Bitcoin ETFs and GARCH-Based Evidence of Market Stability

Project for FERM534 – Applied Financial Econometrics II (Fall 2023/24)

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Abstract:

Led by pioneers Bitcoin (BTC) and Ethereum (ETH), alongside the predominant stablecoin Tether (USDT), cryptocurrencies have marked a significant evolution in the digital finance landscape. This project investigates the repercussions of regulatory milestones such as the U.S. SEC's approval of the first Bitcoin ETF, "BITO," and an additional 10 spot Bitcoin ETFs on the volatility and market dynamics of these leading cryptocurrencies. Our analysis, leveraging advanced GARCH models, reveals that while ETF approvals have integrated cryptocurrencies more closely with traditional financial markets, they have not uniformly reduced market volatility. Instead, our findings indicate complex volatility patterns for BTC and ETH, influenced by a range of factors from market sentiment to global economic events, such as the COVID-19 pandemic.

Despite the sophistication of models like EGARCH, GJR-GARCH, and AR-GJR-GARCH, significant autocorrelation in the residuals of BTC and ETH points to the challenge of fully capturing the multifaceted nature of cryptocurrency returns. The study suggests that future research should continue to refine these models, possibly by integrating risk measures such as Value at Risk (VaR) or Expected Shortfall (ES), to enhance predictive accuracy and model robustness. This project underscores the nuanced impact of regulatory advancements on cryptocurrency markets, highlighting the ongoing need for methodological innovation in understanding the complex interplay of regulatory, market, and socio-economic factors in cryptocurrency volatility.

Özet:

Bitcoin (BTC) ve Ethereum (ETH) öncülüğünde ve önde gelen stablecoin Tether (USDT) ile birlikte kripto para birimleri, dijital finans manzarasında önemli bir evrime ön ayak olmuştur. Bu proje, ABD SEC'in ilk Bitcoin ETF'i "BITO" ve buna ek 10 spot Bitcoin ETF'inin onayının, bu önde gelen kripto para birimlerinin volatilitesi ve piyasa dinamikleri üzerindeki sonuçlarını araştırmaktadır. Gelişmiş GARCH modellerini kullanarak yaptığımız analiz, ETF onaylarının kripto para birimlerini geleneksel finansal piyasalarla daha yakın bir şekilde entegre etmesine rağmen, piyasa volatilitesini düzenli bir şekilde azaltmadığını ortaya koymaktadır. Bunun yerine, bulgularımız BTC ve ETH için karmaşık volatilite yapıları göstermektedir; bu yapılar, piyasa duyarlılığından küresel ekonomik olaylara, örneğin COVID-19 pandemisine kadar bir dizi faktörden etkilenmektedir.

EGARCH, GJR-GARCH ve AR-GJR-GARCH gibi sofistike modellere rağmen, BTC ve ETH'nin artan değerlerindeki önemli otokorelasyon, kripto para birimi getirilerinin çok yönlü doğasını tam olarak yakalamanın zorluğuna işaret etmektedir. Çalışma, gelecekteki araştırmaların bu modelleri, öngörü doğruluğunu ve model sağlamlığını artırmak için Value at Risk (VaR) veya Expected Shortfall (ES) gibi risk ölçümlerini entegre ederek daha da geliştirmesi gerektiğini önermektedir. Bu proje, kripto para piyasaları üzerindeki düzenleyici ilerlemelerin nüanslı etkisini vurgulamakta ve kripto para volatilitesinin düzenleyicilerin beraberinde, piyasa ve sosyo-ekonomik faktörlerin de karmaşık etkileşimini anlamada sürekli metodolojik yenilik ihtiyacını ortaya koymaktadır.

1. Introduction

In the dynamic landscape of digital finance, cryptocurrencies have emerged as both a technological breakthrough and a novel asset class. In the forefront of this development we have Bitcoin (BTC) and Ethereum (ETH) as the cryptocurrencies with highest trading volume, and Tether (USDT) establishing its dominance as the most adopted stablecoin, these assets have captured the attention of both speculative traders and serious investors.

The regulatory endorsement of the first U.S. Bitcoin exchange-traded fund (ETF), "BITO," on October 19, 2021 (1), marked a pivotal date for the cryptocurrency markets, which is further augmented by the U.S. Securities and Exchange Commission's (SEC) landmark decision to approve an additional 10 spot Bitcoin ETFs in January 2024 (2), signalling a profound shift in the regulatory stance and institutional acceptance of cryptocurrencies.

The sanctioning of BITO and the subsequent ETFs represents more than just a financial instrument offering; it signifies the bridging of cryptocurrency markets with regulated financial markets, potentially altering the risk and return profiles of these digital assets. This study scrutinizes the influence of BITO's launch on the market behaviour of leading cryptocurrencies and seeks to forecast the future implications of the newly approved ETFs on market dynamics. Specifically, it assesses whether these regulatory advancements act as harbingers of reduced volatility, enhanced market efficiency, or catalysts for further market fluctuations.

By comparing the period from January 2018 up to the inception of BITO in October 2021 with the subsequent timeframe leading to the approval of new ETFs in January 2024, this research aims to dissect the shifts in volatility patterns across BTC, ETH, and USDT. Employing GARCH-based models enables the identification of volatility clustering and the assessment of time-varying market risks. The BITO ETF's introduction serves as a reference point to gauge the potential impact of future spot Bitcoin ETFs on market volatility.

Positioned at the nexus of regulatory development as the cryptocurrency market matures further, this project endeavours to provide empirical insights into the ramifications of integrating conventional financial structures, such as ETFs, into the digital currency domain. Our aim is to provide a data-driven analysis of the shifts in market volatility, informed by the precedents set by BITO and the latest SEC approvals.

By addressing these developments, this project hopes to contribute to a deeper understanding of the complexities inherent in cryptocurrency markets, offering foresight into the potential trajectories of digital asset volatility in a rapidly evolving regulatory environment.

