repres. on 32 bits, SP, m>1.

C	1	D	E	0	0	0	0
1 | 100	0001 1 |	101 1 | 110	0000	0000	0000	0000	

s c = 127 + e (8 bits) m (23 bits)

$c = 10000011_{(2)} = 2^7 + 2^1 + 2^0 = 128 + 2 + 1 = 131$

$\Rightarrow 127 + e = 131 \Rightarrow e = 131 - 127 \Rightarrow e = 4$

$x = -\underset{\text{hidden bit}}{1},101111_{(2)} \cdot 2^4 = -11011,11_{(2)} =$

$= -(2^4 + 2^3 + 2^1 + 2^0 + 2^{-1} + 2^{-2}) =$

$= -(16 + 8 + 2 + 1 + \frac{1}{2} + \frac{1}{4}) = -(27 + \frac{3}{4}) = -27,75$

- Homework presentations

Ex.1 (2) Check the associativity for the ↓ (nor) operation

$\underbrace{(p \downarrow (q \downarrow r)}_{u} \equiv \underbrace{(p \downarrow q) \downarrow r)}_{v}$

$p \downarrow q \stackrel{def}{=} \neg(p \vee q)$

step 1 $u \equiv v$ iff u, v have identical truth table

step 2 Truth table

i	p	q	r	q↓r	p↓(q↓r) (u)	p↓q	(p↓q)↓r (v)
i1	F	F	F	T	F	T	F
i2	F	F	T	F	T	T	F
i3	F	T	F	F	T	F	T
i4	F	T	T	F	T	F	F
i5	T	F	F	T	F	F	T
i6	T	F	T	F	F	F	F
i7	T	T	F	F	F	F	T
i8	T	T	T	F	F	F	F

step 3 u and v don't have identical truth tables, so they are not logically equivalent : $u \not\equiv v$, "↓" is not associative

Ex. 2.2 $U_2 = \underbrace{\neg p \vee \neg(q \wedge r)}_{u} \rightarrow \underbrace{q \wedge \neg p}_{v}$ } Decide the type of U_2

Step 1 Definitions

1. U is consistent iff $\exists i$, $i(u) = T$; i - model of U
2. U is inconsistent iff $\forall i$, $i(u) = F$
3. U is valid (tautology) iff $\forall i$, $i(u) = T$
4. U is contingent iff $\exists i$, $i(u) = T$, i - model of U

   $\exists j$, $j(u) = F$, j - anti-model of U

Step 2 Truth table

i	p	q	r	$\neg p$	$\neg(q \wedge r)$	$\neg p \vee \neg(q\wedge r)$	$q \wedge \neg p$	$U_2 = u \rightarrow v$
i_1	F	F	F	T	T	T	F	F
i_2	F	F	T	T	T	T	F	F
i_3	F	T	F	T	T	T	T	T
i_4	F	T	T	T	F	T	T	T
i_5	T	F	F	F	T	T	F	F
i_6	T	F	T	F	T	T	F	F
i_7	T	T	F	F	T	T	F	F
i_8	T	T	T	F	F	F	F	T

Step 3

U_2 is a contingent formula having 3 models (i_3, i_4, i_8) and 5 anti-models (i_1, i_2, i_5, i_6, i_7)

$i_3 : \{p, q, r\} \rightarrow \{T, F\}$

$i_3(p) = T$, $i_3(q) = T$, $i_3(r) = F$, $i_3(U_2) = T$ → model

$i_1 : \{p, q, r\} \rightarrow \{T, F\}$

$i_1(p) = F$, $i_1(q) = F$, $i_1(r) = T$, $i_1(U_2) = F$ → anti-model

Ex. 3.2. Step 1 $\overbrace{p \rightarrow q}^{u} \models \overbrace{(q \rightarrow r) \rightarrow (p \rightarrow r)}^{v}$

$u \models v$ (v is a logical consequence of u)

iff $\forall i$, $i(u) = T$, we have $i(v) = T$ (All the models of u are also models of v)

Step 2 Truth table

i	p	q	r	p→q	q→r	p→r	v
i_1	F	F	F	T	T	T	T
i_2	F	F	T	T	T	T	T
i_3	F	T	F	T	F	T	T
i_4	F	T	T	T	T	T	T
i_5	T	F	F	F	T	F	F
i_6	T	F	T	F	T	T	T
i_7	T	T	F	T	F	F	T
i_8	T	T	T	T	F	T	T

Step 3

All the models of u ($i_1, i_2, i_3, i_4, i_7, i_8$) are also models of v, so $u \models v$

Ex. 5.2. ⌈DNF $\vee(\wedge l_{ij})$ cubes $\quad A \to B \equiv \neg A \vee B$

CNF $\wedge(\vee l_{ij})$ clauses⌋

$U_2 = (p \xrightarrow{1} q) \wedge (p \wedge q \xrightarrow{2} r) \xrightarrow{3} (p \xrightarrow{4} r)$. Write DNF, CNF

We apply the normalization alg.

$$U_2 \overset{\text{replace } \xrightarrow{1,2,4}}{\equiv} (\neg p \vee q) \wedge (\neg(p \wedge q) \vee r) \xrightarrow{3} (\neg p \vee r)$$

$$\overset{\text{replace } \xrightarrow{3}}{\equiv} \neg((\neg p \vee q) \wedge (\neg(p \wedge q) \vee r)) \vee (\neg p \vee r)$$

$$\overset{\text{De Morgan's}}{\equiv} (p \wedge \neg q) \vee (p \wedge q \wedge \neg r) \vee \neg p \vee r = \text{DNF with 4 cubes}$$

$$\overset{\text{distrib. law}}{\equiv} (\underbrace{p \vee p \vee \neg p \vee r}_{\equiv T}) \wedge (\underbrace{p \vee q \vee \neg p \vee r}_{\equiv T}) \wedge (\underbrace{p \vee \neg r \vee \neg p \vee r}_{T}) \wedge$$

$$\wedge (\underbrace{\neg q \vee p \vee \neg p \vee r}_{T}) \wedge (\underbrace{\neg q \vee q \vee \neg p \vee r}_{T}) \wedge (\underbrace{\neg q \vee \neg r \vee \neg p \vee r}_{T}) =$$

$= \text{CNF with 6 clauses} \equiv T \wedge T \wedge T \wedge T \wedge T \wedge T \equiv T$

* $p \vee \neg p \vee r \equiv T$

$p \wedge \neg p \wedge \neg q \equiv F$

1. U is valid (tautology) if all the clauses of CNF(U) are valid. A clause is valid if it contains a pair of opposite literals.

