**2 x2 - y2 - 2 x + y = 0**

First of all we must determine the **gcd** of all coefficients but the constant term, that is: **gcd**(2, 0, -1, -2, 1) = 1.

Dividing the equation by the greatest common divisor we obtain:  
 2 x2 - y2 - 2 x + y = 0

We try now to solve this equation module 9, 16 and 25.

There are solutions, so we must continue.

We want to convert this equation to one of the form:  
x´2 + B y2 + C y + D = 0

Multiplying the equation by 8:  
 16 x2 - 8 y2 - 16 x + 8 y = 0

 16 x2 + ( - 16)x + ( - 8 y2 + 8 y) = 0

To complete the square we should add and subtract:  
( - 2)2

Then the equation converts to:  
( 4 x - 2)2 + ( - 8 y2 + 8 y) - ( 4) = 0

( 4 x - 2)2 + ( - 8 y2 + 8 y - 4) = 0

Now we perform the substitution:  
x´ =  4 x - 2

This gives:  
 x´2 - 8 y2 + 8 y - 4 = 0

Multiplying the equation by -1:  
- x´2 + 8 y2 - 8 y + 4 = 0

-x´2 + 2( 4 y2 - 4 y) + 4 = 0

-x´2 + 2((-2)2 y2 + 2\*(-2)\*1 y) + 4 = 0

Adding and subtracting 2 \* 12:  
-x´2 + 2((-2)2 y2 + 2\*(-2)\*1 y + 12) + 4 - 2 \* 12 = 0

-x´2 + 2( - 2 y +1)2 + 2 = 0

Making the substitution y´ = - 2 y +1:  
- x´2 + 2 y´2 + 2 = 0

We have to find the continued fraction expansion of the roots of **1 t2 - 2 = 0**, that is, sqrt(8) /2

Simplifying, sqrt(2)

The continued fraction expansion is:  
1+ //**2//**  
where the periodic part is marked in bold.

We have to find the continued fraction expansion of the roots of **-1 t2 + 2 = 0**, that is, sqrt(8) /(-2)

Simplifying, sqrt(2) / (-1)

The continued fraction expansion is:  
-2+ //1, 1, **2//**  
where the periodic part is marked in bold.

**- x2 + 2 y'2 + 2 = 0**

Let **x' = sy' - f'z**, so **[-(as2 + bs + c)/f']y'2 + (2as + b)y'z - af'z2 = 1**.

So **- s2 + 2** should be multiple of **2**.

This holds for **s** = 0.

* Let s = 0. Replacing in the above equation:  
  -1 y'2 + 2 z2 = 1

We have to find the continued fraction expansion of the roots of **-1 t2 + 2 = 0**, that is, sqrt(8) /(-2)

Simplifying, sqrt(2) / (-1)

The continued fraction expansion is:  
-2+ //1, 1, **2//**  
where the periodic part is marked in bold.

Y0 = NUM(1) = -1  
Z0 = DEN(1) = 1  
Since X'0 = - 2 Z0:  
X'0 = -2  
Y'0 = -1  
Y0 = ( Y'0 -1)/(-2)  
X0 = ( X'0 + 2) / 4

**X0 = 0  
Y0 = 1**Y0 = NUM(0) = -1  
Z0 = DEN(0) = -1  
Since X'0 = - 2 Z0:  
X'0 = 2  
Y'0 = -1  
Y0 = ( Y'0 -1)/(-2)  
X0 = ( X'0 + 2) / 4

**and also:  
X0 = 1  
Y0 = 1**

Xn+1 = P Xn + Q Yn + K  
Yn+1 = R Xn + S Yn + L

In order to find the values of P, Q, R, S we have to find first an integer solution of the equation **m2 + bmn + acn2 =  m2 - 2 n2 = 1**.

We have to find the continued fraction expansion of the roots of **1 t2 - 2 = 0**, that is, sqrt(8) /2

Simplifying, sqrt(2)

The continued fraction expansion is:  
1+ //**2//**  
where the periodic part is marked in bold.

An integer solution of the equation **m2 + bmn + acn2 =  m2 - 2 n2 = 1** is:

m = 3  
n = 2

Using the formulas:  
**P = m  
Q = -Cn**

|  |  |
| --- | --- |
| **K =** | **CD(P+S-2) + E(B-Bm-2ACn)**  **4AC-B2** |

**R = An  
S = m + Bn**

|  |  |  |
| --- | --- | --- |
| **L =** | **D(B-Bm-2ACn) + AE(P+S-2)**  **4AC - B2** | **+ Dn** |

we obtain:

|  |
| --- |
| **P = 3 Q = 2 K = -2 R = 4 S = 3 L = -3** |