



Generative Adversarial Networks

Creative Machine Learning - Course 09

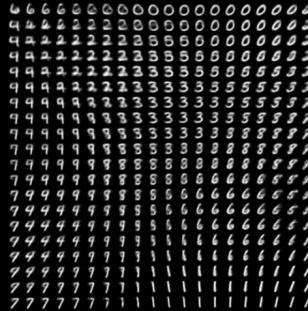
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Brief history of AI

Families of generative models that we will learn

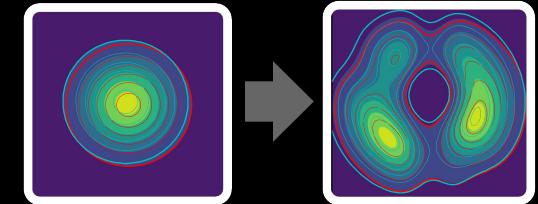
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Variational Auto-Encoder (VAE)

Random variables, distributions, independence

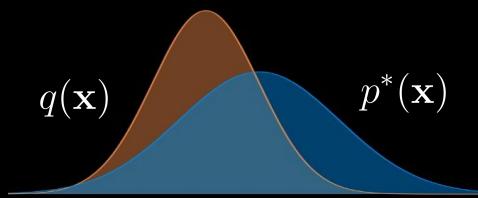
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Normalizing flows

Bayes' theorem, likelihood, conjugate priors

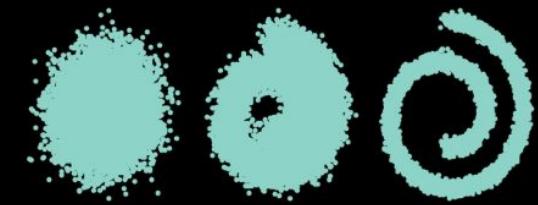
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Generative Adversarial Network (GAN)

Latent variables, probability graphs

4



Diffusion models

Latent variables, probability graphs

2015 - Generative model

First wave of interest in generating data

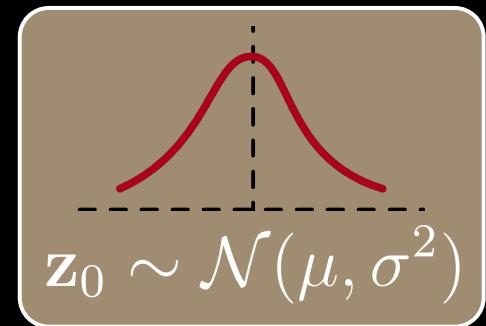
Led to current model craze (VAEs, GANs, Diffusion)



Approximate inference

Probabilistic inference deals with simple distributions

- | Provide easier analytical solutions
- | Implicit assumption of a simple explanation (capacity)
- Issues** | Too simplistic assumption in most cases
Real data follows largely complex distributions
These distributions are usually intractable



The almighty Gaussian

How to model complex distributions but keep analytical simplicity ?

