Two-Stream Graph Convolutional Network-Incorporated Latent Feature Analysis-Supplementary File

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This is the supplementary file for the paper entitled "Two-Stream Graph Convolutional Network-Incorporated Latent Feature Analysis". Additional tables and figures are put into this file and cited by the paper.

1. TABLES

TABLE S1
THE RMSE AND EPOCHS OF TGLFA WITH DIFFERENT NUMBERS OF ATTRIBUTE CHARACTERISTICS EXTRACTION LAYERS ON D1.1-2.4.

No.			Epochs (RMSE)							
INO.	K=1	K=2	K=3	K=4	K=5	K=1	K=2	K=3	K=4	K=5
D1.1	1.2859	1.2732	1.2711	1,2711	1.2760	220	114	102	86	68
D1.2	1.2199	1.2047	1.2133	1.2150	1.2241	432	275	218	216	222
D1.3	1.1747	1.1583	1.1653	1.1632	1.1677	609	326	296	282	257
D1.4	1.1397	1.1255	1.1249	1.1280	1.1321	707	370	342	333	295
D2.1	0.5577	0.5242	0.5286	0.5355	0.5403	1000	290	258	218	210
D2.2	0.4831	0.4673	0.4697	0.4742	0.4771	570	338	318	196	171
D2.3	0.4512	0.4352	0.4344	0.4375	0.4405	547	381	373	307	331
D2.4	0.4287	0.4159	0.4147	0.4165	0.4204	717	425	403	348	375

TABLE S2
THE MAE AND EPOCHS OF TGLFA WITH DIFFERENT NUMBERS OF ATTRIBUTE CHARACTERISTICS EXTRACTION LAYERS ON D1.1-2.4

No.	MAE						Epochs (MAE)					
NO.	K=1	K=2	K=3	K=4	K=5	K=1	K=2	K=3	K=4	K=5		
D1.1	0.5233	0.5124	0.5099	0.5146	0.5169	367	214	204	114	84		
D1.2	0.4769	0.4676	0.4665	0.4657	0.4668	371	222	221	212	189		
D1.3	0.4557	0.4443	0.4449	0.4417	0.4416	383	257	234	222	189		
D1.4	0.4384	0.4288	0.4258	0.4240	0.4235	371	277	259	234	206		
D2.1	0.2067	0.1878	0.1891	0.1914	0.1927	1000	312	272	265	227		
D2.2	0.1727	0.1650	0.1659	0.1653	0.1655	574	293	251	203	209		
D2.3	0.1584	0.1522	0.1523	0.1517	0.1516	526	321	346	290	290		
D2.4	0.1511	0.1455	0.1450	0.1447	0.1447	526	321	324	304	330		

TABLE S3
THE RMSE, WIN/LOSS COUNTS, AND FRIEDMAN TEST RESULTS OF M1-15 ON ALL TESTING CASES.

