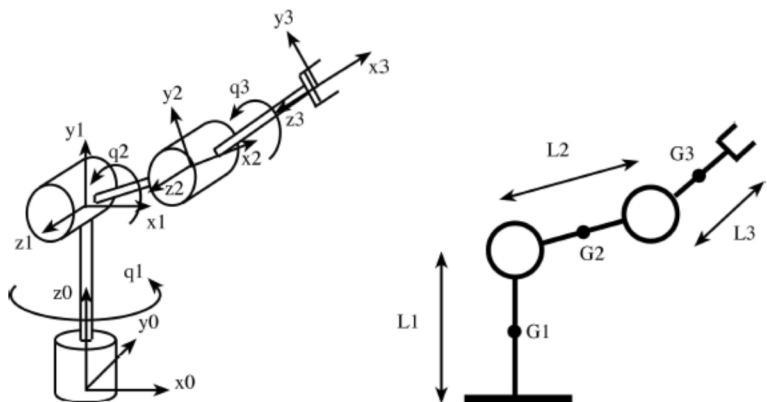```
> restart:with(LinearAlgebra):
```



```
> Mrotztransl:=(theta,point)-><<cos(theta),sin(theta),0,0>|<-sin(theta),cos(theta),0,
  0>|<0,0,1,0>|<point[1],point[2],point[3],1>>;
```

$$Mrotztransl := (\theta,\ point) \mapsto \langle\langle\cos(\theta),\ \sin(\theta),\ 0,\ 0\rangle|\langle-\sin(\theta),\ \cos(\theta),\ 0,\ 0\rangle|\langle0,\ 0,\ 1,\ 0\rangle \tag{1}$$
$$|\langle point_1,\ point_2,\ point_3,\ 1\rangle\rangle$$

```
> Mrotxtransl:=(alpha,point)-><<1,0,0,0>|<0,cos(alpha),sin(alpha),0>|<0,-sin(alpha),cos
  (alpha),0>|<point[1],point[2],point[3],1>>;
```

$$Mrotxtransl := (\alpha,\ point) \mapsto \langle\langle1,\ 0,\ 0,\ 0\rangle|\langle0,\ \cos(\alpha),\ \sin(\alpha),\ 0\rangle|\langle0,\ -\sin(\alpha),\ \cos(\alpha),\ 0\rangle \tag{2}$$
$$|\langle point_1,\ point_2,\ point_3,\ 1\rangle\rangle$$

```
> M01:=Mrotztransl(q1(t),<0,0,L1>).Mrotxtransl(Pi/2,<0,0,0>);
```

$$M01 := \begin{bmatrix} \cos(q1(t)) & 0 & \sin(q1(t)) & 0 \\ \sin(q1(t)) & 0 & -\cos(q1(t)) & 0 \\ 0 & 1 & 0 & L1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

```
> M12:=Mrotztransl(q2(t),<0,0,0>).Mrotxtransl(0,<L2,0,0>);
```

$$M12 := \begin{bmatrix} \cos(q2(t)) & -\sin(q2(t)) & 0 & \cos(q2(t))\ L2 \\ \sin(q2(t)) & \cos(q2(t)) & 0 & \sin(q2(t))\ L2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

```
> M02:=simplify(M01.M12);
```

$$M02 := \tag{5}$$
$$[[\cos(q1(t))\ \cos(q2(t)),\ -\cos(q1(t))\ \sin(q2(t)),\ \sin(q1(t)),$$
$$\cos(q1(t))\ \cos(q2(t))\ L2],$$
$$[\sin(q1(t))\ \cos(q2(t)),\ -\sin(q1(t))\ \sin(q2(t)),\ -\cos(q1(t)),$$
$$\sin(q1(t))\ \cos(q2(t))\ L2],$$
$$[\sin(q2(t)),\ \cos(q2(t)),\ 0,\ \sin(q2(t))\ L2+L1],$$
$$[0,\ 0,\ 0,\ 1]]$$

```
> M23:=Mrotztransl(q3(t),<0,0,0>).Mrotxtransl(0,<L3,0,0>);
```

$$M23 := \begin{bmatrix} \cos(q3(t)) & -\sin(q3(t)) & 0 & \cos(q3(t))\ L3 \\ \sin(q3(t)) & \cos(q3(t)) & 0 & \sin(q3(t))\ L3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{6}$$

```
> M03:=simplify(M02.M23);
```

$M03 := [\,[\cos(q1(t))\ (-\sin(q2(t))\ \sin(q3(t)) + \cos(q2(t))\ \cos(q3(t))),$ (7)
    $-\cos(q1(t))\ (\sin(q2(t))\ \cos(q3(t)) + \cos(q2(t))\ \sin(q3(t))),\ \sin(q1(t)),$
    $\cos(q1(t))\ (L3\ \cos(q2(t))\ \cos(q3(t)) - L3\ \sin(q2(t))\ \sin(q3(t)) + \cos(q2(t))\ L2)$
    $],$
    $[\sin(q1(t))\ (-\sin(q2(t))\ \sin(q3(t)) + \cos(q2(t))\ \cos(q3(t))),$
    $-\sin(q1(t))\ (\sin(q2(t))\ \cos(q3(t)) + \cos(q2(t))\ \sin(q3(t))),\ -\cos(q1(t)),$
    $\sin(q1(t))\ (L3\ \cos(q2(t))\ \cos(q3(t)) - L3\ \sin(q2(t))\ \sin(q3(t)) + \cos(q2(t))\ L2)$
    $],$
    $[\sin(q2(t))\ \cos(q3(t)) + \cos(q2(t))\ \sin(q3(t)),\ -\sin(q2(t))\ \sin(q3(t))$
    $+ \cos(q2(t))\ \cos(q3(t)),\ 0,\ (\cos(q3(t))\ L3 + L2)\ \sin(q2(t))$
    $+ \cos(q2(t))\ \sin(q3(t))\ L3 + L1],$
    $[0,\ 0,\ 0,\ 1]\,]$

```
> E_3:=<0,0,0,1>;
```

$$E\_3 := \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{8}$$

```
> E:=simplify(M03.E_3);
```

$E :=$ (9)
    $[\,[\cos(q1(t))\ (L3\ \cos(q2(t))\ \cos(q3(t)) - L3\ \sin(q2(t))\ \sin(q3(t))$
    $+ \cos(q2(t))\ L2)],$
    $[\sin(q1(t))\ (L3\ \cos(q2(t))\ \cos(q3(t)) - L3\ \sin(q2(t))\ \sin(q3(t)) + \cos(q2(t))\ L2)$
    $],$
    $[(\cos(q3(t))\ L3 + L2)\ \sin(q2(t)) + \cos(q2(t))\ \sin(q3(t))\ L3 + L1],$
    $[1]\,]$

