **0** is a numeral.
  2. If **X** is a numeral, then so is **succ(X)**.

Example: Successor

```
numeral(0).
numeral(succ(X)):- numeral(X).
```

Example: Successor

```
numeral(0).
numeral(succ(X)):- numeral(X).
```

```
?- numeral(succ(succ(succ(0)))).
yes
?-
```

Example: Successor

```
numeral(0).
numeral(succ(X)):- numeral(X).
```

```
?- numeral(X).
```

Example: Successor

```
numeral(0).
numeral(succ(X)):- numeral(X).
```

```
?- numeral(X).
X=0;
X=succ(0);
X=succ(succ(0));
X=succ(succ(succ(0)));
X=succ(succ(succ(succ(0))))
```

Example: Addition

```
?- add(succ(succ(0)),succ(succ(succ(0))), Result).
Result=succ(succ(succ(succ(succ(0)))))
yes
```

Example: Addition

```
add(0,X,X).                                 %%% base clause
```

```
?- add(succ(succ(0)),succ(succ(succ(0))), Result).
Result=succ(succ(succ(succ(succ(0)))))
yes
```

Example: Addition

```
add(0,X,X).                          %%% base clause

add(succ(X),Y,succ(Z)):-             %%% recursive clause
    add(X,Y,Z).
```

```
?- add(succ(succ(0)),succ(succ(succ(0))), Result).
Result=succ(succ(succ(succ(succ(0)))))
yes
```

Examples from the book

descend1.pl

```
child(anna,bridget).
child(bridget,caroline).
child(caroline,donna).
child(donna,emily).

descend(X,Y):- child(X,Y).
descend(X,Y):- child(X,Z), descend(Z,Y).
```

```
?- descend(A,B).
A=anna
B=bridget
```

descend2.pl

```
child(anna,bridget).
child(bridget,caroline).
child(caroline,donna).
child(donna,emily).

descend(X,Y):- child(X,Z), descend(Z,Y).
descend(X,Y):- child(X,Y).
```

```
?- descend(A,B).
A=anna
B=emily
```

descend3.pl

```
child(anna,bridget).
child(bridget,caroline).
child(caroline,donna).
child(donna,emily).

descend(X,Y):- descend(Z,Y), child(X,Z).
descend(X,Y):- child(X,Y).
```

```
?- descend(A,B).
ERROR: OUT OF LOCAL STACK
```

descend4.pl

```
child(anna,bridget).
child(bridget,caroline).
child(caroline,donna).
child(donna,emily).

descend(X,Y):- child(X,Y).
descend(X,Y):- descend(Z,Y), child(X,Z).
```

```
?- descend(A,B).
```

Using `dif` predicate

```
mother(X, Y) :- parent(X, Y), female(X).
sister(X, Y) :-
        parent(Z, X),
        parent(Z, Y),
        female(X).
```

- What is wrong with this rule?
  - Any female is her own sister

- Solution?
  - Use dif (\=) predicate: `…, X \= Y.`

Comments

- Multi-line :

```
/* This is a comment
   This is another comment */
```

- Short :

```
% This is also a comment
```

Notation of Predicate Descriptions

+ Argument must be fully instantiated.

Think of the argument as *input*.

- Argument must be unbound.

Think of the argument as *output*.

? Either instantiated or unbound

Think of the argument as either *input* or *output*

Examples:

Write a predicate `sum(+List, ?Sum)` so that …

Write a predicate `append(?List1, ?List2, ?List1AndList2)` such that …

Side effects

- Some built-in "Predicates" may have side effects.

- Example: The built-in predicate `write(+Term)`

```
print_if_positive(X) :- X > 0, write(X).
```

Logical quantification

- Variables that appear in the head of a rule are universally quantified.
- Variables that only appear in the body of a rule are existentially quantified.

```
path(X,Y) :- edge(X,Z), path(Z,Y).
```

For all nodes X and Y, there is a path from X to Y if *there exists* a node Z such that there is an edge from X to Z and there is path from Z to Y.