No.	D1.1	D1.2	D1.3	D1.4	D2.1	D2.2	D2.3	D2.4	Win/Loss	F-Rank*
M1	1.6989±2.6E-3●	1.4632±6.5E-3 ●	1.3069±1.1E-2●	1.2077±2.2E-3 •	0.6365±1.3E-3•	0.5238±1.4E-3●	0.4743±1.5E-4●	0.4531±2.4E-4•	8/0	8.38
M2	1.4527±7.6E-4 ●	1.2945±2.7E-3 ●	1.2294±4.5E-4 ●	1.1943±2.6E-3 ●	0.6985±1.1E-3●	0.5413±2.6E-3●	0.4884±5.7E-4●	0.4698±8.2E-4●	8/0	7.38
M3	1.7249±5.9E-3 ●	1.6751±4.0E-3 ●	1.6428±3.9E-3●	1.6127±3.8E-3 ●	0.8324±2.1E-3●	0.7844±1.1E-3●	0.7532±1.6E-3 ●	0.7273±3.8E-3●	8/0	13.63
M4	1.2922±2.5E-3 ●	1.2176±1.3E-3 ●	1.1712±5.1E-3 ●	1.1314±1.4E-3●	0.5364±1.9E-3●	0.4879±1.2E-3●	0.4449±1.6E-3●	0.4219±9.7E-4●	8/0	2.56
M5	1.5593±5.7E-3 ●	1.2906±4.2E-3●	1.1952±3.0E-3 •	1.1400±1.4E-3 ●	0.7818±5.3E-3 ●	0.5561±1.6E-2●	0.4773±6.6E-3 ●	0.4394±2.8E-3•	8/0	6.75
M6	1.2842±8.2E-3 ●	1.2146±1.7E-3 ●	1.1725±2.7E-3 •	1.1275±1.1E-3 ●	0.5738±3.3E-3 ●	0.4850±1.2E-3 ●	0.4498±1.1E-3●	0.4298±5.2E-4●	8/0	3.00
M 7	1.5513±2.2E-4 ●	1.4316±3.2E-3●	1.2670±2.5E-3 ●	1.1607±1.2E-3 ●	0.6686±1.6E-3●	0.4930±6.7E-4●	0.4482±5.7E-4●	0.4220±5.2E-4●	8/0	6.38
M8	1.6376±4.1E-3●	1.4952±3.5E-3 ●	1.4264±3.3E-3 •	1.3505±6.6E-3 ●	0.9373±2.5E-3 ●	0.7019±3.1E-3 ●	0.5563±6.0E-3 ●	0.4900±2.8E-3•	8/0	12.75
M9	1.5728±6.9E-4 ●	1.4740±2.9E-3 ●	1.3619±2.8E-3●	1.2397±9.4E-3●	0.9029±3.2E-3•	0.6509±5.5E-3 ●	0.5283±2.3E-3 ●	0.4785±1.5E-3●	8/0	11.00
M10	1.8186±4.3E-4●	1.7981±1.4E-3●	1.7871±1.9E-3 ●	1.7712±2.5E-3 ●	1.0096±2.2E-4●	0.9880±1.9E-4●	0.9680±2.3E-3 ●	0.9467±6.6E-3●	8/0	15.00
M11	1.4873±6.8E-3 ●	1.3014±4.7E-3 ●	1.2179±2.6E-3 ●	1.1601±1.4E-3●	0.6718±1.6E-3 ●	0.5210±2.6E-3 ●	0.4684±9.9E-4●	0.4438±6.9E-4●	8/0	6.13
M12	1.5495±9.5E-5 ●	1.4286±2.0E-3 ●	1.2643±2.3E-3 •	1.1610±8.3E-4 ●	0.6634±2.0E-3 ●	0.4913±4.5E-4●	0.4476±3.6E-4●	0.4219±3.6E-4●	8/0	5.56
M13	1.5975±1.1E-3●	1.4939±7.2E-4●	1.4182±2.7E-3 •	1.3296±1.2E-3●	0.8938±2.5E-3●	0.6308±1.7E-3 ●	0.5268±9.9E-4●	0.4892±1.1E-3•	8/0	11.25
M14	1.4224±1.8E-3 ●	1.3744±8.6E-3●	1.3447±3.6E-3●	1.3289±3.2E-3●	0.6535±4.3E-3●	0.6453±2.3E-4●	0.6430±3.1E-4●	0.6409±3.4E-4●	8/0	9.25
M15	1.2535±9.0E-4	1.1869±1.1E-3	1.1462±5.7E-4	1.1158±9.0E-4	0.5250±7.9E-4	0.4661±8.2E-4	0.4348±8.4E-4	0.4170±5.8E-4	_	1.00

^{*}A lower F-rank value denotes a higher estimation accuracy; and • indicates that M15's RMSE is lower than that of the compared model.

 $\label{thm:condition} TABLE~S4$ The MAE, Win/Loss Counts, and Friedman Test Results of M1-15 on All Testing Cases.

