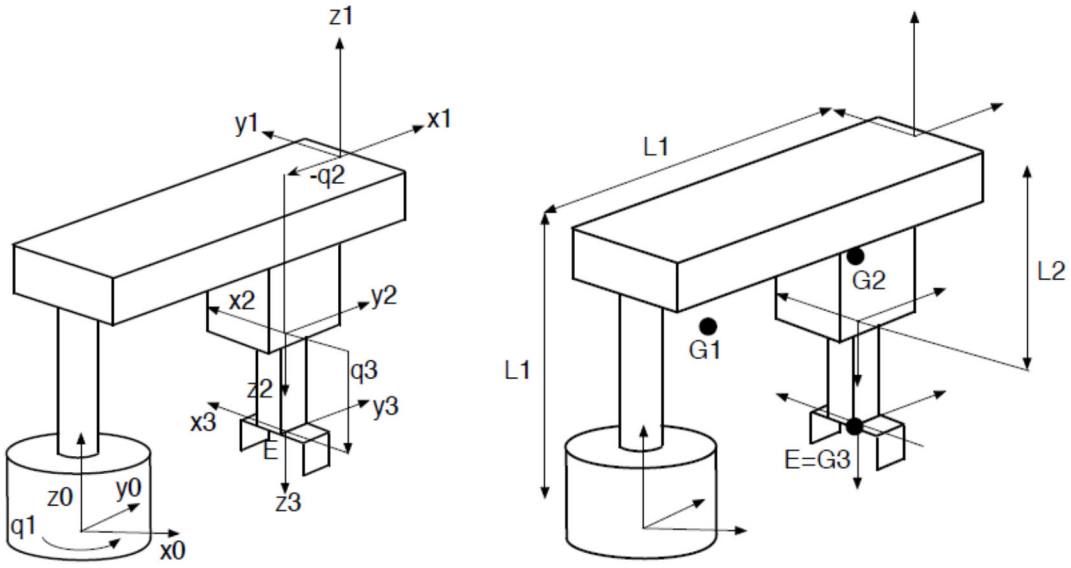
```
> restart:with(LinearAlgebra):
```




```
> data:={L1=1,L2=0.5,m1=20,m2=5,m3=3,K1=1000,K2=1000,K3=1000,g=9.81,
  amax1=2,amax2=2,amax3=3};
```

$$data := \{K1 = 1000,\ K2 = 1000,\ K3 = 1000,\ L1 = 1,\ L2 = 0.5,\ amax1 = 2,\ amax2 = 2,\ amax3 = 3,\ g = 9.81,\ m1 = 20,\ m2 = 5,\ m3 = 3\} \tag{1}$$

```
> config_ini:={q11=Pi/4,q21=-0.5,q31=0.3};
```

$$config\_ini := \left\{q11 = \frac{\pi}{4},\ q21 = -0.5,\ q31 = 0.3\right\} \tag{2}$$

```
> config_fin:={q12=-Pi/4,q22=0,q32=0.1};
```

$$config\_fin := \left\{q12 = -\frac{\pi}{4},\ q22 = 0,\ q32 = 0.1\right\} \tag{3}$$

```
> Mrotztransl:=(theta,point)->&lt;&lt;cos(theta),sin(theta),0,0&gt;|&lt;-sin(theta),
  cos(theta),0,0&gt;|&lt;0,0,1,0&gt;|&lt;point[1],point[2],point[3],1&gt;&gt;;
```

$$Mrotztransl := (\theta,\ point) \mapsto \left\langle \langle \cos(\theta),\ \sin(\theta),\ 0,\ 0\rangle \middle| \langle -\sin(\theta),\ \cos(\theta),\ 0,\ 0\rangle \middle| \langle 0,\ 0,\ 1,\ 0\rangle \middle| \langle point_1,\ point_2,\ point_3,\ 1\rangle \right\rangle \tag{4}$$

```
> Mrotxtransl:=(alpha,point)->&lt;&lt;1,0,0,0&gt;|&lt;0,cos(alpha),sin(alpha),0&gt;|&lt;0,-
  sin(alpha),cos(alpha),0&gt;|&lt;point[1],point[2],point[3],1&gt;&gt;;
```

$$Mrotxtransl := (\alpha,\ point) \mapsto \left\langle \langle 1,\ 0,\ 0,\ 0\rangle \middle| \langle 0,\ \cos(\alpha),\ \sin(\alpha),\ 0\rangle \middle| \langle 0,\ -\sin(\alpha),\ \cos(\alpha),\ 0\rangle \middle| \langle point_1,\ point_2,\ point_3,\ 1\rangle \right\rangle \tag{5}$$

```
> M01:=simplify(Mrotztransl(q1(t),&lt;0,0,L1&gt;).Mrotxtransl(0,&lt;L1,0,0&gt;));
```

$$M01 := \begin{bmatrix} \cos(q1(t)) & -\sin(q1(t)) & 0 & \cos(q1(t))\ L1 \\ \sin(q1(t)) & \cos(q1(t)) & 0 & \sin(q1(t))\ L1 \\ 0 & 0 & 1 & L1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{6}$$

```
> M12:=simplify(Mrotxtransl(Pi,&lt;q2(t),0,0&gt;).Mrotztransl(-Pi/2,&lt;0,0,L2&gt;))
```

;

$$M12 := \begin{bmatrix} 0 & 1 & 0 & q2(t) \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -L2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{7}$$

```
> M02:=simplify(M01.M12);
```

$$M02 := \begin{bmatrix} -\sin(q1(t)) & \cos(q1(t)) & 0 & \cos(q1(t))\ (L1+q2(t)) \\ \cos(q1(t)) & \sin(q1(t)) & 0 & \sin(q1(t))\ (L1+q2(t)) \\ 0 & 0 & -1 & -L2+L1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{8}$$

```
> M23:=Mrotztransl(0,<0,0,q3(t)>);
```

$$M23 := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & q3(t) \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{9}$$

```
> M03:=simplify(M02.M23);
```

$$M03 := \begin{bmatrix} -\sin(q1(t)) & \cos(q1(t)) & 0 & \cos(q1(t))\ (L1+q2(t)) \\ \cos(q1(t)) & \sin(q1(t)) & 0 & \sin(q1(t))\ (L1+q2(t)) \\ 0 & 0 & -1 & -q3(t)-L2+L1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{10}$$

```
> E:=simplify(M03.<0,0,0,1>);
```