```
> Jac_E:=simplify(VectorCalculus[Jacobian](subs(q1(t)=q1,q2(t)=q2,q3(t)=q3,E[1..3]),
  [q1,q2,q3]));
```

$Jac\_E := [\,[-\sin(q1)\ ((\cos(q3)\ L3 + L2)\ \cos(q2) - L3\ \sin(q2)\ \sin(q3)),$ (10)
    $-\cos(q1)\ ((\cos(q3)\ L3 + L2)\ \sin(q2) + \cos(q2)\ \sin(q3)\ L3),$
    $-\cos(q1)\ L3\ (\cos(q3)\ \sin(q2) + \sin(q3)\ \cos(q2))],$
    $[((\cos(q3)\ L3 + L2)\ \cos(q2) - L3\ \sin(q2)\ \sin(q3))\ \cos(q1),\ -((\cos(q3)\ L3$
    $+ L2)\ \sin(q2) + \cos(q2)\ \sin(q3)\ L3)\ \sin(q1),\ -\sin(q1)\ L3\ (\cos(q3)\ \sin(q2)$
    $+ \sin(q3)\ \cos(q2))],$
    $[0,\ (\cos(q3)\ L3 + L2)\ \cos(q2) - L3\ \sin(q2)\ \sin(q3),\ L3\ (\cos(q2)\ \cos(q3)$
    $- \sin(q2)\ \sin(q3))]\,]$

```
> det_Jac_E:=simplify(Determinant(Jac_E));
```

$det\_Jac\_E := -L2\ L3\ \sin(q3)\ ((\cos(q3)\ L3 + L2)\ \cos(q2) - L3\ \sin(q2)\ \sin(q3))$ (11)

```
> singular_config:=simplify(solve(det_Jac_E=0,[q1,q2,q3]));
```

$singular\_config := \Big[\,\Big[q1 = q1,\ q2 = \arctan\Big(\dfrac{-\cos(q3)\ L3 + L2}{\sin(q3)\ L3}\Big),\ q3 = q3\Big],\ \Big[q1 = q1,\ q2$ (12)
    $= \arctan\Big(\dfrac{\cos(q3)\ L3 + L2}{\sin(q3)\ L3}\Big),\ q3 = q3\Big],\ [q1 = q1,\ q2 = q2,\ q3 = 0]\,\Big]$

```
> data:={L1=0.3, L2=0.3, L3=0.3, m1=25, m2=10, m3=5, Tmin_sys=1, g=9.81, q11=-Pi, q12=Pi, q21=0,
  q22=Pi/4, q31=0, q32=-Pi/4};
```

$$data := \left\{ L1 = 0.3,\ L2 = 0.3,\ L3 = 0.3,\ g = 9.81,\ m1 = 25,\ m2 = 10,\ m3 = 5,\ q11 = -\pi,\ q12 = \pi,\ q21 = 0, \right. \tag{13}$$

$$\left. q22 = \frac{\pi}{4},\ q31 = 0,\ q32 = -\frac{\pi}{4},\ Tmin\_sys = 1 \right\}$$

```
> Tmin:=sqrt(4*abs(delteq)/amax);
```

$$Tmin := 2\ \sqrt{\frac{|delteq|}{amax}} \tag{14}$$

```
> amax_rescaled:=4*deltaq/(T^2);
```

$$amax\_rescaled := \frac{4\ deltaq}{T^2} \tag{15}$$

```
> amax1_re:=evalf(subs(T=Tmin_sys, deltaq=q12-q11, data, amax_rescaled));
```

$$amax1\_re := 25.13274123 \tag{16}$$

```
> amax2_re:=evalf(subs(T=Tmin_sys, deltaq=q22-q21, data, amax_rescaled));
```

$$amax2\_re := 3.141592654 \tag{17}$$

```
> amax3_re:=evalf(subs(T=Tmin_sys, deltaq=q32-q31, data, amax_rescaled));
```

$$amax3\_re := -3.141592654 \tag{18}$$

```
> Qbase_profile:=piecewise(t>=0 and t<=T/2, qini+1/2*amax*(t^2), t>T/2 and t<=T, qini+1/4*
  amax*(T^2)-1/2*amax*((T-t)^2));
```

$$Qbase\_profile := \begin{cases} qini + \dfrac{amax\ t^2}{2} & 0 \le t \le \dfrac{T}{2} \\[3mm] qini + \dfrac{amax\ T^2}{4} - \dfrac{amax\ (T-t)^2}{2} & \dfrac{T}{2} < t \le T \end{cases} \tag{19}$$

```
> q1_profile:=evalf(subs(qini=q11, amax=amax1_re, T=Tmin_sys, data, Qbase_profile));
```

$$q1\_profile := \begin{cases} -3.141592654 + 12.56637062\ t^2 & 0. \le t \le 0.5000000000 \\[2mm] 3.141592654 - 12.56637062\ (1.-1.\ t)^2 & 0.5000000000 < t \le 1. \end{cases} \tag{20}$$

```
> q2_profile:=evalf(subs(qini=q21, amax=amax2_re, T=Tmin_sys, data, Qbase_profile));
```

$$q2\_profile := \begin{cases} 1.570796327\ t^2 & 0. \le t \le 0.5000000000 \\[2mm] 0.7853981635 - 1.570796327\ (1.-1.\ t)^2 & 0.5000000000 < t \le 1. \end{cases} \tag{21}$$

```
> q3_profile:=evalf(subs(qini=q31, amax=amax3_re, T=Tmin_sys, data, Qbase_profile));
```

$$q3\_profile := \begin{cases} -1.570796327\ t^2 & 0. \le t \le 0.5000000000 \\[2mm] -0.7853981635 + 1.570796327\ (1.-1.\ t)^2 & 0.5000000000 < t \le 1. \end{cases} \tag{22}$$

```
> q1_Vprofile:=diff(q1_profile, t);
```

$$q1\_Vprofile := \begin{cases} 0. & t < 0. \\ Float(undefined) & t = 0. \\ 25.13274124\ t & t < 0.5000000000 \\ Float(undefined) & t = 0.5000000000 \\ 25.13274124 - 25.13274124\ t & t < 1. \\ Float(undefined) & t = 1. \\ 0. & 1. < t \end{cases} \tag{23}$$

```
> q2_Vprofile:=diff(q2_profile, t);
```

$$q2\_Vprofile := \begin{cases} 0. & t \leq 0. \\ 3.141592654\ t & t \leq 0.5000000000 \\ 3.141592654 - 3.141592654\ t & t < 1. \\ \text{Float(undefined)} & t = 1. \\ 0. & 1. < t \end{cases} \tag{24}$$

```
> q3_Vprofile:=diff(q3_profile,t);
```

$$q3\_Vprofile := \begin{cases} 0. & t \leq 0. \\ -3.141592654\ t & t \leq 0.5000000000 \\ -3.141592654 + 3.141592654\ t & t < 1. \\ \text{Float(undefined)} & t = 1. \\ 0. & 1. < t \end{cases} \tag{25}$$

```
> q1_Accprofile:=diff(q1_profile,t,t);
```

$$q1\_Accprofile := \begin{cases} 0. & t < 0. \\ \text{Float(undefined)} & t = 0. \\ 25.13274124 & t < 0.5000000000 \\ \text{Float(undefined)} & t = 0.5000000000 \\ -25.13274124 & t < 1. \\ \text{Float(undefined)} & t = 1. \\ 0. & 1. < t \end{cases} \tag{26}$$

