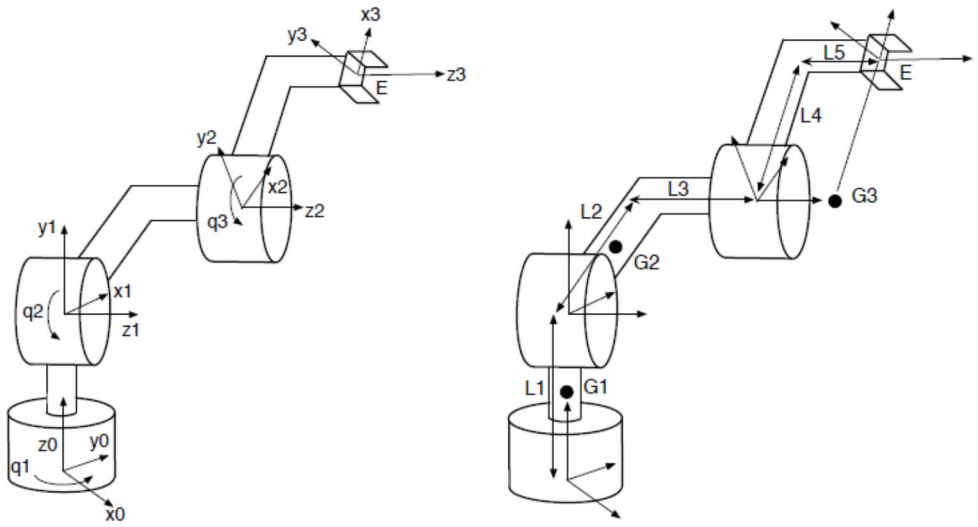
```
> restart:with(LinearAlgebra):
```




```
> data:={L1=0.2,L2=0.5,L3=0.3,L4=0.3,L5=0.2,m1=30,m2=20,m3=10,g=9.81,a=
  3}
```

$$data := \{L1 = 0.2,\ L2 = 0.5,\ L3 = 0.3,\ L4 = 0.3,\ L5 = 0.2,\ a = 3,\ g = 9.81,\ m1 = 30,\ m2 = 20,\ m3 = 10\} \tag{1}$$

1. Position Analysis

```
> Mrotztransl:=(theta,point)->&lt;&lt;cos(theta),sin(theta),0,0&gt;|&lt;-sin(theta),
  cos(theta),0,0&gt;|&lt;0,0,1,0&gt;|&lt;point[1],point[2],point[3],1&gt;&gt;;
```

$$Mrotztransl := (\theta,\ point) \mapsto \left\langle \langle \cos(\theta),\ \sin(\theta),\ 0,\ 0\rangle \middle| \langle -\sin(\theta),\ \cos(\theta),\ 0,\ 0\rangle \middle| \langle 0,\ 0,\ 1,\ 0\rangle \middle| \langle point_1,\ point_2,\ point_3,\ 1\rangle \right\rangle \tag{2}$$

```
> Mrotxtransl:=(alpha,point)->&lt;&lt;1,0,0,0&gt;|&lt;0,cos(alpha),sin(alpha),0&gt;|&lt;0,-
  sin(alpha),cos(alpha),0&gt;|&lt;point[1],point[2],point[3],1&gt;&gt;;
```

$$Mrotxtransl := (\alpha,\ point) \mapsto \left\langle \langle 1,\ 0,\ 0,\ 0\rangle \middle| \langle 0,\ \cos(\alpha),\ \sin(\alpha),\ 0\rangle \middle| \langle 0,\ -\sin(\alpha),\ \cos(\alpha),\ 0\rangle \middle| \langle point_1,\ point_2,\ point_3,\ 1\rangle \right\rangle \tag{3}$$

```
> M01:=Mrotztransl(q1(t),&lt;0,0,L1&gt;).Mrotztransl(Pi/2,&lt;0,0,0&gt;).Mrotxtransl
  (Pi/2,&lt;0,0,0&gt;);
```

$$M01 := \begin{bmatrix} -\sin(q1(t)) & 0 & \cos(q1(t)) & 0 \\ \cos(q1(t)) & 0 & \sin(q1(t)) & 0 \\ 0 & 1 & 0 & L1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

```
> M12:=Mrotztransl(q2(t),&lt;0,0,L3&gt;).Mrotxtransl(0,&lt;L2,0,0&gt;);
```

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$$M12 := \begin{bmatrix} \cos(q2(t)) & -\sin(q2(t)) & 0 & \cos(q2(t))\ L2 \\ \sin(q2(t)) & \cos(q2(t)) & 0 & \sin(q2(t))\ L2 \\ 0 & 0 & 1 & L3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5}$$

```
> M02:=simplify(M01.M12);
```

$$\begin{aligned} M02 := & [[-\sin(q1(t))\ \cos(q2(t)),\ \sin(q1(t))\ \sin(q2(t)),\ \cos(q1(t)), \\ & -\sin(q1(t))\ \cos(q2(t))\ L2+\cos(q1(t))\ L3], \\ & [\cos(q1(t))\ \cos(q2(t)),\ -\cos(q1(t))\ \sin(q2(t)),\ \sin(q1(t)), \\ & \cos(q1(t))\ \cos(q2(t))\ L2+\sin(q1(t))\ L3], \\ & [\sin(q2(t)),\ \cos(q2(t)),\ 0,\ \sin(q2(t))\ L2+L1], \\ & [0,\ 0,\ 0,\ 1]] \end{aligned} \tag{6}$$

```
> M23:=simplify(Mrotztransl(q3(t),<0,0,L5>).Mrotxtransl(0,<L4,0,0>));
```

$$M23 := \begin{bmatrix} \cos(q3(t)) & -\sin(q3(t)) & 0 & \cos(q3(t))\ L4 \\ \sin(q3(t)) & \cos(q3(t)) & 0 & \sin(q3(t))\ L4 \\ 0 & 0 & 1 & L5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{7}$$

```
> M03:=simplify(M02.M23);
```

