Elipsa $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, \quad b=\sqrt{a^2-c^2}.$

focare, situate la distanţa $2c$ unul de altul,

$$\varepsilon=\frac{c}{a}=\sqrt{1-\left(\frac{b^2}{a^2}\right)},$$

raze focale ale unui punct $M(x,y)$ de pe elipsă

$$\begin{cases} r_1 = a+\varepsilon x, \\ r_2 = a-\varepsilon x. \end{cases}$$

1. **Dacă** $\frac{x_0^2}{a^2}+\frac{y_0^2}{b^2}<1$, punctul este **interior** elipsei.
2. **Dacă** $\frac{x_0^2}{a^2}+\frac{y_0^2}{b^2}=1$, punctul este **pe conturul** elipsei.
3. **Dacă** $\frac{x_0^2}{a^2}+\frac{y_0^2}{b^2}>1$, punctul este **exterior** elipsei.

tangenta la elipsa
panta k

$$y=kx\pm\sqrt{a^2k^2+b^2}$$

a - semiaxa mare
b- semiaxa mica

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ec dreptei care trece prin 2 puncte (coarda)

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}$$

$$d(M_0,\Delta)=\frac{|Ax_0+By_0+C|}{\sqrt{A^2+B^2}}.$$

Atunci, distanța d dintre aceste două drepte este:
$d=\frac{|C_2-C_1|}{\sqrt{A^2+B^2}}$

conditia de tangenta este: delta = 0

$A_{\Delta MNP}=\frac{1}{2}\cdot|\Delta| \qquad \Delta = \begin{vmatrix} x_M & y_M & 1 \\ x_N & y_N & 1 \\ x_P & y_P & 1 \end{vmatrix}$

panta $\qquad ax+by+c=0$

$$m=-\frac{a}{b} \qquad m_{\text{perpendiculară}}=-\frac{1}{m}$$

$M_3\left(\frac{3}{2},-\frac{3}{4}\right).$
Prin urmare, aria triunghiului $M_1M_2M_3$ va fi dată de

$$\mathcal{A}=\pm\frac{1}{2}\begin{vmatrix} \frac{6+\sqrt{2}}{2} & \frac{-6-9\sqrt{2}}{4} & 1 \\ \frac{6-\sqrt{2}}{2} & \frac{-6+9\sqrt{2}}{4} & 1 \\ \frac{3}{2} & -\frac{3}{4} & 1 \end{vmatrix}=\pm\frac{1}{128}\Big|$$

Hiperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1, \quad b=\sqrt{c^2-a^2}.$

ec asimptote
$y=\pm\frac{b}{a}x.$

$$\varepsilon=\frac{c}{a}=\sqrt{1+\left(\frac{b^2}{a^2}\right)}.$$

ec tangenta
$y=kx\pm\sqrt{a^2k^2-b^2}.$

Parabola $\quad y^2=2px.$

tg (punct dat) $\quad yy_0=p(x+x_0).$

tg (panta data) $\quad y=kx+\frac{p}{2k}.$

Elipsoid $\quad \frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1,$

elipsoid de rotatie
in jurul Oz $\quad \frac{x^2+y^2}{a^2}+\frac{z^2}{c^2}=1.$

Plan tg cu punctul
A0 pe elipsoid $\quad \frac{xx_0}{a^2}+\frac{yy_0}{b^2}+\frac{zz_0}{c^2}=1.$

Con de gr 2 $\quad \frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=0,$

Generatoarele
intersectează elipsa: $\begin{cases} \frac{x^2}{a^2}+\frac{y^2}{b^2}=1, \\ z=c. \end{cases}$

intersectii cu planele de coordonate:

$$\begin{cases} \frac{x^2}{a^2h^2/c^2}+\frac{y^2}{b^2h^2/c^2}=1 \\ z=h \end{cases}$$

$$\begin{cases} \frac{x}{a}\pm\frac{z}{c}=0, \\ y=0, \end{cases} \qquad \begin{cases} \frac{z^2}{c^2h^2/b^2}-\frac{x^2}{a^2h^2/b^2}=1 \\ y=h \end{cases}$$

$$\begin{cases} \frac{y}{b}\pm\frac{z}{c}=0, \\ x=0, \end{cases} \qquad \begin{cases} \frac{y^2}{b^2h^2/a^2}-\frac{z^2}{c^2h^2/a^2}=1 \\ x=h \end{cases}$$

plan tg in punct $\quad \frac{xx_0}{a^2}+\frac{yy_0}{b^2}-\frac{zz_0}{c^2}=0.$

con de rotatie (o
suprafata conica si o
suprafata de rotatie ın
jurul axei Oz) $\quad \frac{x^2+y^2}{a^2}-\frac{z^2}{c^2}=0,$

