Is it Possible to Beat the Market? Evidence Using a Statistical Arbitrage and Markowitz Optimization Approach

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Abstract

This paper seeks to incorporate a more complex implementation of the Markowitz Optimization approach by using statistical arbitrage in the form of pairs trading to calculate when to long and short sell stocks. Under this trading strategy, we incorporate a mean reversion assumption to try to create an optimal portfolio that has both entry and exit signals. Through selective criteria when choosing which pairs to consider and by comparing our strategy to simply investing within the S&P 500, we found that our model relatively underperformed the index and our expectations, leading us to hypothesize the downfalls of the conjunction of elements we included in our model.

1. Introduction

With the cost of living rising and the potential for the Social Security Fund to become insolvent, it is not surprising that "in 2024, 62 percent of adults in the United States invested in the stock market" [1]. Investing in stocks presents an opportunity for savings to grow over time; however, a significant challenge emerges, as most individuals lack professional investment expertise, making it difficult to determine optimal fund allocation. In the broadest sense, there are two very different strategies one can take when investing: a passive versus an active approach.

Passive investing is a hands-off approach that seeks to build wealth over time by buying securities that mirror stock indices and holding them for a long period of time. Active investing stands in direct contrast, aiming to strategically buy and sell specific stocks on particular days with the goal of outperforming the stock market [2]. The sad reality is that after fees and expenses are paid, "78–97% of actively managed stock funds failed to beat the indexes they were benchmarked against over ten years" [3]. With this in mind, it may seem like a waste of time to try to outperform the market, but we strongly disagree. Through proper analysis and mathematical modeling techniques we *may* be able to accomplish this goal, but more importantly, we can gain insight into what factors drive returns and inspire future work, even if our strategy is unsuccessful at first.

In this project, we employ a statistical arbitrage trading strategy, sometimes referred to as "pairs trading" [4]. We run cointegration tests on a rolling 6-month window to find pairs of stocks whose prices tend to move together and apply filters to ensure the selected pairs have potential for excess returns while managing risk effectively. Our model then uses a Markowitz optimization technique to allocate capital to each pair of stocks, with the undervalued stock being bought and the overvalued stock being shorted. It was then imperative that we validated our findings using historical stock data, aiming for a trading strategy that could outperform the market in the short term through careful assumptions, mean-reversion, and continuous updates. This would be a step in the right direction of trying to "beat the market", and to understand potential drivers of stock returns, while also growing the wealth of non-professional investors such as ourselves.

2. Description of the Model

Our model makes use of the Yahoo Finance API we worked with in class.
For the stocks under analysis, we began with all 503 stocks in the S&P 500 but excluded those that were delisted, established after 2013, or lacked daily adjusted closing price data for the period from 2014 to 2024. We looked at adjusted closing prices to minimize the risk of our returns being affected by factors such as stock splits [5]. To establish an initial set of stock pairs for recurring tests on shorter timeframes, we ran cointegration tests on the past 10 years of data, selecting pairs with a p-value below 0.05. This approach allowed us to focus on analyzing 6,000 stock pairs every six months, rather than the full 100,000. Looking back 10 years provided a substantial dataset while also helping to mitigate the risk of our results being overly influenced by evolving trends in the securities market. The goal of our model is to find pairs of stocks whose prices fluctuate together over time, so that we can

make trades today if the spreads between the stocks are too large. The prices of stocks in the future are unknown, but luckily we can train our model on prior years to see how it would have performed with data we do in fact have.

First, we needed to calculate which pairs of stocks were co-integrated, a property that describes a state of constant residuals between the two stocks. We chose to focus on such a property as we are interested in the potential mean reversion of these cointegrated stocks - namely stating that a deviation from the constancy of the residuals will result in a return to the long-term mean that the stocks previously held. Known as the mean-reversion property, such an assumption allows us to short and long sell the stocks: going long (buying) an asset that we assume will rise back to its mean and going short (selling) the other asset that we assume will fall back to previously stated mean. Every six months, we identified cointegrated stocks by analyzing the previous two years of price data using the Engle-Granger Two-Step Cointegration Test [6] that calculates residuals, which will be stationary if the stocks are cointegrated [7]. The python method we used returns p-values, where a p-value below the commonly used 0.05 threshold signifies that we can reject the null hypothesis that the two stocks are not cointegrated.

