Universitatea Babeş-Bolyai, Facultatea de Matematică şi Informatică

Secţia: Informatică engleză

Curs: Dynamical Systems

Primăvara 2024

Lecture 10 - List of problems

1. We consider the linear planar system

$$\dot{x} = -x + 2y, \quad \dot{y} = 2x - 4y.$$

Find its general solution using two methods:

(a) the reduction method;

(b) the characteristic equation method. $\diamond$

2. We consider the nonlinear planar system

$$\dot{x} = -y - x(x^2 + y^2), \quad \dot{y} = x - y(x^2 + y^2).$$

(a) Justify that the equilibrium point $(0,0)$ is not hyperbolic.

(b) Represent the phase portrait (using polar coordinates).

(c) Reading the phase portrait, deduce that the equilibrium point $(0,0)$ is an attractor. $\diamond$

3. We consider the nonlinear planar system

$$\dot{x} = -y + x(1 - x^2 - y^2), \quad \dot{y} = x + y(1 - x^2 - y^2).$$

(a) Check that $\varphi(t,1,0) = (\cos t, \sin t)$ for all $t \in \mathbb{R}$.

(b) Represent the phase portrait (using polar coordinates).

(c) Reading the phase portrait, deduce that the equilibrium point $(0,0)$ is a repeller. There is an attractor in this phase portrait? $\diamond$

4. Let $\omega, \mu > 0$ be fixed parameters. The second order nonlinear equation $\theta'' + \mu\theta' + \omega^2 \sin\theta = 0$ describes the oscillations of a simple pendulum. Justify that the equilibrium solution $\theta(t) = 0$ for all $t \in \mathbb{R}$ is an attractor. $\diamond$