Hiperboloidul cu o panza

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1,$$

Inttersectii cu planele de coodonate

$$\begin{cases} z=h, \\ \dfrac{x^2}{\left(a\sqrt{\frac{h^2}{c^2}+1}\right)^2}+\dfrac{y^2}{\left(b\sqrt{\frac{h^2}{c^2}+1}\right)^2}=1, \end{cases}$$

<|a| $\begin{cases} x=h, \\ \dfrac{y^2}{\left(b\sqrt{1-\frac{h^2}{a^2}}\right)^2}-\dfrac{z^2}{\left(c\sqrt{1-\frac{h^2}{a^2}}\right)^2}=1. \end{cases}$

=|a| $\begin{cases} x=h, \\ \dfrac{y}{b}\pm\dfrac{z}{c}=0. \end{cases}$

>|a| $\begin{cases} x=h, \\ \dfrac{z^2}{\left(c\sqrt{\frac{h^2}{a^2}-1}\right)^2}-\dfrac{y^2}{\left(b\sqrt{\frac{h^2}{a^2}-1}\right)^2}=1. \end{cases}$

plan tg in punct $\dfrac{xx_0}{a^2}+\dfrac{yy_0}{b^2}-\dfrac{zz_0}{c^2}=1.$

Generatoare rectilinii $\begin{cases} \lambda\left(\dfrac{x}{a}+\dfrac{z}{c}\right)=\mu\left(1+\dfrac{y}{b}\right), \\ \mu\left(\dfrac{x}{a}-\dfrac{z}{c}\right)=\lambda\left(1-\dfrac{y}{b}\right), \end{cases}$

$$\begin{cases} \alpha\left(\dfrac{x}{a}+\dfrac{z}{c}\right)=\beta\left(1-\dfrac{y}{b}\right), \\ \beta\left(\dfrac{x}{a}-\dfrac{z}{c}\right)=\alpha\left(1+\dfrac{y}{b}\right), \end{cases}$$

hip cu 1p de rotatie $\dfrac{x^2}{a^2}+\dfrac{y^2}{a^2}-\dfrac{z^2}{c^2}=1.$

Cilindru hiperbolic $\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1,$

$$\frac{xx_0}{a^2}-\frac{yy_0}{b^2}=1.$$

H cu 2p

=-1

xOy poate fi:
multmea vida, daca |h| < |c|;
un punct, daca h = ±c;
elipsa:

$$\begin{cases} z=h, \\ \dfrac{x^2}{\left(a\sqrt{\frac{h^2}{c^2}-1}\right)^2}+\dfrac{y^2}{\left(b\sqrt{\frac{h^2}{c^2}-1}\right)^2}=1, \end{cases}$$

de rotatie $\dfrac{x^2+y^2}{a^2}-\dfrac{z^2}{c^2}=-1.$

plan tg in pct: -||- = -1

Cilindru eliptic $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1,$

$$\frac{xx_0}{a^2}+\frac{yy_0}{b^2}=1.$$

de rotatie

$$x^2+y^2=a^2.$$

Cilindru parabolic $y^2=2px,$

intersectii cu planele de coord

$$\begin{cases} y^2=2px, \\ z=h. \end{cases}$$

Această intersecție este:

- mulțimea vidă, dacă $h<0$;
- axa Oz, dacă $h=0$;

$$\begin{cases} y=\pm\sqrt{2ph}, \\ x=h, \end{cases}$$

$$\begin{cases} x=\dfrac{h^2}{2p}, \\ y=h. \end{cases}$$

Paraboloid eliptic $\dfrac{x^2}{p}+\dfrac{y^2}{q}=2z,$

intersectia xOy poate fi:
multimea vida, dacaˇ h < 0;
un punct (originea) daca h = 0;
elipsa:

$$\begin{cases} z=h, \\ \dfrac{x^2}{2ph}+\dfrac{y^2}{2qh}=1, \end{cases}$$

$$\begin{cases} x=h, \\ y^2=2qz-\dfrac{qh^2}{p}. \end{cases}$$

de rotatie

$$x^2+y^2=2pz.$$

Plan tg

$$\frac{xx_0}{p}+\frac{yy_0}{q}=p(z+z_0).$$

Paraboloid hiperbolic $\dfrac{x^2}{p}-\dfrac{y^2}{q}=2z.$

plan tg $\dfrac{xx_0}{p}-\dfrac{yy_0}{q}=z+z_0.$

$$\begin{cases} \lambda\left(\dfrac{x}{\sqrt{p}}-\dfrac{y}{\sqrt{q}}\right)=2\mu z, \\ \mu\left(\dfrac{x}{\sqrt{p}}+\dfrac{y}{\sqrt{q}}\right)=\lambda, \end{cases} \qquad \begin{cases} \alpha\left(\dfrac{x}{\sqrt{p}}+\dfrac{y}{\sqrt{q}}\right)=2\beta z, \\ \beta\left(\dfrac{x}{\sqrt{p}}-\dfrac{y}{\sqrt{q}}\right)=\alpha, \end{cases}$$

Suprafete cilindrice

$$(\Delta)\ \begin{cases} P_1(x,y,z)=0, \\ P_2(x,y,z)=0, \end{cases}$$

$$(C)\ \begin{cases} F(x,y,z)=0, \\ G(x,y,z)=0. \end{cases}$$

$$(G_{\lambda,\mu})\ \begin{cases} P_1(x,y,z)=\lambda, \\ P_2(x,y,z)=\mu, \end{cases}$$

$$\begin{cases} P_1(x,y,z)=\lambda, \\ P_2(x,y,z)=\mu, \\ F(x,y,z)=0, \\ G(x,y,z)=0. \end{cases}$$

$$\varphi(\lambda,\mu)=0.$$

Suprafete conice

$$(V)\ \begin{cases} P_1(x,y,z)=0, \\ P_2(x,y,z)=0, \\ P_3(x,y,z)=0, \end{cases}$$

$$(C)\ \begin{cases} F(x,y,z)=0, \\ G(x,y,z)=0. \end{cases}$$

$$(G_{\lambda,\mu})\ \begin{cases} P_1(x,y,z)=\lambda P_3(x,y,z), \\ P_2(x,y,z)=\mu P_3(x,y,z), \end{cases}$$

$$\begin{cases} P_1(x,y,z)=\lambda P_3(x,y,z), \\ P_2(x,y,z)=\mu P_3(x,y,z), \\ F(x,y,z)=0, \\ G(x,y,z)=0. \end{cases}$$

$$\varphi\left(\frac{P_1(x,y,z)}{P_3(x,y,z)},\frac{P_2(x,y,z)}{P_3(x,y,z)}\right).$$

Suprafete conoide

$$(\Delta)\ \begin{cases} P_1(x,y,z)=0, \\ P_2(x,y,z)=0, \end{cases}$$

$$(\Pi)\ P(x,y,z)=0,$$

$$(G_{\lambda,\mu})\ \begin{cases} P_1(x,y,z)=\lambda P_2(x,y,z), \\ P(x,y,z)=\mu, \end{cases}$$

$$(C)\ \begin{cases} F(x,y,z)=0, \\ G(x,y,z)=0. \end{cases}$$

$$\begin{cases} P_1(x,y,z)=\lambda P_2(x,y,z), \\ P(x,y,z)=\mu, \\ F(x,y,z)=0, \\ G(x,y,z)=0. \end{cases}$$

$$\varphi\left(\frac{P_1(x,y,z)}{P_2(x,y,z)},P(x,y,z)\right)=0.$$

Suprefete de rotatie

$$(\Delta)\ \frac{x-x_0}{l}=\frac{y-y_0}{m}=\frac{z-z_0}{n}.$$

$$(G_{\lambda,\mu})\ \begin{cases} (x-x_0)^2+(y-y_0)^2+(z-z_0)^2=\lambda^2, \\ lx+my+cz=\mu. \end{cases}$$

$$(C)\ \begin{cases} F(x,y,z)=0, \\ G(x,y,z)=0, \end{cases}$$

$$\begin{cases} (x-x_0)^2+(y-y_0)^2+(z-z_0)^2=\lambda^2, \\ lx+my+cz=\mu, \\ F(x,y,z)=0, \\ G(x,y,z)=0. \end{cases}$$

$$\varphi\left(\sqrt{(x-x_0)^2+(y-y_0)^2+(z-z_0)^2},lx+my+cz\right)=0.$$