Entering the world of **approximate inference**

Approximating statistics

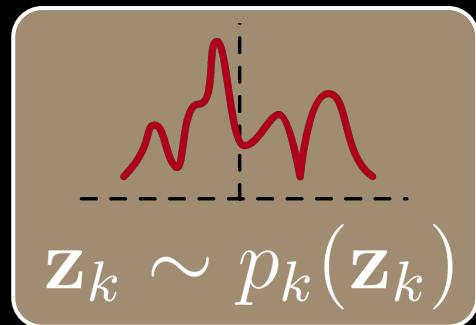
1. Sampling methods

Optimizing simpler distributions

- 1. Variational inference
- 2. Normalizing flows
- 3. Adversarial learning**



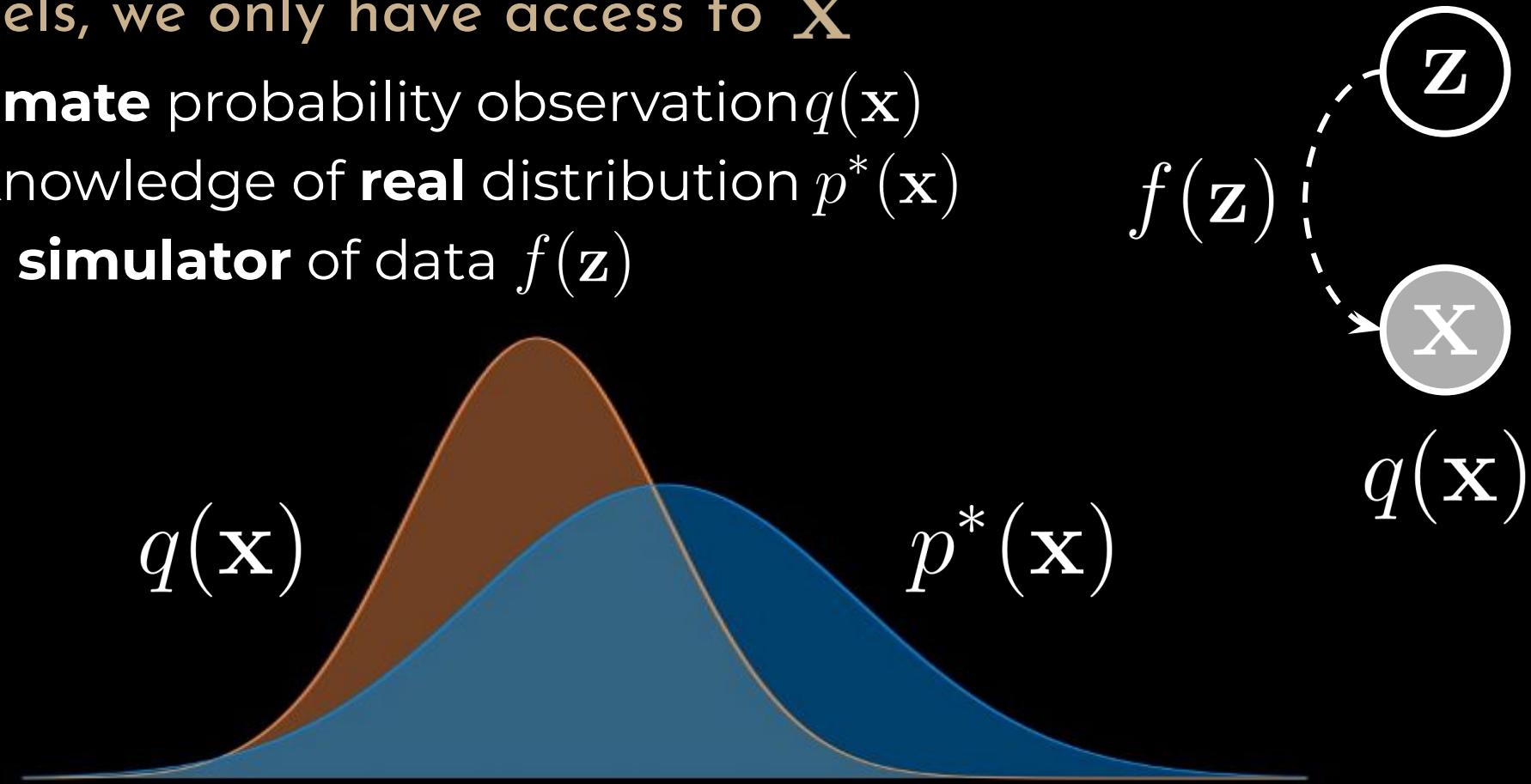
Real data ?



Learning by comparison

For our models, we only have access to \mathbf{X}

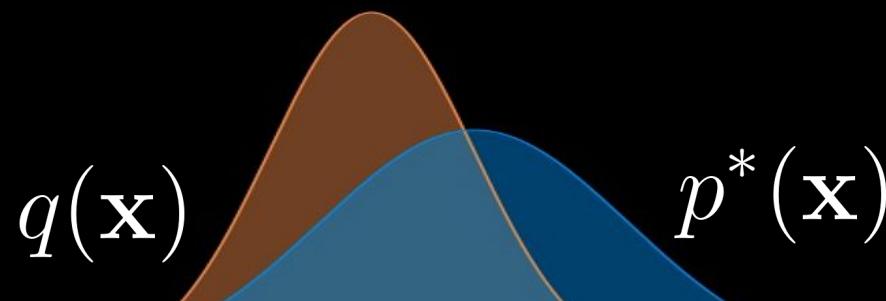
- **Approximate** probability observation $q(\mathbf{x})$
- Partial knowledge of **real** distribution $p^*(\mathbf{x})$
- Define a **simulator** of data $f(\mathbf{z})$



How to compare the estimated distribution $p^*(\mathbf{x})$ to the true distribution $q(\mathbf{x})$ using only **samples**