$$\begin{aligned} M03 := & [[\sin(q1(t))\ (\sin(q2(t))\ \sin(q3(t))-\cos(q2(t))\ \cos(q3(t))), \\ & \sin(q1(t))\ (\sin(q2(t))\ \cos(q3(t))+\cos(q2(t))\ \sin(q3(t))),\ \cos(q1(t)), \\ & ((-\cos(q3(t))\ L4-L2)\ \cos(q2(t))+\sin(q2(t))\ \sin(q3(t))\ L4)\ \sin(q1(t)) \\ & +\cos(q1(t))\ (L3+L5)], \\ & [\cos(q1(t))\ (-\sin(q2(t))\ \sin(q3(t))+\cos(q2(t))\ \cos(q3(t))), \\ & -\cos(q1(t))\ (\sin(q2(t))\ \cos(q3(t))+\cos(q2(t))\ \sin(q3(t))), \\ & \sin(q1(t)),\ ((\cos(q3(t))\ L4+L2)\ \cos(q2(t)) \\ & -\sin(q2(t))\ \sin(q3(t))\ L4)\ \cos(q1(t))+\sin(q1(t))\ (L3+L5)], \\ & [\sin(q2(t))\ \cos(q3(t))+\cos(q2(t))\ \sin(q3(t)),\ -\sin(q2(t))\ \sin(q3(t)) \\ & +\cos(q2(t))\ \cos(q3(t)),\ 0,\ (\cos(q3(t))\ L4+L2)\ \sin(q2(t)) \\ & +\cos(q2(t))\ \sin(q3(t))\ L4+L1], \\ & [0,\ 0,\ 0,\ 1]] \end{aligned} \tag{8}$$

```
> E_3:=<0,0,0,1>;
```

$$E\_3 := \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{9}$$

```
> E:=simplify(M03.E_3);
```

$$E := [[((-\cos(q3(t))\ L4-L2)\ \cos(q2(t)) \tag{10}$$

```
    + sin(q2(t)) sin(q3(t)) L4) sin(q1(t)) + cos(q1(t)) (L3 + L5)],
   [((cos(q3(t)) L4 + L2) cos(q2(t)) - sin(q2(t)) sin(q3(t)) L4) cos(q1(t))
    + sin(q1(t)) (L3 + L5)],
   [(cos(q3(t)) L4 + L2) sin(q2(t)) + cos(q2(t)) sin(q3(t)) L4 + L1],
   [1]]
```

```
> Jac_E:=simplify(VectorCalculus[Jacobian](subs(q1(t)=q1,q2(t)=q2,q3(t)=
  q3,data,E[1..3]),[q1,q1,q3]));
```

$$Jac\_E := \left[\left[\left(\left(-0.3\cos(q3) - 0.5\right)\cos(q2) + 0.3\sin(q2)\sin(q3)\right)\cos(q1) - 0.5\sin(q1),\ \left(\left(-0.3\cos(q3) - 0.5\right)\cos(q2) + 0.3\sin(q2)\sin(q3)\right)\cos(q1) - 0.5\sin(q1),\ 0.3\left(\sin(q2)\cos(q3) + \cos(q2)\sin(q3)\right)\sin(q1)\right],\left[\left(\left(-0.3\cos(q3) - 0.5\right)\cos(q2) + 0.3\sin(q2)\sin(q3)\right)\sin(q1) + 0.5\cos(q1),\ \left(\left(-0.3\cos(q3) - 0.5\right)\cos(q2) + 0.3\sin(q2)\sin(q3)\right)\sin(q1) + 0.5\cos(q1),\ \left(-0.3\cos(q2)\sin(q3) - 0.3\sin(q2)\cos(q3)\right)\cos(q1)\right],\left[0,\ 0,\ -0.3\sin(q2)\sin(q3) + 0.3\cos(q2)\cos(q3)\right]\right] \tag{11}$$

2. Motion Planning

```
> Config1:={q11=-Pi/4,q21=Pi/6,q31=-Pi/4};
```

$$Config1 := \left\{q11 = -\frac{\pi}{4},\ q21 = \frac{\pi}{6},\ q31 = -\frac{\pi}{4}\right\} \tag{12}$$

```
> Config2:={q12=Pi/4,q22=0,q32=Pi};
```

$$Config2 := \left\{q12 = \frac{\pi}{4},\ q22 = 0,\ q32 = \pi\right\} \tag{13}$$

```
> ConfigDelta:=subs(Config1,Config2,{deltaq1=q12-q11,deltaq2=q22-q21,
  deltaq3=q32-q31});
```

$$ConfigDelta := \left\{deltaq1 = \frac{\pi}{2},\ deltaq2 = -\frac{\pi}{6},\ deltaq3 = \frac{5\ \pi}{4}\right\} \tag{14}$$

```
> Tmin:=sqrt(4*abs(deltaq)/amax);
```

$$Tmin := 2\sqrt{\frac{|deltaq|}{amax}} \tag{15}$$

```
> amax_re:=4*deltaq/(T^2);
```

$$amax\_re := \frac{4\ deltaq}{T^2} \tag{16}$$

```
> Tmin_q1:=evalf(subs(deltaq=deltaq1,amax=a,data,ConfigDelta,Tmin));
```

$$Tmin\_q1 := 1.447202509 \tag{17}$$

```
> Tmin_q2:=evalf(subs(deltaq=deltaq2,amax=a,data,ConfigDelta,Tmin));
```

$$Tmin\_q2 := 0.8355427587 \tag{18}$$

```
> Tmin_q3:=evalf(subs(deltaq=deltaq3,amax=a,data,ConfigDelta,Tmin));
```

$$Tmin\_q3 := 2.288228083 \tag{19}$$

```
> Tmin_sys:=Tmin_q3;
```

$$Tmin\_sys := 2.288228083 \tag{20}$$

```
> amaxq1_re:=evalf(subs(deltaq=deltaq1,T=Tmin_sys,ConfigDelta,amax_re));
```

$$amaxq1\_re := 1.199999999 \tag{21}$$

```
> amaxq2_re:=evalf(subs(deltaq=deltaq2,T=Tmin_sys,ConfigDelta,amax_re));
```