No.	D1.1	D1.2	D1.3	D1.4	D2.1	D2.2	D2.3	D2.4	Win/Loss	F-Rank*
M1	0.7037 _{±6.9E-4} •	0.5815±2.6E-3●	0.5186±3.9E-3●	0.4805±9.5E-4●	0.2324 _{±7.0E-4} •	0.1932 _{±6.4E-4} •	0.1755±9.9E-5●	0.1675 _{±1.5E-3} •	8/0	8.00
M2	0.5606±7.5E-4●	0.4852 _{±3.8E-4} •	0.4621±5.2E-4●	0.4468 _{±6.0E-4} •	0.2891±4.6E-4●	0.1947 _{±1.0E-3} •	0.1775 _{±8.0E-4} •	0.1700 _{±6.0E-4} •	8/0	6.13
M3	0.8066±6.4E-3●	0.7777 _{±5.0E-3} •	0.7688±6.4E-3●	0.6991 _{±1.7E-2} •	0.3818±4.8E-3●	0.3550 _{±3.2E-3} •	0.3350 _{±2.6E-3} •	0.3249 _{±2.1E-3} •	8/0	14.25
M4	0.5067±2.4E-3 ●	0.4635±3.1E-3●	0.4456±1.5E-3●	0.4289 _{±1.4E-3} •	0.1993 _{±7.8E-4} •	0.1761 _{±1.1E-3} •	0.1596±5.1E-4●	0.1509 _{±2.5E-4} •	8/0	2.69
M5	0.6081±5.2E-3●	0.5116±2.4E-3●	0.4724±2.9E-3 ●	0.4474±3.2E-3●	0.2830 _{±1.5E-3} •	0.2157 _{±7.5E-3} •	0.1891±9.3E-4●	0.1767 _{±7.3E-4} •	8/0	7.38
M6	0.5687 _{±6.7E-3} •	0.5112 _{±2.3E-3} •	0.4850 _{±1.3E-3} •	0.4607±2.6E-3●	0.2357 _{±1.5E-3} •	0.1878 _{±1.0E-3} •	0.1700 _{±1.1E-3} •	0.1618±9.1E-4●	8/0	5.38
M 7	0.6423±4.9E-4●	0.5620 _{±1.9E-3} •	0.4892 _{±6.3E-4} •	0.4458±3.9E-4●	0.2375 _{±1.0E-3} •	0.1747 _{±2.4E-4} •	0.1589 _{±2.9E-4} •	0.1485±4.6E-4●	8/0	4.69
M8	0.6992±3.3E-3●	0.6273 _{±3.0E-3} •	0.5990 _{±1.9E-3} •	0.5522 _{±2.6E-3} •	0.3872 _{±1.4E-3} •	0.2725 _{±1.3E-3} •	0.2080 _{±1.5E-3} •	0.1819±4.3E-4●	8/0	12.38
M9	0.6634±1.6E-3●	0.6064±1.6E-3 ●	0.5488±1.5E-3●	0.4930±7.3E-3●	0.3469±1.4E-3•	0.2380±1.9E-3 ●	0.1940±6.6E-4●	0.1769±5.1E-4●	8/0	10.25
M10	0.7539±5.7E-5●	0.7532±7.5E-5 ●	0.7530±8.2E-5 ●	0.7530±1.1E-4●	0.4164±6.3E-5●	0.4162±9.2E-5●	0.4162±3.4E-5●	0.4160±3.4E-5●	8/0	14.63
M11	0.5674±5.9E-3 ●	0.4967±2.0E-3 ●	0.4712±9.6E-4●	0.4467±1.6E-3●	0.2555±1.2E-3●	0.1899±1.5E-3●	0.1714±8.1E-4 ●	0.1635±5.4E-4•	8/0	5.25
M12	0.6417±8.5E-4●	0.5654 _{±1.8E-3} •	0.4909 _{±7.7E-4} •	0.4464±9.1E-4●	0.2372 _{±8.9E-4} •	0.1747 _{±1.5E-4} •	0.1596 _{±1.8E-4} •	0.1495±2.0E-4●	8/0	5.13
M13	0.6862 _{±1.2E-3} •	0.6360 _{±1.0E-3} •	0.5988 _{±2.1E-3} •	0.5491±2,2E-3●	0.3628 _{±1.8E-4} •	0.2394 _{±5.6E-4} •	0.1990±3.6E-4●	0.1847±3.5E-4●	8/0	11.75
M14	0.6474±3.4E-4●	0.5968 _{±1.7E-2} •	0.5627 _{±8.5E-4} •	0.5485 _{±1.4E-3} •	0.2868 _{±6.6E-4} •	0.2816±4.1E-4●	0.2787 _{±1.1E-3} •	0.2781 _{±1.6E-4} •	8/0	11.13
M15	0.4948±4.2E-4	0.4551±1.2E-3	0.4334±1.7E-3	0.4199±2.0E-3	0.1873±5.1E-4	0.1628±4.0E-4	0.1500±5.3E-4	0.1436±2.8E-4	_	1.00

