

IZVODI - ZADACI

1. Odrediti jednačinu tangente na grafik funkcije $f(x)$ u tački (x_0, y_0) ako je

a) $f(x) = 2x^3 - 5x$, $x_0 = -1$, b) $f(x) = \sqrt{x}$, $x_0 = 0$,

c) $f(x) = \frac{x-1}{x-2}$, $x_0 = 1$, d) $f(x) = \frac{2x}{(x+1)^2}$, $x_0 = 0$.

2. Naći prvi izvod sledećih funkcija:

(a) 1) $f(x) = x^2 + 2x - 4$, 2) $f(x) = x^5 + 3x^8 + x^{-9} + \frac{4}{x^8}$, 3) $f(x) = \sqrt[3]{x} + \sqrt[3]{3}$,

4) $f(x) = (x-1)(x+1)$, 5) $f(x) = \frac{1}{x} + \frac{1}{\sqrt{x}} + \frac{1}{\sqrt[3]{x}}$,

(b) 1) $f(x) = x^2(2x+3)$, 2) $f(x) = \sqrt{x}(x^3-4)$, 3) $f(x) = (x+1)(\sqrt{x}+2)$,

4) $f(x) = (x^2+1)(x^2+2)$, 5) $f(x) = \frac{\sqrt{x}+1}{\sqrt{x}-1}$, 6) $f(x) = \frac{1+x\sqrt{x}}{1-x\sqrt{x}}$,

(c) 1) $f(x) = \sin x + \cos x$, 2) $f(x) = \operatorname{tg} x - \operatorname{ctg} x$, 3) $f(x) = \sqrt{x} \sin x$, 4) $f(x) = \frac{1}{\sin x}$

5) $f(x) = (\cos x)(1 - \sin x)$, 6) $f(x) = \frac{x \sin x}{1 + \operatorname{tg} x}$, 7) $f(x) = \frac{1 - \cos x}{1 + \cos x}$,

(d) 1) $f(x) = 3 + \ln x$, 2) $f(x) = e^x \ln x$, 3) $f(x) = \sin x \ln x$, 4) $f(x) = x^2 \ln x$

5) $f(x) = \frac{x^2+1}{\ln x}$, 6) $f(x) = \frac{a}{\ln x}$, 7) $f(x) = \frac{1 - \ln x}{1 + \ln x}$,

(e) 1) $f(x) = 2^x + 3^x + 4^x$, 2) $f(x) = \frac{1}{5^x} + \frac{1}{7^x}$, 3) $f(x) = (x^2 - x + 1)e^x$, 4) $f(x) = e^x \sin x$

5) $f(x) = \frac{e^x}{\cos x}$, 6) $f(x) = \frac{a^x}{\ln x}$, 7) $f(x) = \frac{e^x - e^{-x}}{2}$,

(f) 1) $f(x) = x \arcsin x$, 2) $f(x) = \frac{1}{\arcsin x}$, 3) $f(x) = \frac{\arcsin x}{\arccos x}$, 4) $f(x) = \sqrt{x} \operatorname{arctg} x$.

3. Naći prvi izvod sledećih funkcija:

(a) 1) $f(x) = e^{3x}$, 2) $f(x) = \operatorname{arctg} x^2$, 3) $f(x) = \operatorname{arctg}^2 x$, 4) $f(x) = \ln(3x^2 - 4)$

5) $f(x) = 2^{\frac{x}{\ln x}}$, 6) $f(x) = x \sin(\ln x)$, 7) $f(x) = \arcsin \frac{2}{x}$,

(b) 1) $f(x) = \left(x^2 - \frac{1}{x^3} + 2\right)^4$, 2) $f(x) = (1 + \sqrt[3]{x})^3$, 3) $f(x) = \sqrt[3]{\frac{1}{1-x^3}}$,

4) $f(x) = x \sqrt{\frac{1-x}{x^2+1}}$, 5) $f(x) = \frac{1+x}{\sqrt{1-x}}$,

(c) 1) $f(x) = \frac{1}{\sin x}$, 2) $f(x) = \frac{\sin x - x \cos x}{\cos x + x \sin x}$, 3) $f(x) = \sin^2(\cos 3x)$, 4) $f(x) = \sin^2 x \sin x^2$,

4) $f(x) = \frac{1}{4} \ln \frac{x^2-1}{x^2+1}$, 5) $f(x) = \cos^2 \frac{1-\sqrt{x}}{1+\sqrt{x}}$, 6) $f(x) = \ln(x + \sqrt{x^2+1}) - \frac{1}{\sqrt{x^2+1}}$,

4. Izračunati:

(a) $f'(x) + f(x) + f\left(\frac{1}{x}\right)$ ako je $f(x) = \ln x$,

(b) $f'(1) - f'(-1) + 4f(0)$ ako je $f(x) = x^5 + x^3 - 2x - 3$,

(c) $f'(2) - f'(-2)$ ako je $f(x) = x^2 - \frac{1}{2x^2}$.