Ex. 7 2. U is inconsistent if all the cubes of DNF(U) are inconsistent. A cube is inconsistent if it contains a pair of opposite literals.

Models:

i_1 ($i_1 = i_8 = i_4$)

i_2 ($i_2 = i_5 = i_7$)

i_3

i_6

THEORETICAL ASPECTS:

- Normalization algorithm

1) replace $\rightarrow, \leftrightarrow$: $p \rightarrow q \equiv \neg p \vee q$

2) apply DeMorgan's laws

3) apply distributivity

Ex. 8

$\boxed{CNF}$: $(p \vee q) \wedge (\quad \vee \quad \vee \quad) \wedge (\quad \vee \quad) \wedge (\quad)$

Clause 1 : F

$p \vee q \quad F \Rightarrow p, q \rightarrow F$

$r \rightarrow T, F$

Ex. 9 Use definition of deduction to prove the following deductions:

$p \rightarrow r,\ p \vee r \rightarrow q,\ r \vdash q$

THEORETICAL ASPECTS

The sequence $(f_1, \dots, f_m)$ is the deduction of c from the hypothesis (and the axioms)

f_1: $p \rightarrow r$

f_2: $p \vee r \rightarrow q \equiv (\neg p \rightarrow r) \rightarrow q$

f_3: r

f_4: $r \rightarrow (\neg p \rightarrow r)$ obtained from A_1

$f_3, f_4 \vdash_{mp} \neg p \rightarrow r = f_5$ from A_1 with $\begin{cases} U = r \\ V = \neg p \end{cases}$

$f_2, f_5 \vdash_{mp} q = f_6$ $\begin{cases} U = (\neg p \rightarrow r) \\ V = q \end{cases}$

- Axioms:

$A_1: U \to (V \to U)$

$U = r$

$V = \neg p$

MODUS PONENS

$U, U \to V \vdash_{mp} V$

(Ex. 11) Use theorem of deduction & its reverse to prove:

$$\vdash (p \to q) \to \underbrace{((\neg r \vee p) \to (r \to q))}_{U_2}$$

- step 1) APPLY REVERSE OF THEOREM OF DEDUCTION

if $\vdash (p \to q) \to ((\neg r \vee p) \to (r \to q))$

$p \to q \vdash (\neg r \vee p) \to (r \to q)$ then

$p \to q, \neg r \vee p \vdash r \to q$ then

$p \to q, \neg r \vee p, r \vdash q$

- step 2) PROVE DEDUCTION

$f_1: r$

$f_2: \neg r \vee p \equiv r \to p$

$f_3: p \to q$

$f_1, f_2: \vdash_{mp} p$

$f_4: p$

$f_3, f_4: \vdash_{mp} q = f_5$

- Step 3) USE THEOREM OF DEDUCTION

if $U_1, U_2, \dots, U_m \vdash V$

$U_1, U_2, \dots, U_{m-1} \vdash U_m \to V$

if $r, \neg r \vee p, p \to q \vdash q$

then $\neg r \vee p, p \to q \vdash r \to q$

then $p \to q \vdash (\neg r \vee p) \to (r \to q)$

then $\vdash U_2$

* mp = MODUS PONENTS

Ex 12

H_1 : it is not sunny and cold

H_2 : We will go swimming only if sunny.

H_3 : if we don't go swimming, we will take canoe trip.

H_4 : if we take canoe trip, home by sunset.

C : Home by sunset

M - it is sunny   T - it is cold

N - We will go swimming

P - We will take a canoe trip

Q - We will be home by sunset

H_1 : $\neg M \land T$

H_2 : $N \to M \equiv \neg N \lor \neg M$

H_3 : $\neg N \to P$

H_4 : $P \to Q$

C : Q

$f_1 = H_1$

$f_2 = H_2$

$f_3 = H_3$

$f_4 = H_4$

$f_2 \vdash_{mt} \neg N \to \neg M = f_5$

$f_1, f_5 \vdash_{mt} \neg N = f_6$

$f_1 \vdash_{simplified} \neg M$

$f_6, f_3 \vdash_{mp} P = f_7$

$f_7, f_4 \vdash_{mp} Q = C$

MODUS TOLLENS:

$U \to V \vdash \neg V \to \neg U$

SIMPLIFICATION

$U \land V \vdash U$

$U \land V \vdash V$

• PREDICATE LOGIC

Ex 1 Transform the following sentences from natural language into predicate formulas

- VARIABLES
- CONSTANTS
- FUNCTIONS
- PREDICATES

1.2 in a plane there are lines parallel to a constant line d and there are lines perpendicular to d

~~$(\exists x)(\exists y)(parallel(x,d) \wedge perpendicular(y,d))$~~

$(\exists x)(parallel(x,d)) \wedge (\exists y)(perpendicular(y,d))$

- VARIABLES : x, y

CONSTANTS : d

FUNCTIONS : —

$\left(\begin{array}{ll} sum(x,y) = x+y & sum: \mathbb{R} \to \mathbb{R} \\ f: \mathbb{N} \to \mathbb{N}, \ f(x) = x^2 & \end{array} \right)$

PREDICATES :

parallel : $D \times D \to \{T, F\}$

parallel (x,y) = "$x \parallel y$"

perpendicular $D \times D \to \{T, F\}$

perpendicular(x,y): "$x \perp y$"

	$\parallel$	$\perp$
REFLEXIVITY :	$x \parallel x$ ✓	~~$x \perp x$~~
SYMMETRY :	$x \parallel y \Rightarrow y \parallel x$	$x \perp y \Rightarrow y \perp x$
TRANSITIVITY :	$x \parallel y \wedge y \parallel z \Rightarrow x \parallel z$	~~$x \perp y \wedge y \perp z \Rightarrow x \perp z$~~

REFLEXIVITY : $(\forall x)\, parallel(x,x)$

SYMMETRY : $(\forall x)(\forall y)(parallel(x,y) \to parallel(y,x))$

TRANSITIVITY : $(\forall x)(\forall y)(\forall z)(parallel(x,y) \wedge parallel(y,z) \to parallel(x,z))$

(1.8.) The sum of two even numbers is an even number and their product is divisible by 4.

$D = \mathbb{N}$, $4 \in D$, 4 - const.