After running this test we were left with typically 500-800 stock pairs, so to narrow this list down further, we ran a series of filters depending on traits of the stocks. Note that some of the thresholds we use were based on outside research, but with a lack of sufficient documentation for other thresholds, we were forced to make reasonable assumptions through testing. First, we filtered the pairs based on volume and volatility. Each stock had to have a daily volume of at least 500,000 shares traded such that our trades do not move prices, and the daily volatility could not exceed 2.5% (eliminating extremely volatile stocks).

Next, we sorted the stocks by their profit-to-risk ratio, calculated by the average spread return (signifying the average distance between the two spreads) divided by the spread volatility (signifying the average fluctuation of the aforementioned distance between the spreads). Due to our desire to long and short the distance, a greater spread return denotes a higher potential profit, while a greater spread volatility introduces significant risk to mean reversion success. In such a way, we are able to assess profitability through these descending ratios. Another characteristic we assess is the correlation between the percentage changes in daily spreads of stock pairs. This helps us avoid trading on pairs that are overly correlated, reducing the risk of exposure to sector-wide trends, such as downturns affecting a single industry. By assuming a maximum allowable correlation of 0.8, we filter out the risk of similar spread movements between pairs using a correlation matrix, aimed at keeping our pairs well-diversified, eliminating idiosyncratic risks.

We now have a portfolio of cointegrated pairs with balanced characteristics with filtered out significant extraneous variables. We calculate z-scores ((the spread on a given day - average of the spreads) / the standard deviation of the spreads) for each pair and keep only the pairs that exceed a z-score threshold of ± 1.75 between 6 and 30 times over the 6-month time frame. In such a way, by only including the pairs that exceed this threshold, we ensure that there will be enough movement to create consistent profitability. The logic behind the maximum trigger count is to prevent our pairs from including stocks that deviate uncontrollably, and could lead to major capital losses. In the end, we cap the final portfolio at 50 stock pairs (including a qualification that certain stocks do not overwhelmingly dominate the list) that we are able to optimize. For each pair, we calculate the daily spread returns, returning the mean of these returns (profit) and a covariance matrix (detailing the relationship of the returns of the stocks within the pairs, allowing us to calculate expected risks and returns), vital characteristics for portfolio optimization. Drawing from the Markowitz model that we learned in class, we took these stocks, minimized the negative Sharpe ratio, and returned the optimal weights of the stocks that reach this goal. Using these stock weights and an arbitrary total amount of capital, we allocate capital proportionally among the stock pairs according to their weights, filtering out pairs once again that our model didn't allocate capital to.

Our model continues now to predict the trading behavior across the subsequent 6 month period, calculating the z-score for the remaining stock pairs existing once again as a threshold for determining profitable trading behavior. To calculate this, we introduce daily entry and exit signals; entry signals can hint that a mean reversion opportunity will occur (basically stating that it is time for a trader to enter the market), while exit signals serve as the opposite, hinting at a stop-loss condition (indicating that it is time for a trader to exit the market) or a take-profit scenario, signaling that the trade has reached its desired level of profitability. Using chosen thresholds (based on testing and background research) [8], we are able to denote how to behave in relation to each stock in the pair, placing an additional qualification on the maximum number of stop losses a pair can have before we stop making future trades on the pair (preventing a pattern of negative profits). The appropriate stock pairs are logged with their defining characteristics, signals, dates, and more. Using the signals, we are able to return the stocks with their corresponding capital amounts, continuing by calculating their individual profits, their percentage return based on the allocated capital, and the overall profits or arbitrage returns (the returns made off of a single asset in the market). To assess the profitability of our portfolio, we compare it to returns made on the S&P 500 if we had invested the same amount of capital on the same days in the broad market index. The total capital allocated is the sum of the long and short allocations made on that day across all arbitrage trades.

3. Analysis

As briefly mentioned above, the goal of this model is to be able to use it to make real time trading decisions, but in order to see its performance, we must have data on stock prices for the period it aims to predict. Thus, our base case gets mean-reverting pairs from the start of 2022 to the start of 2024, and uses these to make trades in the first six months of 2024. We then validate this trading strategy making trades all the way from 2016 to the end of 2023 and compare our hypothetical returns to those that could have been made elsewhere: the S&P 500.