$$E := \begin{bmatrix} \cos(q1(t))\ (L1+q2(t)) \\ \sin(q1(t))\ (L1+q2(t)) \\ -q3(t)-L2+L1 \\ 1 \end{bmatrix} \tag{11}$$

```
> Jac_E:=simplify(VectorCalculus[Jacobian](subs(q1(t)=q1,q2(t)=q2,q3(t)=
  q3,E[1..3]),[q1,q2,q3]));
```

$$Jac\_E := \begin{bmatrix} -\sin(q1)\ (L1+q2) & \cos(q1) & 0 \\ \cos(q1)\ (L1+q2) & \sin(q1) & 0 \\ 0 & 0 & -1 \end{bmatrix} \tag{12}$$

```
> Det_Jac_E:=simplify(Determinant(Jac_E));
```

$$Det\_Jac\_E := L1+q2 \tag{13}$$

```
> q2_singular:=solve(Det_Jac_E=0,q2);
```

$$q2\_singular := -L1 \tag{14}$$

```
> data
```

$$\{K1 = 1000,\ K2 = 1000,\ K3 = 1000,\ L1 = 1,\ L2 = 0.5,\ amax1 = 2,\ amax2 = 2,\ amax3 = 3,\ g = 9.81,\ m1 = 20,\ m2 = 5,\ m3 = 3\} \tag{15}$$

```
> config_ini
```

$$\left\{q11 = \frac{\pi}{4},\ q21 = -0.5,\ q31 = 0.3\right\} \tag{16}$$

```
> config_fin
```

$$\left\{q12 = -\frac{\pi}{4},\ q22 = 0,\ q32 = 0.1\right\} \tag{17}$$

```
> Tmin:=sqrt(4*abs(deltaq)/amax);
```

$$Tmin := 2\sqrt{\frac{|deltaq|}{amax}} \tag{18}$$

```
> acc_re:=4*deltaq/(T^2);
```

$$acc\_re := \frac{4\ deltaq}{T^2} \tag{19}$$

```
> Qbase_profile:=piecewise(t>=0 and t<=T/2,qini+1/2*amax*(t^2),t>=T/2 and
  t<=T,qini+1/4*amax*(T^2)-1/2*amax*((T-t)^2));
```

$$Qbase\_profile := \begin{cases} qini + \frac{amax\ t^2}{2} & 0 \le t \le \frac{T}{2} \\ qini + \frac{amax\ T^2}{4} - \frac{amax\ (T-t)^2}{2} & \frac{T}{2} \le t \le T \end{cases} \tag{20}$$

```
> Vbase_profile:=diff(Qbase_profile,t);
```

$$Vbase\_profile := \begin{cases} amax\ t & 0 \le t \le \frac{T}{2} \\ amax\ (T-t) & \frac{T}{2} \le t \le T \end{cases} \tag{21}$$

```
> Accbase_profile:=diff(Qbase_profile,t,t);
```

$$Accbase\_profile := \begin{cases} amax & 0 \le t \le \frac{T}{2} \\ -amax & \frac{T}{2} \le t \le T \end{cases} \tag{22}$$

```
> Tmin_q1:=evalf(subs(deltaq=q12-q11,amax=amax1,config_ini,config_fin,
  data,Tmin));
```

$$Tmin\_q1 := 1.772453850 \tag{23}$$

```
> Tmin_q2:=evalf(subs(deltaq=q22-q21,amax=amax2,config_ini,config_fin,
  data,Tmin));
```

$$Tmin\_q2 := 0.9999999998 \tag{24}$$

```
> Tmin_q3:=evalf(subs(deltaq=q32-q31,amax=amax3,config_ini,config_fin,
  data,Tmin));
```

$$Tmin\_q3 := 0.5163977796 \quad (25)$$

```
> Tmin_sys:=Tmin_q1;
```

$$Tmin\_sys := 1.772453850 \quad (26)$$

```
> config_ini;config_fin;
```

$$\left\{q11 = \frac{\pi}{4},\ q21 = -0.5,\ q31 = 0.3\right\}$$

$$\left\{q12 = -\frac{\pi}{4},\ q22 = 0,\ q32 = 0.1\right\} \quad (27)$$

```
> amax1_re:=-2;
```

$$amax1\_re := -2 \quad (28)$$

```
> amax2_re:=evalf(subs(deltaq=q22-q21,T=Tmin_sys,config_ini,config_fin,
  acc_re));
```

$$amax2\_re := 0.6366197730 \quad (29)$$

```
> amax3_re:=evalf(subs(deltaq=q32-q31,T=Tmin_sys,config_ini,config_fin,
  acc_re));
```

$$amax3\_re := -0.2546479092 \quad (30)$$

```
> q1_profile:=evalf(subs(amax=amax1_re,T=Tmin_sys,qini=q11,config_ini,
  Qbase_profile));
```

$$q1\_profile := \quad (31)$$

$$\begin{cases} -1.\ t^2 + 0.7853981635 & 0.\ \le t \le 0.8862269250 \\ -0.7853981615 + (1.772453850 - 1.\ t)^2 & 0.8862269250 \le t \le 1.772453850 \end{cases}$$

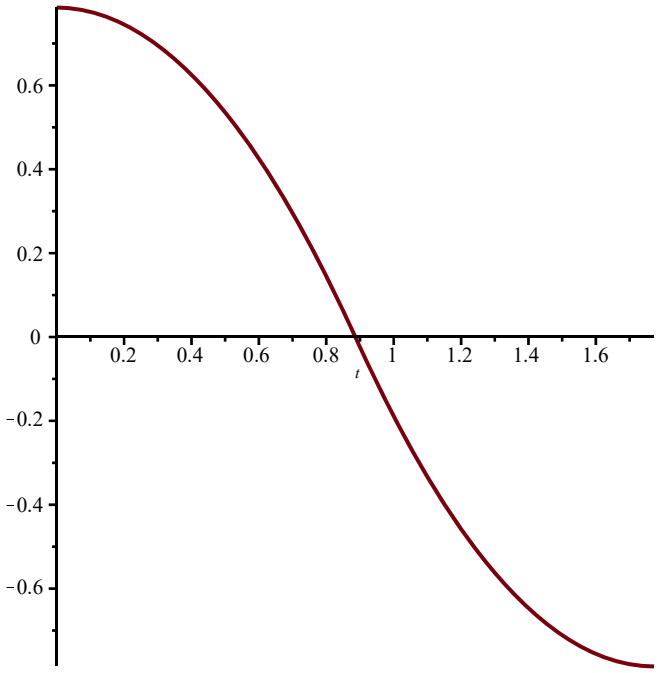
```
> q2_profile:=evalf(subs(amax=amax2_re,T=Tmin_sys,qini=q21,config_ini,
  Qbase_profile));
```

$$q2\_profile := \begin{cases} -0.5 + 0.3183098865\ t^2 & 0.\ \le t \le 0.8862269250 \\ -2.\ 10^{-10} - 0.3183098865\ (1.772453850 - 1.\ t)^2 & 0.8862269250 \le t \le 1.772453 \end{cases}$$