$$amaxq2\_re := -0.3999999998 \tag{22}$$

```
> amaxq3_re:=3;
```

$$amaxq3\_re := 3 \tag{23}$$

```
> base_profile:=piecewise(t>=0 and t<=T/2,qini+amax*(t^2)/2,t>T/2 and t<=
  T,qini+amax*(T^2)/4-amax*((T-t)^2)/2);
```

$$base\_profile := \begin{cases} qini + \frac{amax\ t^2}{2} & 0 \le t \le \frac{T}{2} \\ qini + \frac{amax\ T^2}{4} - \frac{amax\ (T-t)^2}{2} & \frac{T}{2} < t \le T \end{cases} \tag{24}$$

```
> q1_profile:=subs(qini=q11,amax=amaxq1_re,T=Tmin_sys,Config1,
  base_profile);
```

$$q1\_profile := \begin{cases} -\frac{\pi}{4} + 0.5999999995\ t^2 & 0 \le t \le 1.144114042 \\ 0.7853981635 - 0.5999999995\ (2.288228083 - t)^2 & 1.144114042 < t \le 2.2882280 \end{cases}$$

```
> q2_profile:=subs(qini=q21,amax=amaxq2_re,T=Tmin_sys,Config1,
  base_profile);
```

$$q2\_profile := \tag{26}$$

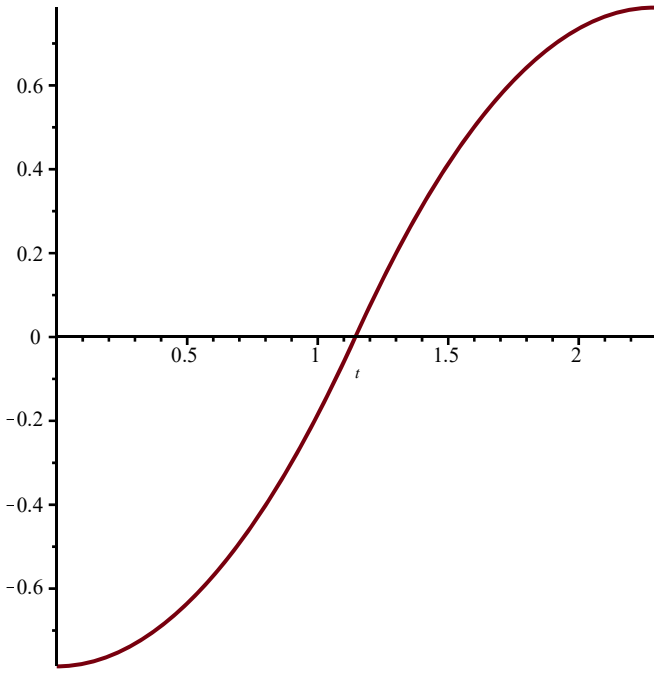
$$\begin{cases} \frac{\pi}{6} - 0.1999999999\ t^2 & 0 \le t \le 1.144114042 \\ 0.1999999999\ (2.288228083 - t)^2 & 1.144114042 < t \le 2.288228083 \end{cases}$$

```
> q3_profile:=subs(qini=q31,amax=amaxq3_re,T=Tmin_sys,Config1,
  base_profile);
```

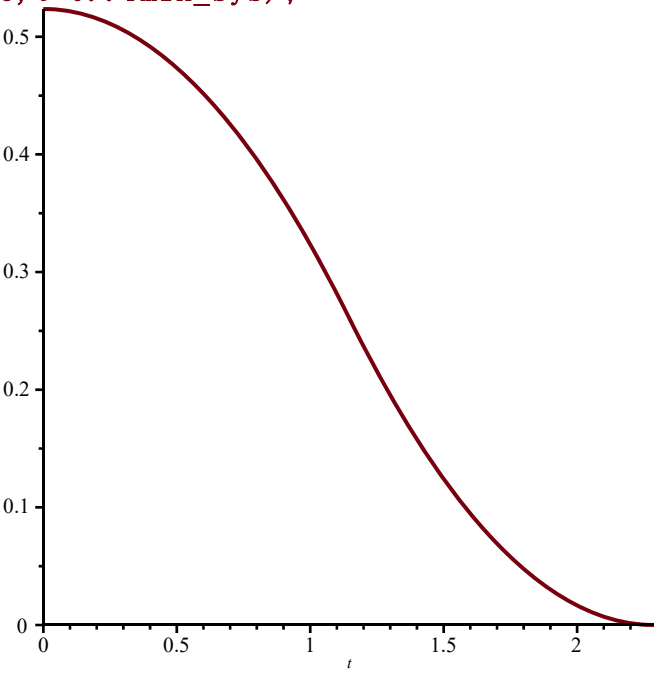
$$q3\_profile := \tag{27}$$

$$\begin{cases} -\frac{\pi}{4} + \frac{3\ t^2}{2} & 0 \le t \le 1.144114042 \\ 3.141592656 - \frac{3\ (2.288228083 - t)^2}{2} & 1.144114042 < t \le 2.288228083 \end{cases}$$

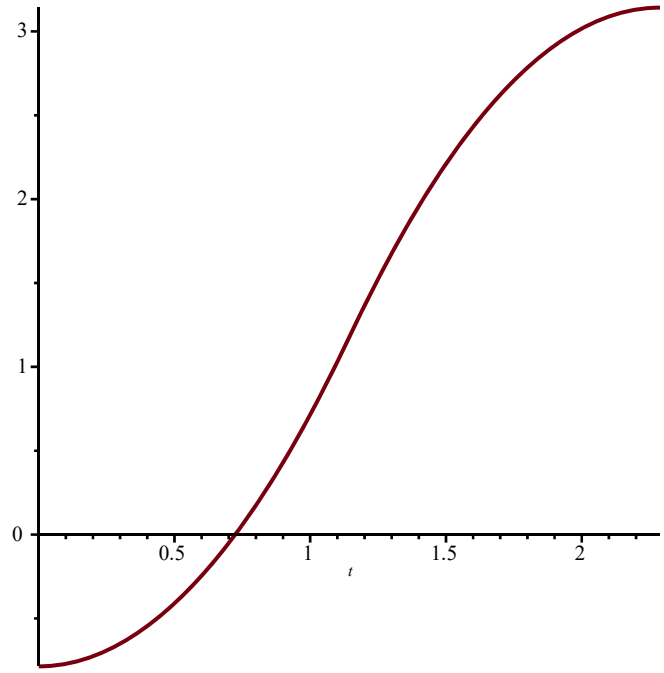
```
> plot(q1_profile,t=0..Tmin_sys);
```




```
> plot(q2_profile,t=0..Tmin_sys);
```




```
> plot(q3_profile,t=0..Tmin_sys);
```



