Universitatea Babeş-Bolyai, Facultatea de Matematică şi Informatică

Secţia: Informatică engleză

Curs: Dynamical Systems

Primăvara 2024

Seminar 7

1. Let $\lambda \in \mathbb{R}^*$ and $\eta \in \mathbb{R}$ be fixed parameters. Find the unique solution $(x_k)_{k\geq 0}$ of the initial value problem $x_{k+1} = \lambda x_k, \quad x_0 = \eta$.

Note that the solution is a geometric progression. What is the long term behavior of this sequence? Discuss with respect to λ and η. $\diamond$

2. (a) Find solutions of the form $x_k = a\,3^k$ of the difference equation $x_{k+1} = 2x_k + 3^k$, $k \geq 0$. Here we look for $a \in \mathbb{R}$.

(b) Find the general solution of $x_{k+1} = 2x_k + 3^k$.

(c) Find the solution of the IVP $x_{k+1} = 2x_k + 3^k$, $x_0 = 0$. $\diamond$

3. (a) Find solutions of the form $x_k = ak + b$ of the difference equation $x_{k+1} = -5x_k - k$, $k \geq 0$. Here we look for $a, b \in \mathbb{R}$.

(b) Find the general solution of $x_{k+1} = -5x_k - k$.

(c) Find the solution of the IVP $x_{k+1} = -5x_k - k$, $x_0 = -1$. $\diamond$

4. Find the general solution of

(a) $x_{k+2} - 6x_{k+1} + 9x_k = 0$.

(b) $x_{k+2} - 2x_{k+1} + x_k = 0$.

(c) $x_{k+2} + x_{k+1} + x_k = 0$. $\diamond$

5. Find the expression of the Fibonacci sequence

$$x_{k+2} = x_{k+1} + x_k,\ x_0 = 0,\ x_1 = 1.$$

6. Find the linear homogeneous difference equation of minimal order that has the solution $(x_k)_{k\geq 0}$ such that

(a) $x_k = \frac{7}{2^k} - \frac{2}{3^k}$, $k \geq 0$.

(b) $x_k = 7Re(i^k) - 2Im(i^k)$, $k \geq 0$.