Universitatea Babeş-Bolyai, Facultatea de Matematică şi Informatică

Secţia: Informatică engleză

Curs: Dynamical Systems

Primăvara 2024

Seminar 4

1. For each $k > 0$ we consider the differential equation

$$\dot{x} = -k\,(x - 21),$$

which is the model of Newton for cooling processes, here $x(t)$ being the temperature of a cup of tea at time t.

(a) Find its flow. (b) An experiment revealed the following fact. A cup of tea with initial temperature of $49°C$ has a temperature of $37°C$ after 10 minutes. Find the initial temperature of a cup of tea such that after 20 minutes the tea has $37°C$. $\diamond$

Theorem 1 *Let $f \in C^1(\mathbb{R})$ and $\eta^* \in \mathbb{R}$ be such that $f(\eta^*) = 0$.*
If $f'(\eta^*) < 0$ then η^* is an attractor equilibrium point of $\dot{x} = f(x)$.
If $f'(\eta^*) > 0$ then η^* is a repeller equilibrium point of $\dot{x} = f(x)$.

2. Let $0 < c < 1$ be a parameter and consider the scalar dynamical system $\dot{x} = x(1 - x) - cx$.

a) Find its equilibria and study their stability using the linearization method.

b) Represent its phase portrait.

c) When $x(t) > 0$ is considered to be the density of fish in a lake, and $0 < c < 1$ to be the rate of fishing, try to predict the fate of the fish from the lake interpreting the theoretical result obtained at a) and b). $\diamond$

3. Represent the phase portrait of the scalar dynamical system $\dot{x} = x - x^3$. Find $\varphi(t, -1)$ and $\varphi(t, 0$ and justify. Specify the properties of the functions $\varphi(t, -2)$, $\varphi(t, 3)$ and, respectively, $\varphi(t, -0.5)$.

4. Represent the phase portrait of the scalar dynamical systems

a) $\dot{x} = x - x^3 + 1$; b) $\dot{x} = -x^3$; c) $\dot{x} = x^3$; d) $\dot{x} = -x^2$. Try to use the linearization method.