3. Equation of Motion and Control Requirement

```
> T_func:=(W,J)->simplify(Trace(1/2*W.J.(W^%T)));
```

$$T\_func := (W,\ J) \mapsto simplify\left(LinearAlgebra:-Trace\left(\left(\frac{W}{2}\right)\cdot J\cdot W^{T}\right)\right) \tag{28}$$

```
> U_func:=(Hg,J)->simplify(Trace(-1*Hg.J));
```

$$U\_func := (Hg,\ J) \mapsto simplify(LinearAlgebra:-Trace(-Hg\cdot J)) \tag{29}$$

```
> W_func:=(M)->simplify(map(diff,M,t).MatrixInverse(M));
```

$$W\_func := M \mapsto simplify(map(diff,\ M,\ t)\cdot LinearAlgebra:-MatrixInverse(M)) \tag{30}$$

```
> J_lumped_func:=(m,G)->m*<<G[1]*G[1],G[1]*G[2],G[1]*G[3],G[1]>|<G[2]*G
  [1],G[2]*G[2],G[2]*G[3],G[2]>|<G[3]*G[1],G[3]*G[2],G[3]*G[3],G[3]>|<G
  [1],G[2],G[3],1>>;
```

$$J\_lumped\_func := (m,\ G) \mapsto m\cdot\left\langle\left\langle G_1\cdot G_1,\ G_1\cdot G_2,\ G_1\cdot G_3,\ G_1\right\rangle\middle|\left\langle G_1\cdot G_2,\ G_2\cdot G_2,\ G_2\cdot G_3,\ G_2\right\rangle\middle|\left\langle G_1\cdot G_3,\ G_2\cdot G_3,\ G_3\cdot G_3,\ G_3\right\rangle\middle|\left\langle G_1,\ G_2,\ G_3,\ 1\right\rangle\right\rangle \tag{31}$$

```
> J_mobile_2_fixed:=(M,J)->simplify(M.J.(M^%T));
```

$$J\_mobile\_2\_fixed := (M,\ J) \mapsto simplify\left(M\cdot J\cdot M^{T}\right) \tag{32}$$

```
> Hg_temp:=<<0,0,0,0>|<0,0,0,0>|<0,0,0,0>|<0,0,-g,0>>;
```

$$Hg\_temp := \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -g \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{33}$$

```
> W01:=W_func(M01);
```

$$W01 := \begin{bmatrix} 0 & -\frac{d}{dt} q1(t) & 0 & 0 \\ \frac{d}{dt} q1(t) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{34}$$

```
> W02:=W_func(M02);
```

$$W02 := \left[\left[0,\ -\frac{d}{dt} q1(t),\ \left(\frac{d}{dt} q2(t)\right) \sin(q1(t)),\ -L1 \left(\frac{d}{dt} q2(t)\right) \sin(q1(t))\right],\right.$$
$$\left[\frac{d}{dt} q1(t),\ 0,\ -\left(\frac{d}{dt} q2(t)\right) \cos(q1(t)),\ L1 \left(\frac{d}{dt} q2(t)\right) \cos(q1(t))\right],$$
$$\left[-\left(\frac{d}{dt} q2(t)\right) \sin(q1(t)),\ \left(\frac{d}{dt} q2(t)\right) \cos(q1(t)),\ 0,\ 0\right],$$
$$\left.\left[0,\ 0,\ 0,\ 0\right]\right] \tag{35}$$

```
> W03:=W_func(M03);
```

$$W03 := \left[\left[0,\ -\frac{d}{dt} q1(t),\ \left(\frac{d}{dt} q3(t) + \frac{d}{dt} q2(t)\right) \sin(q1(t)),\right.\right.$$
$$\left.-\sin(q1(t)) \left((\sin(q2(t))\ L2 + L1) \left(\frac{d}{dt} q3(t)\right) + L1 \left(\frac{d}{dt} q2(t)\right)\right)\right],$$
$$\left[\frac{d}{dt} q1(t),\ 0,\ -\left(\frac{d}{dt} q3(t) + \frac{d}{dt} q2(t)\right) \cos(q1(t)),\right.$$
$$\left.\cos(q1(t)) \left((\sin(q2(t))\ L2 + L1) \left(\frac{d}{dt} q3(t)\right) + L1 \left(\frac{d}{dt} q2(t)\right)\right)\right],$$
$$\left[-\left(\frac{d}{dt} q3(t) + \frac{d}{dt} q2(t)\right) \sin(q1(t)),\ \left(\frac{d}{dt} q3(t) + \frac{d}{dt} q2(t)\right) \cos(q1(t)),\right.$$
$$\left.0,\ -L2 \left(\frac{d}{dt} q3(t)\right) \cos(q2(t))\right],$$
$$\left.\left[0,\ 0,\ 0,\ 0\right]\right] \tag{36}$$

```
> J11:=J_lumped_func(m1,<0,-L1/2,0>);
```