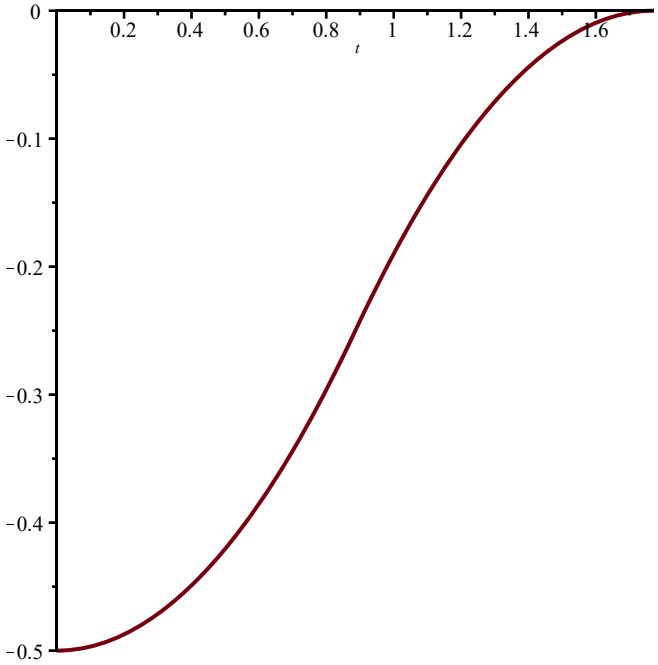
```
> q3_profile:=evalf(subs(amax=amax3_re,T=Tmin_sys,qini=q31,config_ini,
  Qbase_profile));
```

$$q3\_profile := \begin{cases} 0.3 - 0.1273239546\ t^2 & 0.\ \le t \le 0.886226925 \\ 0.1000000000 + 0.1273239546\ (1.772453850 - 1.\ t)^2 & 0.8862269250 \le t \le 1.7724 \end{cases}$$

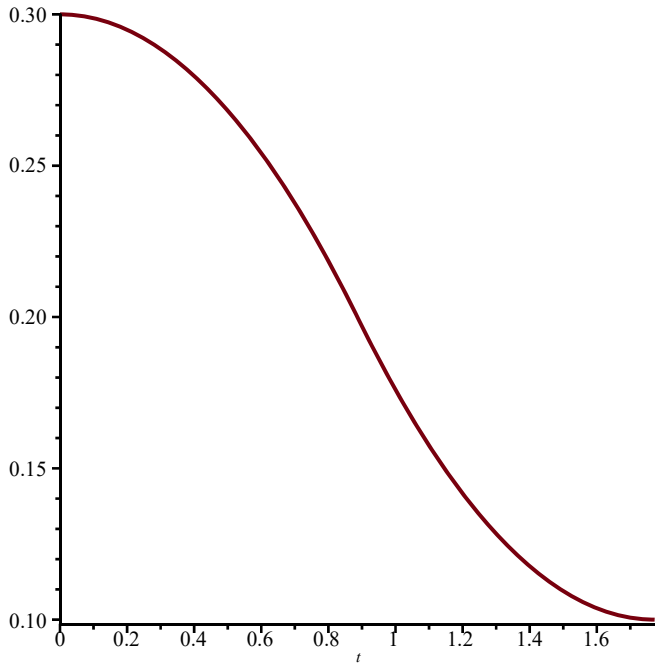
```
> plot(q1_profile,t=0..Tmin_sys);
```




```
> plot(q2_profile,t=0..Tmin_sys);
```




```
> plot(q3_profile,t=0..Tmin_sys);
```




```
> T_Func:=(W,J)->simplify(Trace(1/2*W.J.(W^%T)));
```

$$T\_Func := (W,\ J) \mapsto simplify\left(LinearAlgebra:-Trace\left(\left(\frac{W}{2}\right) \cdot J \cdot W^{T}\right)\right) \tag{34}$$

```
> W_Func:=(M)->simplify(map(diff,M,t).MatrixInverse(M));
```

$$W\_Func := M \mapsto simplify(map(diff,\ M,\ t) \cdot LinearAlgebra:-MatrixInverse(M)) \tag{35}$$

```
> J_Lumped_Func:=(m,G)-><<m*G[1]*G[1],m*G[1]*G[2],m*G[1]*G[3],m*G[1]>|<m*
  G[2]*G[1],m*G[2]*G[2],m*G[2]*G[3],m*G[2]>|<m*G[3]*G[1],m*G[3]*G[2],m*G
  [3]*G[3],m*G[3]>|<m*G[1],m*G[2],m*G[3],m>>;
```

$$J\_Lumped\_Func := (m,\ G) \mapsto \left\langle \left\langle m \cdot G_1 \cdot G_1,\ m \cdot G_1 \cdot G_2,\ m \cdot G_1 \cdot G_3,\ m \cdot G_1 \right\rangle \middle| \left\langle m \cdot G_1 \cdot G_2,\ m \cdot G_2 \cdot G_2,\ m \cdot G_2 \cdot G_3,\ m \cdot G_2 \right\rangle \middle| \left\langle m \cdot G_1 \cdot G_3,\ m \cdot G_2 \cdot G_3,\ m \cdot G_3 \cdot G_3,\ m \cdot G_3 \right\rangle \middle| \left\langle m \cdot G_1,\ m \cdot G_2,\ m \cdot G_3,\ m \right\rangle \right\rangle \tag{36}$$

```
> J_MobileToFixed:=(M,J)->simplify(M.J.(M^%T));
```

$$J\_MobileToFixed := (M,\ J) \mapsto simplify\left(M \cdot J \cdot M^{T}\right) \tag{37}$$

```
> U_Func:=(Hg,J)->simplify(Trace(-Hg.J));
```

$$U\_Func := (Hg,\ J) \mapsto simplify(LinearAlgebra:-Trace(-Hg \cdot J)) \tag{38}$$

```
> Hg_template:=<<0,0,0,0>|<0,0,0,0>|<0,0,0,0>|<0,0,-g,0>>;
```

$$Hg\_template := \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -g \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{39}$$

```
> W01:=W_Func(M01);
```