```
> q2_Accprofile:=diff(q2_profile,t,t);
```

$$q2\_Accprofile := \begin{cases} 0. & t < 0. \\ \text{Float(undefined)} & t = 0. \\ 3.141592654 & t < 0.5000000000 \\ \text{Float(undefined)} & t = 0.5000000000 \\ -3.141592654 & t < 1. \\ \text{Float(undefined)} & t = 1. \\ 0. & 1. < t \end{cases} \tag{27}$$
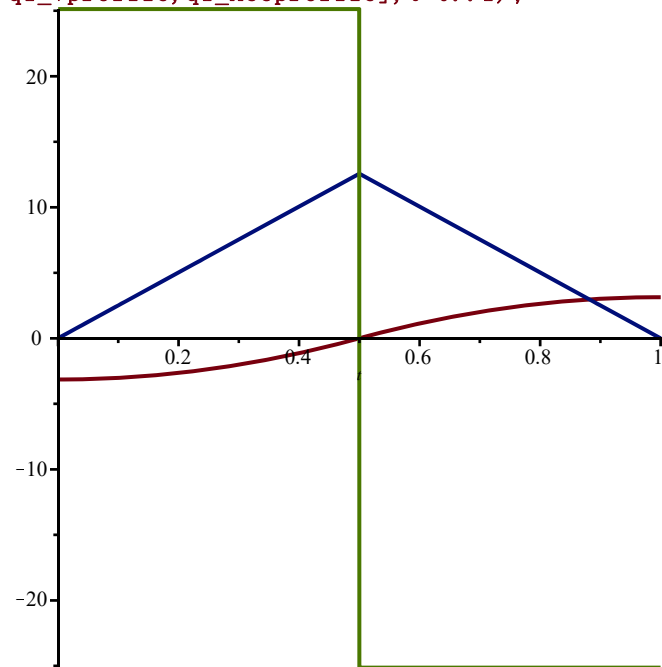
```
> q3_Accprofile:=diff(q3_profile,t,t);
```

$$q3\_Accprofile := \begin{cases} 0. & t < 0. \\ \text{Float(undefined)} & t = 0. \\ -3.141592654 & t < 0.5000000000 \\ \text{Float(undefined)} & t = 0.5000000000 \\ 3.141592654 & t < 1. \\ \text{Float(undefined)} & t = 1. \\ 0. & 1. < t \end{cases} \tag{28}$$
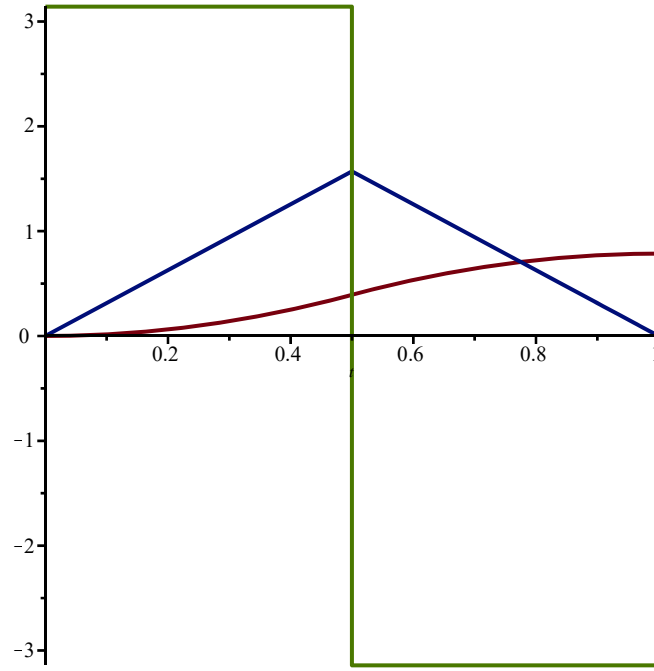
```
> plot([q1_profile,q2_profile,q3_profile], t=0..1);
```
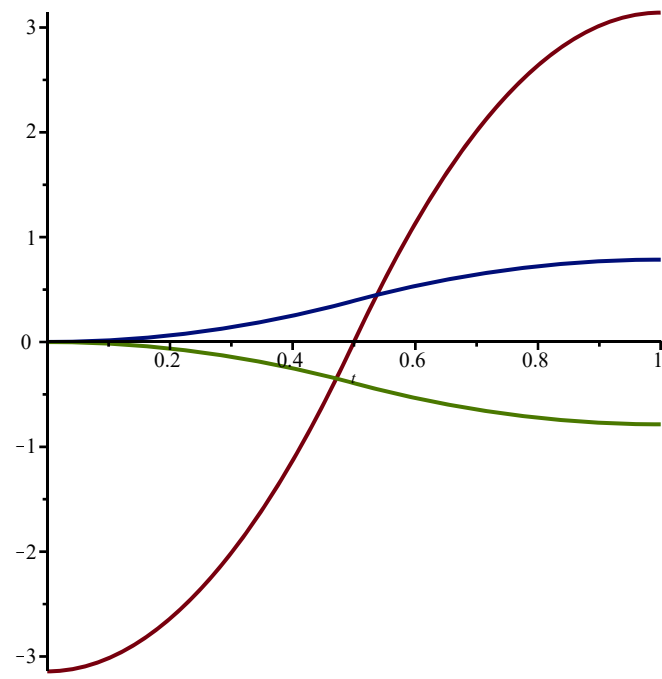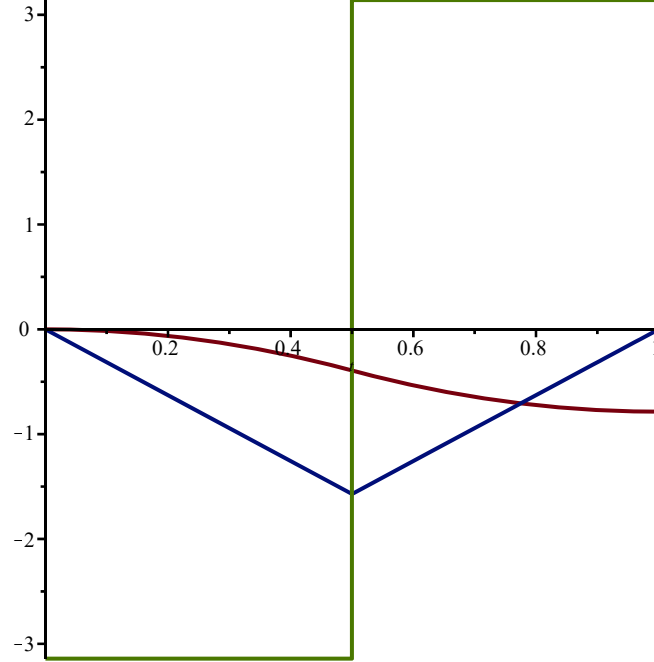
```
> plot([q1_profile, q1_Vprofile, q1_Accprofile], t=0..1);
```



```
> plot([q2_profile, q2_Vprofile, q2_Accprofile], t=0..1);
```

```
> plot([q3_profile, q3_Vprofile, q3_Accprofile], t=0..1);
```



```
> T_func:=(W, J)->simplify(Trace(1/2*W.J.(W^%T)));
```

$$T\_func := (W, J) \mapsto simplify\left(LinearAlgebra{:-}Trace\left(\left(\frac{W}{2}\right) \cdot J \cdot W^T\right)\right) \tag{29}$$

```
> W_func:=(M)->simplify(map(diff, M, t).MatrixInverse(M));
```

$$W\_func := M \mapsto simplify(map(diff, M, t) \cdot LinearAlgebra{:-}MatrixInverse(M)) \tag{30}$$

```
> H_func:=(M)->simplify(map(diff, M, t, t).MatrixInverse(M));
```

$$H\_func := M \mapsto simplify(map(diff, M, t, t) \cdot LinearAlgebra{:-}MatrixInverse(M)) \tag{31}$$

