**How do you work out the sum of divisors?**

Imagine you wish to work out the sum of divisors of the number 72. It would not take long to list the divisors, and then find their sum: 1 + 2 + 3 + 4 + 6 + 8 + 9 + 12 + 18 + 24 + 36 + 72 = 195.

However, this method would become both tedious and difficult for large numbers like 145600. Fortunately, there is a simple and elegant method at hand.

Let σ(*n*) be the sum of divisors of the natural number, *n*.

For any prime, *p*: σ(*p*) = *p* + 1, as the only divisors would be 1 and *p*.

Consider *pa*: σ(*pa*) = 1 + *p* + *p*2 + ... + *pa* (1).

Multiplying by *p*: *p*σ(*pa*) = *p* + *p*2 + *p*3 + ... + *pa*+ 1 (2).

Subtracting (1) from (2): *p*σ(*pa*)−σ(*pa*) = (*p*−1)σ(*pa*) = *pa*+1 − 1.

Hence σ(*pa*) = (*pa*+1 − 1)/(*p* − 1).

For example, σ(34)=(35−1)/(3−1) = 242/2 = 121,  
and checking: 1 + 3 + 9 + 27 + 81 = 121.

Although no proof is supplied here, the usefulness of the function, σ(*n*), is its multiplicativity, which means that σ(*a*×*b*×...)=σ(*a*)×σ(*b*)×..., where *a*, *b*, ..., are relatively prime.

Returning to example, we use the fact that σ(72) = σ(23×32). As 23 and 32 are relatively prime, we can separately work out σ(23) = 24 − 1 = 15 and σ(32) = (33 − 1)/2 = 13. Therefore, σ(72) = 15×13 = 195.