(40)

$$W01 := \begin{bmatrix} 0 & -\frac{d}{dt} q1(t) & 0 & 0 \\ \frac{d}{dt} q1(t) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{40}$$

```
> W02:=W_Func(M02);
```

$$W02 := \begin{bmatrix} 0 & -\frac{d}{dt} q1(t) & 0 & \cos(q1(t)) \left(\frac{d}{dt} q2(t)\right) \\ \frac{d}{dt} q1(t) & 0 & 0 & \sin(q1(t)) \left(\frac{d}{dt} q2(t)\right) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{41}$$

```
> W03:=W_Func(M03);
```

$$W03 := \begin{bmatrix} 0 & -\frac{d}{dt} q1(t) & 0 & \cos(q1(t)) \left(\frac{d}{dt} q2(t)\right) \\ \frac{d}{dt} q1(t) & 0 & 0 & \sin(q1(t)) \left(\frac{d}{dt} q2(t)\right) \\ 0 & 0 & 0 & -\frac{d}{dt} q3(t) \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{42}$$

```
> J11:=J_Lumped_Func(m1,<-3/4*L1,0,-1/4*L1>);
```

$$J11 := \begin{bmatrix} \frac{9\ m1\ L1^2}{16} & 0 & \frac{3\ m1\ L1^2}{16} & -\frac{3\ m1\ L1}{4} \\ 0 & 0 & 0 & 0 \\ \frac{3\ m1\ L1^2}{16} & 0 & \frac{m1\ L1^2}{16} & -\frac{m1\ L1}{4} \\ -\frac{3\ m1\ L1}{4} & 0 & -\frac{m1\ L1}{4} & m1 \end{bmatrix} \tag{43}$$

```
> J22:=J_Lumped_Func(m2,<0,0,-1/2*L2>);
```

(44)

$$J22 := \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{m2\ L2^2}{4} & -\frac{m2\ L2}{2} \\ 0 & 0 & -\frac{m2\ L2}{2} & m2 \end{bmatrix} \tag{44}$$

```
> J33:=J_Lumped_Func(m3,<0,0,0>);
```

$$J33 := \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & m3 \end{bmatrix} \tag{45}$$

```
> J10:=J_MobileToFixed(M01,J11);
```

$$J10 := \left[\left[\frac{\cos(q1(t))^2\ m1\ L1^2}{16},\ \frac{\cos(q1(t))\ m1\ L1^2\ \sin(q1(t))}{16},\ \frac{3\ \cos(q1(t))\ m1\ L1^2}{16},\ \frac{\cos(q1(t))\ m1\ L1}{4}\right],\right.$$
$$\left[\frac{\cos(q1(t))\ m1\ L1^2\ \sin(q1(t))}{16},\ \frac{\sin(q1(t))^2\ m1\ L1^2}{16},\ \frac{3\ \sin(q1(t))\ m1\ L1^2}{16},\ \frac{\sin(q1(t))\ m1\ L1}{4}\right],$$
$$\left[\frac{3\ \cos(q1(t))\ m1\ L1^2}{16},\ \frac{3\ \sin(q1(t))\ m1\ L1^2}{16},\ \frac{9\ m1\ L1^2}{16},\ \frac{3\ m1\ L1}{4}\right],$$
$$\left.\left[\frac{\cos(q1(t))\ m1\ L1}{4},\ \frac{\sin(q1(t))\ m1\ L1}{4},\ \frac{3\ m1\ L1}{4},\ m1\right]\right] \tag{46}$$

```
> J20:=J_MobileToFixed(M02,J22);
```

$$J20 := \left[\left[\cos(q1(t))^2\ (L1+q2(t))^2\ m2,\ \cos(q1(t))\ (L1+q2(t))^2\ m2\ \sin(q1(t)),\ \frac{\cos(q1(t))\ (L1+q2(t))\ m2\ (-L2+2\ L1)}{2},\right.\right.$$
$$\left.\cos(q1(t))\ (L1+q2(t))\ m2\right],$$
$$\left[\cos(q1(t))\ (L1+q2(t))^2\ m2\ \sin(q1(t)),\ \sin(q1(t))^2\ (L1+q2(t))^2\ m2,\right.$$
$$\left.\left.\frac{\sin(q1(t))\ (L1+q2(t))\ m2\ (-L2+2\ L1)}{2},\ \sin(q1(t))\ (L1+q2(t))\ m2\right],\right. \tag{47}$$

$$\left[\frac{\cos(q1(t))\ (L1+q2(t))\ m2\ (-L2+2\ L1)}{2},\ \frac{\sin(q1(t))\ (L1+q2(t))\ m2\ (-L2+2\ L1)}{2},\ \left(-\frac{L2}{2}+L1\right)^2\ m2,\ \frac{m2\ (-L2+2\ L1)}{2}\right],$$

$$\left[\cos(q1(t))\ (L1+q2(t))\ m2,\ \sin(q1(t))\ (L1+q2(t))\ m2,\ \frac{m2\ (-L2+2\ L1)}{2},\ m2\right]\Bigg]$$

```
> J30:=J_MobileToFixed(M03,J33);
```

$$J30 := [[\cos(q1(t))^2\ (L1+q2(t))^2\ m3,\ \cos(q1(t))\ (L1+q2(t))^2\ m3\ \sin(q1(t)),\ \cos(q1(t))\ (L1+q2(t))\ m3\ (-q3(t)-L2+L1),\ \cos(q1(t))\ (L1+q2(t))\ m3],$$
$$[\cos(q1(t))\ (L1+q2(t))^2\ m3\ \sin(q1(t)),\ \sin(q1(t))^2\ (L1+q2(t))^2\ m3,\ \sin(q1(t))\ (L1+q2(t))\ m3\ (-q3(t)-L2+L1),\ \sin(q1(t))\ (L1+q2(t))\ m3],$$
$$[\cos(q1(t))\ (L1+q2(t))\ m3\ (-q3(t)-L2+L1),\ \sin(q1(t))\ (L1+q2(t))\ m3\ (-q3(t)-L2+L1),\ (-q3(t)-L2+L1)^2\ m3,\ (-q3(t)-L2+L1)\ m3],$$
$$[\cos(q1(t))\ (L1+q2(t))\ m3,\ \sin(q1(t))\ (L1+q2(t))\ m3,\ (-q3(t)-L2+L1)\ m3,\ m3]] \tag{48}$$

```
> T1:=T_Func(W01,J10);
```

