```
> with(Student[LinearAlgebra]): with(LinearAlgebra): with(linalg):
> A := Matrix([[0, -2, 0], [1, -2, 0], [0, 0, -2]])
```

$$A := \begin{bmatrix} 0 & -2 & 0 \\ 1 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix} \quad (1)$$

```
> det(A)
```

$$-4 \quad (2)$$

```
> inverse(A)
```

$$\begin{bmatrix} -1 & 1 & 0 \\ -\frac{1}{2} & 0 & 0 \\ 0 & 0 & -\frac{1}{2} \end{bmatrix} \quad (3)$$

```
> Determinant(A);
```

$$-4 \quad (4)$$

```
> CharacteristicPolynomial(A,r);
```

$$r^3 + 4 r^2 + 6 r + 4 \quad (5)$$

```
> Eigenvalues(A);
```

$$\begin{bmatrix} -2 \\ -1 - \mathrm{I} \\ -1 + \mathrm{I} \end{bmatrix} \quad (6)$$

```
> lam,P:=Eigenvectors(A);
```

$$lam, P := \begin{bmatrix} -1 + \mathrm{I} \\ -1 - \mathrm{I} \\ -2 \end{bmatrix}, \begin{bmatrix} 1 + \mathrm{I} & 1 - \mathrm{I} & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (7)$$

```
> lam
```

$$\begin{bmatrix} -1 + \mathrm{I} \\ -1 - \mathrm{I} \\ -2 \end{bmatrix} \quad (8)$$

```
> P
```

$$\begin{bmatrix} 1 + \mathrm{I} & 1 - \mathrm{I} & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (9)$$

```
> u1:=<0,0,1>
```

$$u1 := \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad (10)$$

```
> lam1:=-2
```

$$lam1 := -2 \tag{11}$$

```
> u2:=<1 + I, 1, 0>
```

$$u2 := \begin{bmatrix} 1+\mathrm{I} \\ 1 \\ 0 \end{bmatrix} \tag{12}$$

```
> lam2:= -1+I
```

$$lam2 := -1+\mathrm{I} \tag{13}$$

```
> u3:=<1 - I, 1, 0>
```

$$u3 := \begin{bmatrix} 1-\mathrm{I} \\ 1 \\ 0 \end{bmatrix} \tag{14}$$

```
> lam3:= -1-I
```

$$lam3 := -1-\mathrm{I} \tag{15}$$

```
> A.u1-lam1*u1
```

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{16}$$

```
> A.u2-lam2*u2
```

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{17}$$

```
> A.u3-lam3*u3
```

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{18}$$

```
> P:=<u1|u2|u3>
```

$$P := \begin{bmatrix} 0 & 1+\mathrm{I} & 1-\mathrm{I} \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \tag{19}$$

```
> J:=DiagonalMatrix([lam1,lam2,lam3])
```

$$J := \begin{bmatrix} -2 & 0 & 0 \\ 0 & -1+\mathrm{I} & 0 \\ 0 & 0 & -1-\mathrm{I} \end{bmatrix} \tag{20}$$

```
> A=P.J.P^(-1)
```

(21)

$$\begin{bmatrix} 0 & -2 & 0 \\ 1 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix} = \begin{bmatrix} 0 & -2 & 0 \\ 1 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix} \tag{21}$$

```
> MatrixExponential(t*J)
```

$$\begin{bmatrix} e^{-2t} & 0 & 0 \\ 0 & e^{-t}\cos(t) + I\,e^{-t}\sin(t) & 0 \\ 0 & 0 & e^{-t}\cos(t) - I\,e^{-t}\sin(t) \end{bmatrix} \tag{22}$$

```
> E:=MatrixExponential(t*A)
```

$$E := \begin{bmatrix} e^{-t}\sin(t) + e^{-t}\cos(t) & -2\,e^{-t}\sin(t) & 0 \\ e^{-t}\sin(t) & e^{-t}\cos(t) - e^{-t}\sin(t) & 0 \\ 0 & 0 & e^{-2t} \end{bmatrix} \tag{23}$$

```
> Map(limit,E,t=infinity)
```