Learning by comparison

Density estimation by comparison



$$\mathcal{L}(\theta, \phi)$$

Probability difference

$$r_\phi = p^* - q_\theta$$

Maximum Mean
Discrepancy

Moment
matching

Probability ratio

$$r_\phi = \frac{p^*}{q_\theta}$$

Bregman
Divergence

Class probability
estimation

f-Divergence

Use a hypothesis test or comparison to **build an auxiliary model** to indicate how data simulated from the model differs from observed data

Adjust parameters to match the data distribution by comparison

Density ratios and classification

Objective

Density ratio $\frac{p^*(\mathbf{x})}{q_\theta(\mathbf{x})}$

Bayes' rule
(reminder) $p(\mathbf{x} | y) = \frac{p(y|\mathbf{x})p(\mathbf{x})}{p(y)}$

Goal

Based on real data samples $\{\mathbf{x}_1^{(r)}, \dots, \mathbf{x}_m^{(r)}\}$

Generate data samples $\{\mathbf{x}_1^{(g)}, \dots, \mathbf{x}_n^{(g)}\}$ that should resemble real

1. Combine data

$$\{\mathbf{x}_1, \dots, \mathbf{x}_N\} = \left\{ \mathbf{x}_1^{(r)}, \dots, \mathbf{x}_m^{(r)}, \mathbf{x}_1^{(g)}, \dots, \mathbf{x}_n^{(g)} \right\}$$

2. Assign labels

$$\{y_1, \dots, y_N\} = \left\{ +1, \dots, +1, -1, \dots, -1 \right\}$$

Real data *Generated data*

Equivalence

$$p^*(\mathbf{x}) = p(\mathbf{x}|y = +1) \quad q(\mathbf{x}) = p(\mathbf{x}|y = -1)$$

Density ratios and classification

Our equivalence

$$p^*(\mathbf{x}) = p(\mathbf{x}|y = +1) \text{ real distribution}$$
$$q(\mathbf{x}) = p(\mathbf{x}|y = -1) \text{ generative distribution}$$

Developing the density ratio

$$\frac{p^*(\mathbf{x})}{q(\mathbf{x})} = \frac{p(\mathbf{x}|y = +1)}{p(\mathbf{x}|y = -1)}$$

$$\frac{p(y = +1|\mathbf{x})p(\mathbf{x})}{p(y)}$$

$$\frac{p(y = -1|\mathbf{x})p(\mathbf{x})}{p(y)}$$

Final simple equivalence

$$\frac{p^*(\mathbf{x})}{q(\mathbf{x})} = \frac{p(y = +1|\mathbf{x})}{p(y = -1|\mathbf{x})}$$

Our density estimation task **reduces to a classification task**

Unsupervised-as-supervised learning

With a 2-class problem, we can introduce a simple *scoring function*

$$p(y = +1|\mathbf{x}) = D_\theta(\mathbf{x})$$

$$p(y = -1|\mathbf{x}) = 1 - D_\theta(\mathbf{x})$$

We can define this as a **Bernoulli loss**

$$\mathcal{F}(\mathbf{x}, \theta, \phi) = \mathbb{E}_{p^*(\mathbf{x})} [\log D_\theta(\mathbf{x})] + \mathbb{E}_{q_\phi(\mathbf{x})} [\log 1 - D_\theta(\mathbf{x})]$$