Predicate symbols: $eq : D \times D \to \{T, F\}$, $eq(x,y) = T$ if $x = y$

$div : D \times D \to \{T, F\}$, $div(x,y) = T$ if $x \vdots y$

Function symbols:

$sum : D \times D \to D$, $sum(x,y) = "x+y"$

$prod : D \times D \to D$, $prod(x,y) = "x \cdot y"$

$(\forall x)(\forall y)(div(x,2) \wedge div(y,2) \to div(sum(x,y),2) \wedge div(prod(x,y),4))$

Properties of div:

- reflexive: $(\forall x)\, div(x,x)$
- transitive: $(\forall x)(\forall y)(\forall z)(div(x,y) \wedge div(y,z) \to div(x,z))$

Properties of sum, prod:

* sum:
  - commutativity: $(\forall x)(\forall y)(eq(sum(x,y), sum(y,x)))$
  - associativity: $(\forall x)(\forall y)(\forall z)[eq(sum(x, sum(y,z)), sum(sum(x,y),z))]$
  - distributivity:

$(\forall x)(\forall y)(\forall z)[eq(prod(x, sum(y,z)), sum(prod(x,y), prod(x,z)))]$

(2.12.) Every animal either likes to eat all plants or all animals much smaller than itself that like to eat some plants.

A - animals ; P - plants

- Predicate symbols:

$sm : A \times A \to \{T, F\}$, $sm(x,y) = T$ if x is much smaller than y

$et : A \times (A \cup P) \to \{T, F\}$, $et(x,y) = T$ if x likes to eat y

$(\forall x)_{x \in A}[(\forall y)_{y \in P}\, et(x,y) \vee (\forall z)_{z \in A}(sm(z,x) \wedge (\exists t)_{t \in P}\, et(z,t) \to et(x,z))]$

- Propr:

- transitivity for sm:

$(\forall x)(\forall y)(\forall z)[sm(x,y) \wedge sm(y,z) \to sm(x,z)]$

Ex. 7.2. $U_2 = (\neg p \vee q) \wedge \neg(\neg q \xrightarrow{1} \neg p)$

We apply the normalization algorithm.

$\overset{\text{replace } \xrightarrow{1}}{\equiv} (\neg p \vee q) \wedge \neg(q \vee \neg p)$

$\overset{\text{De Morgan's}}{\equiv} (\neg p \vee q) \wedge \neg q \wedge p$ = CNF with 3 clauses

$\overset{\text{distrib. laws}}{\equiv} \underbrace{(\neg p \vee \neg q \wedge p)}_{\equiv F} \vee \underbrace{(q \wedge \neg q \wedge p)}_{\equiv F}$ = DNF with 2 cubes

$\equiv F \vee F = F \Rightarrow U_2$-inconsistent

Homework (in 2 weeks) → ex. 5.2

SEMINAR 6

Ex. 8 Write all the anti-models of U_1 using the appropriate THEORETICAL NF.

$U_2 = (q \vee r \to p) \to (p \to r) \wedge q$

Ex. 6 Using the appropriate NF, write all models of U_2

$U_2 = \neg(\neg p \vee q) \vee r \to \neg p \wedge \neg(q \wedge r)$

$U_2 = \neg(\neg(\neg p \vee q) \vee r) \vee \neg p \wedge \neg(q \wedge r)$

$U_2 = ((\neg p \vee q) \wedge \neg r) \vee (\neg p \wedge (\neg q \vee \neg r))$

$U_2 = (\neg p \wedge \neg r) \vee (q \wedge \neg r) \vee (\neg p \wedge \neg q) \vee (\neg p \wedge \neg r)$

Disjunctive form with 4 cubes

- DNF

$i : \{p, q, r\} \to \{T, F\}$

Cube 1 : $\underset{T}{\neg p} \wedge \underset{T}{\neg r}$

$\Downarrow \quad \Downarrow$

$p = F \quad r = F$

i_1	i_2
$i_1(p) = F$	F
$i_1(q) = T$	F
$i_1(r) = F$	F

Cube 2 : $q \wedge \neg r$

i_3	i_4
$i_3(p) = T$	F
$i_3(q) = T$	T
$i_3(r) = F$	F

Cube 3 : $\neg p \wedge \neg q$

i_5	i_6
$i_5(p) = F$	F
$i_5(q) = F$	F
$i_5(r) = F$	T

Cube 4 : $\neg p \wedge \neg r$:

i_7	i_8
$i(p) = F$	F
$i(q) = F$	T
$i(r) = F$	F

(4.2) Using the given interpretations, evaluate the following formulas:

$U = (\exists x)(P(x) \land Q(x)) \to (\exists x)P(x) \lor Q(12)$

interpretation: $i = \langle D, m \rangle$, where $D = \mathbb{N}$

$m(P): \mathbb{N} \to \{T,F\}$, $m(P)(x) = $ "$x \vdots 5$"

$m(Q): \mathbb{N} \to \{T,F\}$, $m(Q)(x) = $ "$x \vdots 7$"

$$v^i(U) = v^i((\exists x)(P(x) \land Q(x))) \to v^i((\exists x)P(x) \lor Q(12))$$

$$= v^i((\exists x)(P(x) \land Q(x))) \to v^i(\exists(x)P(x)) \lor v^i(Q(12))$$

$$= \underset{x \in \mathbb{N}}{(\exists x)}((x \vdots 5) \land (x \vdots 7)) \to \underset{x \in \mathbb{N}}{(\exists x)}(x \vdots 5) \lor (12 \vdots 7)$$

$$= (T \land T) \to T \lor F$$

$$= T \to T = T$$

$\Rightarrow$ i is a model of U

(6.2) $U = (\exists x)(P(x) \to Q(x)) \to ((\exists x)P(x) \to (\exists x)Q(x))$

- Find an anti-model:

$i = \langle D, m \rangle$, where $D = \{2n \mid n \in \mathbb{N}\} \cup \{3\}$

$m(P): D \to \{T,F\}$, $m(P)(x) = $ "x is odd"