5. Naći prvi izvod sledećih funkcija:

$$\begin{array}{ll} \text{a)} & x = \frac{1+t^3}{t^2-1}, y = \frac{t}{t^2-1} \quad \text{b)} \quad x = 2\operatorname{tg} t, y = 2\sin^2 t + \sin 2t \\ \text{c)} & y = (\sin x)^x \quad \text{d)} \quad x = \left(\frac{x}{1+x}\right)^x + \ln x \\ \text{d)} & x y = e^{x+y} \quad \text{e)} \quad \operatorname{arctg} \frac{y}{x} = \ln \sqrt{x^2 + y^2} \end{array}$$

6. Naći drugi izvod funkcije $y = f(x)$:

$$\begin{array}{llll} \text{(a) 1)} & y = x e^{x^2}, & 2) & y = e^{\sqrt{x}}, & 3) & y = (1+x^2) \operatorname{arctg} x, & 4) & y = \cos^2 x, \\ \text{(b) 1)} & x = \cos t, \quad y = \sin t, & 2) & x = t^2 e^t, \quad y = \frac{\ln t}{t}, & 3) & x = \sqrt{t} \cos t, \quad y = \sqrt{t} \sin t, \\ \text{(c) 1)} & \sin(x+y) = \cos xy, & 2) & \frac{xy}{x+y} = (1+\frac{y}{x}), & 3) & e^{xy} = \ln(x^2 + y^2), \\ \text{(d) 1)} & y = x^x, & 2) & y = x^{\ln x}, & 3) & y = (\ln x)^x, & 4) & y = x^{\sin x}. \end{array}$$

7. Neka je $p(x)$ polinom četvrtog stepena. Ako je $p(2) = -1$, $p'(2) = 0$, $p''(2) = 2$, $p'''(2) = -1$ i $p^{IV}(2) = 24$ izračunati $p(-1)$, $p'(0)$ i $p''(1)$.

8. Dokazati da

- (a) funkcija $y = e^x \sin x$ zadovoljava jednačinu $y'' - 2y' + 2y = 0$;
- (b) funkcija $y = \sqrt{2x - x^2}$ zadovoljava jednačinu $y^3 y'' + 1 = 0$;
- (c) funkcija $y = \sin(x^2 + 1)$ zadovoljava jednačinu $xy'' - y' + 4x^3 y = 0$.

9. (Lopitalovo pravilo) Odrediti sledeće granične vrednosti:

$$\begin{array}{llll} \text{(a) 1)} & \lim_{x \rightarrow 0} \frac{\sin x - x}{e^x - 1}, & 2) & \lim_{x \rightarrow 2} \frac{2^{x-2} - 3^{x-2}}{\operatorname{arctg}(x^2 - 4)}, & 3) & \lim_{x \rightarrow 0} \frac{e^x - e^{-x} - 2x}{x - \sin x} \\ \text{(b) 1)} & \lim_{x \rightarrow \infty} \frac{\ln x}{x^{10}}, & 2) & \lim_{x \rightarrow 0^+} \frac{\operatorname{ctg} x - x}{\sin x + \ln x}, & 3) & \lim_{x \rightarrow \infty} \frac{\sqrt{x^2 + 1}}{1 - x}, \\ \text{(c) 1)} & \lim_{x \rightarrow \infty} \frac{1}{x^2} e^{x^2}, & 2) & \lim_{x \rightarrow 0^+} \ln(x+1) \operatorname{ctg} x, & 3) & \lim_{x \rightarrow +\infty} x \left(\frac{\pi}{2} - \operatorname{arctg} x \right), \\ \text{(d) 1)} & \lim_{x \rightarrow 0^+} \left(\frac{1}{x} - \frac{1}{e^x - 1} \right) & 2) & \lim_{x \rightarrow \infty} (\sqrt{x+3} - \sqrt{x}), & 3) & \lim_{x \rightarrow 0} \left(\operatorname{ctg}^2 x - \frac{1}{x^2} \right), \\ \text{(e) 1)} & \lim_{x \rightarrow \infty} \left(\frac{1}{x} \right)^{\frac{1}{x}}, & 2) & \lim_{x \rightarrow \infty} \left(\frac{\pi}{2} - \operatorname{arctg} x \right)^{\frac{1}{\ln x}}, \\ \text{(f) 1)} & \lim_{x \rightarrow 0^+} (e^{2x} + x)^{\frac{1}{x}}, & 2) & \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^{\frac{1}{1 - \cos x}}, \\ \text{(g) 1)} & \lim_{x \rightarrow 0^+} \left(\frac{1}{x} \right)^x & 2) & \lim_{x \rightarrow 0} (\ln x)^{\frac{1}{x}}. \end{array}$$