3.1 Trading Performance in Our Base Case

After running our cointegration test, we were left with 780 cointegrated stock pairs from 2022 to the start of 2024. We then performed all of the filters described above and were left with 89 potential stock pairs that we used in our Markowitz Optimization. To keep our trading strategy diversified, we capped the maximum weight of any stock pair in our portfolio at 12.50%. Starting with an initial bankroll of $1,000,000, Figure 1 shows the fraction of capital that was allocated to trades on each stock pair upon entering an arbitrage opportunity. Moreover, in Figure 2, we show the cointegrated nature of a pair of stocks we trade on during this period. Once we had our final pairs and capital allocations, we could then start analyzing the trading behavior that would occur over the next 6 months.

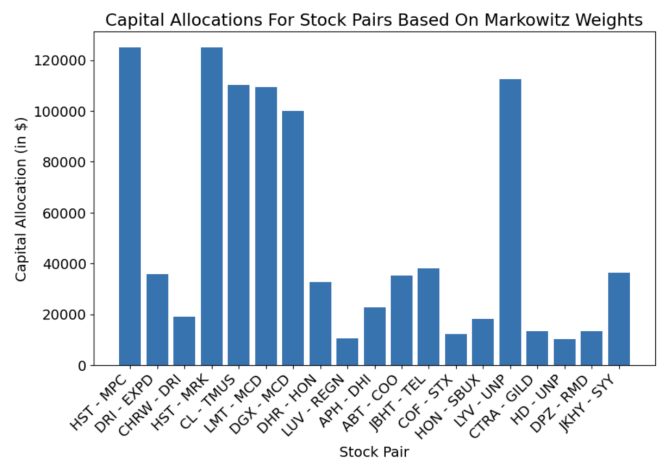

Figure 1: Fraction of our Total Portfolio Cash that will be Invested Upon Entering An Arbitrage Opportunity for each Stock Pair.

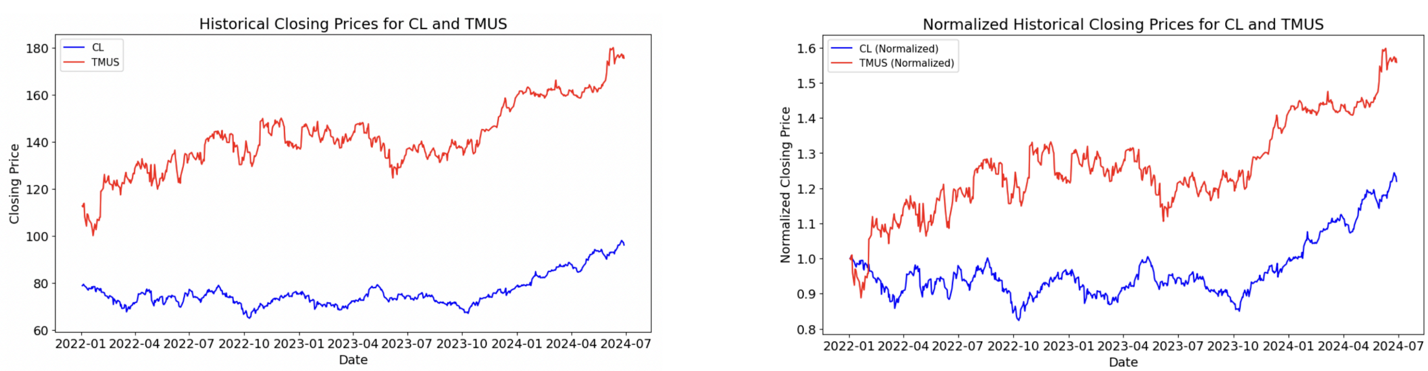

Figure 2: The Adjusted Closing Prices for CL and TMUS from 2022-2024. Notice that the prices of the two stocks tend to fluctuate with one another, suggesting that they are in fact cointegrated stocks allowing the potential for arbitrage opportunities.

After doing extensive testing and evaluations, we found that the optimal threshold for entering an arbitrage opportunity is when the magnitude of the z-score was larger than 1.75 but smaller than 2.75, which acted as our stop loss point, which signified that the spread between the stock prices has become too large and we should exit the trade to prevent further financial losses. Our exit threshold was ±1.5, and with this we simulated trades that our model would have made during the 6-month period at the start of 2024 (see Figure 3).