^{*}A lower F-rank value denotes a higher estimation accuracy; and • indicates that M15's MAE is lower than that of the compared model.

TABLE S5
THE TRAINING TIME COST IN RMSE (Sec.), WIN/LOSS COUNTS, AND FRIEDMAN TEST RESULTS OF M1-15 ON ALL TESTING CASES.

No.	D1.1	D1.2	D1.3	D1.4	D2.1	D2,2	D2.3	D2.4	Win/Loss	F-Rank*
M1	11954±4092.98 ●	12107±4769.09 ●	11156±5460.32●	8583±2421.37 •	17267±2272.97 •	13982±2368.04 •	8882±2224.26•	5175±1532.91 •	8/0	10.38
M2	235±7.12 •	325±17.97 ●	299±17.51 •	327±22.12 •	184±7.78	433±18.45 ●	368±22.09	$379_{\pm 26.10}$	5/3	1.63
M3	1070±24.43 ●	1002±116.82 ●	1032±21.07 ●	1246±19.00 ●	1336±98.60 ●	1615±122.60 ●	1624±125.01 ●	1834±220.93 ●	8/0	3.25
M4	881±87.55 ●	1015±53.73 ●	2194±121.24 ●	2419±251.35 ●	14625±2342.49 ●	2647±314.09 •	3643±833.96 ●	4446±763.50 ●	8/0	6.25
M 5	2416±115.26 ●	2402±55.87 ●	2591±90.47 •	2823±80.26 ●	2462±36.55 ●	2252±55.95 ●	2499±168.56 ●	2743±72.51 ●	8/0	6.50
M 6	1537±52.96 ●	1833±93.52 ●	2277±129.40 ●	2650±95.35 ●	2328±767.43 ●	1748±50.33 ●	1853±118.35 ●	2171±107.51 •	8/0	5.00
M7	3242±136.54 •	9331±193.59•	12299±234.04 ●	14211±302.65 ●	4964±106.70 ●	8086±144.00 ●	10019±283.43 ●	11812±156.58 ●	8/0	8.88
M 8	5122±448.81 ●	11377±662.84 ●	17593±316.03 ●	24323±1609.49 ●	3382±375.85 ●	9568±730.37 ●	17055±1170.95 ●	23215±2260.42 •	8/0	10.75
M 9	5679±178.29 ●	11356±658.53 ●	18836±888.19 ●	24886±193.27 ●	3823±153.40 ●	7954±216.53 ●	10724±386.23 ●	12689±434.26•	8/0	10.38
M10	2151±403.94 ●	1977±260.81 ●	1327±219.04 •	1288±141.65 ●	1682±492.23 ●	1381±488.28 ●	2085±655.38 ●	2306±1044.22 •	8/0	4.63
M11	36992±1391.36 ●	33108±1317.36 ●	35478±702.79 ●	37621±760.63 ●	21164±492.46 •	26404±409.56 ●	25938±766.47 ●	29208±644.28 •	8/0	14.38
M12	3272±204.54 •	9714±83.95 ●	12415±121.75 ●	14405±119.67 ●	5249±103.26 ●	8445±155.70 ●	10353±149.50 ◆	12006±276.26 ●	8/0	9.88
M13	6611±483.08 ●	12931±310.82 ●	20614±541.37 ●	28265±511.37 ●	4572±162.75 ●	13219±585.96 ●	19211±436.56 ●	22050±828.51 ●	8/0	12.13
M14	22562±3206.86 •	29119±4938.11 ●	33585±6431.56 ●	42091±11482.16 ●	22139±3590.56 •	28474±3703.66 •	36034±1900.76 ●	40132±6234.69 ●	8/0	14.63
M15	109±13.87	253±48.28	266±17.92	297±11.55	441±74.81	411±31.99	445±42.42	447±30.80	_	1.38

^{*}A lower F-rank value denotes a higher computational efficiency; and • indicates that M15's time cost to converge in RMSE is less than that of the compared model.

