

Dynamical Systems Laboratory Test

1. Find a polynomial solution of the differential equation

$$u'' + 5u' - 7u = x^2 + 5x - 7,$$

then plot its graph on the interval $[-10, 5]$, and finally compute, for it and for its first order derivative, approximate values in $\pi\sqrt{2}$. Note that the unknown is the function denoted by $u(x)$.

2. a) Plot the planar curve of parametric equations $x = \cos(2t) + 3\sin(2t)$, $y = \sin(2t)$ for $t \in [0, 4]$.

b)* Can $\varphi(t) = (\cos(2t) + 3\sin(2t), \sin(2t))$, $t \in \mathbb{R}$, be a solution of a linear planar system $\dot{X} = AX$?

3. Introduce the matrix A corresponding to the linear system $x' = -x - 3y$, $y' = 3x - y$. Compute its determinant and eigenvalues. Compute e^{tA} . Specify the type and stability of the linear system.

4. Consider the nonlinear system $x' = x - xy$, $y' = -y + xy$. Is $(0, 0)$ the unique equilibrium point? Is $(0, 0)$ a hyperbolic equilibrium point?

5. Consider the map $f : [0, 1] \rightarrow \mathbb{R}$, $f(x) = \frac{2x}{1+x}$. Find its fixed points. Describe the behavior of the sequence of iterations starting with 1, 0.2 and 0.7.