Seminar 2
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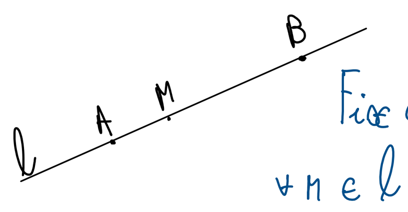
① Intersection of lines
② The vector equation of a line



$A, B \in l$

Fix an arbitrary point O

$\forall M \in l, \vec{OM} = \vec{OA} + \vec{AM} \Rightarrow \exists \lambda \in \mathbb{R}, \vec{AM} = \lambda \vec{AB} \Rightarrow$

$\Rightarrow \vec{OM} = \vec{OA} + \lambda \vec{AB} = \vec{OA} + \lambda(\vec{AO} + \vec{OB}) = \lambda \vec{OB} + (1-\lambda)\vec{OA}$

2.5) BOB' be a proper angle (B, O, B') not colinear

$A \in [OB]$, $A' \in [O, B']$

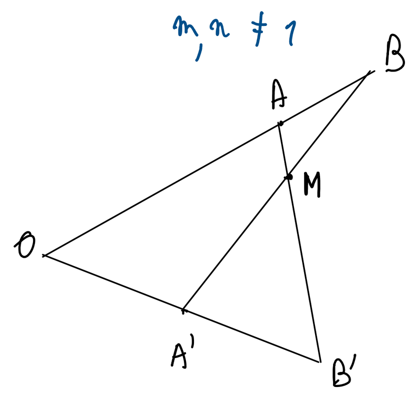
Let $m, n \in \mathbb{R}$, $\vec{OB} = m\vec{OA}$

$\vec{OB'} = n\vec{OA'}$

Let $AB' \cap A'B = \{M\}$

$AA' \cap BB' = \{N\}$

$m, n \neq 1$



Show that $\vec{OM} = m\frac{1-n}{1-mn}\vec{OA} + n\frac{1-m}{1-mn}\vec{OA'}$;

$\vec{ON} = m\frac{n-1}{nm}\vec{OA} + n\frac{m-1}{nm}\vec{OA'}$;

$M \in AB' \Rightarrow \exists \lambda \in \mathbb{R}, \vec{OM} = \lambda\vec{OA} + (1-\lambda)\vec{OB'}$

$\vec{OA} = \vec{v}, \vec{OA'} = \vec{w}$ $\quad \vec{OM} = \lambda\vec{v} + (1-\lambda)n\vec{w}$

$\vec{v}, \vec{w}$ lin. indep

$M \in A'B \Rightarrow \exists \mu \in \mathbb{R}, \vec{OM} = \mu\vec{OA'} + (1-\mu)\vec{OB}$

$\vec{OM} = \mu\vec{w} + (1-\mu)m\vec{v}$

$\lambda\vec{v} + (1-\lambda)n\vec{w} = \mu\vec{w} + (1-\mu)m\vec{v} \Rightarrow \begin{cases} \lambda = (1-\mu)m \\ \mu = (1-\lambda)n \end{cases} \Leftrightarrow \begin{cases} \lambda = m - \mu m \\ \mu = n - \lambda n \end{cases} \Leftrightarrow \begin{cases} \lambda = m - \mu m \\ \mu = n - mn + \mu mn \end{cases} \Leftrightarrow \begin{cases} \lambda = m - \mu m \\ \mu = n\frac{1-m}{mn} \end{cases}$

$N \in [AA'] \Rightarrow \exists \alpha \in \mathbb{R}, \vec{OM} = \alpha\vec{OA} + (1-\alpha)\vec{OA'} = \alpha\vec{v} + (1-\alpha)\vec{w}$

$N \in [BB'] \Rightarrow \exists \beta \in \mathbb{R}, \vec{OM} = \beta\vec{OB} + (1-\beta)\vec{OB'} = \beta m\vec{v} + (1-\beta)n\vec{w}$

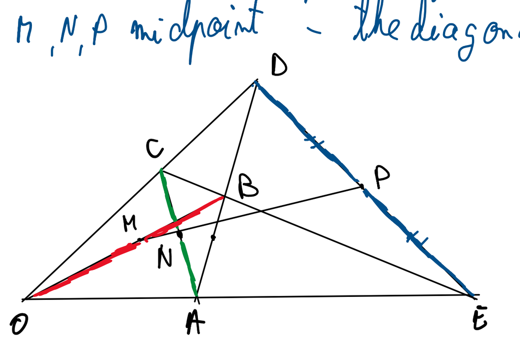
$\alpha\vec{v} + (1-\alpha)\vec{w} = \beta m\vec{v} + (1-\beta)n\vec{w} \Rightarrow \begin{cases} \alpha = m\beta \\ 1-\alpha = (1-\beta)n \end{cases} \Leftrightarrow \begin{cases} \alpha = m\beta \\ 1 - m\beta = n - n\beta \end{cases} \Leftrightarrow \begin{cases} \alpha = m\beta \\ \beta(n-m) = n-1 \end{cases} \Leftrightarrow \begin{cases} \alpha = m\beta \\ \beta = \frac{n-1}{n-m} \end{cases} \Leftrightarrow \begin{cases} \alpha = m\frac{n-1}{n-m} \\ \beta = \frac{1-n}{mn} \end{cases}$, $1 - \alpha = 1 - \frac{mn-m}{n-m} = \frac{n-m-mn+m}{n-m} = \frac{n-mn}{n-m} = n\left(\frac{m-1}{m-n}\right)$