$m(Q): D \to \{T,F\}$, $m(Q)(x) = T$ if $x = 5$

$$v^i(U) = v^i((\exists x)(P(x) \to Q(x))) \to v^i((\exists x)P(x) \to (\exists x)Q(x))$$

$$= v^i((\exists x)(P(x) \to Q(x))) \to (v^i(\exists x)P(x) \to v^i(\exists x)Q(x))$$

$$= \underset{x \in D}{(\exists x)}(\neg P(x) \lor Q(x)) \to (\underset{x \in D}{(\exists x)} P(x) \to \underset{x \in D}{(\exists x)} Q(x))$$

$$= (\exists x)_{x \in D}(\text{not "x is odd"} \lor x = 5) \to ((\exists x)\text{"x is odd"} \to (\exists x)\text{"}x = 5\text{"})$$

$$= (T \lor F) \to (T \to F)$$

$$= T \to F = F$$

$\Rightarrow$ i is an anti-model of U

SEMINAR 9

- First order predicate formulas

$H_1: (\forall x)(\overset{ch}{child}(x) \to loves(x, Santa))$

$H_2: (\forall x)(\forall y)(loves(x, \overset{Sa}{Santa}) \wedge reindeer(y) \to loves(x,y))$

$H_3: \overset{re}{reindeer}(Rudolf) \wedge red\_nose(\overset{R}{Rudolf})$

$H_4: (\forall z)(red\_\overset{rn}{nose}(z) \to weird(z) \vee \overset{cl}{clown}(z))$

$H_5: (\forall s)(reindeer(s) \to \neg clown(s))$

$H_6: (\forall t)(weird(t) \to \neg loves(\overset{Sc}{Scrooge}, t))$

$C: \neg child(Scrooge)$

$H_3 \vdash_{simplif} re(R) : f_7$

$H_3 \vdash_{simpl} rn(R) : f_8$

$H_1 \vdash^{[x \leftarrow Sc]}_{univ.inst.} ch(Sc) \to loves(Sc, Sa) \; f_9$

$H_2 \vdash^{[x \leftarrow Sc]}_{univ.inst.} (\forall y)(loves(Sc, Sa) \wedge re(y) \to loves(Sc, y)) \; f_{10}$

$f_{10} \vdash^{[y \leftarrow R]}_{univ.inst.} loves(Sc, Sa) \wedge re(R) \to loves(Sc, R) \; f_{11}$

$H_4 \vdash^{[z \leftarrow R]}_{univ.inst.} rn(R) \to weird(R) \vee cl(R) : f_{12}$

$H_5 \vdash^{[s \leftarrow R]}_{univ.inst.} re(R) \to \neg cl(R) : f_{13}$

$H_6 \vdash^{[t \leftarrow R]}_{univ.inst.} weird(R) \to \neg loves(Sc, R) \; f_{14}$

$f_8, f_{12} \vdash_{mp} weird(R) \vee cl(R) : f_{15}$

$f_7, f_{13} \vdash_{mp} \neg cl(R) : f_{16}$

$f_{15}, f_{16} \vdash_{res.} weird(R) : f_{17}$

$f_{17}, f_{14} \vdash_{mp} \neg loves(Sc, R) : f_{18}$

$f_{11}, f_{18} \vdash_{mt} \neg(loves(Sc, Sa) \wedge re(R)) \equiv \neg loves(Sc, Sa) \vee \neg re(R) : f_{19}$

$f_{19}, f_7 \vdash_{res.} \neg loves(Sc, Sa) : f_{20}$

$f_9, f_{20} \vdash_{mt} \neg ch(Sc) : f_{21}$

The sequence $(H_1, \dots, H_6, f_7, \dots, f_{21} = C)$ is the proof (deduction) of C from the hypothesis.

★ inference rules

$U, U \to V \vdash_{mp} V$

$\neg V, U \to V \vdash_{mt} \neg U$

$U \to V, V \to Z \vdash_{syllog} U \to Z$

$U \wedge V \vdash_{simplif.} U$

$(\forall x)\, U(x) \vdash_{univ. inst.} U(t)$

$\neg U, U \vee V \vdash_{res} V$

Exercises - Semantic Tableaux Method

(1.2.) Decide what kind (consistent, inconsistent, valid) of formula is U_2.

$$U_2 = (p \vee q \to r) \to (p \vee r \to q)$$

★ Decomposition rules

α rules

$$\begin{array}{ccc} A \wedge B & \neg(A \vee B) & \neg(A \to B) \\ | & | & | \\ A & \neg A & A \\ | & | & | \\ B & \neg B & \neg B \end{array}$$

β rules

$$\begin{array}{ccc} A \vee B & & \overset{(\neg A \vee B)}{A \to B} \\ / \quad \backslash & & / \quad \backslash \\ A \qquad B & & \neg A \qquad B \end{array}$$

★ Theoretical result

1. A branch is closed if it contains a pair of opposite literals. $(p, \neg p)$

2. U is consistent if it has an open semantic tableaux, at least one open branch.

3. The open branches of U provide the models of U.

$U_2 = (p \vee q \to r) \to (p \vee r \to q)$

$\beta(1)$

(2) $\neg(p \vee q \to r)$ — $p \vee r \to q$ (3)

$\alpha(2)$: $p \vee q$ (4), $\neg r$ — β for (3): (5) $\neg(p \vee r)$, q ⊙

β for (4): p ⊙, q ⊙ — α for (5): $\neg p$, $\neg r$ ⊙

The semantic tableaux of U_2 is open with 4 open branches, so U_2 is consistent.

$$DNF(U_2) = (p \land \neg r) \lor (q \land r) \lor (\neg r \land \neg p) \lor q$$

$$\equiv (p \land \neg r) \lor (\neg r \land \neg p) \lor q \equiv [\neg r \land \underbrace{(p \lor \neg p)}_{T}] \lor q$$

$$\equiv \neg r \lor q$$

- Cube $\neg r \equiv T$ provides 4 models:

$i_1, i_2, i_3, i_4 : \{p, q, r\} \to \{T, F\}$

$i_1(p) = T \quad i_1(q) = T$

$i_2(p) = T \quad i_2(q) = F \qquad i_{1,2,3,4}(r) = F$

$i_3(p) = F \quad i_3(q) = T$

$i_4(p) = F \quad i_4(q) = F$

- Cube $q \equiv T$ provides 4 models:

$i_5, i_6, i_7, i_8 : \{p, q, r\} \to \{T, F\}$

$i_5(p) = T \quad i_5(r) = T$

$i_6(p) = T \quad i_6(r) = F \qquad i_{5,6,7,8}(q) = T$

$i_7(p) = F \quad i_7(r) = T$

$i_8(p) = F \quad i_8(r) = F$

$i_1 = i_6, \; i_3 = i_8$

U_2 has 6 models:

$$i_1(U_2) = i_2(U_2) = i_3(U_2) = i_4(U_2) = i_5(U_2) = i_7(U_2) = T$$

★ Theoretical result:

4. U is inconsistent if it has a closed semantic tableaux, all branches are closed.

5. U is valid ($\models U$) if $\neg U$ has a closed semantic tableaux.

so

2.2. $U = (p \wedge q \to r) \to (p \to (q \to r))$ (CHECK VALIDITY)

$\neg U = \neg[(p \wedge q \to r) \to (p \to (q \to r))]$ (1) ✓

$\alpha(1)$

$p \wedge q \to r$ (2) ✓

$\alpha(2)$

$\neg(p \to (q \to r))$ (3) ✓

$\alpha(3)$

p

$\neg(q \to r)$ (4) ✓

$\alpha(4)$

q

$\neg r$

$\beta(2)$

$\neg(p \wedge q)$ (5) ✓ | r ⊗

$\beta(5)$

$\neg p$ ⊗ | $\neg q$ ⊗

$\neg U$ has a closed sem. table. with 3 closed branches $(p, \neg p), (q, \neg q), (r, \neg r)$, so $\neg U$ is inconsistent and U is a tautology

3.2.

$\overbrace{\neg p \to (\neg q \to r)}^{U_1}, \overbrace{r \vee q}^{U_2} \models \overbrace{(\neg p \to q) \vee r}^{V}$

$U_1 \wedge U_2 \wedge \neg V$ (1)

$\alpha(1)$

$U_1: \neg p \to (\neg q \to r)$

$U_2: r \vee q$

$\neg V: \neg((\neg p \to q) \vee r)$ (4)

$\alpha(4)$

$\neg(\neg p \to q)$ (5)

$\neg r$

$\alpha(5)$

$\neg p$

$\neg q$

$\beta(3)$

r | q ⊗

$\beta(2)$

p ⊗ | $\neg q \to r$

q ⊗ | r ⊗

$U_1 \wedge U_2 \wedge \neg V$ has a closed semantic tableaux with 4 closed branches $(p, \neg p), (q, \neg q), (r, \neg r), (q, \neg q)$, so $U_1, U_2 \vDash V$

SEMINAR 10

- α rules:

$A \wedge B$	$\neg(A \vee B)$	$\neg(A \to B)$
$\mid$	$\mid$	$\mid$
A	$\neg A$	A
$\mid$	$\mid$	$\mid$
B	$\neg B$	$\neg B$

- β rules:

$A \vee B$	$A \to B$	$\neg(A \wedge B)$
/ \	/ \	/ \
$A \quad B$	$\neg A \quad B$	$\neg A \quad \neg B$

- δ rule:

$(\exists x)\, U(x)$
$\mid$
$U(c)$
c - new const.

- γ rule:

$(\forall x)\, U(x)$
$\mid$
$U(c_1)$ $\quad$ c_1, c_n are all constants on the branch
$\mid$
$U(c_n)$
$\mid$
$(\forall x)\, U(x)$ copy

(7.2.) '$\forall$' - semi distributive over '$\vee$'

Let $U_1 = (\forall x) A(x) \vee (\forall x) B(x) \to (\forall x)(A(x) \vee B(x))$

$U_2 = (\forall x)(A(x) \vee B(x)) \to (\forall x) A(x) \vee (\forall x) B(x)$

$\neg U_1 = \neg\big((\forall x) A(x) \vee (\forall x) B(x) \to (\forall x)(A(x) \vee B(x))\big)$ (1)

$\mid$ α, γ rules for (1)

$(\forall x) A(x) \vee (\forall x) B(x)$ (2)

$\mid$

$\neg((\forall x)(A(x) \vee B(x))) \equiv (\exists x)(\neg A(x) \wedge \neg B(x))$ (3)

$\mid$ δ for (3)
a - new constant

$\neg A(a) \wedge \neg B(a)$ (4)

$\mid$ α rules for (4)

$\neg A(a)$

$\mid$

$\neg B(a)$

$\mid$

β rules for (2)

$(\forall x)A(x)$ (5) $\quad$ $(\forall x)B(x)$ (6)

γ rules for (5) $(x \leftarrow a)$ $\quad$ γ rule for (6)

$A(a)$ $\quad$ $B(a)$

(5) copy $\quad$ (6) copy

$\otimes$ $\quad$ $\otimes$

$\neg U_1$ has a closed semantic tableaux with 2 closed branches $(A(a), \neg A(a))$; $(B(a), \neg B(a))$, so, according to T.s.c. $\models U_1$

★ T.s.c. sem. table:

$\models \mathcal{U}$ iff $\neg \mathcal{U}$ has a closed sem. table.

$\neg U_2 = \neg[(\forall x)(A(x) \vee B(x)) \rightarrow (\forall x)A(x) \vee (\forall x)B(x)]$ (1)

α rules for (1)

$(\forall x)(A(x) \vee B(x))$ (2)

$\neg((\forall x)A(x) \vee (\forall x)B(x)]$ (3) ✓

α rules for (3)

$\neg(\forall x)A(x) \equiv (\exists x)\neg A(x)$ (4) ✓

$\neg(\forall x)B(x) \equiv (\exists x)\neg B(x)$ (5) ✓

δ rules for (4)
a - new const.

$\neg A(a)$ (6)

δ rules for (5)
b - new const.

$\neg B(b)$ (7)

γ rules for (2)
a, b - used for inst.

$A(a) \vee B(a)$ (8)

$A(b) \vee B(b)$ (9)

(2) copy

β rules for (8)

$A(a)$ $\quad$ $B(a)$

$\otimes$

β rules for (9)

$A(b)$ $\quad$ $B(b)$

$\odot$ $\quad$ $\otimes$

$\neg U_2$ has an open semantic tableaux, with 2 closed branches $(A(a), \neg A(a))$, $(B(b), \neg B(b))$, so $\neg U_2$ is inconsistent and $\not\models U_2$.

The open branch provides a model of $\neg U_2$ which is anti model of U_2.

$i = \langle D, m \rangle$

$D = \{a, b\}$

$m(A)(a) = F$, $m(A)(b) = T$

$m(B)(a) = T$, $m(B)(b) = F$

$v^I(M_2) = T$, $v^I(U_2) = F$

Based on the generic model we build a concrete model.