10. Izračunati sledeće granične vrednosti:

$$\begin{array}{lll} \text{a)} & \lim_{x \rightarrow a} \frac{\cos x \ln(x-a)}{\ln(e^x - e^a)}, & \text{b)} & \lim_{x \rightarrow \infty} \left(\sqrt{1 + x^2 \ln \frac{ex}{x+1}} - x \right), & \text{c)} & \lim_{x \rightarrow 0} \left(\frac{(1+x)^{\frac{1}{x}}}{e} \right)^{\frac{1}{x}} \\ & & \text{d)} & \lim_{x \rightarrow 0^+} (\operatorname{ctg} x)^{\frac{1}{\ln x}}, & \text{e)} & \lim_{x \rightarrow \infty} \left(x - x^2 \ln(1 + \frac{1}{x}) \right). \end{array}$$

11. Napisati jednačine tangente i normale na grafike funkcija $y = f(x)$ u tački $A(x_0, y_0)$.

$$\begin{array}{ll} \text{(a)} & y = x^3 + \frac{1}{x} - \ln x, \quad A(1, y_0) \\ \text{(b)} & x = \frac{2}{1+t}, \quad y = 1-t, \quad t_0 = (2, y_0) \end{array}$$

- (c) $y = \sqrt{x}$, $A(4, y_0)$
- (d) $y = e^{x^2-1}$, $A(1, y_0)$
- (e) $y = \operatorname{arctg} x^2$, $A(0, y_0)$
- (f) $y = 3^x + 3^{-2x}$, $A(1, y_0)$
- (g) $y = \frac{2x+1}{x+1}$ u tački preseka sa y -osom.

12. Odrediti ugao pod kojim uglom se seku krive

- (a) $y = x^2$ i $y^2 = x$,
- (b) $y = \sqrt{x}$ i $y = \frac{1}{x}$.

13. Detaljno ispitati i nacrtati grafik funkcija:

$$a) \quad y = \frac{x^3+1}{x^2}, \quad b) \quad y = x e^{\frac{1}{x-2}}$$

14. Za sledeće funkcije odrediti domen, ispitati parnost i ispitati postojanje asimptota:

$$a) \quad y = \frac{x^2+3}{x}, \quad b) \quad y = \sqrt{\frac{(x-2)^3}{x}}, \quad c) \quad y = \frac{x^2}{x-1} e^{\frac{1}{x}}$$

$$d) \quad y = \ln \frac{x+3}{1-x}, \quad e) \quad y = \frac{1-\ln x}{1+\ln x}, \quad f) \quad y = \operatorname{arctg} \frac{2x}{x^2-1}$$

15. Za sledeće funkcije odrediti domen, ispitati parnost i odrediti intervale monotonosti i ispitati postojanje ekstremnih vrednosti:

$$a) \quad y = \sqrt[3]{\frac{x^2}{x+1}}, \quad b) \quad y = \ln \frac{x^3}{x^2-3}, \quad c) \quad y = \frac{1-\ln x^2}{1+\ln x^2}$$

16. Aproksimirati sledeće funkcije Maklorenovim polinomom četvrtog stepena

$$a) \quad f(x) = e^x, \quad b) \quad f(x) = \sin x, \quad c) \quad f(x) = \cos x, \quad d) \quad f(x) = \ln(1+x),$$

$$e) \quad f(x) = e^{5x+1}, \quad f) \quad f(x) = \ln(x+e), \quad g) \quad f(x) = \sin^2 x, \quad h) \quad f(x) = (x^2+5)e^{3x}.$$

17. Napisati Maklorenov polinom drugog stepena za sledeće funkcije i izračunati (oceniti gresku?!)

- (a) $f(x) = \ln(x^2+x+1)$, $P_2(\frac{1}{2})=?$
- (b) $f(x) = \sin^3 x + x^2 + x + 1$, $P_2(\frac{1}{3})=?$

Dodati bar jednu primenu, npr $\sqrt[3]{15}$, $\sin 1$ ili nesto slično..... :-)