```
> J_lumped_func:=(m,G)->< <m*G[1]*G[1],m*G[1]*G[2],m*G[1]*G[3],m*G[1]>|<m*G[2]*G[1],m*G
  [2]*G[2],m*G[2]*G[3],m*G[2]>|<m*G[3]*G[1],m*G[3]*G[2],m*G[3]*G[3],m*G[3]>|<m*G[1],m*G
  [2],m*G[3],m>>;
```

$$J\_lumped\_func := (m, G) \mapsto \langle\langle m \cdot G_1 \cdot G_1, \ m \cdot G_1 \cdot G_2, \ m \cdot G_1 \cdot G_3, \ m \cdot G_1\rangle | \langle m \cdot G_1 \cdot G_2, \ m \cdot G_2 \cdot G_2, \ m \cdot G_2 \cdot G_3, \ m \cdot G_2\rangle | \langle m \cdot G_1 \tag{32}$$

$$\cdot G_3, \; m \cdot G_2 \cdot G_3, \; m \cdot G_3 \cdot G_3, \; m \cdot G_3 \rangle | \langle m \cdot G_1, \; m \cdot G_2, \; m \cdot G_3, \; m \rangle \rangle$$

> `J_mobile_to_fixed:=(M,J)->simplify(M.J.(M^%T));`

$$J\_mobile\_to\_fixed := (M, \; J) \mapsto simplify(M \cdot J \cdot M^T) \tag{33}$$

> `U_func:=(Hg,J)->simplify(Trace(-Hg.J));`

$$U\_func := (Hg, \; J) \mapsto simplify(LinearAlgebra{:}-Trace(-Hg \cdot J)) \tag{34}$$

> `Hg_template:=<<0,0,0,0>|<0,0,0,0>|<0,0,0,0>|<0,0,-g,0>>;`

$$Hg\_template := \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -g \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{35}$$

> `W01:=W_func(M01);`

$$W01 := \begin{bmatrix} 0 & -\dfrac{d}{dt}\,q1(t) & 0 & 0 \\ \dfrac{d}{dt}\,q1(t) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{36}$$

> `W02:=W_func(M02);`

$$W02 := \left[\left[0, \; -\frac{d}{dt}\,q1(t), \; -\left(\frac{d}{dt}\,q2(t)\right)\cos(q1(t)), \; L1\left(\frac{d}{dt}\,q2(t)\right)\cos(q1(t)) \right], \tag{37}$$
$$\left[\frac{d}{dt}\,q1(t), \; 0, \; -\left(\frac{d}{dt}\,q2(t)\right)\sin(q1(t)), \; L1\left(\frac{d}{dt}\,q2(t)\right)\sin(q1(t)) \right],$$
$$\left[\left(\frac{d}{dt}\,q2(t)\right)\cos(q1(t)), \; \left(\frac{d}{dt}\,q2(t)\right)\sin(q1(t)), \; 0, \; 0 \right],$$
$$\left[0, \; 0, \; 0, \; 0 \right] \right]$$

> `W03:=W_func(M03);`

$$W03 := \left[\left[0, \; -\frac{d}{dt}\,q1(t), \; -\left(\frac{d}{dt}\,q3(t) + \frac{d}{dt}\,q2(t)\right)\cos(q1(t)), \; \cos(q1(t))\left((\sin(q2(t))\,L2 \right.\right.\right. \tag{38}$$
$$\left.\left. + L1)\left(\frac{d}{dt}\,q3(t)\right) + L1\left(\frac{d}{dt}\,q2(t)\right)\right)\right],$$
$$\left[\frac{d}{dt}\,q1(t), \; 0, \; -\left(\frac{d}{dt}\,q3(t) + \frac{d}{dt}\,q2(t)\right)\sin(q1(t)), \; \sin(q1(t))\left((\sin(q2(t))\,L2 \right.\right.$$
$$\left.\left. + L1)\left(\frac{d}{dt}\,q3(t)\right) + L1\left(\frac{d}{dt}\,q2(t)\right)\right)\right],$$
$$\left[\left(\frac{d}{dt}\,q3(t) + \frac{d}{dt}\,q2(t)\right)\cos(q1(t)), \; \left(\frac{d}{dt}\,q3(t) + \frac{d}{dt}\,q2(t)\right)\sin(q1(t)), \; 0, \right.$$
$$\left. -L2\left(\frac{d}{dt}\,q3(t)\right)\cos(q2(t))\right],$$
$$\left[0, \; 0, \; 0, \; 0 \right] \right]$$

> `J11:=J_lumped_func(m1,<0,-L1/2,0>);`

$$\tag{39}$$

$$J11 := \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \dfrac{m1\ L1^2}{4} & 0 & -\dfrac{m1\ L1}{2} \\ 0 & 0 & 0 & 0 \\ 0 & -\dfrac{m1\ L1}{2} & 0 & m1 \end{bmatrix} \tag{39}$$

> `J22:=J_lumped_func(m2,<-L2/2,0,0>);`

$$J22 := \begin{bmatrix} \dfrac{m2\ L2^2}{4} & 0 & 0 & -\dfrac{m2\ L2}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\dfrac{m2\ L2}{2} & 0 & 0 & m2 \end{bmatrix} \tag{40}$$

> `J33:=J_lumped_func(m3,<-L3/2,0,0>);`

$$J33 := \begin{bmatrix} \dfrac{m3\ L3^2}{4} & 0 & 0 & -\dfrac{m3\ L3}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\dfrac{m3\ L3}{2} & 0 & 0 & m3 \end{bmatrix} \tag{41}$$

> `J10:=J_mobile_to_fixed(M01,J11);`

$$J10 := \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \dfrac{m1\ L1^2}{4} & \dfrac{m1\ L1}{2} \\ 0 & 0 & \dfrac{m1\ L1}{2} & m1 \end{bmatrix} \tag{42}$$

> `J20:=J_mobile_to_fixed(M02,J22);`

$$J20 := \left[\left[\frac{\cos(q1(t))^2\ \cos(q2(t))^2\ m2\ L2^2}{4}, \ \frac{\cos(q1(t))\ \cos(q2(t))^2\ m2\ L2^2\ \sin(q1(t))}{4}, \right. \right. \tag{43}$$

$$\frac{\cos(q1(t))\ \cos(q2(t))\ m2\ L2\ (\sin(q2(t))\ L2 + 2\ L1)}{4}, \ \left. \frac{\cos(q1(t))\ \cos(q2(t))\ m2\ L2}{2} \right],$$

$$\left[\frac{\cos(q1(t))\ \cos(q2(t))^2\ m2\ L2^2\ \sin(q1(t))}{4}, \ \frac{\sin(q1(t))^2\ \cos(q2(t))^2\ m2\ L2^2}{4}, \right.$$

$$\frac{\sin(q1(t))\ \cos(q2(t))\ m2\ L2\ (\sin(q2(t))\ L2 + 2\ L1)}{4}, \ \left. \frac{\sin(q1(t))\ \cos(q2(t))\ m2\ L2}{2} \right],$$

$$\left[\frac{\cos(q1(t))\ \cos(q2(t))\ m2\ L2\ (\sin(q2(t))\ L2 + 2\ L1)}{4}, \right.$$

$$\frac{\sin(q1(t))\ \cos(q2(t))\ m2\ L2\ (\sin(q2(t))\ L2 + 2\ L1)}{4}, \ m2\ \left(\frac{\sin(q2(t))\ L2}{2} + L1 \right)^2,$$