$$T1 := \frac{\left(\frac{d}{dt}\ q1(t)\right)^2\ m1\ L1^2}{32} \tag{49}$$

```
> T2:=T_Func(W02,J20);
```

$$T2 := \frac{\left((L1+q2(t))^2\ \left(\frac{d}{dt}\ q1(t)\right)^2+\left(\frac{d}{dt}\ q2(t)\right)^2\right)\ m2}{2} \tag{50}$$

```
> T3:=T_Func(W03,J30);
```

$$T3 := \frac{\left((L1+q2(t))^2\ \left(\frac{d}{dt}\ q1(t)\right)^2+\left(\frac{d}{dt}\ q2(t)\right)^2+\left(\frac{d}{dt}\ q3(t)\right)^2\right)\ m3}{2} \tag{51}$$

```
> Hg1:=Hg_template;
```

$$Hg1 := \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -g \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{52}$$

```
> Hg2:=Hg_template;
```

$$Hg2 := \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -g \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{53}$$

```
> Hg3:=Hg_template;
```

$$Hg3 := \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -g \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{54}$$

```
> U1:=0*U_Func(Hg1,J10);
```

$$U1 := 0 \tag{55}$$

```
> U2:=0*U_Func(Hg2,J20);
```

$$U2 := 0 \tag{56}$$

```
> U3:=0*U_Func(Hg3,J30);
```

$$U3 := 0 \tag{57}$$

```
> Lagr:=simplify(T1+T2+T3-U1-U2-U3);
```

$$Lagr := \frac{1}{32}\left(\left((16\ m2+16\ m3)\ q2(t)^2+32\ L1\ (m2+m3)\ q2(t)+L1^2\ (m1+16\ m2+16\ m3)\right)\left(\frac{\mathrm{d}}{\mathrm{d}t}\ q1(t)\right)^2\right)+\frac{(16\ m2+16\ m3)\left(\frac{\mathrm{d}}{\mathrm{d}t}\ q2(t)\right)^2}{32}+\frac{\left(\frac{\mathrm{d}}{\mathrm{d}t}\ q3(t)\right)^2\ m3}{2} \tag{58}$$

```
> P1:=<-1/3*L1,0,0,1>;
```

$$P1 := \begin{bmatrix} -\frac{L1}{3} \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{59}$$

```
> F1:=<0,-F,0,0>;
```

$$F1 := \begin{bmatrix} 0 \\ -F \\ 0 \\ 0 \end{bmatrix} \tag{60}$$

```
> Lrot:=<<0,1,0,0>|<-1,0,0,0>|<0,0,0,0>|<0,0,0,0>>;
```

$$Lrot := \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{61}$$

```
> Phi11:=simplify(F1.(P1^%T)-P1.(F1^%T));
```

$$\Phi 11 := \begin{bmatrix} 0 & -\frac{F\ L1}{3} & 0 & 0 \\ \frac{F\ L1}{3} & 0 & 0 & -F \\ 0 & 0 & 0 & 0 \\ 0 & F & 0 & 0 \end{bmatrix} \tag{62}$$

```
> Phi10:=simplify(M01.Phi11.(M01^%T));
```

$$\Phi 10 := \tag{63}$$

$$\begin{bmatrix} 0 & \frac{2\ F\ L1}{3} & \sin(q1(t))\ F\ L1 & \sin(q1(t))\ F \\ -\frac{2\ F\ L1}{3} & 0 & -\cos(q1(t))\ F\ L1 & -\cos(q1(t))\ F \\ -\sin(q1(t))\ F\ L1 & \cos(q1(t))\ F\ L1 & 0 & 0 \\ -\sin(q1(t))\ F & \cos(q1(t))\ F & 0 & 0 \end{bmatrix}$$

```
> PSS:=(matrix1,matrix2)->matrix1[3,2]*matrix2[3,2]+matrix1[1,3]*matrix2
  [1,3]+matrix1[2,1]*matrix2[2,1]+matrix1[1,4]*matrix2[1,4]+matrix1[2,4]*
  matrix2[2,4]+matrix1[3,4]*matrix2[3,4];
```

$$PSS := (matrix1,\ matrix2) \mapsto matrix1_{1,\ 3} \cdot matrix2_{1,\ 3} + matrix1_{1,\ 4} \cdot matrix2_{1,\ 4} + matrix1_{2,\ 1} \cdot matrix2_{2,\ 1} + matrix1_{2,\ 4} \cdot matrix2_{2,\ 4} + matrix1_{3,\ 2} \cdot matrix2_{3,\ 2} + matrix1_{3,\ 4} \cdot matrix2_{3,\ 4} \tag{64}$$

```
> fq1:=PSS(Phi10,Lrot);
```

$$fq1 := -\frac{2\ F\ L1}{3} \tag{65}$$

```
> diffF := (f,x)->subs(y=x,diff(subs(x=y,f),y));
```

$$diffF := (f,\ x) \rightarrow subs\left(y = x,\ \frac{\partial}{\partial y}\ subs(x = y,\ f)\right) \tag{66}$$

```
> EQM_Left:=(Lagr,qt)->simplify(diff(diffF(Lagr,diff(qt,t)),t)-diffF
  (Lagr,qt));
```

$$EQM\_Left := (Lagr,\ qt) \rightarrow simplify\left(\frac{\partial}{\partial t}\ diffF\left(Lagr,\ \frac{\partial}{\partial t}\ qt\right) - diffF(Lagr,\ qt)\right) \tag{67}$$

```
> EQM_1:=EQM_Left(Lagr,q1(t))-fq1-C1;
```

$$EQM\_1 := \frac{1}{16}\left(\left((16\ m2 + 16\ m3)\ q2(t)^2 + 32\ L1\ (m2 + m3)\ q2(t) + L1^2\ (m1 + 16\ m2 + 16\ m3)\right)\left(\frac{d^2}{dt^2}\ q1(t)\right)\right) + 2\left(\frac{d}{dt}\ q1(t)\right)\left(\frac{d}{dt}\ q2(t)\right)(m2 + m3)\ (L1 + q2(t)) + \frac{2\ F\ L1}{3} - C1 \tag{68}$$

```
> EQM_2:=EQM_Left(Lagr,q2(t))-F2;
```