Requires an alternate optimisation

$$\min_{\phi} \max_{\theta} \mathcal{F}(\mathbf{x}, \theta, \phi)$$

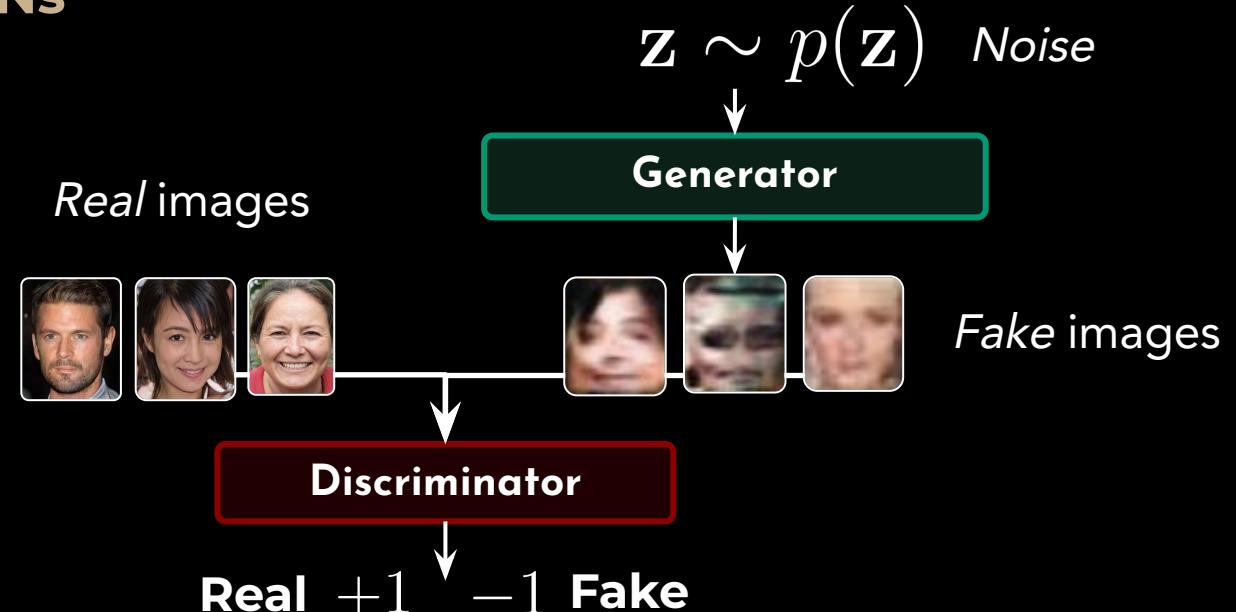
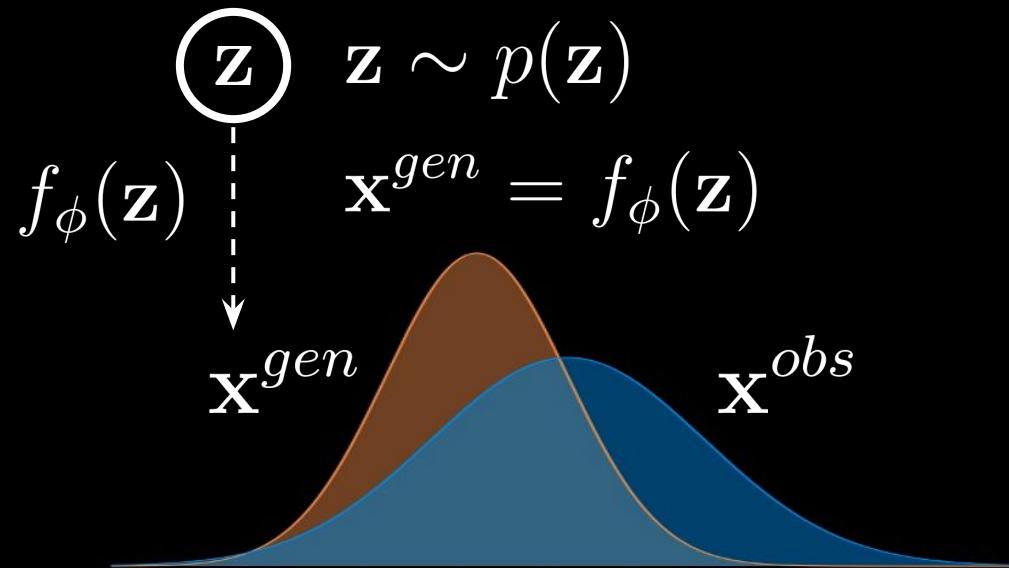
Minimize the number of examples classified as fake
(improve our generation)

Maximize the quality of our discrimination
(improve our classification)

Paradoxical ?

Generative Adversarial Networks (GAN)

Understanding the structure of GANs



Alternating optimization objective

$$\min_{\phi} \max_{\theta} \mathcal{F}(\mathbf{x}, \theta, \phi) = \mathbb{E}_{p^*(\mathbf{x})} [\log D_\theta(\mathbf{x})] + \mathbb{E}_{q_\phi(\mathbf{x})} [\log 1 - D_\theta(\mathbf{x})]$$

Discriminator loss

$$\theta \propto \nabla_\theta \mathbb{E}_{p^*(\mathbf{x})} [\log D_\theta(\mathbf{x})] + \mathbb{E}_{q_\phi(\mathbf{x})} [\log 1 - D_\theta(\mathbf{x})]$$

