Adversarial Divergences are Good Task Losses for Generative Modeling

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Overview

Summary

Generative modeling of high dimensional data, like images, is notoriously **difficult** and **ill-defined**. It is not obvious how to specify **relevant evaluation metrics** and **meaningful objectives** to optimize. In this work, we give arguments why **adversarial divergences** are **good objectives for generative modeling**, and perform experiments to better understand their properties.

Contributions

- Unify structured prediction and generative adversarial networks using **statistical decision theory**. **Relate theoretical results** on structured losses with the notion of **weak** and **strong** divergences.
- Show that compared to traditional divergences, adversarial divergences are a **good objective** in terms of sample complexity, computation, ability to integrate prior knowledge, flexibility and ease of optimization.
- Show experimentally the importance of choosing a divergence that **reflects the final task**.

Context and Motivation

Problems with KL divergence

Maximimum Likelihood Estimation (MLE), or minimizing the Kullback-Leibler divergence $\mathbf{KL}(p||q_\theta) = \mathbf{E}_{\boldsymbol{x}\sim p}[\log \frac{p(\boldsymbol{x})}{q_\theta(\boldsymbol{x})}]$ have several drawbacks, including:

- No meaningful **training signal** when p and q_θ are far away. Workarounds generally involve smoothing q_θ, which makes it hard to learn sharp distributions.
- Requires evaluating $q_\theta(x)$, so **cannot be directly used with implicit models**.
- **Teacher-forcing** on autoregressive models.
- **Hard** to enforce properties that **characterize the final task**.

Adversarial Divergences

We define **(neural) adversarial divergences** as

$$\boxed{\mathbf{Adv\Delta}(p||q_\theta) \widehat{=} \sup_{\phi\in\Phi} \mathbf{E}_{(\boldsymbol{x},\boldsymbol{x}')\sim p\otimes q_\theta}[\Delta(f_\phi(\boldsymbol{x}), f_\phi(\boldsymbol{x}'))]}$$

where the choice of the discriminator neural network f_ϕ and function Δ determine properties of the adversarial divergence. For instance, the adversarial Jensen-Shannon from GANs writes

$$\boxed{\mathbf{AdvJS}(p||q_\theta) \widehat{=} \sup_{\phi\in\Phi} \mathbf{E}_{\boldsymbol{x}\sim p}[\log f_\phi(\boldsymbol{x})] + \mathbf{E}_{\boldsymbol{x}'\sim q_\theta}[\log(1 - f_\phi(\boldsymbol{x}'))]}$$

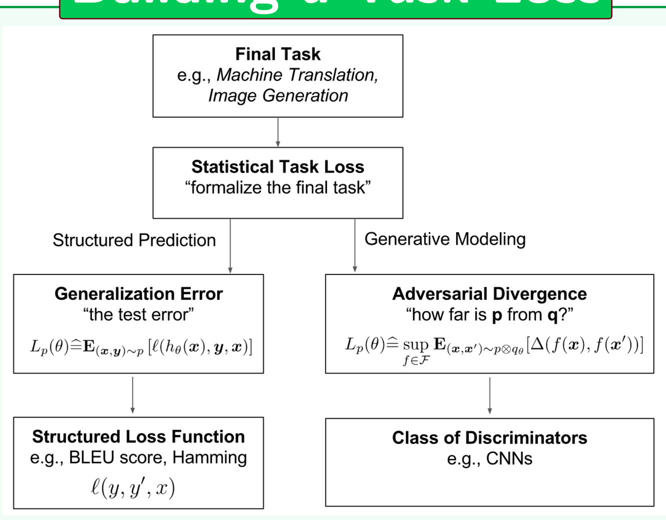
Other adversarial divergences: adversarial Wasserstein, MMD-GANs, ...

Statistical Decision Theory Framework

General Framework

- $\mathcal{P}$: set of possible states of world.
- $\mathcal{A}$: set of actions available.
- $L_p(a)$: cost of playing action $a \in \mathcal{A}$ when the current state is $p \in \mathcal{P}$.
- **Goal**: find $a \in \mathcal{A}$ minimizing the **(statistical) task loss** $L_p(a)$.

Building a Task Loss



MLE, Structured Prediction (SP) and GANs

	$\mathcal{P}$	$\mathcal{A}$	$L_p(a)$
MLE	$\{p(\boldsymbol{x})\}$	$\{q_\theta\,;\,\theta\in\Theta\}$	$\mathbf{E}_{\boldsymbol{x}\sim p}[-\log(q_\theta(\boldsymbol{x}))]$
SP	$\{p(\boldsymbol{x},\boldsymbol{y})\}$	$\{h_\theta\,;\,\theta\in\Theta\}$	$\mathbf{E}_{(\boldsymbol{x},\boldsymbol{y})\sim p}[\ell(h_\theta(\boldsymbol{x}), \boldsymbol{y}, \boldsymbol{x})]$
GAN	$\{p(\boldsymbol{x})\}$	$\{q_\theta\,;\,\theta\in\Theta\}$	$\sup_{f\in\mathcal{F}} \mathbf{E}_{(\boldsymbol{x},\boldsymbol{x}')\sim p\otimes q_\theta}[\Delta(f(\boldsymbol{x}), f(\boldsymbol{x}'))]$

where $\ell : \mathcal{Y}\times\mathcal{Y}\times\mathcal{X} \to \mathbb{R}$ is a structured loss function, while the class of discriminators $\mathcal{F}$ and $\Delta : \mathbb{R}^{d'}\times\mathbb{R}^{d'} \to \mathbb{R}$ determine properties of the adversarial divergence.