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$$J11 := \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{m1\ L1^2}{4} & 0 & -\frac{m1\ L1}{2} \\ 0 & 0 & 0 & 0 \\ 0 & -\frac{m1\ L1}{2} & 0 & m1 \end{bmatrix} \tag{37}$$

```
> J22:=J_lumped_func(m2,<-L2/2,0,-L3>);
```

$$J22 := \begin{bmatrix} \frac{m2\ L2^2}{4} & 0 & \frac{m2\ L2\ L3}{2} & -\frac{m2\ L2}{2} \\ 0 & 0 & 0 & 0 \\ \frac{m2\ L2\ L3}{2} & 0 & m2\ L3^2 & -m2\ L3 \\ -\frac{m2\ L2}{2} & 0 & -m2\ L3 & m2 \end{bmatrix} \tag{38}$$

```
> J33:=J_lumped_func(m3,<-L4,0,0>);
```

$$J33 := \begin{bmatrix} m3\ L4^2 & 0 & 0 & -m3\ L4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -m3\ L4 & 0 & 0 & m3 \end{bmatrix} \tag{39}$$

```
> J10:=J_mobile_2_fixed(M01,J11);
```

$$J10 := \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{m1\ L1^2}{4} & \frac{m1\ L1}{2} \\ 0 & 0 & \frac{m1\ L1}{2} & m1 \end{bmatrix} \tag{40}$$

```
> J20:=J_mobile_2_fixed(M02,J22);
```

$$J20 := \left[\left[\frac{\sin(q1(t))^2\ \cos(q2(t))^2\ m2\ L2^2}{4}, \right.\right. \tag{41}$$

$$\left. -\frac{\cos(q1(t))\ \cos(q2(t))^2\ \sin(q1(t))\ m2\ L2^2}{4}, \right.$$

$$-\frac{\cos(q2(t))\ m2\left(\frac{\sin(q2(t))\ L2}{2}+L1\right)\sin(q1(t))\ L2}{2},$$

$$-\frac{\sin(q1(t))\cos(q2(t))\ m2\ L2}{2}\Bigg],$$

$$\Bigg[-\frac{\cos(q1(t))\cos(q2(t))^2\sin(q1(t))\ m2\ L2^2}{4},$$

$$\frac{\cos(q1(t))^2\cos(q2(t))^2\ m2\ L2^2}{4},$$

$$\frac{\cos(q1(t))\cos(q2(t))\ m2\ L2\ (\sin(q2(t))\ L2+2\ L1)}{4},$$

$$\frac{\cos(q1(t))\cos(q2(t))\ m2\ L2}{2}\Bigg],$$

$$\Bigg[-\frac{\cos(q2(t))\ m2\left(\frac{\sin(q2(t))\ L2}{2}+L1\right)\sin(q1(t))\ L2}{2},$$

$$\frac{\cos(q1(t))\cos(q2(t))\ m2\ L2\ (\sin(q2(t))\ L2+2\ L1)}{4},\ \left(\frac{\sin(q2(t))\ L2}{2}+L1\right)^2\ m2,\ \frac{m2\ (\sin(q2(t))\ L2+2\ L1)}{2}\Bigg],$$

$$\Bigg[-\frac{\sin(q1(t))\cos(q2(t))\ m2\ L2}{2},\ \frac{\cos(q1(t))\cos(q2(t))\ m2\ L2}{2},$$

$$\frac{m2\ (\sin(q2(t))\ L2+2\ L1)}{2},\ m2\Bigg]\Bigg]$$

```
> J30:=J_mobile_2_fixed(M03,J33);
```

$$J30 := \Bigg[\Bigg[-2\ m3\left(\left(\frac{L2^2\cos(q2(t))^2}{2}-\frac{(L3+L5)^2}{2}\right)\cos(q1(t))^2 + \sin(q1(t))\cos(q2(t))\ L2\ (L3+L5)\cos(q1(t)) - \frac{L2^2\cos(q2(t))^2}{2}\right),\ -(-2\ L2\cos(q2(t))\ (L3+L5)\cos(q1(t))^2 + \sin(q1(t))\ (\cos(q2(t))\ L2+L3+L5)\ (\cos(q2(t))\ L2-L3-L5)\cos(q1(t)) + L2\cos(q2(t))\ (L3+L5))\ m3,$$
$$-((-L3-L5)\cos(q1(t)) + \sin(q1(t))\cos(q2(t))\ L2)\ m3\ (\sin(q2(t))\ L2+L1),\ -m3\ ((-L3-L5)\cos(q1(t)) + \sin(q1(t))\cos(q2(t))\ L2)\Bigg], \tag{42}$$

$$\left[-\left(-2\ L2\cos(q2(t))\ (L3+L5)\cos(q1(t))^2+\sin(q1(t))\ (\cos(q2(t))\ L2+L3+L5)\ (\cos(q2(t))\ L2-L3-L5)\cos(q1(t))+L2\cos(q2(t))\ (L3+L5)\right)\right.$$

$$m3,\ 2\left(\left(\frac{L2^2\cos(q2(t))^2}{2}-\frac{(L3+L5)^2}{2}\right)\cos(q1(t))^2+\sin(q1(t))\cos(q2(t))\ L2\ (L3+L5)\cos(q1(t))+\frac{(L3+L5)^2}{2}\right)\ m3,$$

$$(\sin(q1(t))\ (L3+L5)+\cos(q1(t))\cos(q2(t))\ L2)\ m3\ (\sin(q2(t))\ L2+L1),\ m3\ (\sin(q1(t))\ (L3+L5)+\cos(q1(t))\cos(q2(t))\ L2)\Big],$$

$$\left[-((-L3-L5)\cos(q1(t))+\sin(q1(t))\cos(q2(t))\ L2)\ m3\ (\sin(q2(t))\ L2+L1),\ (\sin(q1(t))\ (L3+L5)\right.$$

$$+\cos(q1(t))\cos(q2(t))\ L2)\ m3\ (\sin(q2(t))\ L2+L1),$$

$$m3\ (\sin(q2(t))\ L2+L1)^2,\ m3\ (\sin(q2(t))\ L2+L1)\Big],$$

$$\left[-m3\ ((-L3-L5)\cos(q1(t))+\sin(q1(t))\cos(q2(t))\ L2),\right.$$

$$m3\ (\sin(q1(t))\ (L3+L5)+\cos(q1(t))\cos(q2(t))\ L2),\ m3\ (\sin(q2(t))\ L2+L1),\ m3\Big]\Big]$$

```
> T1:=T_func(W01,J10);
```