TABLE S6
THE TRAINING TIME COST IN MAE (Sec.), WIN/LOSS COUNTS, AND FRIEDMAN TEST RESULTS OF M1-15 ON ALL TESTING CASES.

No.	D1.1	D1.2	D1.3	D1.4	D2.1	D2.2	D2.3	D2.4	Win/Loss	F-Rank*
M1	10556±4357.46 ●	12148±4143.32 ●	10872±5402.71 ●	8185±1562.90 ●	17170±2273.43 ●	14120±2646.15 ●	8130±2213.19•	4619±1212.11 ●	8/0	9.75
M2	293±33.87●	356±30.67 ●	351±11.37●	356±28.45 ●	257±21.59	492±27.73●	434±40.41 ●	441±44.99 ●	7/1	1.88
M3	1180±229.11 ●	1058±306.69 ●	1224±97.29 ●	23764±11859.43 •	2054±219.49 •	1868±112.63 ●	1920±181.61 •	2125±311.95 ●	8/0	4.75
M4	990±111.53 ●	1290±275.14 •	2338±185.49 ●	3311±974.27 ●	14513±2480.57 ●	3243±660.04 ●	5220±1312.60 ●	5666±699.69 ●	8/0	6.75
M5	1949±98.10 ●	1928±47.91 ●	2014±110.80 •	2084±83.22 ●	2463±39.64 •	2124±49.26 ●	2206±80.28 ●	2423±73.80 ●	8/0	5.50
M 6	1680±168.65 ●	1832±113.62 ●	2055±111.77 •	2219±200.02 ●	2342±829.11 ●	1701±50.05 ●	1732±172.04 ●	1642±78.67 ●	8/0	4.63
M7	8249±201.41 •	11615±141.02 ●	12947±149.46 ●	13713±141.48 ●	6626±100.16 ●	8107±162.61 ●	9309±162.23 ●	10492±64.46●	8/0	8.63
M8	6830±574.10 ●	13351±1072.35 ●	20042±1105.05 ●	28866±773.37 •	4363±173.32 ●	11372±442.11 •	17796±617.42 ●	21347±778.06 ●	8/0	10.50
M9	8943±227.51 ●	15719±495.71 ●	21659±730.30 ●	25554±395.15 ●	5037±110.10 •	8236±209.58 •	9837±157.11 •	11222±250.25 ●	8/0	10.75
M10	4813±191.31 ●	2344±89.84 ●	1426±60.30 ●	1079±18.84 ●	2475±186.80 ●	1211±58.33 ●	737±31.32 ●	556±25.01 ●	8/0	4.50
M11	39764±1490.50 ●	32747±1230.10 ●	33187±429.97 ●	34077±483.32 ●	25385±612.62 ●	24920±581.60 ●	23243±484.09 •	24361±324.07●	8/0	14.50
M12	8518±182.16 ●	11926±97.50 ●	13210±107.78 ●	13968±72.74●	6768±150.34 ●	8223±179.02 ●	9509±138.70 •	10772±267.13 ●	8/0	9.63
M13	9250±203.44 •	16228±640.70 ●	24379±335.21 ●	31689±470.50 ●	7372±173.82 ●	14798±289.76 ●	19726±747.49 ●	22922±955.79•	8/0	12.75
M14	23004±3830.51 ●	30423±5773.42 ●	33634±6202.17 ●	31525±2403.30 ●	21042±3438.79•	26048±4605.46 •	27398±5724.56 •	36247±3486.86 ●	8/0	14.38
M15	196±33.27	228±41.61	221±12.83	208±10.76	451±63.60	331±39.80	343±34.54	328±64.10	_	1.13

^{*}A lower F-rank value denotes a higher computational efficiency; and • indicates that M15's time cost to converge in MAE is less than that of the compared model.