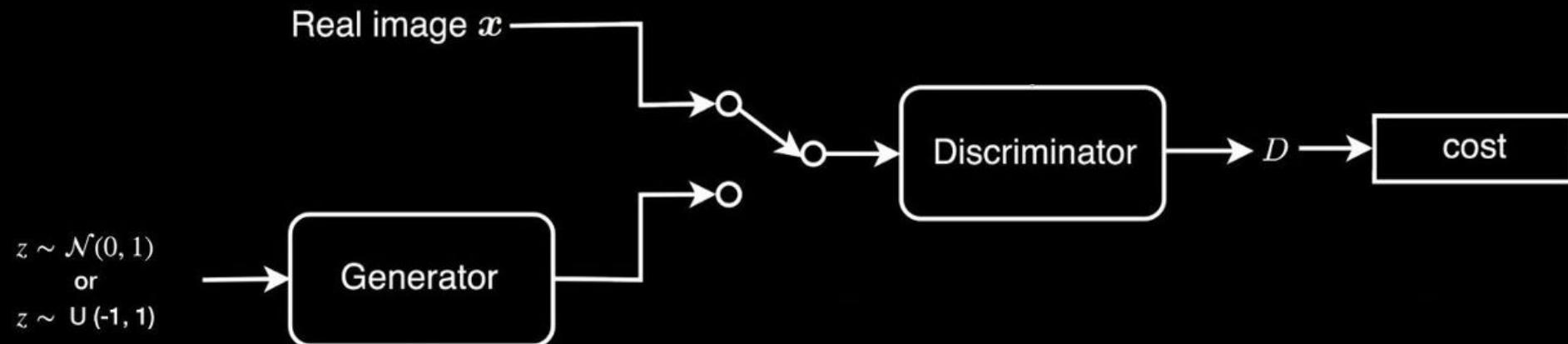
Generator loss

$$\phi \propto -\nabla_\phi \mathbb{E}_{q_\phi(\mathbf{x})} [\log D_\theta(f_\phi(\mathbf{z}))]$$

Understanding gradient flow

Can be linked to *game theory* (minmax game)

- Generator tries to create outputs that resemble real data
- Discriminator tries to recognize them
- Defines a **zero-sum game**



$$\mathcal{F}(\mathbf{x}, \theta, \phi) = \mathbb{E}_{p^*(\mathbf{x})} [\log D_\theta(\mathbf{x})] + \mathbb{E}_{q_\phi(\mathbf{x})} [\log 1 - D_\theta(\mathbf{x})]$$

Understanding the objective

The discriminator D tries to maximize the discrimination

Changing his weights to better separate fake from real

$$\min_{\phi} \max_{\theta} \mathcal{F}(\mathbf{x}, \theta, \phi) = \underbrace{\mathbb{E}_{p^*(\mathbf{x})} [\log D_{\theta}(\mathbf{x})]}_{\text{real data}} + \underbrace{\mathbb{E}_{q_{\phi}(\mathbf{x})} [\log 1 - D_{\theta}(\mathbf{x})]}_{\text{fake data}}$$

↑ 1 ↓ 0

The generator G tries to minimize the discrimination

Generating better examples to confound the classification

$$\min_{\phi} \max_{\theta} \mathcal{F}(\mathbf{x}, \theta, \phi) = \underbrace{\mathbb{E}_{p^*(\mathbf{x})} [\log D_{\theta}(\mathbf{x})]}_{\text{real data}} + \underbrace{\mathbb{E}_{q_{\phi}(\mathbf{x})} [\log 1 - D_{\theta}(\mathbf{x})]}_{\text{fake data}}$$

↓ 0 ↑ 1

Alternating and competing objectives

Issues

- **Differences in strength** of discriminator and generator
- Leads to **unstable training dynamic**