$$T1 := 0 \tag{43}$$

```
> T2:=T_func(W02,J20);
```

$$T2 := \frac{m2\ L2^2\left(\left(\frac{\mathrm{d}}{\mathrm{d}t}\ q1(t)\right)^2\cos(q2(t))^2+\left(\frac{\mathrm{d}}{\mathrm{d}t}\ q2(t)\right)^2\right)}{8} \tag{44}$$

```
> T3:=T_func(W03,J30);
```

$$T3 := \frac{1}{2}\left(m3\left(\left(L2^2\cos(q2(t))^2+(L3+L5)^2\right)\left(\frac{\mathrm{d}}{\mathrm{d}t}\ q1(t)\right)^2-2\left(\frac{\mathrm{d}}{\mathrm{d}t}\ q2(t)\right)L2\sin(q2(t))\ (L3+L5)\left(\frac{\mathrm{d}}{\mathrm{d}t}\ q1(t)\right)+\left(\frac{\mathrm{d}}{\mathrm{d}t}\ q2(t)\right)^2 L2^2\right)\right) \tag{45}$$

```
> Hg1:=Hg_temp;
```

$$Hg1 := \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -g \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{46}$$

```
> Hg2:=Hg_temp;
```

$$Hg2 := \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -g \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{47}$$

```
> Hg3:=Hg_temp;
```

$$Hg3 := \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -g \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{48}$$

```
> U1:=U_func(Hg1,J10);
```

$$U1 := \frac{g\ m1\ L1}{2} \tag{49}$$

```
> U2:=U_func(Hg2,J20);
```

$$U2 := \frac{g\ m2\ (\sin(q2(t))\ L2 + 2\ L1)}{2} \tag{50}$$

```
> U3:=U_func(Hg3,J30);
```

$$U3 := g\ m3\ (\sin(q2(t))\ L2 + L1) \tag{51}$$

```
> diffF := (f,x)->subs(y=x,diff(subs(x=y,f),y));
```

$$diffF := (f,\ x) \rightarrow subs\left(y = x,\ \frac{\partial}{\partial y}\ subs(x = y,\ f)\right) \tag{52}$$

```
> EQM_left:=(Lagr,qt)->simplify(diff(diffF(Lagr,diff(qt,t)),t)-diffF
  (Lagr,qt));
```

$$EQM\_left := (Lagr,\ qt) \rightarrow simplify\left(\frac{\partial}{\partial t}\ diffF\left(Lagr,\ \frac{\partial}{\partial t}\ qt\right) - diffF(Lagr,\ qt)\right) \tag{53}$$

```
> Lagr:=simplify(T1+T2+T3);
```

$$Lagr := \frac{\left(L2^2\ (m2 + 4\ m3)\ \cos(q2(t))^2 + 4\ m3\ (L3 + L5)^2\right)\ \left(\frac{d}{dt}\ q1(t)\right)^2}{8} - \sin(q2(t))\ \left(\frac{d}{dt}\ q2(t)\right)\ L2\ m3\ (L3 + L5)\ \left(\frac{d}{dt}\ q1(t)\right) + \frac{\left(\frac{d}{dt}\ q2(t)\right)^2\ L2^2\ (m2 + 4\ m3)}{8} \tag{54}$$

```
> EQM_q1:=EQM_left(Lagr,q1(t))-C1;
```

$$EQM\_q1 := \frac{\left(L2^2\ (m2 + 4\ m3)\ \cos(q2(t))^2 + 4\ m3\ (L3 + L5)^2\right)\ \left(\frac{d^2}{dt^2}\ q1(t)\right)}{4} \tag{55}$$

$$-\frac{1}{2}\left(\left(\left(2\ \sin(q2(t))\ m3\ (L3+L5)\ \left(\frac{d^2}{dt^2}\ q2(t)\right)+\left(\frac{d}{dt}\ q2(t)\right)\ \left(2\ m3\ (L3+L5)\ \left(\frac{d}{dt}\ q2(t)\right)+\sin(q2(t))\ \left(\frac{d}{dt}\ q1(t)\right)\ L2\ (m2+4\ m3)\right)\ \cos(q2(t))\right)\right)\right)$$

$$L2\Bigg)-C1$$

```
> EQM_q2:=EQM_left(Lagr,q2(t))-C2;
```

$$EQM\_q2 := \frac{1}{4}\left(L2\ \left(-4\ \sin(q2(t))\ m3\ (L3+L5)\ \left(\frac{d^2}{dt^2}\ q1(t)\right)+L2\ (m2+4\ m3)\ \left(\sin(q2(t))\ \left(\frac{d}{dt}\ q1(t)\right)^2\ \cos(q2(t))+\frac{d^2}{dt^2}\ q2(t)\right)\right)\right)-C2 \tag{56}$$

```
> EQM_q3:=EQM_left(Lagr,q3(t))-C3;
```

$$EQM\_q3 := -C3 \tag{57}$$

```
> data
```

$$\{L1=0.2,\ L2=0.5,\ L3=0.3,\ L4=0.3,\ L5=0.2,\ a=3,\ g=9.81,\ m1=30,\ m2=20,\ m3=10\} \tag{58}$$

```
> c1:=evalf(solve(subs(data,EQM_q1)=0,C1));
```

$$c1 := 3.750000000\ \cos(q2(t))^2\ \left(\frac{d^2}{dt^2}\ q1(t)\right)+2.500000000\ \frac{d^2}{dt^2}\ q1(t)-2.500000000\ \sin(q2(t))\ \left(\frac{d^2}{dt^2}\ q2(t)\right)-7.500000000\ \cos(q2(t))\ \left(\frac{d}{dt}\ q2(t)\right)\ \sin(q2(t))\ \left(\frac{d}{dt}\ q1(t)\right)-2.500000000\ \cos(q2(t))\ \left(\frac{d}{dt}\ q2(t)\right)^2 \tag{59}$$

```
> c2:=evalf(solve(subs(data,EQM_q2)=0,C2))
```

$$c2 := -2.500000000\ \sin(q2(t))\ \left(\frac{d^2}{dt^2}\ q1(t)\right)+3.750000000\ \sin(q2(t))\ \left(\frac{d}{dt}\ q1(t)\right)^2\ \cos(q2(t))+3.750000000\ \frac{d^2}{dt^2}\ q2(t) \tag{60}$$

```
> c3:=evalf(solve(subs(data,EQM_q3)=0,C3));
```