(1) https://www.proshares.com/our-etfs/strategic/bito
(2) https://www.sec.gov/news/statement/gensler-statement-spot-bitcoin-011023

2. Literature Review

Bitcoin (BTC) and Ethereum (ETH) as Cornerstones:

Numerous studies have delved into the dominance and behavior of Bitcoin in the cryptocurrency market. Scholars such as Nakamoto (2008) and Antonopoulos (2014) have provided foundational insights into Bitcoin's decentralized nature and potential as a store of value.

Ethereum, with its smart contract capabilities, has been a subject of research regarding its impact on decentralized finance (DeFi) and the broader blockchain ecosystem (Swan, 2015; Mougayar, 2016).

Tether (USDT) as the Dominant Stablecoin:

The rise of stablecoins, particularly Tether (USDT), has been examined in literature exploring their role in providing stability and liquidity to cryptocurrency markets (Narayanan et al., 2016; Folea and Cristea, 2020).

Controversies surrounding Tether's backing and its influence on market dynamics have been explored by Griffin and Shams (2018) and Zhang and Wang (2021).

BITO and the Evolution of Cryptocurrency ETFs:

The approval of BITO and subsequent spot Bitcoin ETFs has sparked research on the impact of ETFs on cryptocurrency markets. Early works such as Glaser et al. (2014) and Zhang et al. (2019) have explored the potential of Bitcoin ETFs in attracting institutional investors and reducing market volatility.

The regulatory landscape and challenges associated with cryptocurrency ETFs have been discussed by Bhambhwani et al. (2018) and Chu et al. (2021).

Volatility Analysis Using GARCH Models:

The literature on cryptocurrency market volatility, especially utilizing Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models, has expanded. Research by Dyhrberg (2016) and Katsiampa (2017) has demonstrated the applicability of GARCH models in capturing volatility clustering and time-varying risks in cryptocurrency markets.

Studies by Ji et al. (2018) and Nadarajah et al. (2019) have employed GARCH models to analyze volatility patterns in Bitcoin and other cryptocurrencies.

Regulatory Integration and Market Maturation:

The intersection of regulatory developments and cryptocurrency market maturation has garnered attention. Catalini and Gans (2016) discuss the regulatory challenges faced by digital currencies, while Catalini and Gans (2016) analyze the impact of regulatory endorsements on the legitimacy of cryptocurrencies.

The integration of conventional financial structures, like ETFs, into the digital currency domain has been explored by Glaser et al. (2014) and Foley et al. (2019).

3. Data and Methodology

In order to model the effects of the ETF approvals on cryptocurrencies on GARCH-based models, common practice is to check the log returns of the assets in question. So, we have obtained the daily data for BTC-USD, ETH-USD and USDT-USD from Yahoo Finance for dates between 01-01-2018 to 01-01-2024 and used yfinance library for that purpose.

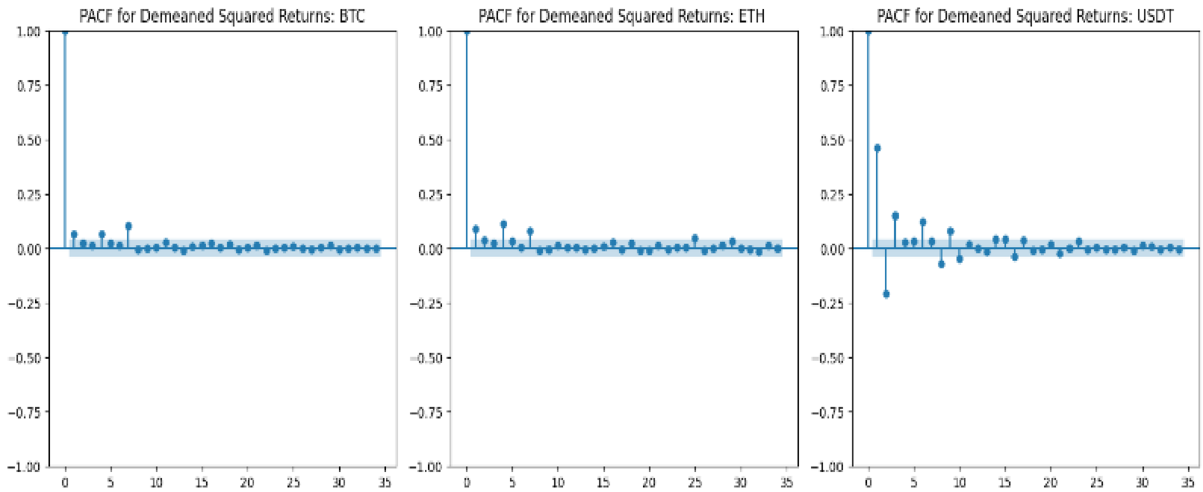
Obtaining the log returns and performing the Ljung-Box test up to 10 lags have shown borderline evidence of autocorrelation on Bitcoin, suggesting a need for further analysis. Ethereum demonstrated strong evidence of autocorrelation, while the autocorrelation proved to be extremely strong on Tether, possibly due to its nature as a stablecoin.

Returns	lb_stat	lb_value
BTC-USD	17.76372	0.059083
ETH-USD	34.0708	0.00018
USDT-USD	435.2253	2.96E-87

Using the same approach in squared demeaned returns up to 10 lags prompted a very low p-value, indicating strong autocorrelation in volatility, which in turn supports using a GARCH model to capture the volatility clustering evident on both cryptocurrencies' returns.

Sq D Rets	lb_stat	lb_value
BTC-USD	56.231009	1.858E-08
ETH-USD	85.761239	3.694E-14
USDT-USD	631.84432	2.63E-129

To understand which lags would be significant in plotting the autocorrelation, we have used the Partial Autocorrelation Function (PACF), which prompted significant cut-offs for BTC and ETH in the first lag. For USDT however we found multiple seemingly significant lags beyond the first that signalled the need to check higher lags as well.