$$\left. \frac{m2\ (\sin(q2(t))\ L2 + 2\ L1)}{2} \right],$$

$$\left[\frac{\cos(q1(t))\ \cos(q2(t))\ m2\ L2}{2}, \ \frac{\sin(q1(t))\ \cos(q2(t))\ m2\ L2}{2}, \right.$$

$$\frac{m2 \ (\sin(q2(t)) \ L2 + 2 \ L1)}{2}, \quad m2 \Big]\Big]$$

`> J30:=J_mobile_to_fixed(M03,J33);`

$$J30 := \Bigg[\Bigg[\frac{1}{2}\Bigg(\Bigg(\Bigg(L3^2 \ \cos(q3(t))^2 + 2 \ L2 \ L3 \ \cos(q3(t)) + 2 \ L2^2 - \frac{L3^2}{2}\Bigg) \cos(q2(t))^2 \tag{44}$$

$$- \sin(q2(t)) \ \sin(q3(t)) \ L3 \ (\cos(q3(t)) \ L3 + 2 \ L2) \ \cos(q2(t)) - \frac{L3^2 \ \cos(q3(t))^2}{2}$$

$$+ \frac{L3^2}{2}\Bigg) \ m3 \ \cos(q1(t))^2\Bigg), \ \frac{1}{2}\Bigg(m3 \ \cos(q1(t)) \ \Bigg(\Bigg(L3^2 \ \cos(q3(t))^2$$

$$+ 2 \ L2 \ L3 \ \cos(q3(t)) + 2 \ L2^2 - \frac{L3^2}{2}\Bigg) \cos(q2(t))^2$$

$$- \sin(q2(t)) \ \sin(q3(t)) \ L3 \ (\cos(q3(t)) \ L3 + 2 \ L2) \ \cos(q2(t)) - \frac{L3^2 \ \cos(q3(t))^2}{2}$$

$$+ \frac{L3^2}{2}\Bigg) \ \sin(q1(t))\Bigg), \ \frac{1}{2}\Bigg(\cos(q1(t)) \ \Bigg(\sin(q3(t)) \ L3 \ (\cos(q3(t)) \ L3$$

$$+ 2 \ L2) \ \cos(q2(t))^2 + \Bigg(L3^2 \ \cos(q3(t))^2 \ \sin(q2(t)) + 2 \ \Bigg(\sin(q2(t)) \ L2$$

$$+ \frac{L1}{2}\Bigg) \ L3 \ \cos(q3(t)) + \Bigg(2 \ L2^2 - \frac{L3^2}{2}\Bigg) \ \sin(q2(t)) + 2 \ L1 \ L2\Bigg) \ \cos(q2(t))$$

$$- \frac{\sin(q3(t)) \ L3 \ (\cos(q3(t)) \ L3 + 2 \ \sin(q2(t)) \ L1 + 2 \ L2)}{2}\Bigg) \ m3\Bigg),$$

$$\frac{((\cos(q3(t)) \ L3 + 2 \ L2) \ \cos(q2(t)) - L3 \ \sin(q2(t)) \ \sin(q3(t))) \ m3 \ \cos(q1(t))}{2}\Bigg],$$

$$\Bigg[\frac{1}{2}\Bigg(m3 \ \cos(q1(t)) \ \Bigg(\Bigg(L3^2 \ \cos(q3(t))^2 + 2 \ L2 \ L3 \ \cos(q3(t)) + 2 \ L2^2$$

$$- \frac{L3^2}{2}\Bigg) \ \cos(q2(t))^2 - \sin(q2(t)) \ \sin(q3(t)) \ L3 \ (\cos(q3(t)) \ L3 + 2 \ L2) \ \cos(q2(t))$$

$$- \frac{L3^2 \ \cos(q3(t))^2}{2} + \frac{L3^2}{2}\Bigg) \ \sin(q1(t))\Bigg), \ -\Bigg(\Bigg(-\frac{L3^2 \ \cos(q3(t))^2}{2} - L2 \ L3 \ \cos(q3(t))$$

$$- L2^2 + \frac{L3^2}{4}\Bigg) \ \cos(q2(t))^2 + L3 \ \sin(q2(t)) \ \sin(q3(t)) \ \Bigg(\frac{\cos(q3(t)) \ L3}{2}$$

$$+ L2\Bigg) \ \cos(q2(t)) + \frac{L3^2 \ \cos(q3(t))^2}{4} - \frac{L3^2}{4}\Bigg) \ m3 \ \sin(q1(t))^2,$$

$$\frac{1}{2}\Bigg(\Bigg(\sin(q3(t)) \ L3 \ (\cos(q3(t)) \ L3 + 2 \ L2) \ \cos(q2(t))^2 + \Bigg(L3^2 \ \cos(q3(t))^2 \ \sin(q2(t))$$

$$+ 2 \ \Bigg(\sin(q2(t)) \ L2 + \frac{L1}{2}\Bigg) \ L3 \ \cos(q3(t)) + \Bigg(2 \ L2^2 - \frac{L3^2}{2}\Bigg) \ \sin(q2(t)) + 2 \ L1 \ L2\Bigg)$$

$$\cos(q2(t)) - \frac{\sin(q3(t)) \ L3 \ (\cos(q3(t)) \ L3 + 2 \ \sin(q2(t)) \ L1 + 2 \ L2)}{2}\Bigg)$$

$$m3 \ \sin(q1(t))\Big),$$

$$-\frac{\big((-\cos(q3(t)) \ L3-2 \ L2) \ \cos(q2(t)) + L3 \ \sin(q2(t)) \ \sin(q3(t))\big) \ m3 \ \sin(q1(t))}{2}\Bigg],$$

$$\Bigg[\frac{1}{2}\Bigg(\cos(q1(t)) \ \Bigg(\sin(q3(t)) \ L3 \ (\cos(q3(t)) \ L3+2 \ L2) \ \cos(q2(t))^2$$

$$+\Big(L3^2 \ \cos(q3(t))^2 \ \sin(q2(t)) + 2 \ \Big(\sin(q2(t)) \ L2+\frac{L1}{2}\Big) \ L3 \ \cos(q3(t)) + \Big(2 \ L2^2$$

$$-\frac{L3^2}{2}\Big) \ \sin(q2(t)) + 2 \ L1 \ L2\Big) \ \cos(q2(t))$$

$$-\frac{\sin(q3(t)) \ L3 \ (\cos(q3(t)) \ L3+2 \ \sin(q2(t)) \ L1+2 \ L2)}{2}\Bigg) \ m3\Bigg),$$

$$\frac{1}{2}\Bigg(\Big(\sin(q3(t)) \ L3 \ (\cos(q3(t)) \ L3+2 \ L2) \ \cos(q2(t))^2 + \Big(L3^2 \ \cos(q3(t))^2 \ \sin(q2(t))$$

$$+2 \ \Big(\sin(q2(t)) \ L2+\frac{L1}{2}\Big) \ L3 \ \cos(q3(t)) + \Big(2 \ L2^2 -\frac{L3^2}{2}\Big) \ \sin(q2(t)) + 2 \ L1 \ L2\Big)$$

$$\cos(q2(t)) - \frac{\sin(q3(t)) \ L3 \ (\cos(q3(t)) \ L3+2 \ \sin(q2(t)) \ L1+2 \ L2)}{2}\Big)$$

$$m3 \ \sin(q1(t))\Bigg), \ m3 \ \Bigg(\Big(-\frac{L3^2 \ \cos(q3(t))^2}{2} - L2 \ L3 \ \cos(q3(t)) - L2^2 + \frac{L3^2}{4}\Big) \ \cos(q2(t))^2$$

$$+\sin(q3(t)) \ L3 \ \Big(\frac{\sin(q2(t)) \ \cos(q3(t)) \ L3}{2} + \sin(q2(t)) \ L2+L1\Big) \ \cos(q2(t))$$

$$+\frac{L3^2 \ \cos(q3(t))^2}{4} + L3 \ (\sin(q2(t)) \ L1+L2) \ \cos(q3(t)) + 2 \ \sin(q2(t)) \ L1 \ L2+L1^2$$

$$+L2^2\Bigg),$$

$$\frac{m3 \ (\cos(q2(t)) \ \sin(q3(t)) \ L3+\sin(q2(t)) \ \cos(q3(t)) \ L3+2 \ \sin(q2(t)) \ L2+2 \ L1)}{2}$$

$$\Bigg],$$

$$\Bigg[\frac{\big((\cos(q3(t)) \ L3+2 \ L2) \ \cos(q2(t)) - L3 \ \sin(q2(t)) \ \sin(q3(t))\big) \ m3 \ \cos(q1(t))}{2},$$

$$-\frac{\big((-\cos(q3(t)) \ L3-2 \ L2) \ \cos(q2(t)) + L3 \ \sin(q2(t)) \ \sin(q3(t))\big) \ m3 \ \sin(q1(t))}{2},$$

$$\frac{m3 \ (\cos(q2(t)) \ \sin(q3(t)) \ L3+\sin(q2(t)) \ \cos(q3(t)) \ L3+2 \ \sin(q2(t)) \ L2+2 \ L1)}{2},$$

$$m3\Bigg]\Bigg]$$