 $\label{thm:thm:thm:coxon} TABLE~S7\\ WILCOXON~SIGNED-RANKS~TEST~RESULTS~IN~RMSE~AND~MAE~CORRESPONDING~TO~TABLES~S3~AND~S4.$

Comparison	R+*	R-	<i>p</i> -value**
M15 vs M1	136	0	2.41E-4
M15 vs M2	136	0	2.41E-4
M15 vs M3	136	0	2.41E-4
M15 vs M4	136	0	2.41E-4
M15 vs M5	136	0	2.41E-4
M15 vs M6	136	0	2.41E-4
M15 vs M7	136	0	2.41E-4
M15 vs M8	136	0	2.41E-4
M15 vs M9	136	0	2.41E-4
M15 vs M10	136	0	2.41E-4
M15 vs M11	136	0	2.41E-4
M15 vs M12	136	0	2.41E-4
M15 vs M13	136	0	2.41E-4
M15 vs M14	136	0	2.41E-4

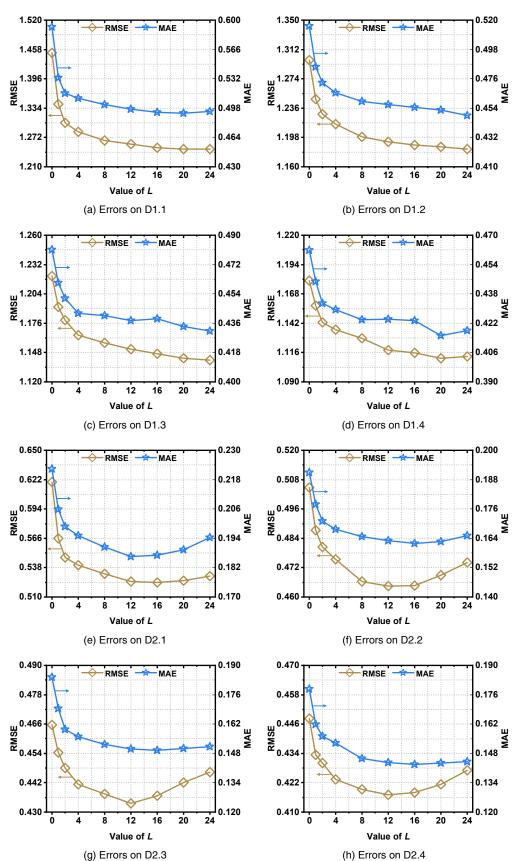
^{*}For M15, a higher R+ value indicates a higher estimation accuracy; **With the significance level of 0.1, the accepted hypotheses are highlighted.

TABLE S8
WILCOXON SIGNED-RANKS TEST RESULTS ON CONVERGING TIME COST IN RMSE AND MAE CORRESPONDING TO TABLES S5 AND S6.

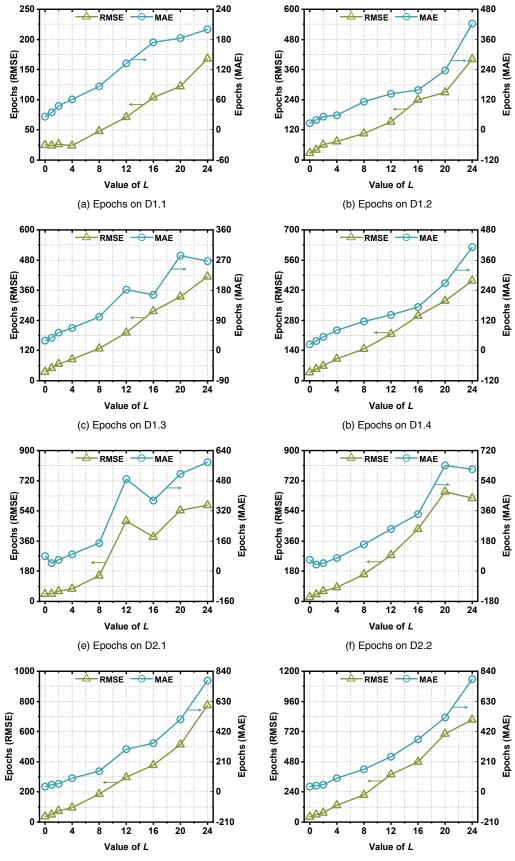
Comparison	R+*	R-	<i>p</i> -value**
M15 vs M1	136	0	2.41E-4
M15 vs M2	95	41	8.53E-2
M15 vs M3	136	0	2.41E-4
M15 vs M4	136	0	2.41E-4
M15 vs M5	136	0	2.41E-4
M15 vs M6	136	0	2.41E-4
M15 vs M7	136	0	2.41E-4
M15 vs M8	136	0	2.41E-4
M15 vs M9	136	0	2.41E-4
M15 vs M10	136	0	2.41E-4
M15 vs M11	136	0	2.41E-4
M15 vs M12	136	0	2.41E-4
M15 vs M13	136	0	2.41E-4
M15 vs M14	136	0	2.41E-4