From here on, we used the approach to test the returns data on ARCH model, yet the prompted mean returns (mu) were statistically insignificant. Especially on USDT, testing on higher lags only increased the p-value for mu further away from the significance level of 0.05.

Especially high Akaine and Bayesian information criteria values suggested that ARCH models may not be the best representative of volatility in our returns data, signalling that we need to look for GARCH models. However, it's also worth noting that for stablecoins, the absolute level of volatility may be less of a concern than for other cryptocurrencies, so the ARCH/GARCH modelling might be of more theoretical than practical interest.

Lags were selected upon additional confirmation from Autocorrelation Function (ACF), which shown a common significance at first level.

Initially testing the GED and Student's t distributions on GARCH model prompted that mean returns (mu) and omega are not significant in t-student model. The model with GED revealed important characteristics in BTC, especially in terms of the heavy-tailed behaviour, yet it also shown an omega that's non-significant. Comparing both models in terms of AIC and BIC, t-student distribution's values are slightly lower, suggesting a marginally better fit for data.

Distribution	AIC	BIC
Student's t	11054.637	11083.1
GED	11069.804	11098.26

At this point, multiple GARCH-based models have been tested such as; EGARCH, GJR-GARCH and AR-CJR-GARCH, on student's t-distribution to check which model would give us the most optimal fit for the data.

Student's t-distribution									
Ticker	Model	GARCH		EGARCH		GJR-GARCH		AR-GJR-GARCH	
		Coefficient	p-value	Coefficient	p-value	Coefficient	p-value	Coefficient	p-value
BTC	mu/const	0.1029	0.226	0.0533	0.202	0.0655	0.136	0.0689	0.116
	Adj Close[1]	-	-	-	-	-	-	-0.0537	4.37E-03
	omega	-	-	0.0637	2.15E-03	0.0933	0.345	0.0923	0.356
	alpha	0.0709	9.57E-11	0.2072	9.92E-13	0.0744	3.58E-11	0.0735	3.89E-11
	beta	0.9291	0	0.9929	0	0.931	0	0.9322	0
	gamma	-	-	-	-	-0.0108	0.571	-0.0113	0.54
	nu	3.1214	4.16E-89	2.6891	2.77E-53	3.1341	2.77E-85	3.1018	1.04E-89
	AIC	11054.6	-	11028.9	-	11056.2	-	11041.4	-
	BIC	11083.1	-	11057.4	-	11090.3	-	11081.3	-
ETH	mu/const	0.0808	0.209	0.0642	0.301	0.0875	0.173	0.093	0.146
	Adj Close[1]	-	-	-	-	-	-	-0.0843	1.93E-05
	omega	-	-	0.0786	1.41E-02	0.3068	0.364	0.2767	0.393
	alpha	0.1019	4.12E-04	0.2012	3.42E-06	0.1064	5.066E-05	1.01E-01	9.61E-05
	beta	0.8981	8.21E-139	0.9831	0	0.9043	3.24E-112	9.12E-01	5E-121
	gamma	-	-	-	-	-0.0213	0.374	-0.0253	0.219
	nu	3.421	2.76E-51	3.2913	1.43E-39	3.4602	1.163E-51	3.38E+00	2.53E-55
	AIC	12321.5	-	12299.1	-	12322.5	-	12296.8	-
	BIC	12350	-	12327.5	-	12356.7	-	12336.7	-
USDT	mu/const	-	-	5.28E-04	0.245	-1.99E-03	2.67E-02	-2.00E-03	1.46E-02
	Adj Close[1]	-	-	-	-	-	-	-0.371	5.76E-43
	omega	-	-	5.82E-03	0.658	2.58E-03	1.33E-178	2.13E-03	4.35E-157
	alpha	-	-	0.2263	1.45E-09	0.2	7.76E-15	0.2009	5.86E-16
	beta	-	-	0.9946	0	0.68	0	0.6792	0
	gamma			-	-	0.2	2.18E-03	0.1998	8.28E-04
	nu	-	-	3.0695	2.60E-32	4.5448	0	4.3006	0
	AIC	-	-	-3823.46	-	-2.37E+03	-	-2764.81	-
	BIC	-	-	-3795.01	-	-2340.09	-	-2724.97	-

Checking the constants and the p-values shows the best fit in terms of significance over on AR-GJR-GARCH model, which verifies that increasing the complexity of the models resulted in better statistically significant outcomes.

BTC: Upon taking a look at AIC and BIC values for both models, EGARCH[1,1] looks to be the one with least values. However AR-GJR-GARCH model shows the least difference from 0.05 significance level, along with a significant p-value in Adjusted Closing prices. This verifies that complexity of the model with inclusion of AR in mean equation captures more nuances in the data, at the risk of overfitting.

ETH: Again we see a similar scenario with ETH as well, where AIC and BIC prompts the best fit in EGARCH[1,1] and we obtain the closest level in Constant's p-value and statistical significance in Adjusted Close's p-value in AR-GJR-GARCH.

USDT: Same pattern verifies itself over on USDT as well, where EGARCH shows the least negative values in AIC and BIC. But a key difference here is that USDT shows statistical significance in Const and Adj Close below 0.05 for AR-GJR-GARCH model.