$i_1 = \langle D_1, m_1 \rangle$, $D_1 = \{3, 4\}$

$m_1(A)(x) =$ "x is a perfect square"

$m_1(B)(x) =$ "x is a prime"

5. $H_1, H_2, H_3 \vDash C$ iff $H_1 \wedge H_2 \wedge H_3 \wedge \neg C$ has a closed sem. tabl.

$H_1 : (\forall x)(H(x) \to RC(x))$ — H(x) - x is a hummingbird

$H_2 : \neg(\exists x)(L(x) \wedge LOH(x))$ — RC(x) - x is richly colored

$H_3 : (\forall x)(\neg LOH(x) \to \neg RC(x))$ — LOH(x) - x lives on honey

$C : (\forall x)(H(x) \to \neg L(x))$ — L(x) - x is a large bird

$H_1 \wedge H_2 \wedge H_3 \wedge \neg C$ (1) ✓

| α rule (1)

$H_1 : (\forall x)(H(x) \to RC(x))$ (2)

|

$H_2 : \neg(\exists x)(L(x) \wedge LOH(x)) \equiv (\forall x)\neg(L(x) \wedge LOH(x))$ (3)

|

$H_3 : (\forall x)(\neg LOH(x) \to \neg RC(x))$ (4)

|

$\neg C : (\forall x)(H(x) \to \neg L(x)) \equiv (\exists x)\neg(H(x) \to \neg L(x))$ (5) ✓

| δ rule for (5)
| a - new const.

$\neg(H(a) \to \neg L(a))$ (6) ✓

| α rule for (6)

$H(a)$

|

$L(a)$

|

| γ for (2), $[x \leftarrow a]$

$H(a) \to RC(a)$ (7)

(2) copy

| β rule for (7)

$\neg H(a)$ ⊗ $RC(a)$

| γ for (3), $[x \leftarrow a]$

$\neg(L(a) \wedge LOH(a))$ (8)

(3) copy

| β rule for (8)

$\neg L(a)$ ⊗ $\neg LOH(a)$

| γ rule for (4), $[x \leftarrow a]$

$(\neg LOH(a) \to \neg RC(a))$ (9)

(4) copy

| β rule for (9)

$LOH(a)$ ⊗ $\neg RC(a)$ ⊗

$H_1 \wedge H_2 \wedge H_3 \wedge \neg C$ has a closed semantic tableaux $\Rightarrow$ C is a logical consequence of H_1, H_2, H_3

4) $S_1 = \{ \underbrace{p \vee q \vee r}_{C_1}, \underbrace{\neg q \vee r}_{C_2}, \underbrace{\neg r}_{C_3}, \underbrace{\neg p \vee r}_{C_4} \}$

(T) S is inconsistent if $S \vdash_{Res} \square$

top clause: $\boxed{p \vee q \vee r}$ $\boxed{C_3 = \neg r}$, C_2, C_4

central clauses:

$\boxed{C_5 = p \vee q}$ $\boxed{C_4 = \neg p \vee r}$, C_2

$\boxed{C_6 = q \vee r}$ $\boxed{C_2 = \neg q \vee r}$, C_3

$\boxed{C_7 = r}$ $\boxed{C_3 = \neg r}$

0

side clauses

[Resolution

$C_1 = p \vee \neg q$

$C_2 = \neg p \vee r$

$C_3 = Res_p(C_1, C_2) = \neg q \vee r$

$C_1 = p$

$C_2 = \neg p$

$Res_p(C_1, C_2) = \square$

Linear resolution

$S_1 \vdash^{lin}_{Res} \square$, sv

S_1 is inconsist.

SEMINAR 11

• Resolution proposition

(5.2.) $S = \{ \overbrace{p \vee q \vee r}^{C_1}, \overbrace{\neg p \vee q}^{C_2}, \overbrace{\neg p \vee \neg q}^{C_3} \}$

I. central clauses

top clause

$\boxed{C_1 = p \vee q \vee r}$ $\boxed{C_2 = \neg p \vee q}$, C_3, C_3 — side clauses

$\boxed{C_4 = Res_p(C_1, C_2) = q \vee r}$, $\boxed{C_3 = \neg p \vee \neg q}$

$\boxed{C_5 = Res_q(C_3, C_4) = \neg p \vee r}$ $\boxed{C_1 = p \vee q \vee r}$

$\boxed{C_6 = Res_p(C_5, C_1) = q \vee r = C_4}$ → Process is blocked

II. $\boxed{C_1 = p \vee q \vee r}$, C_2, $\boxed{C_3 = \neg p \vee \neg q}$, C_3

$\boxed{C_7 = Res_p(C_1, C_3) = q \vee \neg q \vee r}$ → Tautology

process blocked

III. $\boxed{C_1 = p \vee q \vee r}$, C_2, C_3, $\boxed{C_3 = \neg p \vee \neg q}$

$\boxed{C_8 = Res_q(C_1, C_3) = p \vee r \vee \neg p}$

process is blocked

C : After a linear complete search using the backtracking algorithm, without the derivation of the empty clause, we conclude that S is consistent.

(1.2.) $U_2 = (B \to A) \wedge (C \to A) \to (B \wedge C \to A)$

• Using general resolution prove that the following formula is a theorem.

(T2) $\vdash U$ iff $CNF(\neg U) \vdash_{Res} \square$

$\neg U_2 = \neg[(B \overset{1}{\to} A) \wedge (C \overset{2}{\to} A) \overset{3}{\to} (B \wedge C \overset{4}{\to} A)] \equiv$ We apply the normalization algorithm

$\overset{replace\ 1,2,4}{\equiv} \neg[(\neg B \vee A) \wedge (\neg C \vee A) \to (\neg(B \wedge C) \vee A)]$

$\overset{replace\ 3}{\equiv} \neg[\neg[(\neg B \vee A) \wedge (\neg C \vee A)] \vee [\neg(B \wedge C) \vee A]]$

$\equiv \underbrace{(\neg B \vee A)}_{C_1} \wedge \underbrace{(\neg C \vee A)}_{C_2} \wedge \underbrace{B}_{C_3} \wedge \underbrace{C}_{C_4} \wedge \underbrace{\neg A}_{C_5}$ => CNF