$\vec{OM} = m\frac{n-1}{nm}\vec{v} + n\frac{mn-n+1}{mn}\vec{w}$

$\vec{OM} = m\frac{n-1}{n-m}\vec{OA} + n\frac{m-1}{m-n}\vec{OA'}$

② $OAEBDC$ complete quadrilateral

M, N, P midpoint - the diagonals of $[OB], [AC], [ED]$



Let $\vec{OC} = \vec{v}, \vec{OA} = \vec{w}$

$\vec{OD} = m\vec{OC} = m\vec{v}$

$\vec{OE} = n\vec{OA} = n\vec{w}$

$\vec{OB} = m\frac{1-n}{1-mn}\vec{OC} + n\frac{(1-m)}{1-mn}\vec{OA}$

$\vec{MN} = \vec{MO} + \vec{ON} = -\frac{1}{2}\vec{OB} + \frac{1}{2}\vec{OA} + \frac{1}{2}\vec{OC}$

$\vec{MN} = -\frac{1}{2}\left(m\frac{(1-n)}{1-mn}\vec{OC} + \frac{n(1-m)}{1-mn}\vec{OA}\right) + \frac{1}{2}\vec{OA} + \frac{1}{2}\vec{OC}$

$\vec{MN} = \frac{1}{2}\left(1 - \frac{m(1-n)}{1-mn}\right)\vec{OC} + \frac{1}{2}\left(1 - \frac{n(1-m)}{1-mn}\right)\vec{OA}$

$= \frac{1}{2}\frac{1-\cancel{mn}-m+\cancel{mn}}{1-mn}\vec{v} + \frac{1}{2}\frac{1-\cancel{mn}-n+\cancel{nm}}{1-mn}\vec{w}$

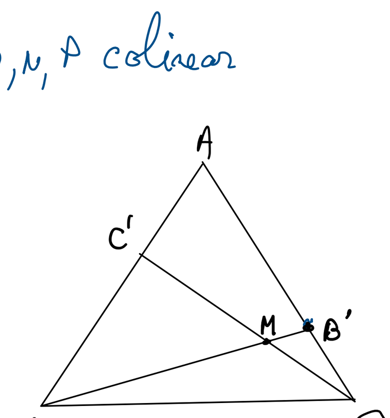
$= \frac{1}{2}\frac{1-m}{1-mn}\vec{v} + \frac{1}{2}\frac{1-n}{1-mn}\vec{w}$

$\vec{NP} = \vec{NO} + \vec{OP} = -\frac{1}{2}\vec{OA} + \frac{1}{2}(\vec{OE} + \vec{OD})$

$\vec{NP} = -\frac{1}{2}\vec{w} + \frac{1}{2}m\vec{v} + \frac{1}{2}n\vec{w} = \frac{1}{2}\vec{v}(m-1) + \frac{1}{2}\vec{w}(n-1)$

$\vec{MN} = \frac{1}{mn-1}\vec{NP} \Rightarrow M, N, P$ colinear

③ ΔABC



$C' \in [AB]$

$B' \in [AC]$

$\vec{AC'} = \lambda\vec{BC'}$

$\vec{AB'} = \mu\vec{CB'}$

$M = BB' \cap CC'$

$\vec{OM} = \frac{\vec{OA} - \lambda\vec{OB} - \mu\vec{OC}}{1-\lambda-\mu}$

$\left.\begin{array}{l} \vec{AB} = \vec{v} \\ \vec{AC} = \vec{w} \\ \exists \Delta ABC \end{array}\right\} \Rightarrow \vec{v}, \vec{w}$ lin. indep

$M \in BB' \Rightarrow \exists \alpha \in \mathbb{R}, \quad \vec{OM} = \alpha\vec{OB} + (1-\alpha)\vec{OB'}$

$\vec{OB} = \vec{OA} + \vec{AB} = \vec{OA} + \vec{v}$

$\vec{OB'} = \vec{OA} + \vec{AB'} = \vec{OA} + \vec{w}$

$\vec{OM} = \alpha\vec{OA} + \alpha\vec{v} + (1-\alpha)\vec{OA} + (1-\alpha)\vec{w}$

$\vec{OM} = \vec{OA} + \alpha\vec{v} + (1-\alpha)\vec{w}$