$$EQM\_2 := -\left(-\frac{d^2}{dt^2}\ q2(t) + \left(\frac{d}{dt}\ q1(t)\right)^2\ (L1 + q2(t))\right)(m2 + m3) - F2 \tag{69}$$

```
> EQM_3:=EQM_Left(Lagr,q3(t))-F3;
```

$$EQM\_3 := \left(\frac{d^2}{dt^2}\ q3(t)\right)\ m3 - F3 \tag{70}$$

```
> EQM_1_controlled:=subs(C1=K1*(q1_profile-q1(t)),data,F=2*q1(t),EQM_1);
```

$$EQM\_1\_controlled := \frac{(128\ q2(t)^2 + 256\ q2(t) + 148)\left(\frac{d^2}{dt^2}\ q1(t)\right)}{16} + 16\left(\frac{d}{dt}\ q1(t)\right)\left(\frac{d}{dt}\ q2(t)\right)(1 + q2(t)) + \frac{3004\ q1(t)}{3} - 1000\left(\begin{cases} -1.\ t^2 + 0.7853981635 & 0. \le t \le 0.8862269250 \\ -0.7853981615 + (1.772453850 - 1.\ t)^2 & 0.8862269250 \le t \le 1.772453850 \end{cases}\right) \tag{71}$$

```
> EQM_2_controlled:=subs(F2=K2*(q2_profile-q2(t)),data,EQM_2);
```

$$EQM\_2\_controlled := 8\ \frac{d^2}{dt^2}\ q2(t) - 8\left(\frac{d}{dt}\ q1(t)\right)^2\ (1 + q2(t)) - 1000\left(\begin{cases} -0.5 + 0.31 \\ -2.\ 10^{-10} - 0.3183098 \end{cases}\right.$$

$$\left.\right) + 1000\ q2(t)$$

```
> EQM_3_controlled:=subs(F3=K3*(q3_profile-q3(t)),data,EQM_3);
```

$$EQM\_3\_controlled := 3\ \frac{d^2}{dt^2}\ q3(t) - 1000\ \left(\left\{ \begin{array}{c} 0.3 - 0.1273239546\ t^2 \\ 0.1000000000 + 0.1273239546\ (1.772453850 - 1.\ \ t)^2 \end{array} \right.\right.$$

$$\left.\right) + 1000\ q3(t)$$

```
> Qcontrolled:=dsolve({EQM_1_controlled,EQM_2_controlled,
  EQM_3_controlled,q1(0)=Pi/4,q2(0)=-0.5,q3(0)=0.3,D(q1)(0)=0,D(q2)(0)=0,
  D(q3)(0)=0},{q1(t),q2(t),q3(t)},numeric,output=listprocedure);
```

$$Qcontrolled := \left[t = \mathtt{proc}(t)\ \ldots\ \mathtt{end\ proc},\ q1(t) = \mathtt{proc}(t)\ \ldots\ \mathtt{end\ proc},\ \frac{d}{dt}\right. \qquad (74)$$

$$q1(t) = \mathtt{proc}(t)\ \ldots\ \mathtt{end\ proc},\ q2(t) = \mathtt{proc}(t)\ \ldots\ \mathtt{end\ proc},\ \frac{d}{dt}\ q2(t) =$$

$$\mathtt{proc}(t)$$

$$\ldots$$

$$\left.\mathtt{end\ proc},\ q3(t) = \mathtt{proc}(t)\ \ldots\ \mathtt{end\ proc},\ \frac{d}{dt}\ q3(t) = \mathtt{proc}(t)\ \ldots\ \mathtt{end\ proc}\right]$$

```
> q1_controlled:=rhs(Qcontrolled[2]);
```

$$q1\_controlled := \mathtt{proc}(t)\ \ldots\ \mathtt{end\ proc} \qquad (75)$$

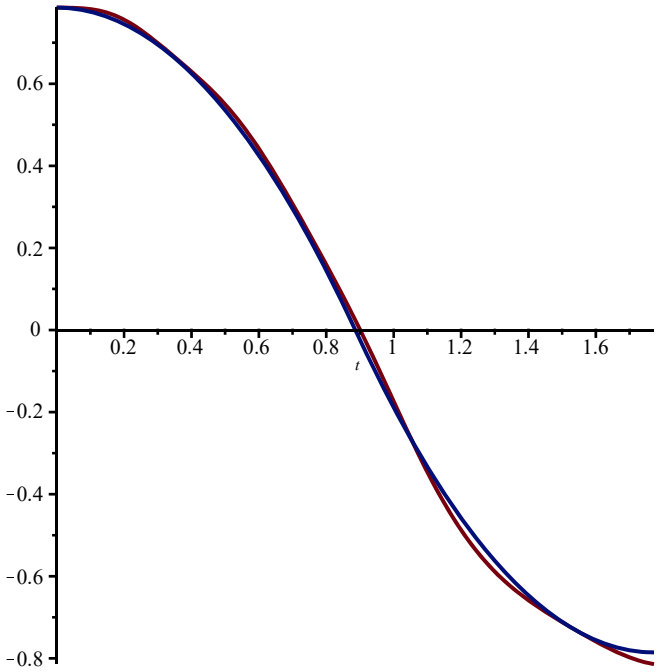
```
> q2_controlled:=rhs(Qcontrolled[4]);
```

$$q2\_controlled := \mathtt{proc}(t)\ \ldots\ \mathtt{end\ proc} \qquad (76)$$

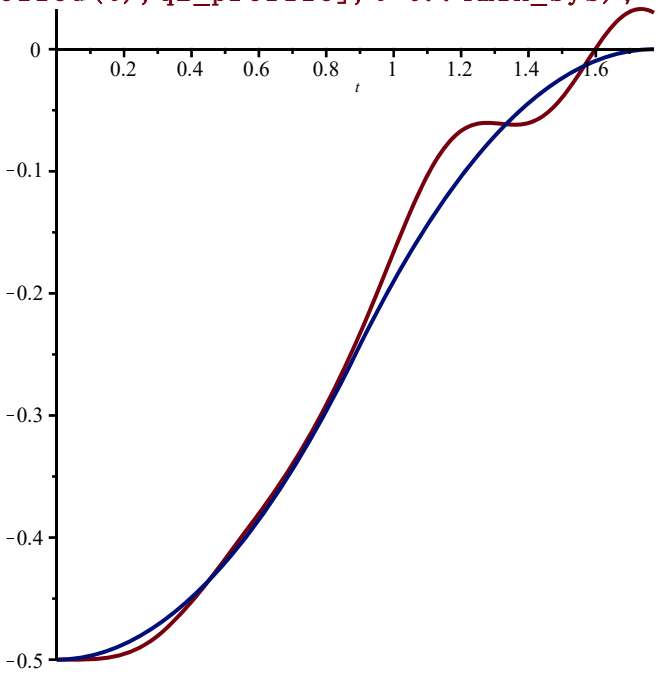
```
> q3_controlled:=rhs(Qcontrolled[6]);
```