$S = \{ C_1, \dots, C_5 \}$, $S \overset{?}{\vdash}_{Res} \square$

$C_6 = Res_A (C_1, C_5) = \neg B$

$C_7 = Res_B (C_3, C_6) = \square$

(3.2.) $H_1 : L \wedge \neg G \to M \equiv \neg L \vee G \vee M : C_1$

$H_2 : J \to L \equiv \neg J \vee L : C_2$

$H_3 : J_t \to L \equiv \neg J_t \vee L : C_3$

$H_4 : G_s \wedge \neg G \equiv \overset{C_4}{G_s} \wedge \neg G : C_5$

$H_5 : J_t \equiv J_t \quad C_6$

$C : M$, $\neg C = \neg M \quad C_7$

$H_1, H_2, H_3, H_4, H_5 \overset{?}{\vdash} C$

$S = \{ C_1, C_2, \dots, C_7 \}$

$S \overset{?}{\vdash}_{Res} \square$

$C_8 = Res_{J_t} (C_3, C_6) = L$

$C_9 = Res_L (C_1, C_2) = \neg J \vee G \vee M$

$C_{10} = Res_G (C_1, C_5) = \neg L \vee M$

$C_{11} = Res_L (C_8, C_{10}) = M$

$C_{12} = Res_M (C_{11}, C_7) = \square$

$CNF(H_1 \wedge \dots \wedge H_5 \wedge \neg C) \vdash_{Res} \square$

so $H_1, \dots, H_5 \vdash C$

[(6.2.)] Prove the inconsistency using lock resolution.

$S = \{ \underbrace{q \vee \neg r}_{C_1}, \underbrace{\neg q \vee \neg p \vee \neg r}_{C_2}, \underbrace{\neg q \vee p \vee \neg r}_{C_3}, \underbrace{r}_{C_4} \}$

a)

$C_1 = \underset{(1)}{q} \vee \underset{(2)}{\neg r}$

$C_2 = \underset{(3)}{\neg q} \vee \underset{(4)}{\neg p} \vee \underset{(5)}{\neg r}$

$C_3 = \underset{(6)}{\neg q} \vee \underset{(7)}{p} \vee \underset{(8)}{\neg r}$

$C_4 = \underset{(9)}{r}$

$C_5 = Res_q^{lock} (C_1, C_2) = \underset{(2)}{\neg r} \vee \underset{(4)}{\neg p}$

$C_6 = Res_r^{lock} (C_4, C_5) = \neg p_{(4)}$

$C_7 = Res_q^{lock} (C_1, C_3) = \underset{(7)}{p} \vee \underset{(2)}{\neg r}$

$C_8 = Res_p^{lock} (C_7, C_4) = p$

$C_9 = Res_p^{lock} (C_8, C_6) = \square \Rightarrow S$ - inconsistent

b) $C_1 = \underset{(2)}{q} \vee \underset{(1)}{\neg r}$

$C_2 = \underset{(5)}{\neg q} \vee \underset{(4)}{\neg p} \vee \underset{(3)}{\neg r}$

$C_3 = \underset{(8)}{\neg q} \vee \underset{(7)}{p} \vee \underset{(6)}{\neg r}$

$C_4 = \underset{(9)}{r}$

$S^0 = \{ C_1, \dots, C_4 \}$

$S^1 = \{ Res^{lock}(C_i, C_j),\ C_i \in S^0,\ C_j \in S^0 \}$

$C_5 = Res_r^{lock}(C_1, C_4) = \underset{(2)}{q}$

$C_6 = Res_r^{lock}(C_2, C_4) = \underset{(5)}{\neg q} \vee \underset{(4)}{\neg p}$

$C_7 = Res_r^{lock}(C_3, C_4) = \underset{(8)}{\neg q} \vee \underset{(7)}{p}$

$S^1 = \{ C_5, C_6, C_7 \}$

$S^2 = \{ Res^{lock}(C_i, C_j) \mid C_i \in S^1,\ C_j \in S^0 \cup S^1 \}$

$C_8 = Res_p^{lock}(C_6, C_7) = \neg q$

$S^3 = \{ Res^{lock}(C_i, C_j) \mid C_i \in S_2,\ C_j \in S^0 \cup S^1 \cup S^2 \}$

$C_9 = Res_q^{lock}(C_8, C_5) = \square$

$\square \in S^3 \Rightarrow S$ is inconsistent

7.2. ★ Check consistency:

$S_2 = \{ \overbrace{p \vee \neg r}^{C_1}, \overbrace{q \vee r}^{C_2}, \overbrace{\neg p \vee \neg r}^{C_3}, \overbrace{\neg q \vee \neg r}^{C_4} \}$

I. $S^1 = \{ Res_p^{lock}(C_i, C_j),\ C_i \in S^0,\ C_j \in S^0 \} = \{ C_5, C_6 \}$

$C_5 = Res_p^{lock}(C_1, C_3) = \neg r$

$C_6 = Res_r^{lock}(C_2, C_4) = q \vee \neg q \equiv T$

$S^2 = \{ Res^{lock}(C_i, C_j),\ C_i \in S^1,\ C_j \in S^0 \cup S^1 \}$

$C_7 = Res_r^{lock}(C_5, C_2) = q$

$S^3 = \{ Res^{lock}(C_i, C_j) \mid C_i \in S^2,\ C_j \in S^0 \cup S^1 \cup S^2 \}$

$S^3 = \emptyset \Rightarrow S_2$ - consist.

After a complete search, using the level saturation strategy, without the derivation of the empty clause, we conclude that S_2 is consistent.

SEMINAR 12 Boolean fct. (ex. 2.2)

(1.2.) Write the premex, Skolem and clausal forms

$U_2 = (\exists x)(\forall y)\{(\exists z)\,\neg P(z) \vee (\exists u)[R(x,u) \to (\forall z)\neg Q(u,z)]\}$

replace→ $\equiv (\exists x)(\forall y)\{(\exists z)\,\neg P(z) \vee (\exists u)[\neg R(x,u) \vee (\forall \underset{t}{z})\neg Q(u,\underset{t}{z})]\}$

$\equiv (\exists x)(\forall y)\{(\exists z)\,\neg P(z) \vee (\exists u)[\neg R(x,u) \vee (\forall t)\neg Q(u,t)]\}$

extract quantifiers $\equiv (\exists x)(\forall y)\{(\exists z)\,\neg P(z) \vee (\exists u)(\forall t)[\neg R(x,u) \vee \neg Q(u,t)]\}$

• Premex forms:

$U_2^{P_1} = \underbrace{(\exists x)(\forall y)(\exists z)(\exists u)(\forall t)}_{prefix}\underbrace{\{\neg P(z) \vee [\neg R(x,u) \vee \neg Q(u,t)]\}}_{matrix}$