Consequences

- Analogy between choice of structured loss ℓ and class of discriminators $\mathcal{F}$ in order to build a statistical task losses that **reflect the final task**.
- Insights from **theoretical structured prediction** (Osokin et al. [1]).

Results by Osokin et al. [1]

Intuition

- Strong losses such as the 0-1 loss are **hard to learn** because they do **not** give any **flexibility** on the prediction. We roughly need as many training examples as $|\mathcal{Y}|$, which is **exponential** in the dimension of y.
- Conversely, weaker losses like the Hamming loss have **more flexibility**; because they tell us how close a prediction is to the ground truth, **less example** are needed to generalize well.

Theory to Back the Intuition

Formalize the intuition and compare the 0-1 loss to the Hamming loss,

$$\ell_{0-1}(\boldsymbol{y}, \boldsymbol{y}') \widehat{=} \mathbf{1}\{\boldsymbol{y} \neq \boldsymbol{y}'\}, \qquad \ell_{Ham}(\boldsymbol{y}, \boldsymbol{y}') \widehat{=} \frac{1}{T}\sum_{t=1}^{T} \mathbf{1}\{\boldsymbol{y}_t \neq \boldsymbol{y}'_t\}$$

when $\boldsymbol{y}$ decomposes as $T = \log_2|\mathcal{Y}|$ binary variables $(y_t)_{1\leq t\leq T}$. They derive a **worst case** sample complexity to get an error $\epsilon > 0$ and obtain,

- For 0-1 loss: $O(|\mathcal{Y}|/\epsilon^2)$ (**exponential**). $\Rightarrow$ **BAD!**
- For Hamming loss[a]: $O(\log_2|\mathcal{Y}|/\epsilon^2)$ (**polynomial**) $\Rightarrow$ **GOOD!**

[a]under certain constraints, see [1]

Insights

Flexible statistical task losses, which can "smoothly" distinguish between good and bad models, are easier to optimize in the context of structured prediction, which can be related to the belief that **weaker adversarial divergences** are **easier to optimize** in generative modeling.

Adversarial vs. Traditional Divergences

Statistical and computational properties

Divergence	Sample Comp.	Computation	Integrate Final Loss
f-Div (EXPL)	$O(1/\epsilon^2)$	MC, $O(n)$	no
f-Div (IMPL)	N/A	N/A	N/A
Wasserstein	$O(1/\epsilon^{d+1})$	Sinkhorn, $O(n^2)$	in base distance
MMD	$O(1/\epsilon^2)$	analytic, $O(n^2)$	in kernel
Adversarial	$O(p/\epsilon^2)$	SGD	in discriminator

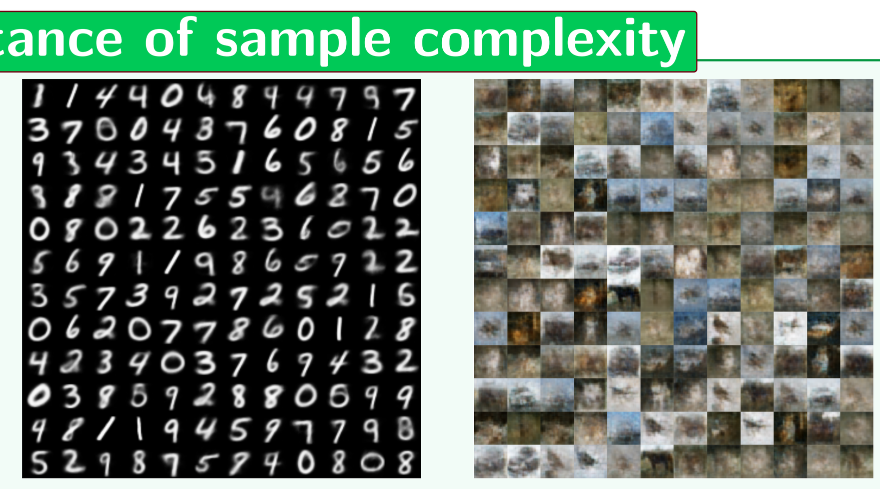
EXPL and IMPL stand for explicit and implicit models, and p is the VC- dimension/number of parameters of the discriminator.