Understanding the objective

Objective function to optimize

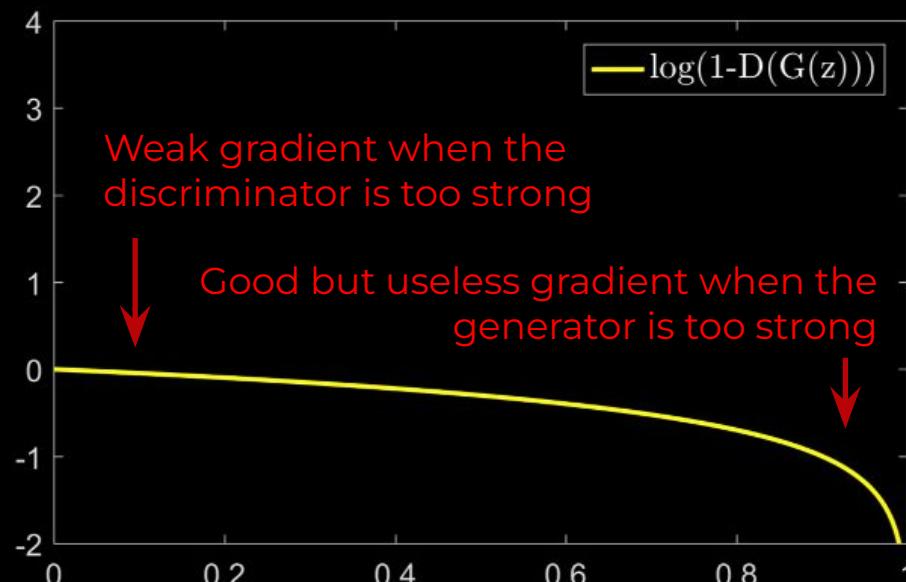
$$\min_{\phi} \max_{\theta} \mathcal{F}(\mathbf{x}, \theta, \phi) = \mathbb{E}_{p^*(\mathbf{x})} [\log D_{\theta}(\mathbf{x})] + \mathbb{E}_{q_{\phi}(\mathbf{x})} [\log 1 - D_{\theta}(G_{\phi}(\mathbf{z}))]$$

– Gradient **ascent** for the discriminator

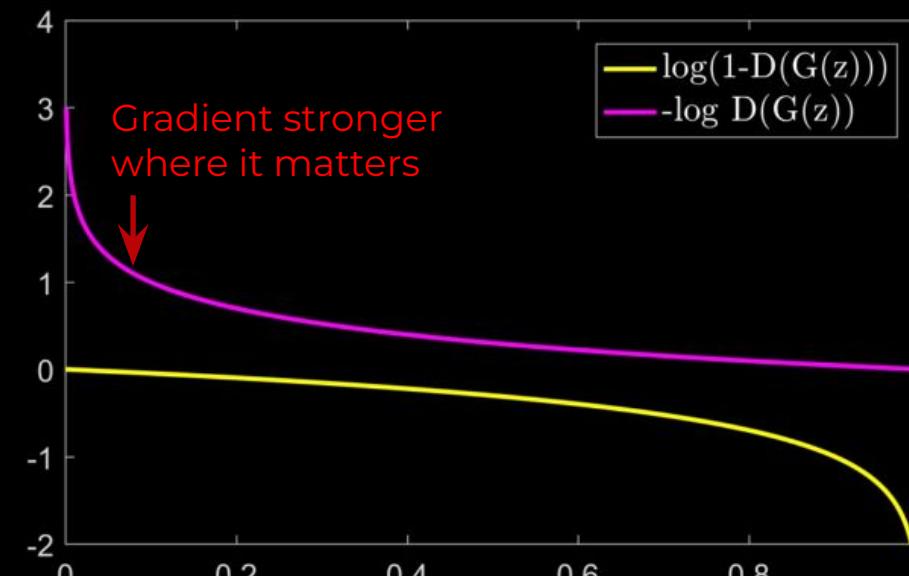
$$\max_{\theta} \mathbb{E}_{p^*(\mathbf{x})} [\log D_{\theta}(\mathbf{x})] + \mathbb{E}_{q_{\phi}(\mathbf{x})} [\log 1 - D_{\theta}(G_{\phi}(\mathbf{z}))]$$

– Gradient **descent** for the generator

$$\min_{\phi} \mathbb{E}_{q_{\phi}(\mathbf{x})} [\log 1 - D_{\theta}(G_{\phi}(\mathbf{z}))]$$