	Long	Short	Z-Score	Signal	Date	Long Allocation	Short Allocation	Trade Profits	Percentage Return
0	HST	MPC	-2.080034	Enter	2024-01-30	62499.999963	62499.999963	-9139.596957	-7.311678
1	HST	MPC	-2.766588	Exit (Stop Loss)	2024-04-01	N/A	N/A	0.000000	0.000000
2	HST	MPC	-2.712287	Enter	2024-04-12	62499.999963	62499.999963	5817.816770	4.654253
3	HST	MPC	-1.475829	Exit (Take Profit)	2024-05-15	N/A	N/A	0.000000	0.000000
4	EXPD	DRI	1.940600	Enter	2024-02-20	17915.015879	17915.015879	1149.768182	3.208951
5	EXPD	DRI	0.982669	Exit (Take Profit)	2024-03-21	N/A	N/A	0.000000	0.000000
6	CHRW	DRI	-2.396220	Enter	2024-02-01	17915.015879	17915.015879	986.562719	2.753452
7	CHRW	DRI	-1.469000	Exit (Take Profit)	2024-04-11	N/A	N/A	0.000000	0.000000
8	HST	MRK	-1.990848	Enter	2024-02-01	62499.999963	62499.999963	6164.727896	4.931782
9	HST	MRK	-1.460934	Exit (Take Profit)	2024-03-05	N/A	N/A	0.000000	0.000000

Figure 3: A DataFrame providing signals on trading activity, including the signal for the trade (enter, exit to prevent further losses, or exit to take profits), the date of the trade, the capital that will be allocated to each stock, the profits made off of the potential arbitrage opportunity, and the percentage return of the trade. This is just a snapshot of the full trades list, but we see the risk involved with this strategy: if the spreads do not revert back to the mean, then there is the potential to lose significant amounts of capital. Nevertheless, a substantial portion of the trades do seem to lead to positive profits.

During this 6-month time frame, our trading strategy would have made a total profit of roughly $18,049. This profit is heavily influenced by entering trades on pairs that do not exhibit mean reversion in practice, which is an inherent risk involved in this strategy. Figure 4 shows the total returns that are made off of trading each pair in our portfolio. The stocks that did exhibit mean reversion yielded profits, while the stocks that did not follow the underlying assumption of our model lost us money. Some pairs had mixed results, leading to total returns being close to 0 over the entire trading period.

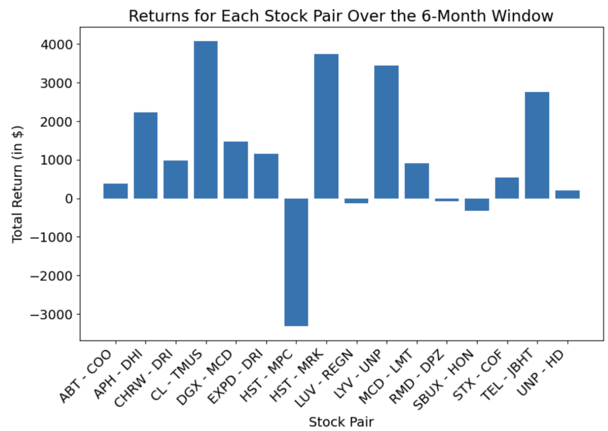

Figure 4: The total returns made by trading various stock pairs across the first 6 months of 2024.

In order to evaluate how well our model performed, we wanted to compare the 6-month returns that we would have made with the returns that would have been made if we instead invested in the S&P 500 on the same active trading days, with the same amount of capital that was otherwise allocated to the pairs. It turns out that the total profit that could have been made investing in the S&P 500 is approximately $60,348. Thus, our trading strategy underperformed the market by roughly $42,299. Upon learning these results, we were disappointed, but then we noticed that the S&P 500 experienced rapid returns (Figure 5) for the first half of 2024. This is a great motivation as to why we ended up running much more extensive backtesting (explained in Section 3.2). Moreover, we also compared the profits we made to profits that could have been made if we invested in a portfolio formed using Markowitz Optimization with randomly chosen stocks from the S&P 500. This alternative strategy would have yielded total returns of $55,456. While this is higher than our returns, we must also consider that with an arbitrage strategy, there are often many days where no trades will be made. Therefore, if we assumed that the capital not being used for an active trade was instead invested in a risk-free security or an index fund, the overall underperformance of our strategy compared to these other baselines would be lowered significantly.

