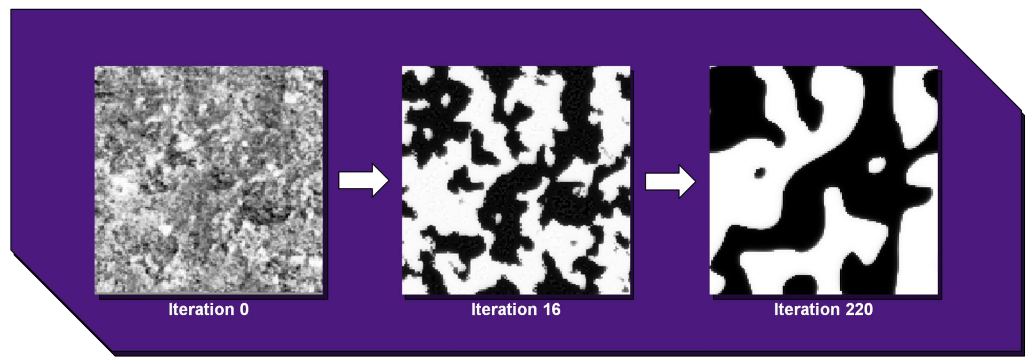
Bayesian Optimization of Image Inpainting Models

Abstract

The Doubly Nonlocal Cahn-Hilliard Equation (dnCHE) is used to model phase separation, which takes in information from the entire domain to dictate phase separation dynamics. With the use of Bayesian optimization (BO) techniques, we can tune the model's parameters to tackle image inpainting, the process of reconstructing missing or damaged regions in an image by leveraging information from surrounding pixels.

Doubly Nonlocal Cahn-Hilliard Model

The dnCHE models phase separation, similar to how an emulsion of oil and water separates over time. In the images, black and white pixels represent two immiscible materials, while gray indicates their mixture. The dnCHE aims to clearly separate these mixed states by considering interactions between pixels throughout the entire domain, influenced by their previous states and distant neighbors.



To apply dnCHE specifically for image inpainting, we introduce an additional inpainting force, F_{IP} . This force actively restores damaged regions by minimizing the discrepancy between the original damaged image and its evolving reconstruction.

$$\begin{cases} \partial_t^{\alpha} \phi(x,t) = L_J \mu(x,t) + \lambda(f - \phi(x,t)), \\ \mu(x,t) = -L_K \phi(x,t) + F'(\phi) \end{cases} \tag{1}$$

where:

- f : The corrupted input image.
- $\phi(x,t)$: The recovered image over time.
- $F'(\phi(x,t))$: Helmholtz Free Energy.
- $L_J \mu(x,t)$ and $-L_K \phi(x,t)$: Nonlocal operators applied to μ and ϕ, respectively.
- $\lambda(f - \phi(x,t))$: F_{IP} The inpainting force

Bayesian Optimization

BO is used to find local (ideally global) optimizers using fewer runs by choosing where to sample next based on previous results. The batched version selects several samples at once, speeding up runtime and convergence.

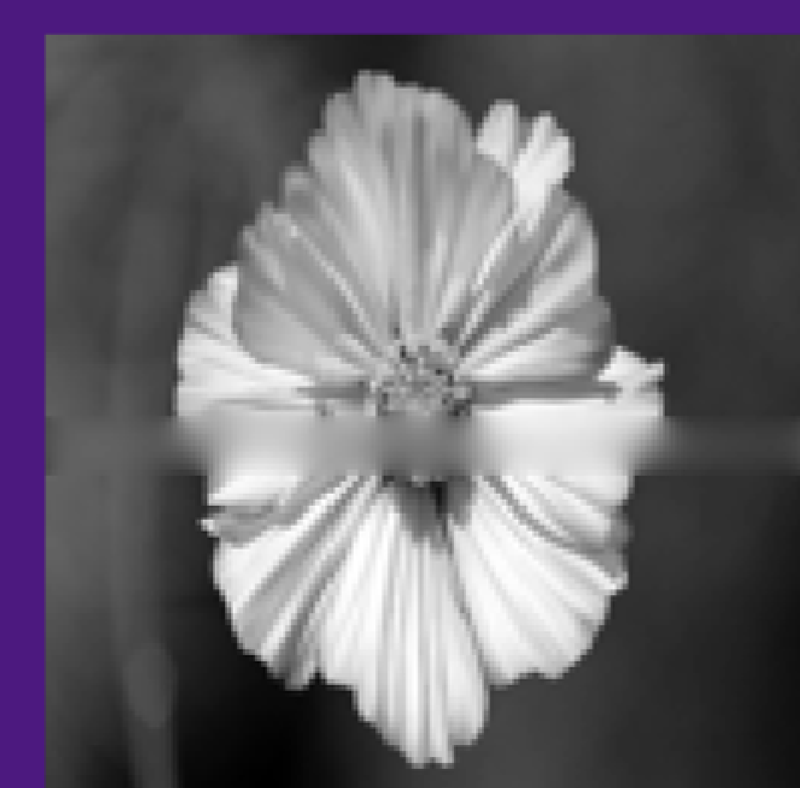
Achieved PSNR Improvement of **10.35 dB** using **Batched Bayesian Optimization**

Original Image

PSNR = 18.69

Damaged Image

PSNR = 29.75



Optimized Image

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q-Knowledge Gradient

BO balances exploring new areas and exploiting the best-known ones. To do this, we use an acquisition function that scores each option. We use the q-Knowledge Gradient (qKG), which selects a batch that is expected to improve our results the most.

$$\mathrm{qKG}(X) = \mathbb{E}[\max \mu_{n+q}(x') \mid X] - \max \mu_n(x') \tag{2}$$

Here, $\mu_n(x')$ and $\mu_{n+q}(x')$ represent Gaussian Process mean predictions before and after evaluating a batch.

PSNR Objective Function

The optimized objective function is the Peak Signal-to-Noise Ratio (PSNR), which compares the undamaged image with the in-painted image, with higher values indicating a closer match to the undamaged image. ϵ is included as a numerical stabilizer to avoid divide-by-zero errors when the images are identical, and it scales PSNR to a 0–100 range— which makes it easier to normalize PSNR to a 0–1 scale for BO.

$$\mathrm{PSNR} = 10 \cdot \log_{10} \frac{\mathrm{DynamicRange}^{\,2}}{\mathrm{MSE} + \epsilon} \tag{3}$$

$$\epsilon = \frac{\mathrm{DynamicRange}^2}{2.5 \times 10^9}. \tag{4}$$

Maximizing PSNR enhances pixel-level similarity. PSNR is measured in decibels (dB), where every whole-number increase corresponds to a tenfold improvement in pixel similarity.

Future Work

- Combining our model with a damage segmentation model to be able to handle inpainting problems with less pre-processing
- Extending optimization methods to handle larger and more complex image datasets.
- Incorporating other metrics such as SSIM in our objective function.

References

1. De Jesus, M., Gal, C. G., & Shomberg, J. L. (2025). *A general paradigm of binary phase-segregation processes through the lens of four critical mechanisms*. Discrete and Continuous Dynamical Systems, 45(8), 2591–2627. https://doi.org/10.3934/dcds.2024177
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