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{24}$$

```
> limit(E[1,1],t=infinity)
```

$$0 \tag{25}$$

```
> limit(E[1,2],t=infinity)
```

$$0 \tag{26}$$

```
> limit(E[1,3],t=infinity)
```

$$0 \tag{27}$$

> $limit(E[2, 1], t = \text{infinity})$

$$0 \tag{28}$$

> $limit(E[2, 2], t = \text{infinity})$

$$0 \tag{29}$$

> $limit(E[2, 3], t = \text{infinity})$

$$0 \tag{30}$$

```
> limit(E[3, 1], t = infinity)
```

$$0 \tag{31}$$

> $limit(E[3, 2], t = \text{infinity})$

$$0 \tag{32}$$

> $limit(E[3, 3], t = \text{infinity})$

$$0 \tag{33}$$

>

Exercitiul 2.*******************

```
> P:=Matrix([[-1,0,0,3],[0,5,2,1],[2,0,0,0],[-3,1,-2,-1]]);
```

$$P := \begin{bmatrix} -1 & 0 & 0 & 3 \\ 0 & 5 & 2 & 1 \\ 2 & 0 & 0 & 0 \\ -3 & 1 & -2 & -1 \end{bmatrix} \quad (34)$$

```
> det(P)
```

$$-72 \quad (35)$$

```
> J:=DiagonalMatrix([2,2,-1,0])
```

$$J := \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (36)$$

```
> P.J.P^(-1)
```

$$\begin{bmatrix} 0 & 0 & -1 & 0 \\ \frac{1}{3} & \frac{3}{2} & \frac{47}{12} & \frac{5}{2} \\ 0 & 0 & 2 & 0 \\ -\frac{1}{3} & \frac{1}{2} & -\frac{47}{12} & -\frac{1}{2} \end{bmatrix} \quad (37)$$

```
>
```

Exercitiul 3! **
****

```
> solve(1-x^2=0,x)
```

$$-1, 1 \quad (38)$$

```
> phim2:=rhs(dsolve({diff(x(t),t)=1-x(t)^2,x(0)=-2},x(t)))
```

$$phim2 := \tanh(t - \operatorname{arctanh}(2)) \quad (39)$$

```
> convert(convert(phim2,exp),exp)
```

$$\frac{e^{2t} + 3}{e^{2t} - 3} \quad (40)$$

```
> eval(phim2,t=0)
```

$$-2 \quad (41)$$

```
> solm2:=unapply((exp(2*t)+3)/(exp(2*t)-3),t)
```

$$solm2 := t \to \frac{e^{2t} + 3}{e^{2t} - 3} \quad (42)$$

```
> simplify(diff(solm2(t),t)-1+solm2(t)^2)
```

$$0 \quad (43)$$

```
> solm2(0)
```

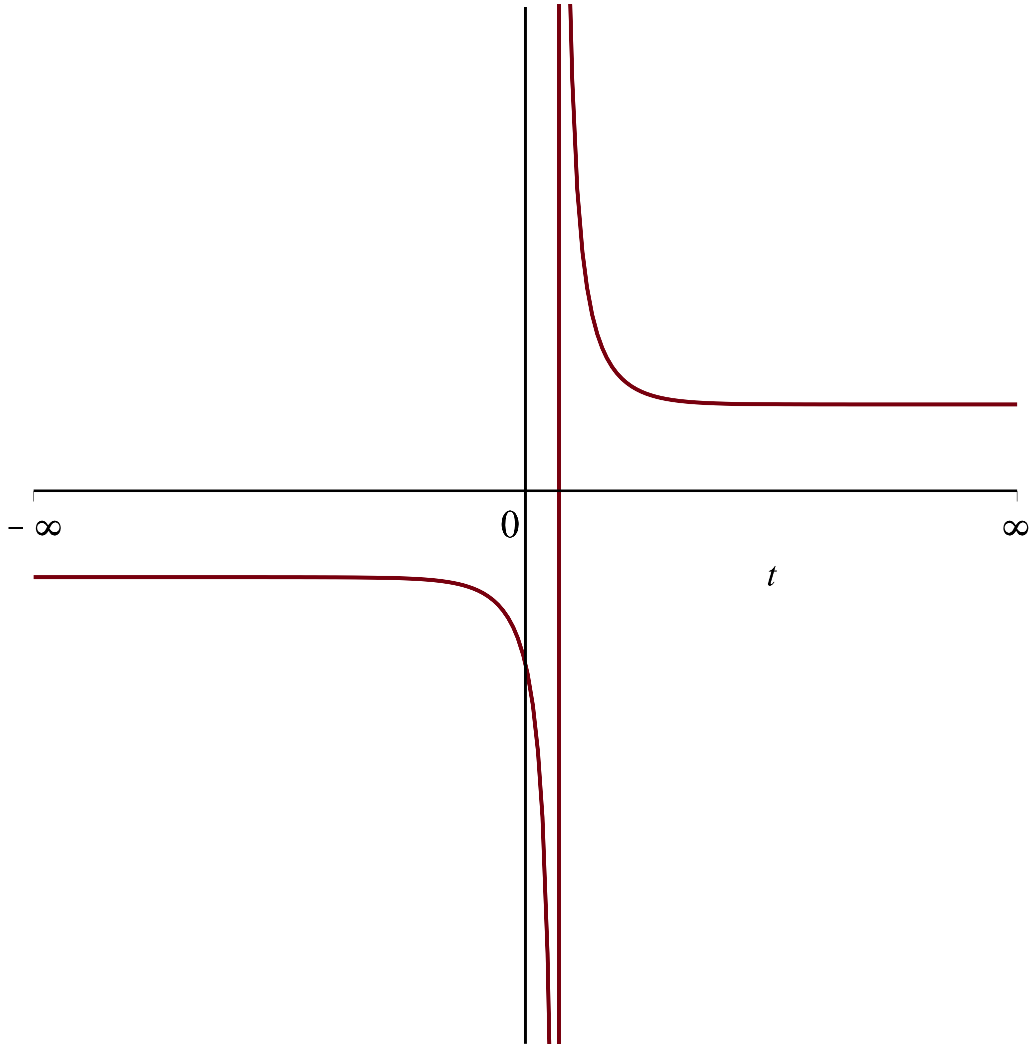
$$-2 \quad (44)$$

```
> solve(3*cosh(t)^2-4,t)
```