```
> T1:=T_func(W01,J10);
```
$$T1 := 0 \tag{45}$$

```
> T2:=T_func(W02,J20);
```
$$T2 := \frac{m2 \ L2^2 \ \Big(\Big(\frac{d}{dt} \ q1(t)\Big)^2 \ \cos(q2(t))^2 + \Big(\frac{d}{dt} \ q2(t)\Big)^2\Big)}{8} \tag{46}$$

```
> T3:=T_func(W03,J30);
```

$$T3 := -\frac{1}{2}\left(\left(\left(\left(-\frac{L3^2\,\cos(q3(t))^2}{2} - L2\,L3\,\cos(q3(t)) - L2^2 + \frac{L3^2}{4}\right)\cos(q2(t))^2\right.\right.\right.$$

$$+ L3\,\sin(q2(t))\,\sin(q3(t))\left(\frac{\cos(q3(t))\,L3}{2} + L2\right)\cos(q2(t)) + \frac{L3^2\,\cos(q3(t))^2}{4}$$

$$\left.- \frac{L3^2}{4}\right)\left(\frac{\mathrm{d}}{\mathrm{d}t}\,q1(t)\right)^2 + \left(-L2\,L3\,\cos(q3(t)) - L2^2 - \frac{L3^2}{4}\right)\left(\frac{\mathrm{d}}{\mathrm{d}t}\,q2(t)\right)^2$$

$$\left.\left.- L3\left(\cos(q3(t))\,L2 + \frac{L3}{2}\right)\left(\frac{\mathrm{d}}{\mathrm{d}t}\,q3(t)\right)\left(\frac{\mathrm{d}}{\mathrm{d}t}\,q2(t)\right) - \frac{\left(\frac{\mathrm{d}}{\mathrm{d}t}\,q3(t)\right)^2 L3^2}{4}\right)m3\right) \tag{47}$$

```
> Hg1:=Hg_template;
```

$$Hg1 := \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -g \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{48}$$

```
> Hg2:=Hg_template;
```

$$Hg2 := \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -g \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{49}$$

```
> Hg3:=Hg_template;
```

$$Hg3 := \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -g \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{50}$$

```
> U1:=U_func(Hg1,J10);
```

$$U1 := \frac{g\,m1\,L1}{2} \tag{51}$$

```
> U2:=U_func(Hg2,J20);
```

$$U2 := \frac{g\,m2\,(\sin(q2(t))\,L2 + 2\,L1)}{2} \tag{52}$$

```
> U3:=U_func(Hg3,J30);
```

$$U3 := \frac{g\,((\cos(q3(t))\,L3 + 2\,L2)\,\sin(q2(t)) + \cos(q2(t))\,\sin(q3(t))\,L3 + 2\,L1)\,m3}{2} \tag{53}$$

```
> Lagr:=simplify(T1+T2+T3-0*(U1+U2+U3));
```

$$Lagr := \frac{1}{8}\left(\left(\left(2\,\cos(q3(t))^2\,L3^2\,m3 + 4\,\cos(q3(t))\,L2\,L3\,m3 + \left(4\,L2^2 - L3^2\right)m3\right.\right.\right.$$

$$+ m2\,L2^2\Big)\cos(q2(t))^2 - 2\,\sin(q2(t))\,\sin(q3(t))\,L3\,m3\,(\cos(q3(t))\,L3$$

$$\left.+ 2\,L2)\cos(q2(t)) - \cos(q3(t))^2\,L3^2\,m3 + m3\,L3^2\right)\left(\frac{\mathrm{d}}{\mathrm{d}t}\,q1(t)\right)^2\Bigg)$$

$$+ \frac{\left(4\,\cos(q3(t))\,L2\,L3\,m3 + \left(4\,L2^2 + L3^2\right)m3 + m2\,L2^2\right)\left(\frac{\mathrm{d}}{\mathrm{d}t}\,q2(t)\right)^2}{8} \tag{54}$$

$$+ \frac{L3\left(\cos(q3(t))\ L2 + \frac{L3}{2}\right)\ m3\left(\frac{\mathrm{d}}{\mathrm{d}t}\ q3(t)\right)\left(\frac{\mathrm{d}}{\mathrm{d}t}\ q2(t)\right)}{2} + \frac{m3\left(\frac{\mathrm{d}}{\mathrm{d}t}\ q3(t)\right)^2 L3^2}{8}$$

```
> diffF := (f,x)->subs(y=x,diff(subs(x=y,f),y));
```
$$diffF := (f,\ x) \rightarrow subs\left(y = x,\ \frac{\partial}{\partial y}\ subs(x = y,\ f)\right) \tag{55}$$

```
> EQM_Left:=(Lagr,qt)->simplify(diff(diffF(Lagr,diff(qt,t)),t) - diffF(Lagr,qt));
```
$$EQM\_Left := (Lagr,\ qt) \rightarrow simplify\left(\frac{\partial}{\partial t}\ diffF\left(Lagr,\ \frac{\partial}{\partial t}\ qt\right) - diffF(Lagr,\ qt)\right) \tag{56}$$