Figure 5: S&P 500 closing prices over time; notice the rapid growth in the first half of 2024.

The entry and exit thresholds we chose, signifying when to exploit arbitrage opportunities, was the largest assumption we had to make and arguably the most important. The defaults we set seemed to yield the highest profits, but we can explore how our profits would have been altered if we used different thresholds instead [9]. Figure 6 does exactly this: we compare the 6-month profits of our model and the S&P 500 when using our default thresholds, when the entry threshold is increased to 2 and the stop-loss threshold is increased to 3, and when the entry threshold is lowered to 1.5, the stop-loss threshold is lowered to 2.5, and the exit threshold is lowered to 1.

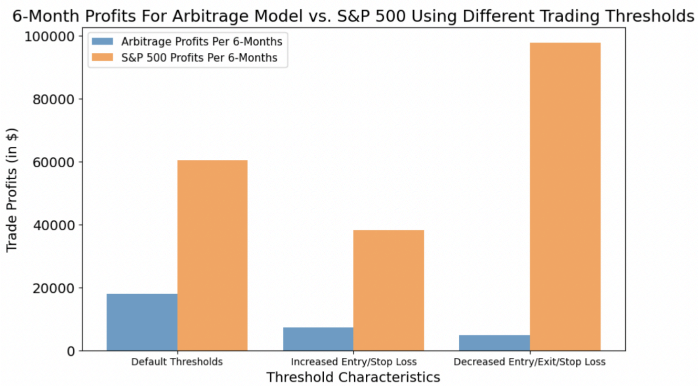

Figure 6: First, notice that the arbitrage profits during the 6-month trading period were the highest under the default thresholds we decided to use. With increased and decreased entry/exit signals, profits went down. This is largely due to trades being made or not being made when an arbitrage opportunity did or did not confidently exist in reality. The S&P 500 profits vary widely, but this is also a direct result of more or less trades being made each day. When the thresholds are increased, there are less trades overall and the opposite is true when the thresholds are lowered. When more trades are made, the S&P 500 profits are higher on average because the S&P 500 performed very well overall during this time period. The opposite is true when fewer trades were made. Nevertheless, our chosen thresholds yield the highest profits throughout this time frame. We use these default thresholds in our analysis across all 6-month periods from 2016-2023 discussed below.

3.2 Trading Performance Across 2016 - End of 2023

Our analysis above was extremely useful in getting preliminary results and helping us determine the optimal parameters to use for our model, but it became obvious that we must test the results over more years. The more hypothetical trading periods we can evaluate, the more we can feel confident about how our model is truly performing. Essentially, by computing profits in 6-month windows from 2016-2023, we minimize the risk of drawing conclusions based off of one period where our cointegrated pairs do not perform well. Figure 7 compares the profits that our strategy would have made during various trading periods compared to the profits that would have been made if we instead invested the same fractional funds on the same days in the S&P 500.