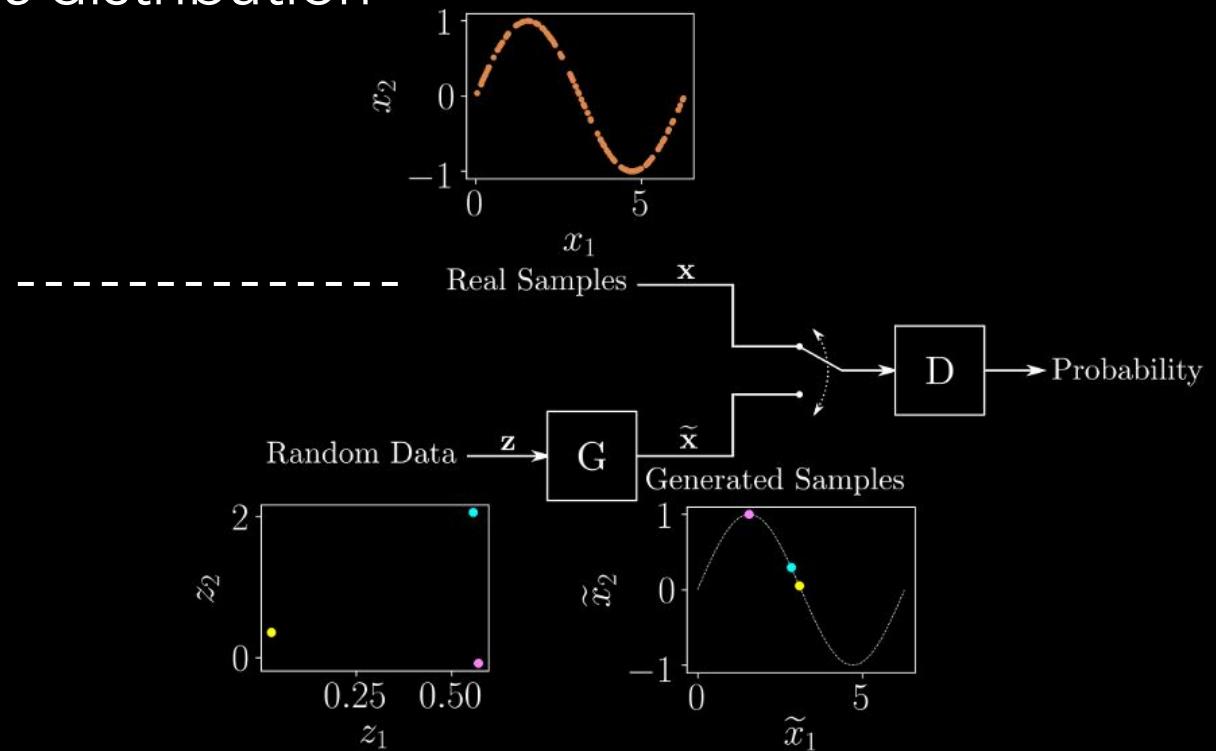
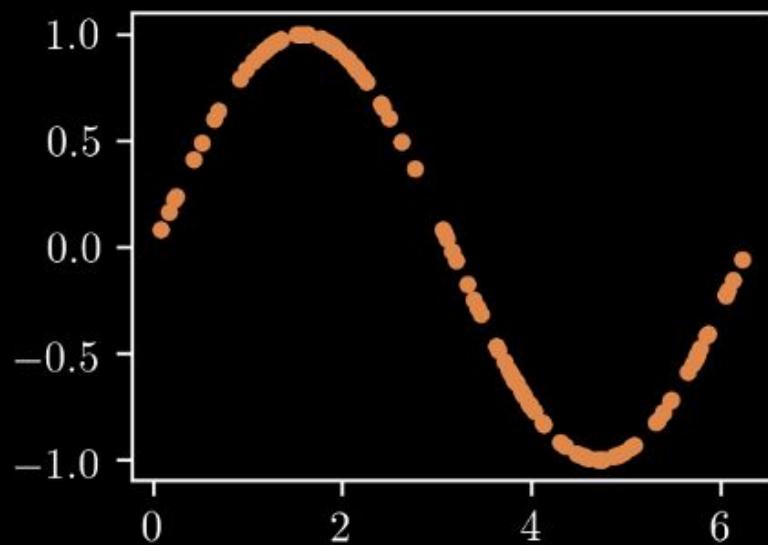
$$\max_{\phi} \mathbb{E}_{q_{\phi}(\mathbf{x})} [D_{\theta}(G_{\phi}(\mathbf{z}))]$$



Adversarial toy exercise

Notebook toy exercise

- We will work out toy example (cf. notebook)
- Imagine we want to optimize a *sinus* distribution



- The generator draws random samples
- *Tries to optimize the function to resemble a sinus*

Adversarial attacks

Vulnerability of machine learning models to adversarial examples

- Adding a small amount of structured (learned) noise
- Leads to very large error rates (with large confidence on errors)
- Can be applied to any type of model