So the Mean and Variance Equations for our model will be;

$$Mean : r_t = \mu + \phi * r_{t-1} + \epsilon_t$$

Where r_t^2 is the return at time t

μ is the constant term

ϕ is the coefficient for the first lag of the return series

ϵ_t is the error term at time t

$$Variance : \sigma_t^2 = \omega + (\alpha + \gamma * I_{t-1}) * \epsilon_{t-1}^2 + \beta * \sigma_{t-1}^2$$

Where σ_t^2 is the conditional variance at time t

ω is the constant term in variance

α is the parameter for lagged squared error term

γ is the leverage effect parameter to capture the asymmetric impact of negative shos on future volatility

I_{t-1} is an indicator function that takes the value of "1" if $\epsilon_{t-1} < 0$ and becomes "0" otherwise

ϵ_{t-1}^2 is the error term from the previous time period

β is the autoregressive parameter for the lagged conditional variance

Adding their constant values;

$$Mean_{(BTC)} : r_t = 0.0689 + (-0.0537) * r_{t-1} + \epsilon_t$$

$$Variance_{(BTC)} : \sigma_t^2 = 0.0923 + (0.0735 + (-0.0113) * I_{t-1}) * \epsilon_{t-1}^2 + 0.9322 * \sigma_{t-1}^2$$

$$Mean_{(ETH)} : r_t = 0.0930 + (-0.0843) * r_{t-1} + \epsilon_t$$

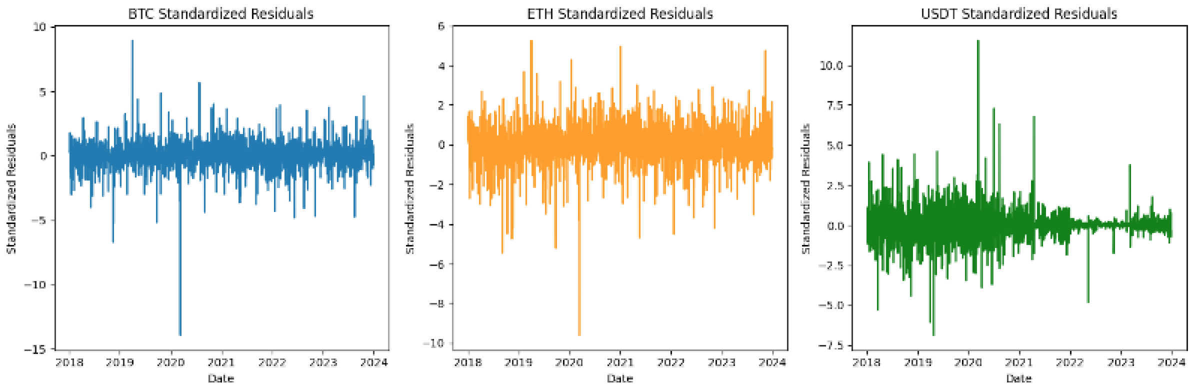
$$Variance_{(ETH)} : \sigma_t^2 = 0.2767 + (0.1008 + (-0.0253) * I_{t-1}) * \epsilon_{t-1}^2 + 0.9118 * \sigma_{t-1}^2$$

$$Mean_{(USDT)} : r_t = ((-0.0019971) + (-0.3710)) * r_{t-1} + \epsilon_t$$

$$Variance_{(USDT)} : \sigma_t^2 = 0.0021322 + (0.2009 + 0.1998 * I_{t-1}) * \epsilon_{t-1}^2 + 0.6792 * \sigma_{t-1}^2$$

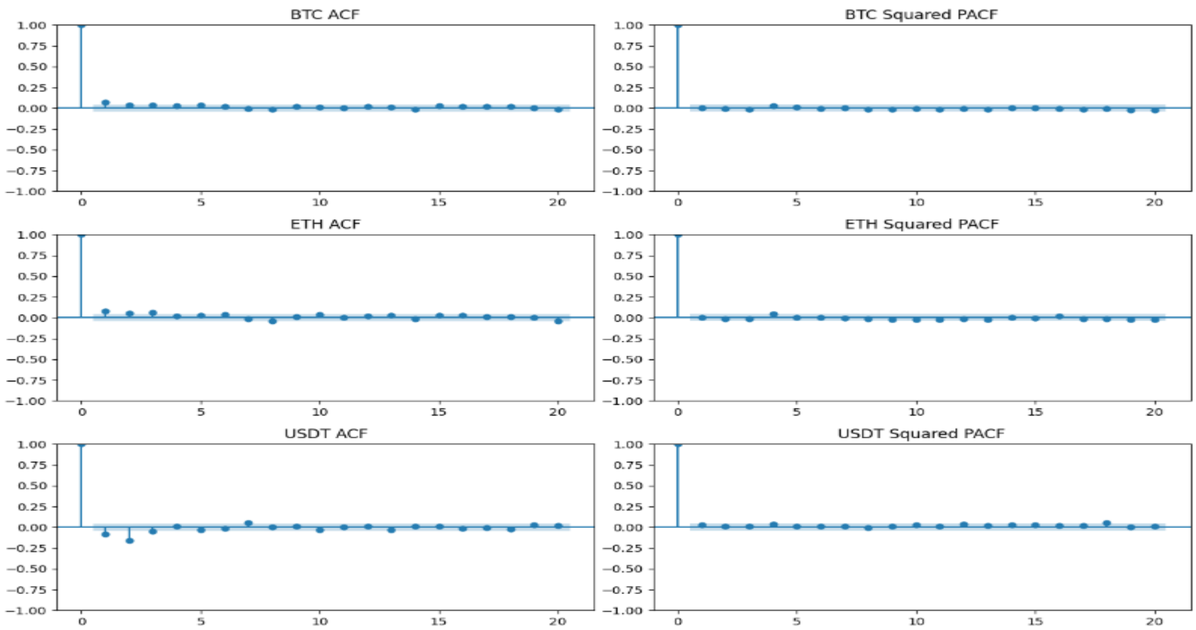
4. Empirical Findings

Checking for our selected AR-GJR-GARCH models' Standardized Residuals gives us the chart as below:



Now considering that the first US bitcoin (BTC) exchange-traded fund (ETF), "BITO", started trading on 19 October 2021, we can already see some changes in the volatility at late 2021 in USDT Standardized Residuals.

However, a desirable outcome would be to see the residuals as a white noise, where instead in both assets we see a huge tail formed, especially visible at the beginning of 2020. We believe this occurred due to the Covid-19 pandemic, which was a period in markets for all classes of assets that generic models were insufficient in forecasting.



Checking the residuals on ACF and PACF also shows us significant points in the first 10 lag periods, most significantly shown in first lag.