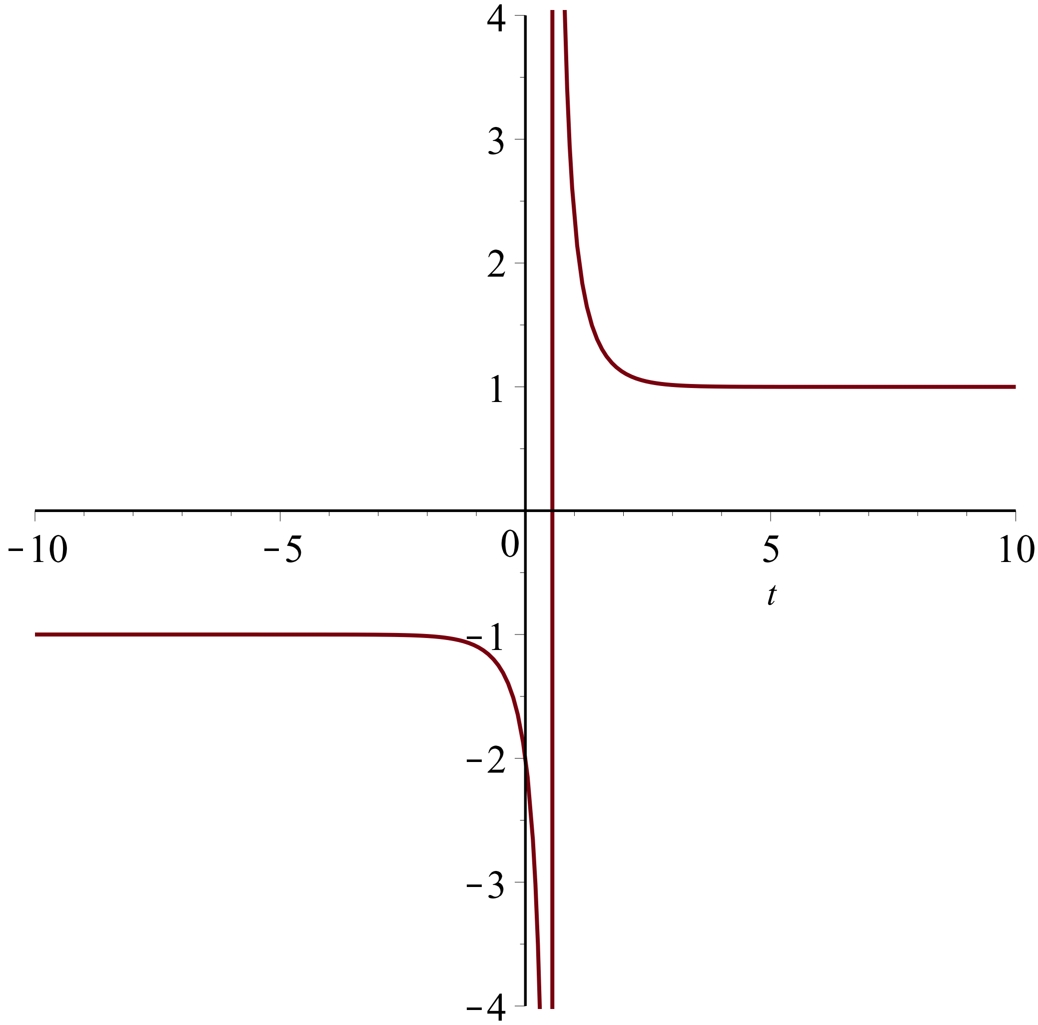
$$(45)$$

$$\operatorname{arccosh}\left(\frac{2}{3}\sqrt{3}\right), \operatorname{arccosh}\left(-\frac{2}{3}\sqrt{3}\right) \tag{45}$$

```
> plot(phim2,t=-infinity..infinity)
```




```
> plot(phim2)
```