```
> EQM_1:=EQM_Left(Lagr,q1(t))-C1;
```
$$EQM\_1 := \frac{1}{4}\Bigg(\Big(\big(2\ \cos(q3(t))^2\ L3^2\ m3 + 4\ \cos(q3(t))\ L2\ L3\ m3 + (4\ L2^2 - L3^2)\ m3 \tag{57}$$

$$+ m2\ L2^2\big)\cos(q2(t))^2 - 2\ \sin(q2(t))\ \sin(q3(t))\ L3\ m3\ (\cos(q3(t))\ L3$$

$$+ 2\ L2)\cos(q2(t)) - \cos(q3(t))^2\ L3^2\ m3 + m3\ L3^2\Big)\left(\frac{\mathrm{d}^2}{\mathrm{d}t^2}\ q1(t)\right)\Bigg)$$

$$-\Bigg(\bigg(\Big(\sin(q3(t))\ L3\ m3\ (\cos(q3(t))\ L3 + 2\ L2)\cos(q2(t))^2 + \Big(\cos(q3(t))^2\ L3^2\ m3$$

$$+ 2\ \cos(q3(t))\ L2\ L3\ m3 + \left(2\ L2^2 - \frac{L3^2}{2}\right)m3 + \frac{m2\ L2^2}{2}\Big)\sin(q2(t))\ \cos(q2(t))$$

$$- \frac{\sin(q3(t))\ L3\ m3\ (\cos(q3(t))\ L3 + 2\ L2)}{2}\bigg)\left(\frac{\mathrm{d}}{\mathrm{d}t}\ q2(t)\right) + L3\ m3\left(\frac{\mathrm{d}}{\mathrm{d}t}\right.$$

$$\left.q3(t)\right)\bigg(\sin(q3(t))\ (\cos(q3(t))\ L3 + L2)\cos(q2(t))^2 + \sin(q2(t))\Big(\cos(q3(t))^2\ L3$$

$$+ \cos(q3(t))\ L2 - \frac{L3}{2}\Big)\cos(q2(t)) - \frac{\cos(q3(t))\ \sin(q3(t))\ L3}{2}\bigg)\Bigg)\left(\frac{\mathrm{d}}{\mathrm{d}t}\ q1(t)\right) - C1$$

```
> EQM_2:=EQM_Left(Lagr,q2(t))-C2;
```
$$EQM\_2 := \frac{\left(4\ \cos(q3(t))\ L2\ L3\ m3 + (4\ L2^2 + L3^2)\ m3 + m2\ L2^2\right)\left(\frac{\mathrm{d}^2}{\mathrm{d}t^2}\ q2(t)\right)}{4} \tag{58}$$

$$+ \frac{L3\left(\cos(q3(t))\ L2 + \frac{L3}{2}\right)\ m3\left(\frac{\mathrm{d}^2}{\mathrm{d}t^2}\ q3(t)\right)}{2}$$

$$+ \frac{1}{4}\Bigg(\bigg(2\ \sin(q3(t))\ L3\ m3\ (\cos(q3(t))\ L3 + 2\ L2)\cos(q2(t))^2 + 2\Big(\cos(q3(t))^2\ L3^2\ m3$$

$$+ 2\ \cos(q3(t))\ L2\ L3\ m3 + \left(2\ L2^2 - \frac{L3^2}{2}\right)m3 + \frac{m2\ L2^2}{2}\Big)\sin(q2(t))\ \cos(q2(t))$$

$$- \sin(q3(t))\ L3\ m3\ (\cos(q3(t))\ L3 + 2\ L2)\bigg)\left(\frac{\mathrm{d}}{\mathrm{d}t}\ q1(t)\right)^2\Bigg) - \sin(q3(t))\ L3\ m3\ L2\left(\frac{\mathrm{d}}{\mathrm{d}t}\right.$$

$$\left.q3(t)\right)\Bigg(\frac{\mathrm{d}}{\mathrm{d}t}\ q2(t) + \frac{\frac{\mathrm{d}}{\mathrm{d}t}\ q3(t)}{2}\Bigg) - C2$$