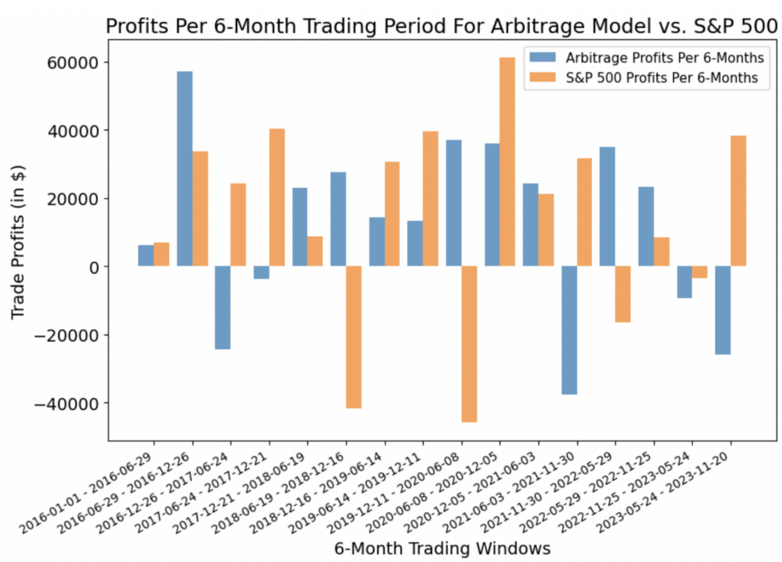

Figure 7: The blue bars represent the profits made during each successive 6-month trading period using our model and the orange represent the profits we could have made by instead investing these funds into the S&P 500. The results were very mixed. We see that during some periods (mainly the second half of 2016, the second half of 2018, the first half of 2020, and the first half of 2022) our model significantly outperforms what would have been made on the index fund. However, in other periods, such as 2017, the second half of 2021, and the second half of 2023 our model is outperformed substantially by the S&P 500. Outside of these extremes, it seems to be neck and neck in terms of what strategy yields higher profits. Finding out why some periods work substantially better for our model could lead to future modifications that would make our strategy more profitable in the long-run. As for right now, the results vary widely, so we would not feel confident implementing this strategy to make trades today, but nevertheless backtesting across multiple years offers more insight into the overall performance of our model compared to just looking at one singular time period.

Overall, across the entire time period when we sum up the profits that were made using our arbitrage strategy we find that we made roughly $196,149 compared to the $237,199 that would have been made by investing in the S&P 500. This would suggest that our model underperformed overall, but however, if we instead look at the cumulative profits at the end of 2022 (excluding 2023), then we actually outperformed the market. Our strategy had amassed profits of $231,337, while the S&P 500 made only $202,389. Therefore, the results of our arbitrage trading strategy are inconclusive. It may be the case that sometimes it allows an investor to outperform the market, but the returns vary too widely for it to be confidently labeled a superior strategy when investing in the stock market.

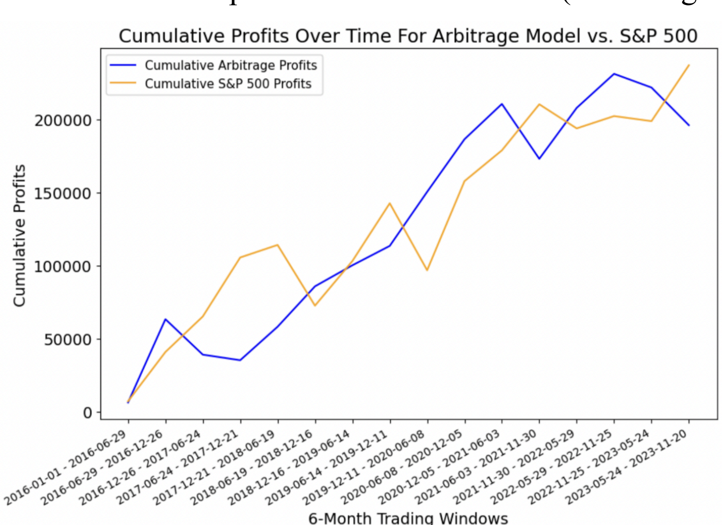

Figure 8: Cumulative profits across the entire validation period for both our model and the alternative profits made on the S&P 500. We see that our model started off

strong, but then experienced a few bad periods. However, from the mid-2019 to mid-2021 our model was outperforming the market. In the most recent years (2023 and the start of 2024) our model has relatively underperformed, bringing into question its practicality and causing us to lose confidence in its overall effectiveness as a strategy that can truly outperform the market.

4. Discussion

As stated above, we see that our statistical arbitrage underperforms compared to S&P 500 overall, although on closer inspection, we see that the S&P 500 is not consistently beating our model. We can brainstorm different reasons for this gap in profitability. Overwhelmingly, the S&P 500 is used for long-term profits - benefiting primarily from long-term growth compounded over the 503 stocks. In contrast, statistical arbitrage and our model shines with short-term trends (looking at a differentiation between two stocks and assuming they will return again). Thus, when looking at long-term profits, it makes sense that our model pales in comparison to the general S&P 500. In addition, the S&P 500 does have risk, but benefits from the immense diversification of the portfolio (an aspect our model lets go as well in our retrieval of pairs trading). Including the lack of benefits we get from diversification, our model also has significantly more risks, such as too highly correlated pairs, unpredictable market effects, incorrect mean reversion assumption, and more. More broadly, the S&P 500 simply works with the market - relying on consistent long-term market growth to have profitable returns; a model that attempts to beat the market and attain excess returns is much more difficult. Lastly, the S&P 500 notably thrives as the market grows, but our model does not take into account such expansion, working in a market-neutral environment that does not parallel real life. A factor we noticed in our model were considerably low profit-to-risk ratios, attaining a maximum of 0.109601, something that correlates with our underperformance related to S&P 500. With such a ratio, it must be paired with very high win rates, with market strategists recommending the ideal risk/reward ratio of around 1:3 (around ⅓) [10]. We see clearly that even the highest pair did not reach this threshold, indicating a high amount of risk associated with our methodology, with correspondingly low returns.