$$q3\_controlled := \mathtt{proc}(t)\ \ldots\ \mathtt{end\ proc} \qquad (77)$$

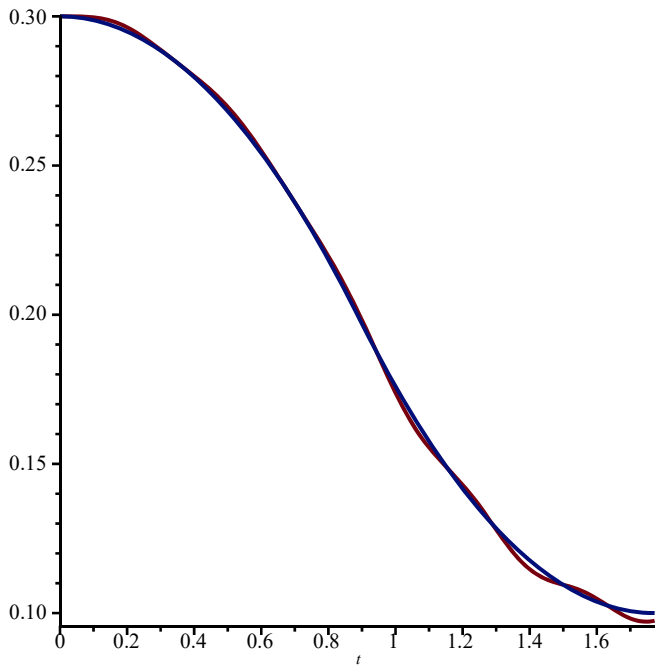
```
> plot([q1_controlled(t),q1_profile],t=0..Tmin_sys);
```



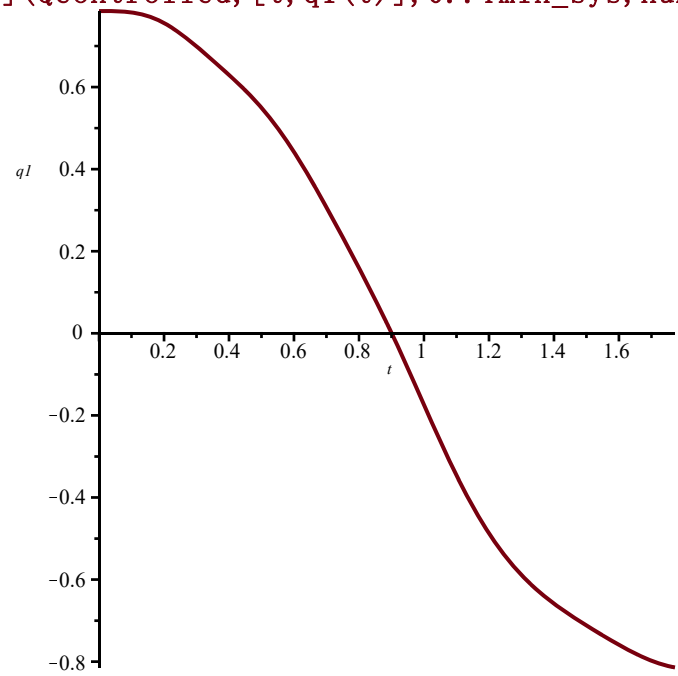

```
> plot([q2_controlled(t),q2_profile],t=0..Tmin_sys);
```



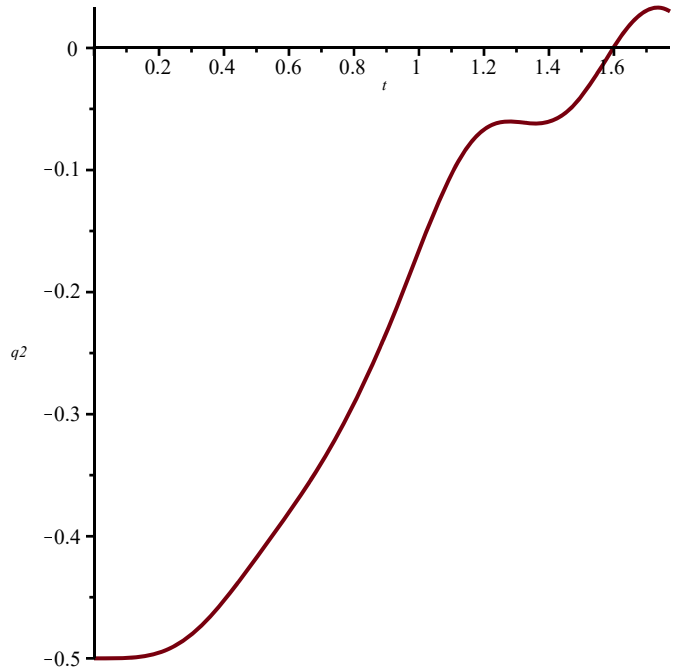

```
> plot([q3_controlled(t),q3_profile],t=0..Tmin_sys);
```



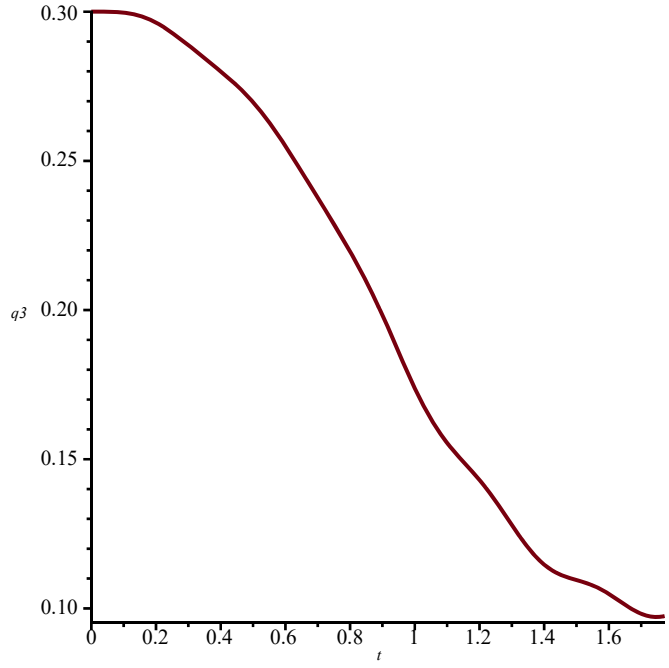

```
> plots[odeplot](Qcontrolled,[t,q1(t)],0..Tmin_sys,numpoints=1000);
```