$U_2^{P_2} = (\exists x)(\forall y)(\exists u)(\forall t)(\exists z)\{\neg P(z) \vee [\neg R(x,u) \vee \neg Q(u,t)]\}$

• Skolem forms: $[x \leftarrow a,\ z \leftarrow f(y),\ u \leftarrow g(y)]$

$U_2^{P_1}$ provides $U_2^{S_1} = (\forall y)(\forall t)\{\neg P(f(y)) \vee [\neg R(a, g(y)) \vee \neg Q(g(y),t)]\}$

$[x \leftarrow a,\ u \leftarrow f(y),\ z \leftarrow g(y)]$

$U_2^{P_2}$ provides $U_2^{S_2} = (\forall y)(\forall t)\{\neg P(g(y,t)) \vee [\neg R(a, f(y)) \vee \neg Q(f(y),t)]\}$

• Clausal forms:

$U_2^{C_1} = \neg P(f(y)) \vee [\neg R(a, g(y)$

$U_2^{C_2} =$

(2.2.) Are the literals from the following pair unifiable? if yes, find their most general unifier.

$x,y,z \in Var$, $a,b \in Const.$, $f,g \in F_1$, $h \in F_2$, $P \in P_3$

★ $l_1 = P(a, x, f(g(y)))$, $l_2 = P(y, f(z), f(z))$

$\theta_i = \varepsilon$

• iteration 1:

$\lambda_i = [y \leftarrow a]$, $\theta = \theta\lambda \cdot [y \leftarrow a]$

$\theta(l_1) = P(a, x, f(g(a)))$, $\theta(l_2) = P(a, f(z), f(z))$

• iteration 2:

$\lambda := [x \leftarrow f(z)]$, $\theta = \theta\lambda = [y \leftarrow a][x \leftarrow f(z)] = [y \leftarrow a, x \leftarrow f(z)]$

$\theta(l_1) = P(a, f(z), f(g(a)))$

$\theta(l_2) = P(a, f(z), f(z))$

• iteration 3:

$\lambda := [z \leftarrow g(a)]$

$\theta = \theta\lambda = [y \leftarrow a, x \leftarrow f(z)][z \leftarrow g(a)] = [y \leftarrow a, x \leftarrow f(z), z \leftarrow g(a)] =$

$= mgu(l_1, l_2)$

$\theta(l_1) = \theta(l_2) = P(a, f(g(a)), f(g(a)))$

common sintactic forms of l_1 and l_2

★ $l_1 = P(x, g(f(a)), f(b))$

$l_2 = P(f(y), z, z)$

$\theta := \varepsilon$

• it 1:

$\lambda := [x \leftarrow f(y)]$

$\theta := \theta\lambda = [x \leftarrow f(y)]$

$\theta(l_1) = P(f(y), g(f(a)), f(b))$

$\theta(l_2) = P(f(y), z, z)$

• it 2:

$\lambda := [z \leftarrow g(f(a))]$

$\theta := \theta\lambda = [x \leftarrow f(y)][z \leftarrow g(f(a))] = [x \leftarrow f(y), z \leftarrow g(f(a))]$

$\theta(l_1) = P($

$\theta(l_2) = P($

The terms $f(b)$ and $g(f(a))$ are not unifiable because none of them is a variable. We cannot unify the arg. on the last pos., so l_1 and l_2 are not unifiable.

(3.2.) Prove the inconsistency using lock resolution:

$$S_2 = \{ \overbrace{{}_{(2)}P(x) \lor {}_{(1)}\neg Q(x)}^{C_1}, \overbrace{{}_{(4)}\neg P(a) \lor {}_{(3)}R(x)}^{C_2}, \overbrace{{}_{(5)}Q(x)}^{C_3}, \overbrace{{}_{(6)}W(z)}^{C_4}, \overbrace{{}_{(8)}\neg R(y) \lor {}_{(7)}\neg W(y)}^{C_5} \}$$

$$C_6 = Res^{lock}(C_1, C_3) = {}_{(2)}P(x)$$

$$C_7 = Res^{lock}_{\theta_1 = [z \leftarrow y]}(C_4, C_5) = {}_{(8)}\neg R(y) \quad , \ \theta_1 = mgu(W(z), W(y)) = [z \leftarrow y]$$

$$C_8 = Res^{lock}_{\theta_2 = [x \leftarrow y]}(C_2, C_7) = \neg P(a) \quad , \ \theta_2 = mgu(R(x), R(y)) = [x \leftarrow y]$$

$$C_9 = Res^{lock}_{\theta_3 = [x \leftarrow a]}(C_6, C_8) = \square \quad , \ \theta_3 = mgu(P(a), P(x)) = [x \leftarrow a]$$

$S_2 \vdash^{lock}_{Res} \square$, so S_2 is inconsistent

(4.2.) Using a refinement of predicate resolution, prove the semidistributivity:

$$\vdash U_1 : (\exists x)(P(x) \land Q(x)) \to (\exists x)P(x) \land (\exists x)\, Q(x)$$

$$\nvdash U_2 : (\exists x)P(x) \land (\exists x)Q(x) \to \exists(x)(P(x) \land Q(x))$$

(T) $\vdash U_1$ iff $(\neg U_1) \vdash_{Res} \square$

$$\neg U_1 = \neg\left((\exists x)(P(x) \land Q(x)) \xrightarrow{1} (\exists x)P(x) \land (\exists x)Q(x)\right)$$

$$\overset{\text{replace } \neg(\to)}{\equiv} (\exists x)(P(x) \land Q(x)) \land \neg[\underset{y}{(\exists x)} \underset{y}{P(x)} \land \underset{z}{(\exists x)} \underset{z}{Q(x)}]$$

$$\equiv (\exists x)(P(x) \land Q(x)) \land \left((\forall y)\neg P(y) \lor (\forall z)\neg Q(z)\right)$$

$$\equiv (\exists x)(\forall y)(\forall z)(P(x) \land Q(x) \land (\neg P(y) \lor \neg Q(z)))$$

$$(\neg U_1)^S = (\forall y)(\forall z)(P(a) \land Q(a) \land (\neg P(y) \lor \neg Q(z)))$$

$[x \leftarrow a]$, a - skolem const.

$$(\neg U_1)^C = P(a) \land Q(a) \land (\neg P(y) \lor \neg Q(z))$$

$S_1 = \quad C_1 = P(a)$

$C_2 = Q(a)$

$C_3 = \neg P(y) \lor \neg Q(z)$