Experiments

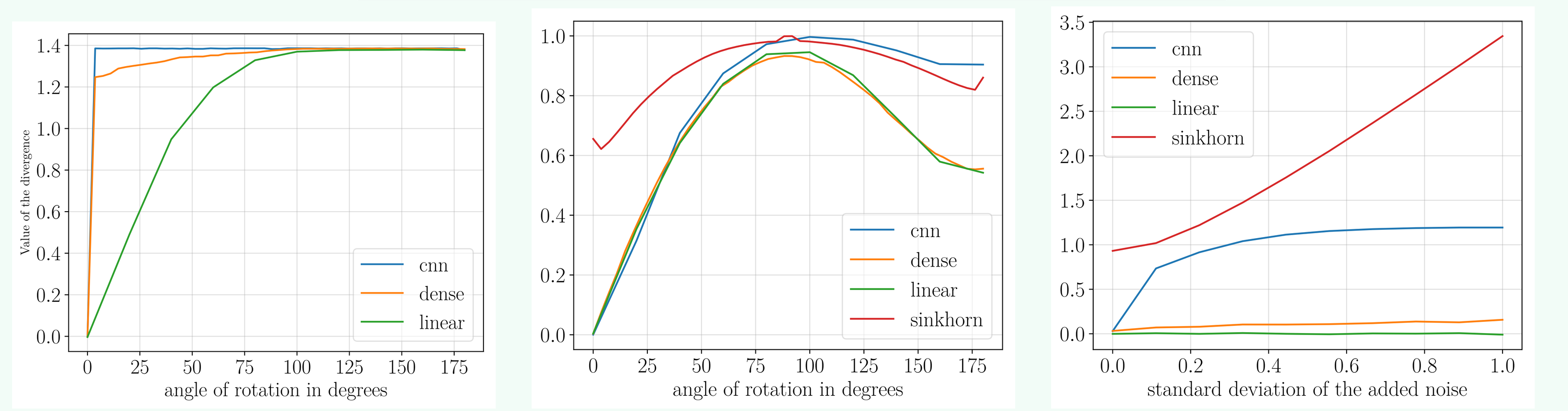
1. Importance of sample complexity

Images generated by the network after minimizing the actual Wasserstein distance[a] on MNIST (left) and CIFAR-10 (right).

[a]Using Sinkhorn-Autodiff to compute minibatch-wise regularized Wasserstein.

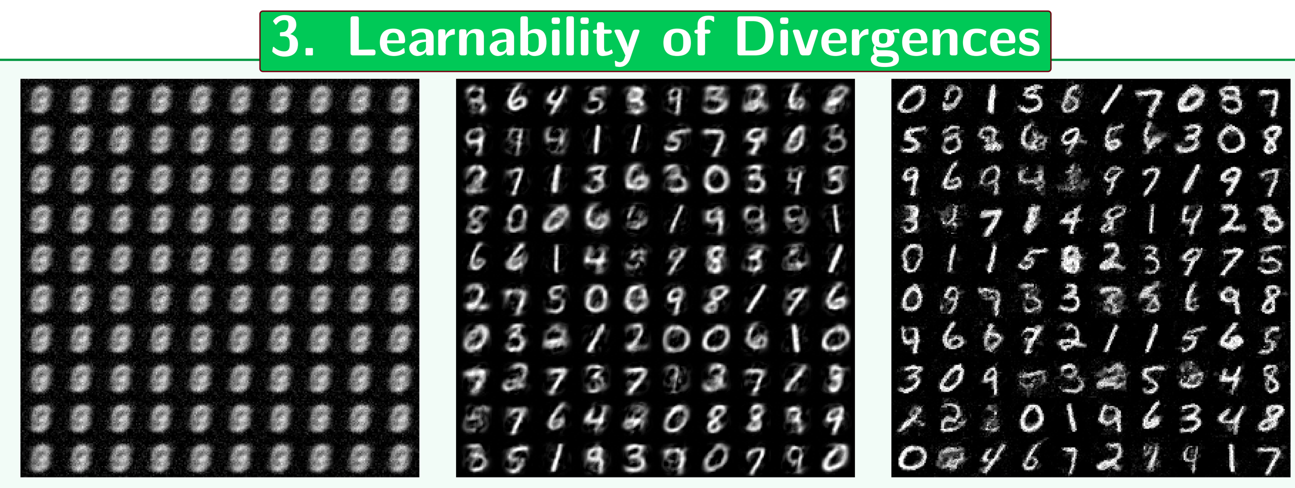


2. Robustness to Transformations



JSD(rotation) **WGAN**(rotation) **WGAN**(noise)

3. Learnability of Divergences



Linear **Dense** **CNN**

References

[1] A. Osokin, F. Bach, and S. Lacoste-Julien. On structured prediction theory with calibrated convex surrogate losses. In *NIPS*, 2017. (to appear).