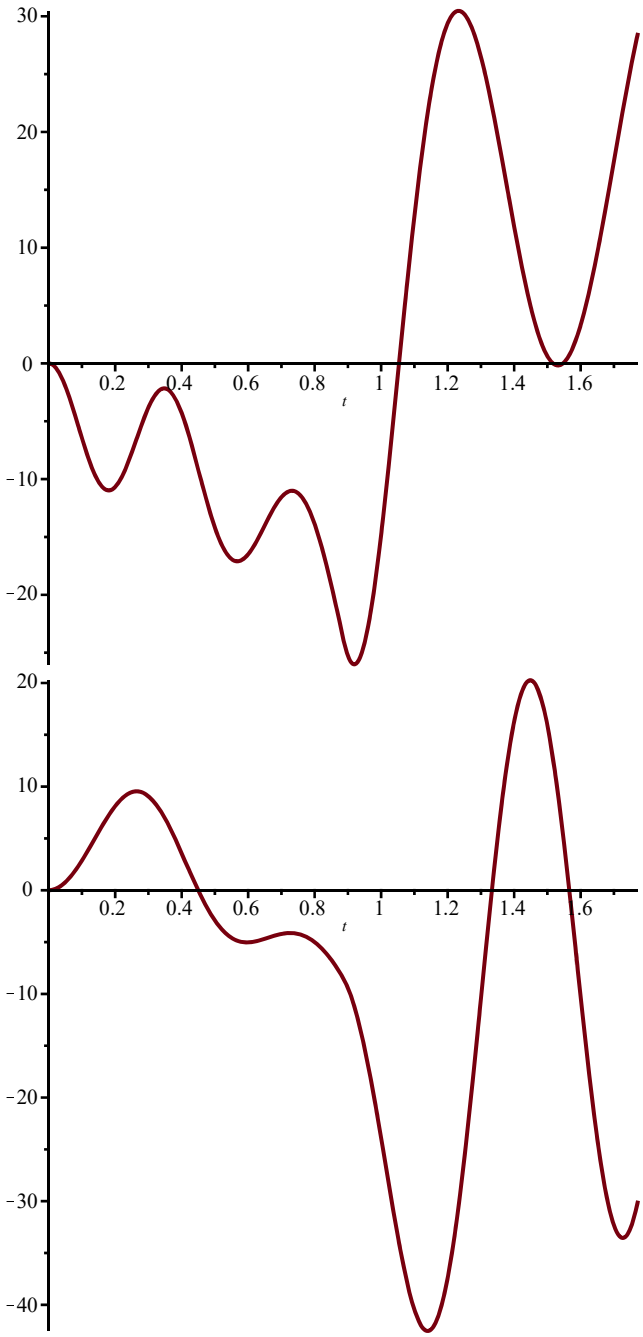

```
> plots[odeplot](Qcontrolled,[t,q2(t)],0..Tmin_sys,numpoints=1000);
```

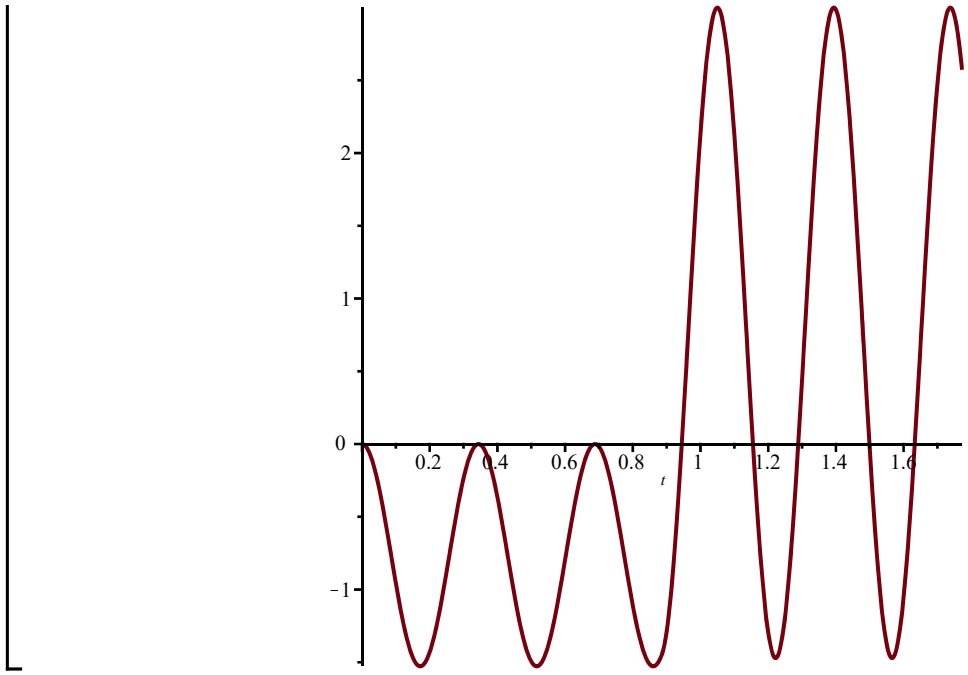



```
> plots[odeplot](Qcontrolled,[t,q3(t)],0..Tmin_sys,numpoints=1000);
```




```
> plots[odeplot](Qcontrolled,[t,1000*(q1_profile-q1(t))],0..Tmin_sys,
  numpoints=1000);
  plots[odeplot](Qcontrolled,[t,1000*(q2_profile-q2(t))],0..Tmin_sys,
  numpoints=1000);
  plots[odeplot](Qcontrolled,[t,1000*(q3_profile-q3(t))],0..Tmin_sys,
  numpoints=1000);
```


30
20
10
0
-10
-20
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
t
20
10
0
-10
-20
-30
-40
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
t


2
1
0
-1
0.2
0.4
0.6
0.8
t
1
1.2
1.4
1.6