$$c3 := 0. \tag{61}$$

```
> c1_profile:=subs(q1(t)=q1_profile,q2(t)=q2_profile,q3(t)=q3_profile,c1)
  ;
```

$$c1\_profile :=$$

$$3.750000000$$

$$\cos\left(\begin{cases} \frac{\pi}{6} - 0.1999999999\ t^2 & 0 \le t \le 1.144114042 \\ 0.1999999999\ (2.288228083 - t)^2 & 1.144114042 < t \le 2.288228083 \end{cases}\right)^2$$

$$\left(\frac{d^2}{dt^2}\begin{cases} -\frac{\pi}{4} + 0.5999999995\ t^2 & 0 \le t \le 1.144114042 \\ 0.7853981635 - 0.5999999995\ (2.288228083 - t)^2 & 1.144114042 < t \le 2.288228083 \end{cases}\right) + 2.500000000\ \frac{d^2}{dt^2}\begin{cases} -\frac{\pi}{4} + 0.5999999995\ t^2 & 0 \le t \le 1.14 \\ 0.7853981635 - 0.5999999995\ (2.288228083 - t)^2 & 1.144114042 < t \le \end{cases}$$

$$-2.500000000\ \sin\left(\begin{cases} \frac{\pi}{6} - 0.1999999999\ t^2 & 0 \le t \le 1.144114042 \\ 0.1999999999\ (2.288228083 - t)^2 & 1.144114042 < t \le 2.288228083 \end{cases}\right)\left(\frac{d^2}{dt^2}\begin{cases} \frac{\pi}{6} - 0.1999999999\ t^2 & 0 \le t \le 1.144114042 \\ 0.1999999999\ (2.288228083 - t)^2 & 1.144114042 < t \le 2.288228083 \end{cases}\right)$$

$$-7.500000000\ \cos\left(\begin{cases} \frac{\pi}{6} - 0.1999999999\ t^2 & 0 \le t \le 1.144114042 \\ 0.1999999999\ (2.288228083 - t)^2 & 1.144114042 < t \le 2.288228083 \end{cases}\right)\left(\frac{d}{dt}\begin{cases} \frac{\pi}{6} - 0.1999999999\ t^2 & 0 \le t \le 1.144114042 \\ 0.1999999999\ (2.288228083 - t)^2 & 1.144114042 < t \le 2.288228083 \end{cases}\right)$$

$$\sin\left(\begin{cases} \frac{\pi}{6} - 0.1999999999\ t^2 & 0 \le t \le 1.144114042 \\ 0.1999999999\ (2.288228083 - t)^2 & 1.144114042 < t \le 2.288228083 \end{cases}\right)$$

$$\left(\frac{d}{dt}\begin{cases} -\frac{\pi}{4} + 0.5999999995\ t^2 & 0 \le t \le 1.144114042 \\ 0.7853981635 - 0.5999999995\ (2.288228083 - t)^2 & 1.144114042 < t \le 2.288228083 \end{cases}\right)$$

$$-2.500000000\ \cos\left(\begin{cases} \frac{\pi}{6} - 0.1999999999\ t^2 & 0 \le t \le 1.144114042 \\ 0.1999999999\ (2.288228083 - t)^2 & 1.144114042 < t \le 2.288228083 \end{cases}\right)\left(\frac{d}{dt}\right.$$

$$\left.\begin{cases} \frac{\pi}{6} - 0.1999999999\ t^2 & 0 \le t \le 1.144114042 \\ 0.1999999999\ (2.288228083 - t)^2 & 1.144114042 < t \le 2.288228083 \end{cases}\right)^2$$

```
> c2_profile:=subs(q1(t)=q1_profile,q2(t)=q2_profile,q3(t)=q3_profile,c2)
  ;
```

c2_profile :=

$$-2.500000000\ \sin\left(\begin{cases} \frac{\pi}{6} - 0.1999999999\ t^2 & 0 \le t \le 1.144114042 \\ 0.1999999999\ (2.288228083 - t)^2 & 1.144114042 < t \le 2.288228083 \end{cases}\right)\left(\frac{d^2}{dt^2}\begin{cases} -\frac{\pi}{4} + 0.5999999995\ t^2 & 0 \le t \le 1.144114042 \\ 0.7853981635 - 0.5999999995\ (2.288228083 - t)^2 & 1.144114042 < t \le 2.288228083 \end{cases}\right)$$

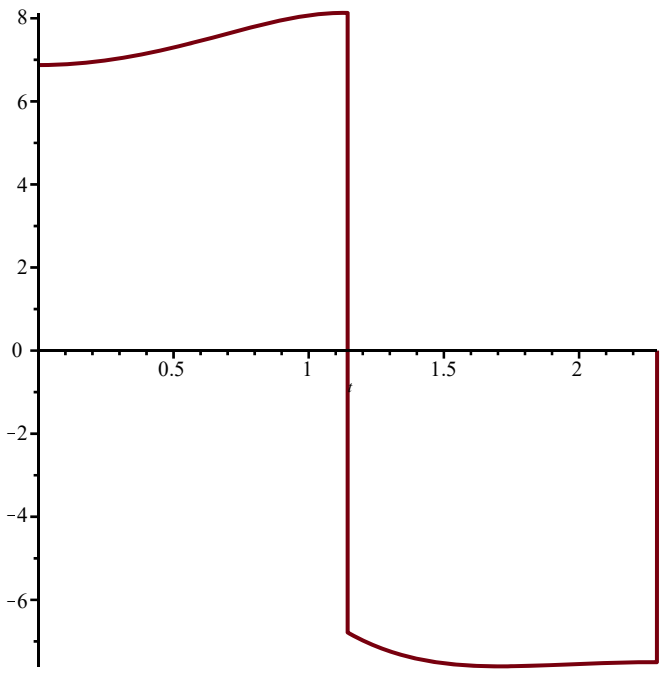
$$+3.750000000\ \sin\left(\left\{\begin{array}{ll}\frac{\pi}{6}-0.1999999999\ t^2 & 0\le t\le 1.144114042\\ 0.1999999999\ (2.288228083-t)^2 & 1.144114042<t\le 2.288228083\end{array}\right.\right)\left(\frac{d}{dt}\left\{\begin{array}{ll}-\frac{\pi}{4}+0.5999999995\ t^2 & 0\le t\le 1.144114042\\ 0.7853981635-0.5999999995\ (2.288228083-t)^2 & 1.144114042<t\le 2.288228083\end{array}\right.\right)^2$$