When presented with such results as described, we turned to evaluate our basis of evaluation cointegration, and how successful of a test it is. While there is sound logic and theoretical backing for this concept, we noticed that a key issue with such a test could be its proficiency on shorter time periods. Experts state that "[e]ssentially, cointegration is a long-run concept and hence requires long spans of data to give tests for cointegration much power rather than merely large numbers of observations [11]." Reasons for such a conclusion include that with a shorter time period, it could be harder to identify legitimate cointegrated pairs versus chance market trends and volatility that incorrectly classify the measure of two stocks' cointegration. A change in market dynamics (like market sentiment and liquidity), stocks moving in correlation for a short period of time, and the requirement for a non-stationary market for optimal returns for statistical arbitrage are additional factors that challenge the profitability of our model.

Moving forward, we can think of ways to improve our strategy. Most immediately, we can use guess-and-check and potential available economic literature to adjust the thresholds that we use to filter our stock pairs. Our estimates were made using reasonable assumptions, but are subject to debate, and thus, we may be able to improve our model's performance by adjusting such factors. On the other hand, we could include even more thresholds we had not included, like liquidity, in an attempt to simulate an even more realistic market. We specifically used the Engle-Granger Test to find p-values that correspond to the cointegration of stock pairs, but we can reevaluate using other tests that measure cointegration (one that could even potentially fix the previous issue that illustrated our insufficiencies regarding short vs. long term results). We could test/ensure the stationary nature of our stocks within the time frame we are testing to validate the requirement of cointegrated pairs to be within a stationary time series [12]. We could do further measurements beyond minimizing the Sharpe Ratio to find an even more optimized set of stocks, using other ratios or use an alternative way of optimizing the portfolio. In an attempt to improve profit as well, we could have within our model a place for the extra money to go - investing perhaps in the index fund rather than staying stationary within an account. In a sense, our model and its findings shed light on common claims made by investors stating that specific strategies or intelligence are the sole reasons for the success of their portfolios. We can see here that while there are better strategies, there are additionally a large number of extraneous market variables that affect the overall results.

5. Summary

Trying to beat the stock market will always be a goal of many passionate investors looking to earn excess returns, but as we have shown here, this is not an easy task to accomplish. Here specifically, our pairs trading model sought to identify pairs of stocks whose prices tend to move with one another, opening the door for potential arbitrage opportunities when the spreads between the stocks were unusually large. We were forced to make many assumptions, which we attempted to optimize through testing, but they undoubtedly played a role in influencing our hypothetical returns. In order to get a clearer understanding of our model's performance, we compared the profits it would have made in 6-month trading periods from 2016 to 2023 with that of the S&P 500. Overall, the results varied widely across time periods, but we see that on average, our model tends to slightly underperform in comparison to allocating funds into the S&P 500. Future work on this topic could include using different techniques to calculate cointegrated pairs, further adjusting our chosen parameters, or transitioning to an entirely new model of active investing such as a factor investing model. Nevertheless, by conducting this analysis, we learned the importance of considering risks when investing, making sure the parameters chosen in the model are optimal, and the necessity to concretely validate the results across long time periods. While we may not have found the groundbreaking strategy that will consistently outperform the market yet, we do in fact feel confident that we are one step closer to doing so in the future.

Attribution of Effort

Once again, for this second mini-project we worked really awesome as a team. This time around we really focused on planning out our model and making it clear what we wanted to accomplish. All three of us took turns working on the code, presentation, and report whenever we were available. Specifically, one technique we tried was after getting our initial pairs, we worked in separate Colab notebooks to test what parameter assumptions seemed to work best within the structure of our model. While this section was done individually amongst ourselves, it provided a unique opportunity to compare results and see what was working best. We were then able to combine our work together nicely and met frequently to keep each other updated on the progress we made over the course of the project. Each group member contributed substantially in the completion of this project, and we are all very proud of how much we were able to accomplish given the time constraints. We look forward to continuing to work together as a team on the final project, which very well may relate to the model we have discussed above!

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