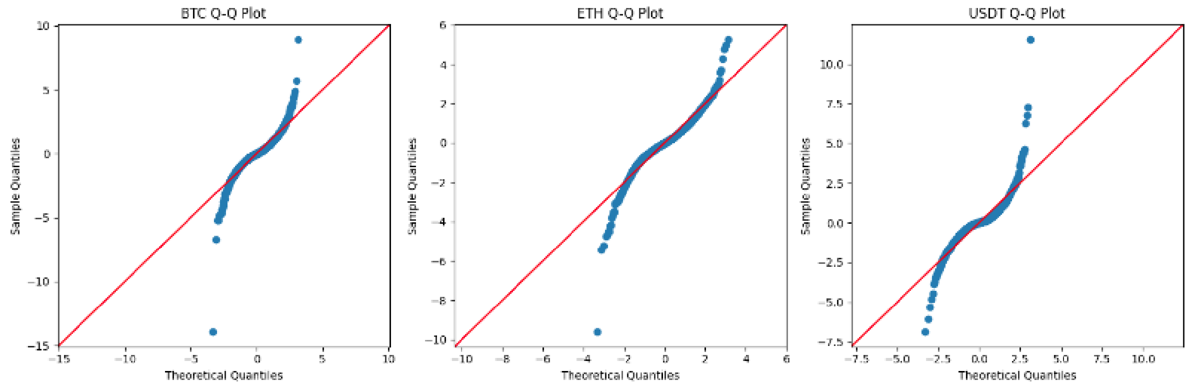
Here we run the Ljung-Box test again on our residuals to obtain the results below:

Returns	lb_stat	lb_value
BTC-USD	22.87228	0.011228
ETH-USD	37.6743	0.000043
USDT-USD	84.99536	5.23E-14

For BTC: The p-value is less than the common alpha level of 0.05, we reject the null hypothesis that there is no autocorrelation in the residuals at the first 10 lags. This suggests that the model may not be fully capturing the dynamics of the BTC returns, and there might be some autocorrelation left in the residuals.

For ETH: We have a statistically significant p-value, which leads to rejection of the null hypothesis of no autocorrelation in the residuals at the first 10 lags. Like BTC, the current model may not be adequate in explaining the ETH returns, as there appears to be significant autocorrelation remaining in the residuals.

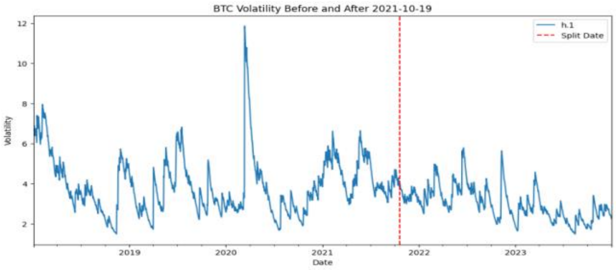
For USDT: We have an extremely small p-value that once again allows us to reject the null hypothesis.

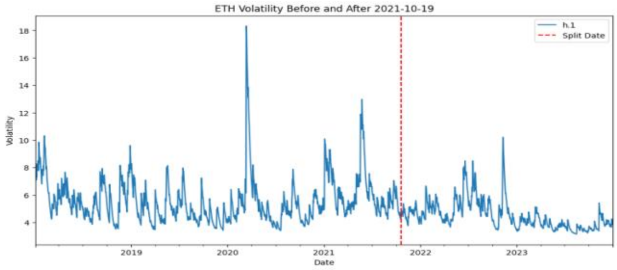


Using a Q-Q Plot to compare the distribution of the standardized residuals from our AR-GJR-GARCH model to a theoretical normal distribution, we can see that the middle points of our data falls over the line, yet we see clear deviations at especially the lower tails. This means that the residuals have heavier tails than the normal distribution, which point out to a higher occurrence of smaller or larger extreme values.

Such heavy tails mean that addition of risk measures such as Value at Risk (VaR) or Expected Shortfall(ES) would improve the model further. Overall we can see that our model falls short on explaining the huge tails forming in big market events, crashes or sudden increases and requires further complexity to achieve a statistical significance that would be used in out-of-sample testing to verify.

5. Conclusion

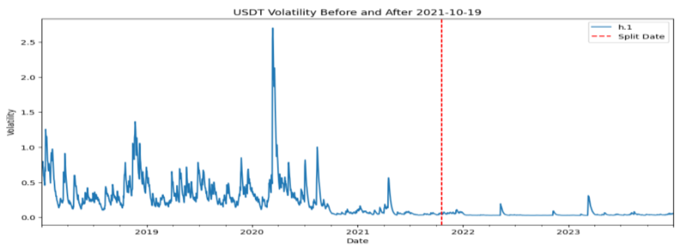




Our findings highlighted the nuanced effects of ETF approvals on cryptocurrency volatility. The introduction of BITO and subsequent ETFs, while representing a bridge between cryptocurrency and regulated financial markets, did not straightforwardly translate to reduced volatility across the board for BTC and ETH. Instead, the analysis revealed complex volatility patterns, influenced by a myriad of factors including market sentiment, investor behaviour, and global economic events such as the COVID-19 pandemic.

The employment of advanced GARCH models, including EGARCH, GJR-GARCH and AR-GJR-GARCH, illuminated the presence of volatility clustering and the significance of leveraging models that account for asymmetries in market conditions. Particularly, AR-GJR-GARCH model emerged as a superior tool in capturing the nuances of cryptocurrency volatility, underscoring the importance of model complexity and the inclusion of autoregressive components in the mean equation.

However, the persistence of significant autocorrelation in the residuals, especially for BTC and ETH, suggests that even sophisticated models may fall short of fully encapsulating the multifaceted nature of cryptocurrency returns. The heavy tails observed in the standardized residuals indicate the prevalence of extreme market movements, which are not adequately addressed by the current model framework.



The implications of these findings are twofold. Firstly, they underscore the ongoing challenges in modelling cryptocurrency volatility, which remains intricately linked to a wide array of market, regulatory, and socio-economic factors. Secondly, they highlight the potential need for further methodological innovations, including the incorporation of risk measures like Value at Risk (VaR) or Expected Shortfall (ES), to enhance model robustness and predictive accuracy.