```
> EQM_3:=EQM_Left(Lagr,q3(t))-C3;
```
$$EQM\_3 := \frac{1}{2}\Bigg(L3\Bigg(\left(\cos(q3(t))\ L2 + \frac{L3}{2}\right)\left(\frac{\mathrm{d}^2}{\mathrm{d}t^2}\ q2(t)\right) + \frac{L3\left(\frac{\mathrm{d}^2}{\mathrm{d}t^2}\ q3(t)\right)}{2} \tag{59}$$

$$+ \left(\sin(q3(t)) \ (\cos(q3(t)) \ L3 + L2) \ \cos(q2(t))^2 + \sin(q2(t)) \ \Big(\cos(q3(t))^2 \ L3 \right.$$

$$\left. + \cos(q3(t)) \ L2 - \frac{L3}{2} \right) \cos(q2(t)) - \frac{\cos(q3(t)) \ \sin(q3(t)) \ L3}{2} \Bigg) \left(\frac{d}{dt} \ q1(t) \right)^2$$

$$\left. + \sin(q3(t)) \ L2 \ \left(\frac{d}{dt} \ q2(t) \right)^2 \right) \ m3 \right) - C3$$

> `data`

$$\{ L1 = 0.3, \ L2 = 0.3, \ L3 = 0.3, \ g = 9.81, \ m1 = 25, \ m2 = 10, \ m3 = 5, \ q11 = \text{-}\pi, \ q12 = \pi, \ q21 = 0, \ q22$$

$$= \frac{\pi}{4}, \ q31 = 0, \ q32 = -\frac{\pi}{4}, \ Tmin\_sys = 1 \}$$

(60)

> `c1:=simplify(evalf(solve(subs(data,EQM_1)=0,C1)));`

$$c1 := \left(\left(0.2250000000 \ \cos(q3(t))^2 + 0.4500000000 \ \cos(q3(t)) + 0.5625000000 \right) \cos(q2(t))^2 + \Big(\right.$$

(61)

$$-0.2250000000 \ \sin(q2(t)) \ \sin(q3(t)) \ \cos(q3(t))$$

$$- 0.4500000000 \ \sin(q2(t)) \ \sin(q3(t)) \Big) \cos(q2(t)) - 0.1125000000 \ \cos(q3(t))^2$$

$$+ 0.1125000000 \Big) \left(\frac{d^2}{dt^2} \ q1(t) \right) + \Bigg(\left((-0.4500000000 \ \cos(q3(t)) \ \sin(q3(t)) \right)$$

$$- 0.9000000000 \ \sin(q3(t)) \Big) \cos(q2(t))^2 + \left(-0.4500000000 \ \sin(q2(t)) \ \cos(q3(t))^2 \right.$$

$$- 0.9000000000 \ \sin(q2(t)) \ \cos(q3(t)) - 1.125000000 \ \sin(q2(t)) \Big) \cos(q2(t))$$

$$+ 0.2250000000 \ \cos(q3(t)) \ \sin(q3(t)) + 0.4500000000 \ \sin(q3(t)) \Big) \left(\frac{d}{dt} \ q2(t) \right) + \Bigg(\Big($$

$$-0.4500000000 \ \cos(q3(t)) \ \sin(q3(t)) - 0.4500000000 \ \sin(q3(t)) \Big) \cos(q2(t))^2 + \Big($$

$$-0.4500000000 \ \sin(q2(t)) \ \cos(q3(t))^2 - 0.4500000000 \ \sin(q2(t)) \ \cos(q3(t))$$

$$+ 0.2250000000 \ \sin(q2(t)) \Big) \cos(q2(t)) + 0.2250000000 \ \cos(q3(t)) \ \sin(q3(t)) \Big) \left(\frac{d}{dt} \right.$$

$$q3(t) \Bigg) \Bigg) \left(\frac{d}{dt} \ q1(t) \right)$$

> `c2:=simplify(evalf(solve(subs(data,EQM_2)=0,C2)));`

$$c2 := \left(0.4500000000 \ \cos(q3(t)) + 0.7875000000 \right) \left(\frac{d^2}{dt^2} \ q2(t) \right) + \left(0.2250000000 \ \cos(q3(t)) \right.$$

(62)

$$+ 0.1125000000 \Big) \left(\frac{d^2}{dt^2} \ q3(t) \right) + \left(0.2250000000 \ \sin(q2(t)) \ \cos(q2(t)) \ \cos(q3(t))^2 \right.$$

$$+ \left(\left(0.2250000000 \ \cos(q2(t))^2 - 0.1125000000 \right) \sin(q3(t)) \right.$$

$$+ 0.4500000000 \ \cos(q2(t)) \ \sin(q2(t)) \Big) \cos(q3(t)) + \left(0.4500000000 \ \cos(q2(t))^2 \right.$$

$$- 0.2250000000 \Big) \sin(q3(t)) + 0.5625000000 \ \cos(q2(t)) \ \sin(q2(t)) \Bigg) \left(\frac{d}{dt} \ q1(t) \right)^2$$

$$- 0.4500000000 \ \sin(q3(t)) \left(\frac{d}{dt} \ q3(t) \right) \left(\frac{d}{dt} \ q2(t) \right)$$

$$- 0.2250000000 \ \sin(q3(t)) \ \left(\frac{d}{dt} \ q3(t) \right)^2$$

> `c3:=simplify(evalf(solve(subs(data,EQM_3)=0,C3)));`

(63)

$$c3 := \left(0.2250000000 \, \cos(q3(t)) + 0.1125000000\right) \left(\frac{d^2}{dt^2} \, q2(t)\right) + 0.1125000000 \, \frac{d^2}{dt^2} \, q3(t) \tag{63}$$

$$+ \Big((0.2250000000 \, \cos(q3(t)) \, \sin(q3(t)) + 0.2250000000 \, \sin(q3(t))) \, \cos(q2(t))^2$$

$$+ \left(0.2250000000 \, \sin(q2(t)) \, \cos(q3(t))^2 + 0.2250000000 \, \sin(q2(t)) \, \cos(q3(t))\right.$$

$$\left. - 0.1125000000 \, \sin(q2(t))\right) \, \cos(q2(t)) - 0.1125000000 \, \cos(q3(t)) \, \sin(q3(t))\Big)$$

$$\left(\frac{d}{dt} \, q1(t)\right)^2 + 0.2250000000 \, \sin(q3(t)) \left(\frac{d}{dt} \, q2(t)\right)^2$$

> `c1_profile:=simplify(evalf(subs(q1(t)=q1_profile, q2(t)=q2_profile, q3(t)=q3_profile, c1)));`

$$c1\_profile := \Bigg\{$$

$$\left(-106.5917276 \; t^2 \, \cos\left(1.570\ldots\right.\right.$$

$$-16.96460034 \, \cos\left(0.7853981635 - 3.141592654 \, t + 1.570796327 \, t^2\right)^2 + \left(-11.30973356 + 106.5917276 \ldots\right.$$

> `c2_profile:=simplify(evalf(subs(q1(t)=q1_profile, q2(t)=q2_profile, q3(t)=q3_profile, c2)));`

$$c2\_profile := \Bigg\{$$

$$\left(426.3669105 \; t^2 \, \cos\ldots\right.$$

$$\Big((-426.3669105 + 852.7338207 \, t - 426.3669105 \, t^2) \, \cos\left(0.7853981635 - 3.141592654 \, t + 1.570796327 \ldots\right.$$
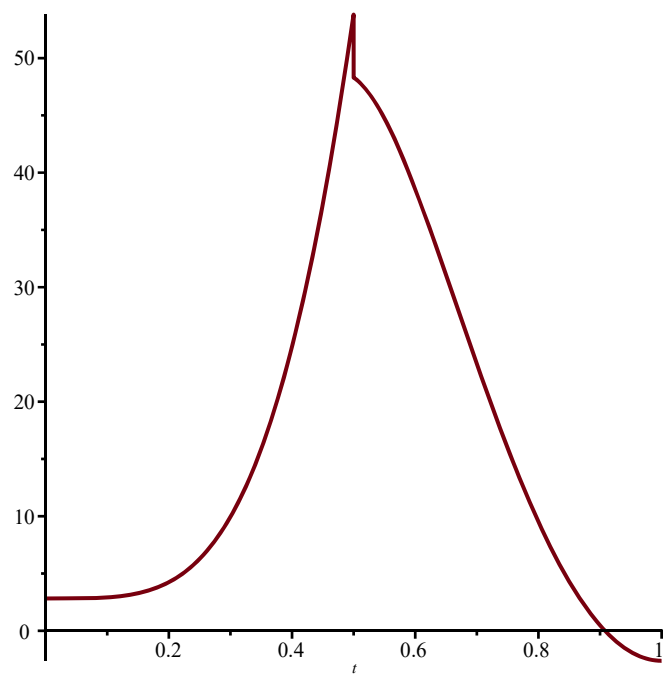
```
> c3_profile:=simplify(evalf(subs(q1(t)=q1_profile, q2(t)=q2_profile, q3(t)=q3_profile,
  c3)));
```

$$c3\_profile := \begin{cases} 0. \\ undefined \\ 0.7068583472\ \cos\left(1.570796327\ t^2\right) - 2.220660991\ t^2\ \sin\left(1.57079\ \cdots\right. \\ undefined \\ 2.220660991\ (t-1.)^2\ \sin\left(0.7853981635 - 3.141592654\ t + 1.570796327\ t^2\right) - 0.7068583472\ \cos\left(0.7\ \cdots\right. \\ undefined \\ 0. \end{cases}$$

```
> plot(c1_profile, t=0..1);
```
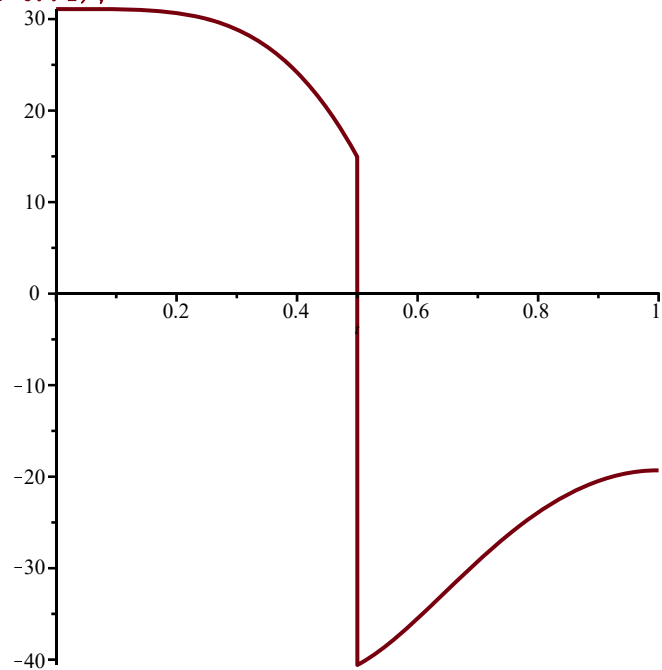


```
> plot(c2_profile, t=0..1);
```

> `plot(c3_profile, t=0..1);`