^{*}For M15, a higher R+ value indicates a higher computational efficiency; **With the significance level of 0.1, the accepted hypotheses are highlighted.

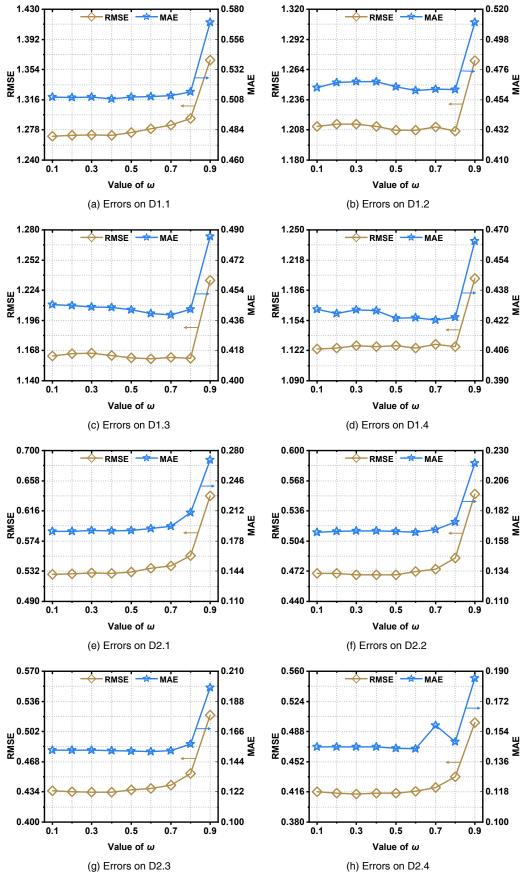
2. FIGURES



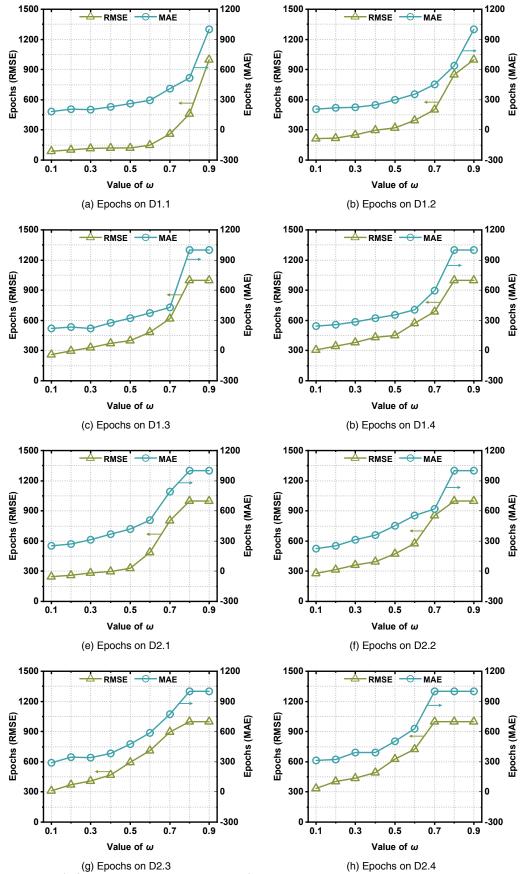
(g) Errors on D2.3 (h) Errors on D2. Fig. S1. Errors of TGLFA with different numbers of light graph convolutional layers on all testing cases.