In conclusion, while regulatory advancements such as the approval of Bitcoin ETFs mark significant milestones in the integration of cryptocurrencies into mainstream financial markets, their impact on market volatility is complex and multifaceted. Our recommendation for future research is to continue the exploration for the intersection of regulatory developments, market structure, and investor behaviour, employing an ever-evolving toolkit of quantitative models to unravel the complexities of this novel asset class.

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Annex – Python Codes:

```
import yfinance as yf
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import scipy.stats as stats
from statsmodels.tsa.stattools import adfuller
from statsmodels.stats.diagnostic import acorr_ljungbox
from statsmodels.graphics.tsaplots import plot_acf, plot_pacf
from arch import *

import warnings
warnings.filterwarnings("ignore")

# Download the data
BTC = yf.download("BTC-USD", start='2018-01-01', end='2024-01-01')['Adj Close']
ETH = yf.download("ETH-USD", start='2018-01-01', end='2024-01-01')['Adj Close']
USDT = yf.download("USDT-USD", start='2018-01-01', end='2024-01-01')['Adj Close']

# Obtain Log Returns
rBTC=(np.log(BTC).diff().dropna())*100
rETH=(np.log(ETH).diff().dropna())*100
rUSDT=(np.log(USDT).diff().dropna())*100

# t-test for the mean. test value=0
t_rBTC, p_rBTC = stats.ttest_1samp(rBTC, 0)
print(t_rBTC , p_rBTC)
t_rETH, p_rETH = stats.ttest_1samp(rETH, 0)
print(t_rETH , p_rETH)
t_rUSDT, p_rUSDT = stats.ttest_1samp(rUSDT, 0)
print(t_rUSDT , p_rUSDT)

# Checked up to 10 lags
acorr_ljungbox(rBTC, lags=[10])
acorr_ljungbox(rETH, lags=[10])
acorr_ljungbox(rUSDT, lags=[10])

# Demeaned Returns
rD_BTC = rBTC - np.mean(rBTC)
rD_ETH = rETH - np.mean(rETH)
rD_USDT = rUSDT - np.mean(rUSDT)

acorr_ljungbox(rD_BTC**2, lags=[10])
acorr_ljungbox(rD_ETH**2, lags=[10])
acorr_ljungbox(rD_USDT**2, lags=[10])

# Create subplots
fig, axs = plt.subplots(1, 3, figsize=(15, 5))
# Plot PACF for each dataset
plot_pacf(rD_BTC**2, ax=axs[0], title='PACF for Demeaned Squared Returns: BTC')
plot_pacf(rD_ETH**2, ax=axs[1], title='PACF for Demeaned Squared Returns: ETH ')
plot_pacf(rD_USDT**2, ax=axs[2], title='PACF for Demeaned Squared Returns: USDT')
# Adjust layout and display the plots
plt.tight_layout()
plt.show()
```

```
# ARCH(1) model estimation
BTC_AM1 = arch_model(rBTC, p=1, q=0, rescale=False)
BTC_res1 = BTC_AM1.fit(update_freq=0, disp='off')
print(BTC_res1.summary())

ETH_AM1 = arch_model(rETH, p=1, q=0, rescale=False)
ETH_res1 = ETH_AM1.fit(update_freq=0, disp='off')
print(ETH_res1.summary())

rUSDT_AM1 = arch_model(rUSDT, p=1, q=0, rescale=False)
rUSDT_res1 = rUSDT_AM1.fit(update_freq=0, disp='off')
print(rUSDT_res1.summary())

rUSDT_AM2 = arch_model(rUSDT, p=2, q=0, rescale=False)
rUSDT_res2 = rUSDT_AM2.fit(update_freq=0, disp='off')
print(rUSDT_res2.summary())

rUSDT_AM6 = arch_model(rUSDT, p=6, q=0, rescale=False)
rUSDT_res6 = rUSDT_AM6.fit(update_freq=0, disp='off')
print(rUSDT_res6.summary())

print(rUSDT_res1.aic, rUSDT_res1.bic)
print(rUSDT_res2.aic, rUSDT_res2.bic)
print(rUSDT_res6.aic, rUSDT_res6.bic)

# Create subplots
fig, axs = plt.subplots(1, 3, figsize=(15, 5))
# Plot PACF for each dataset
plot_acf(rD_BTC**2, ax=axs[0], title='ACF for Demeaned Squared Returns: BTC')
plot_acf(rD_ETH**2, ax=axs[1], title='ACF for Demeaned Squared Returns: ETH ')
plot_acf(rD_USDT**2, ax=axs[2], title='ACF for Demeaned Squared Returns: USDT')
# Adjust layout and display the plots
plt.tight_layout()
plt.show()

BTC_GARCH = arch_model(rBTC, vol='Garch', p=1, q=1, rescale=False)
BTC_GARCH_res = BTC_GARCH.fit(update_freq=5, disp='off')
print(BTC_GARCH_res.summary())

ETH_GARCH = arch_model(rETH, vol='Garch', p=1, q=1, rescale=False)
ETH_GARCH_res = ETH_GARCH.fit(update_freq=5, disp='off')
print(ETH_GARCH_res.summary())

USDT_GARCH = arch_model(rUSDT, vol='Garch', p=1, q=1, rescale=False)
USDT_GARCH_res = USDT_GARCH.fit(update_freq=5, disp='off')
print(USDT_GARCH_res.summary())

BTC_GARCH_t = arch_model(rBTC, vol='Garch', p=1, q=1, dist='t', rescale=False)
BTC_GARCH_t_res = BTC_GARCH_t.fit(update_freq=5, disp='off')