$$\cos\left(\left\{\begin{array}{ll}\frac{\pi}{6}-0.1999999999\ t^2 & 0\le t\le 1.144114042\\ 0.1999999999\ (2.288228083-t)^2 & 1.144114042<t\le 2.288228083\end{array}\right.\right)+3.750000000\ \frac{d^2}{dt^2}\left\{\begin{array}{ll}\frac{\pi}{6}-0.1999999999\ t^2 & 0\le t\le 1.144114042\\ 0.1999999999\ (2.288228083-t)^2 & 1.144114042<t\le 2.288228083\end{array}\right.$$

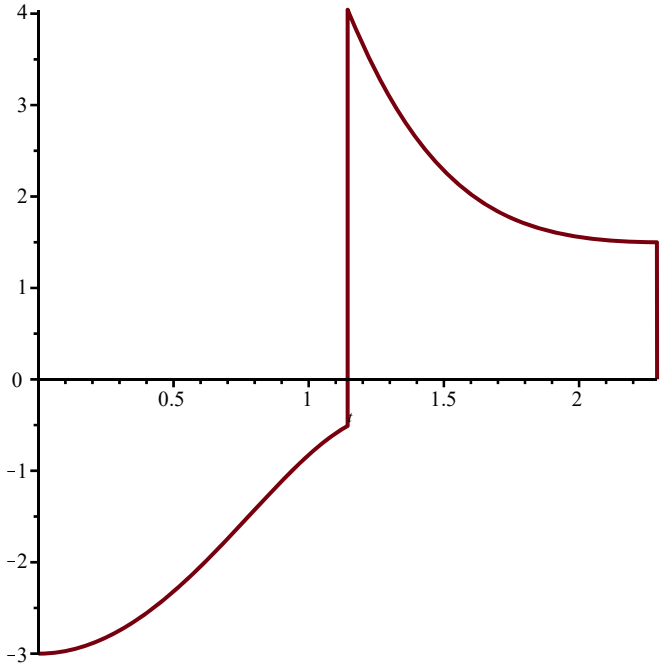
```
> c3_profile:=subs(q1(t)=q1_profile,q2(t)=q2_profile,q3(t)=q3_profile,c3)
  ;
```

$$c3\_profile := 0. \qquad (64)$$

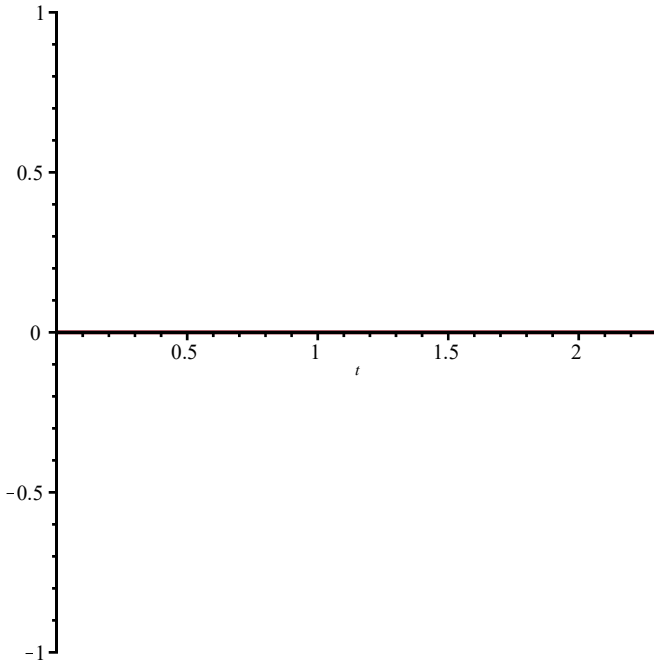
```
> plot(c1_profile,t=0..Tmin_sys);
```




```
> plot(c2_profile,t=0..Tmin_sys);
```




```
> plot(c3_profile,t=0..Tmin_sys);
```




```
> t1:=0.5;
  t2:=1;
  t3:=1.5;
  t4:=2;
```

$$t1 := 0.5$$
$$t2 := 1$$
$$t3 := 1.5$$
$$t4 := 2 \tag{65}$$

```
> c1_t1:=evalf(subs(t=t1,c1_profile));
  c2_t1:=evalf(subs(t=t1,c2_profile));
  c3_t1:=evalf(subs(t=t1,c3_profile));
```

$$c1\_t1 := 7.296311415$$
$$c2\_t1 := -2.320322732$$
$$c3\_t1 := 0. \tag{66}$$

```
> c1_t2:=evalf(subs(t=t2,c1_profile));
  c2_t2:=evalf(subs(t=t2,c2_profile));
  c3_t2:=evalf(subs(t=t2,c3_profile));
```

$$c1\_t2 := 8.069054716$$
$$c2\_t2 := -0.8259685750$$
$$c3\_t2 := 0. \tag{67}$$

```
> c1_t3:=evalf(subs(t=t3,c1_profile));
  c2_t3:=evalf(subs(t=t3,c2_profile));
  c3_t3:=evalf(subs(t=t3,c3_profile));
```

$$c1\_t3 := -7.526339178$$
$$c2\_t3 := 2.284444750$$
$$c3\_t3 := 0. \tag{68}$$

```
> c1_t4:=evalf(subs(t=t4,c1_profile));
  c2_t4:=evalf(subs(t=t4,c2_profile));
  c3_t4:=evalf(subs(t=t4,c3_profile));
```

$$c1\_t4 := -7.543629559$$
$$c2\_t4 := 1.557295240$$
$$c3\_t4 := 0. \tag{69}$$