 $\hbox{(g) Epochs on D2.3} \qquad \hbox{(h) Epochs on D2.4}$ Fig. S2. Converging epochs of TGLFA with different numbers of light graph convolutional layers on all testing cases.} \\



(g) Errors on D2.3 Fig. S3. Errors of TGLFA as ω varies while others being fixed on all testing cases.



(g) Epochs on D2.3 (h) Epochs of TGLFA as ω varies while others being fixed on all testing cases.

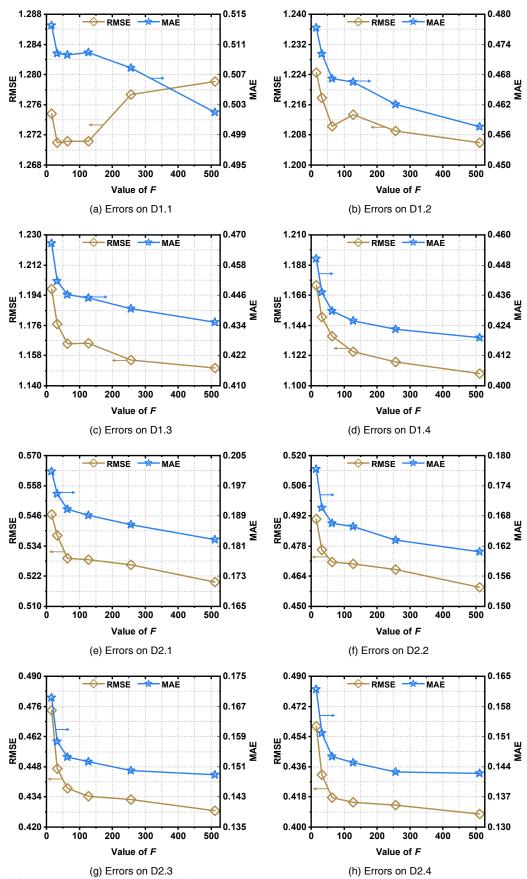
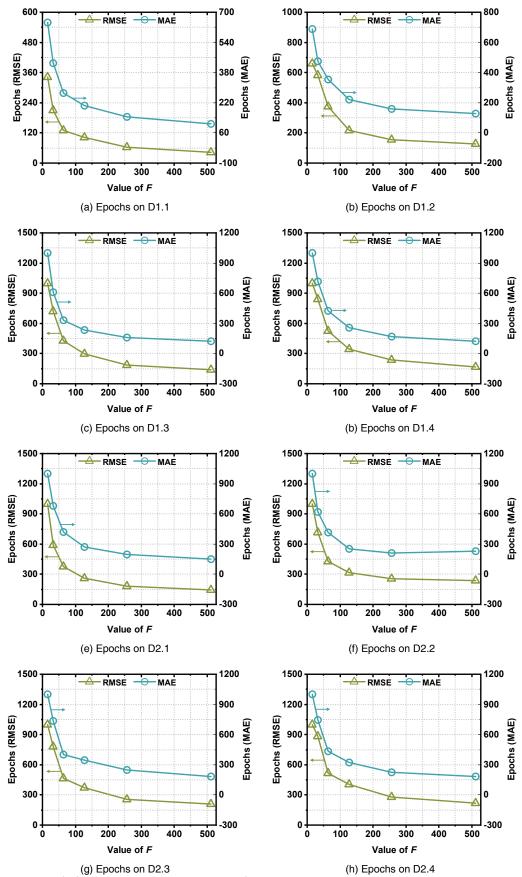


Fig. S5. Errors of TGLFA as *F* varies while others being fixed on all testing cases.



(g) Epochs on D2.3 (h) Epochs S6. Converging epochs of TGLFA as $\it F$ varies while others being fixed on all testing cases.