print(BTC_GARCH_t_res.summary())
BTC_GARCH_ged = arch_model(rBTC, vol='Garch', p=1, q=1, dist='ged', rescale=False)
BTC_GARCH_ged_res = BTC_GARCH_ged.fit(update_freq=5, disp='off')
print(BTC_GARCH_ged_res.summary())

print(BTC_GARCH_t_res.aic, BTC_GARCH_t_res.bic)
print(BTC_GARCH_ged_res.aic, BTC_GARCH_ged_res.bic)

ETH_GARCH_t = arch_model(rETH, vol='Garch', p=1, q=1, dist='t', rescale=False)
ETH_GARCH_t_res = ETH_GARCH_t.fit(update_freq=5, disp='off')
print(ETH_GARCH_t_res.summary())
ETH_GARCH_ged = arch_model(rETH, vol='Garch', p=1, q=1, dist='ged', rescale=False)
ETH_GARCH_ged_res = ETH_GARCH_ged.fit(update_freq=5, disp='off')
print(ETH_GARCH_ged_res.summary())

print(ETH_GARCH_t_res.aic, ETH_GARCH_t_res.bic)
print(ETH_GARCH_ged_res.aic, ETH_GARCH_ged_res.bic)
```

```
# EGARCH
BTC_EGARCH_t = arch_model(rBTC, vol='EGARCH', p=1, q=1, dist='t', rescale=False)
BTC_EGARCH_t_res = BTC_EGARCH_t.fit(update_freq=5, disp='off')
print(BTC_EGARCH_t_res.summary())

# AR-CGARCH
BTC_AR_GARCH = arch_model(rBTC, mean='AR', lags=1, vol='Garch', p=1, q=1, o=1, dist='t', rescale=False)
BTC_AR_GARCH_res = BTC_AR_GARCH.fit(update_freq=5, disp='off')
print(BTC_AR_GARCH_res.summary())

# GJR-GARCH
BTC_GJR_GARCH = arch_model(rBTC, mean='Constant', p=1, o=1, q=1, dist='t')
BTC_GJR_GARCH_res = BTC_GJR_GARCH.fit(update_freq=0, disp='off')
print(BTC_GJR_GARCH_res.summary())

print(BTC_EGARCH_t_res.aic, BTC_EGARCH_t_res.bic)
print(BTC_AR_GARCH_res.aic, BTC_AR_GARCH_res.bic)
print(BTC_GJR_GARCH_res.aic, BTC_GJR_GARCH_res.bic)

print(BTC_EGARCH_t_res.pvalues['mu'])
print(BTC_AR_GARCH_res.pvalues['Const'], BTC_AR_GARCH_res.pvalues['Adj Close[1]'])
print(BTC_GJR_GARCH_res.pvalues['mu'])

# EGARCH
ETH_EGARCH_t = arch_model(rETH, vol='EGARCH', p=1, q=1, dist='t', rescale=False)
ETH_EGARCH_t_res = ETH_EGARCH_t.fit(update_freq=5, disp='off')
print(ETH_EGARCH_t_res.summary())

# AR-CGARCH
ETH_AR_GARCH = arch_model(rETH, mean='AR', lags=1, vol='Garch', p=1, q=1, o=1, dist='t', rescale=False)
ETH_AR_GARCH_res = ETH_AR_GARCH.fit(update_freq=5, disp='off')
print(ETH_AR_GARCH_res.summary())

# GJR-GARCH
ETH_GJR_GARCH = arch_model(rETH, mean='Constant', p=1, o=1, q=1, dist='t')
ETH_GJR_GARCH_res = ETH_GJR_GARCH.fit(update_freq=0, disp='off')
print(ETH_GJR_GARCH_res.summary())

print(ETH_EGARCH_t_res.aic, ETH_EGARCH_t_res.bic)
print(ETH_AR_GARCH_res.aic, ETH_AR_GARCH_res.bic)
print(ETH_GJR_GARCH_res.aic, ETH_GJR_GARCH_res.bic)

print(ETH_EGARCH_t_res.pvalues['mu'])
print(ETH_AR_GARCH_res.pvalues['Const'], ETH_AR_GARCH_res.pvalues['Adj Close[1]'])
print(ETH_GJR_GARCH_res.pvalues['mu'])

# EGARCH
USDT_EGARCH_t = arch_model(rUSDT, vol='EGARCH', p=1, q=1, dist='t', rescale=False)
USDT_EGARCH_t_res = USDT_EGARCH_t.fit(update_freq=5, disp='off')
print(USDT_EGARCH_t_res.summary())

# AR-CGARCH
USDT_AR_GARCH = arch_model(rUSDT, mean='AR', lags=1, vol='Garch', p=1, q=1, o=1, dist='t', rescale=False)
USDT_AR_GARCH_res = USDT_AR_GARCH.fit(update_freq=5, disp='off')
print(USDT_AR_GARCH_res.summary())

# GJR-GARCH
USDT_GJR_GARCH = arch_model(rUSDT, mean='Constant', p=1, o=1, q=1, dist='t')
USDT_GJR_GARCH_res = USDT_GJR_GARCH.fit(update_freq=0, disp='off')
print(USDT_GJR_GARCH_res.summary())

print(USDT_EGARCH_t_res.aic, USDT_EGARCH_t_res.bic)
print(USDT_AR_GARCH_res.aic, USDT_AR_GARCH_res.bic)
print(USDT_GJR_GARCH_res.aic, USDT_GJR_GARCH_res.bic)
```

```
print(USDT_EGARCH_t_res.pvalues['mu'])
print(USDT_AR_GARCH_res.pvalues['Const'], USDT_AR_GARCH_res.pvalues['Adj Close[1]'])
print(USDT_GJR_GARCH_res.pvalues['mu'])

# Standardized residuals for each cryptocurrency
BTC_resid = BTC_AR_GARCH_res.resid / BTC_AR_GARCH_res.conditional_volatility
ETH_resid = ETH_AR_GARCH_res.resid / ETH_AR_GARCH_res.conditional_volatility
USDT_resid = USDT_AR_GARCH_res.resid / USDT_AR_GARCH_res.conditional_volatility

tdx_btc = pd.date_range(start='2018-01-01', periods=len(BTC_resid), freq='D')
tdx_eth = pd.date_range(start='2018-01-01', periods=len(ETH_resid), freq='D')
tdx_usdt = pd.date_range(start='2018-01-01', periods=len(USDT_resid), freq='D')

fig, axs = plt.subplots(1, 3, figsize=(15, 5))