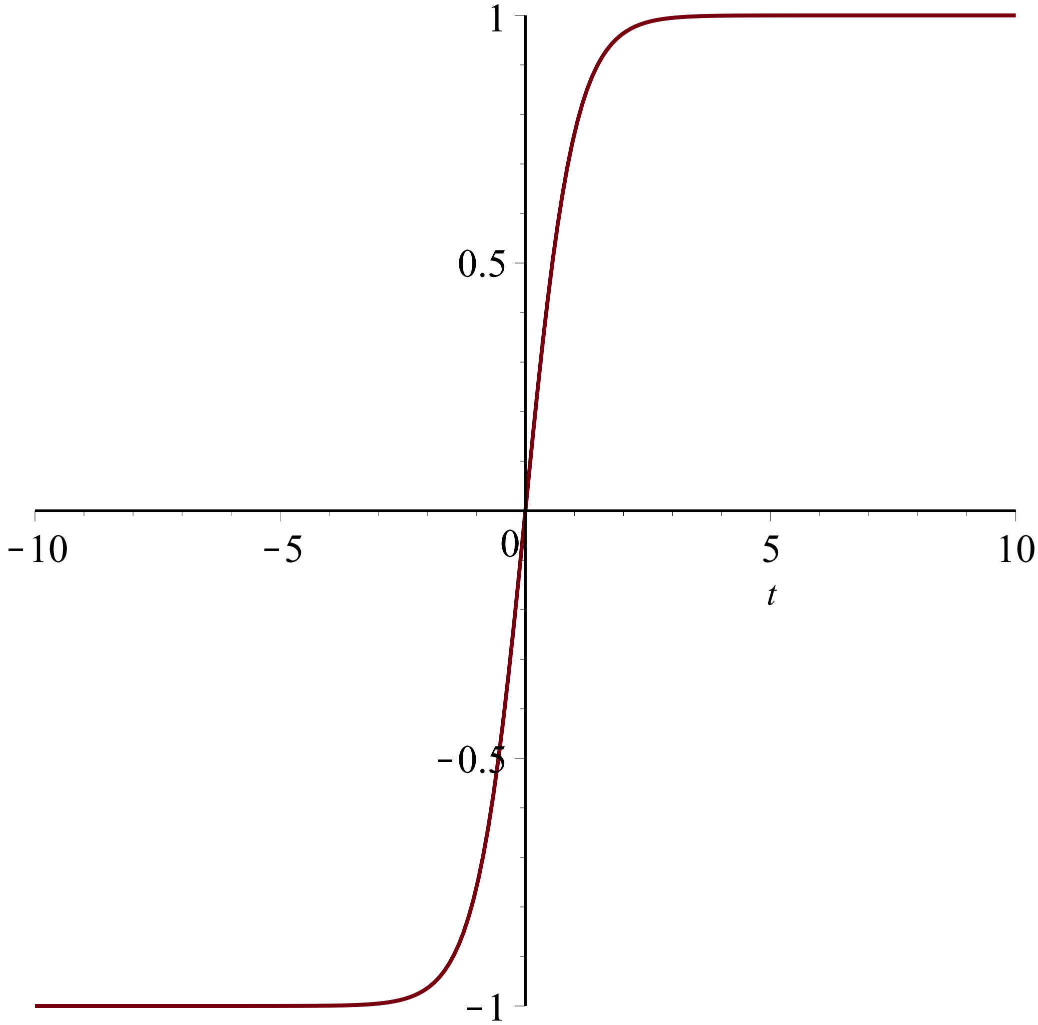

```
> limit(phim2,t=-infinity)
```

$$-1 \tag{46}$$

```
> phi0:=rhs(dsolve({diff(x(t),t)=1-x(t)^2,x(0)=0},x(t)))
```

$$\phi 0 := \tanh(t) \tag{47}$$

```
> plot(phi0)
```



1
0.5
−0.5
−1
−10
−5
0
5
10
t