# Plot standardized residuals for each cryptocurrency
axs[0].plot(tdx_btc, BTC_resid, label='BTC')
axs[0].set_title('BTC Standardized Residuals')
axs[0].set_xlabel('Date')
axs[0].set_ylabel('Standardized Residuals')

axs[1].plot(tdx_eth, ETH_resid, label='ETH', color='orange')
axs[1].set_title('ETH Standardized Residuals')
axs[1].set_xlabel('Date')
axs[1].set_ylabel('Standardized Residuals')

axs[2].plot(tdx_usdt, USDT_resid, label='USDT', color='green')
axs[2].set_title('USDT Standardized Residuals')
axs[2].set_xlabel('Date')
axs[2].set_ylabel('Standardized Residuals')

plt.tight_layout()
plt.show()

BTC_std_resid = BTC_resid.dropna()
ETH_std_resid = ETH_resid.dropna()
USDT_std_resid = USDT_resid.dropna()

fig, axes = plt.subplots(3, 2, figsize=(12, 9))

# Plot ACF and PACF for BTC Standardized Residuals
plot_acf(BTC_std_resid, lags=20, ax=axes[0, 0], title='BTC ACF')
plot_pacf(BTC_std_resid**2, lags=20, ax=axes[0, 1], title='BTC Squared PACF')

plot_acf(ETH_std_resid, lags=20, ax=axes[1, 0], title='ETH ACF')
plot_pacf(ETH_std_resid**2, lags=20, ax=axes[1, 1], title='ETH Squared PACF')

plot_acf(USDT_std_resid, lags=20, ax=axes[2, 0], title='USDT ACF')
plot_pacf(USDT_std_resid**2, lags=20, ax=axes[2, 1], title='USDT Squared PACF')

plt.tight_layout()
plt.show()

lags = 10
BTC_lb = acorr_ljungbox(BTC_std_resid, lags=[lags], return_df=True)
print(BTC_lb)
ETH_lb = acorr_ljungbox(ETH_std_resid, lags=[lags], return_df=True)
print(ETH_lb)
USDT_lb = acorr_ljungbox(USDT_std_resid, lags=[lags], return_df=True)
print(USDT_lb)
```

```
import statsmodels.api as sm

fig, axes = plt.subplots(1, 3, figsize=(15, 5))

sm.qqplot(BTC_AR_GARCH_res.std_resid, line='45', ax=axes[0])
axes[0].set_title('BTC Q-Q Plot')

sm.qqplot(ETH_AR_GARCH_res.std_resid, line='45', ax=axes[1])
axes[1].set_title('ETH Q-Q Plot')

sm.qqplot(USDT_AR_GARCH_res.std_resid, line='45', ax=axes[2])
axes[2].set_title('USDT Q-Q Plot')

plt.tight_layout()
plt.show()

BTC_AR_GARCH_fit = BTC_AR_GARCH.fit(last_obs='2021-10-19', update_freq=5, disp='off')

BTC_forecast = BTC_AR_GARCH_fit.forecast(start='2021-10-20', method='simulation')

BTC_forecast = BTC_AR_GARCH_fit.forecast(start='2018-01-01', horizon=1, method='simulation')

# Extract the forecasted variance and take the square root for volatility
BTC_vol_f = BTC_forecast.variance**0.5

# Create a combined volatility series
BTC_vol_combined = BTC_vol_f['h.1'].dropna()

# Plot the combined volatility
plt.figure(figsize=(12, 6))
BTC_vol_combined.plot(title='BTC Volatility Before and After 2021-10-19')
plt.axvline(x='2021-10-19', color='red', linestyle='--', label='Split Date')
plt.xlabel('Date')
plt.ylabel('Volatility')
plt.legend()
plt.show()

ETH_AR_GARCH_fit = ETH_AR_GARCH.fit(last_obs='2021-10-19', update_freq=5, disp='off')

ETH_forecast = ETH_AR_GARCH_fit.forecast(start='2021-10-20', method='simulation')

ETH_forecast = ETH_AR_GARCH_fit.forecast(start='2018-01-01', horizon=1, method='simulation')

# Extract the forecasted variance and take the square root for volatility
ETH_vol_f = ETH_forecast.variance**0.5

# Create a combined volatility series
ETH_vol_combined = ETH_vol_f['h.1'].dropna()

# Plot the combined volatility
plt.figure(figsize=(12, 6))
ETH_vol_combined.plot(title='ETH Volatility Before and After 2021-10-19')
plt.axvline(x='2021-10-19', color='red', linestyle='--', label='Split Date')
plt.xlabel('Date')
plt.ylabel('Volatility')
plt.legend()
plt.show()
```

```
USDT_AR_GARCH_fit = USDT_AR_GARCH.fit(last_obs='2021-10-19', update_freq=5, disp='off')

USDT_forecast = USDT_AR_GARCH_fit.forecast(start='2021-10-20', method='simulation')

USDT_forecast = USDT_AR_GARCH_fit.forecast(start='2018-01-01', horizon=1, method='simulation')

# Extract the forecasted variance and take the square root for volatility
USDT_vol_f = USDT_forecast.variance**0.5

# Create a combined volatility series
USDT_vol_combined = USDT_vol_f['h.1'].dropna()

# Plot the combined volatility
plt.figure(figsize=(12, 6))
USDT_vol_combined.plot(title='USDT Volatility Before and After 2021-10-19')
plt.axvline(x='2021-10-19', color='red', linestyle='--', label='Split Date')
plt.xlabel('Date')
plt.ylabel('Volatility')
plt.legend()
plt.show()
```