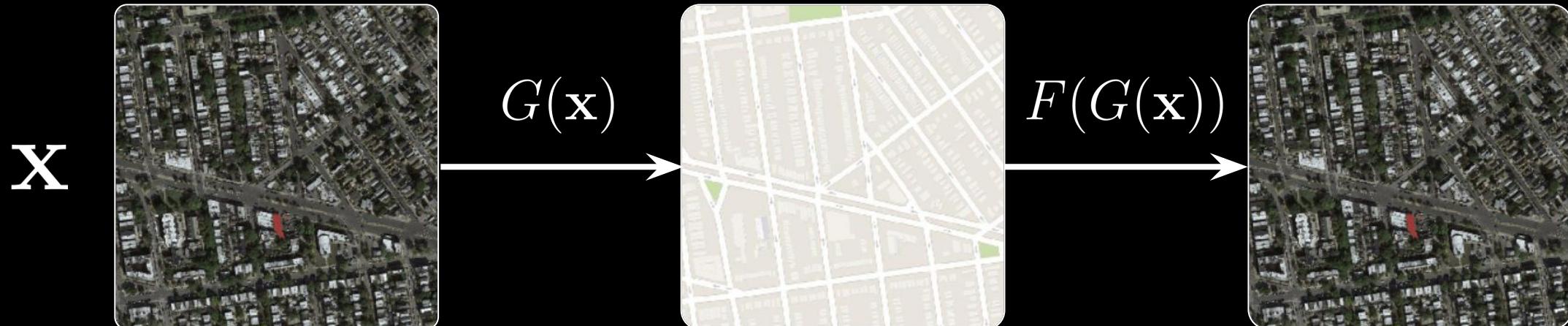
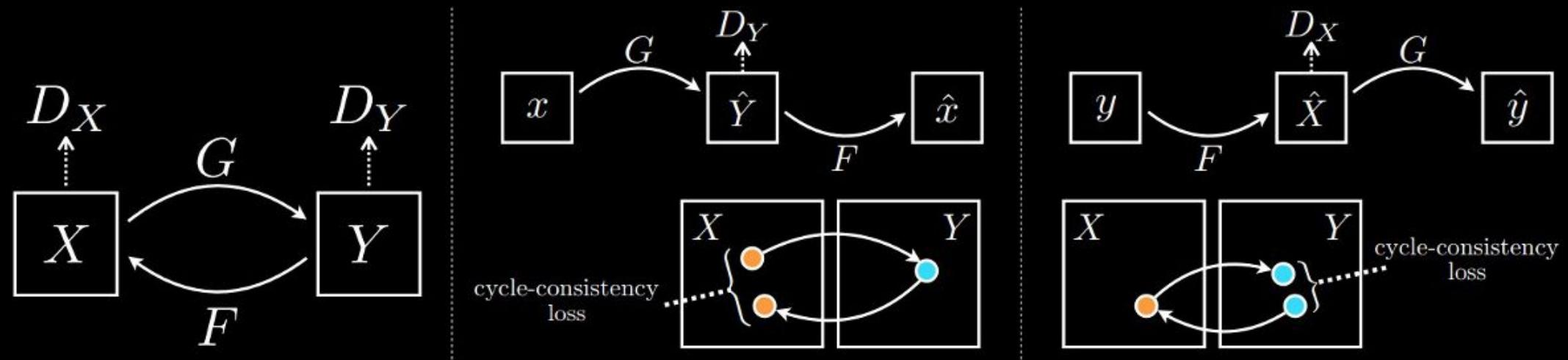
$$x \quad + .007 \times \text{sign}(\nabla_x J(\theta, x, y)) = x + \epsilon \text{sign}(\nabla_x J(\theta, x, y))$$

x “panda” 57.7% confidence *x* + ϵ sign($\nabla_x J(\theta, x, y)$) “gibbon” 99.3 % confidence

- We can directly add adversarial examples in training
 - Adversarial objective function with the *fast gradient sign method*

CycleGAN

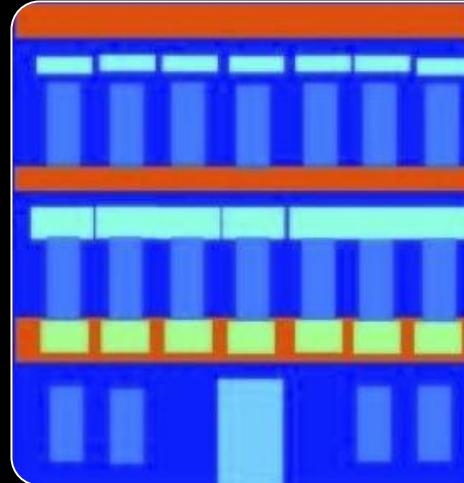
Key idea of CycleGAN is the **cycle loss**



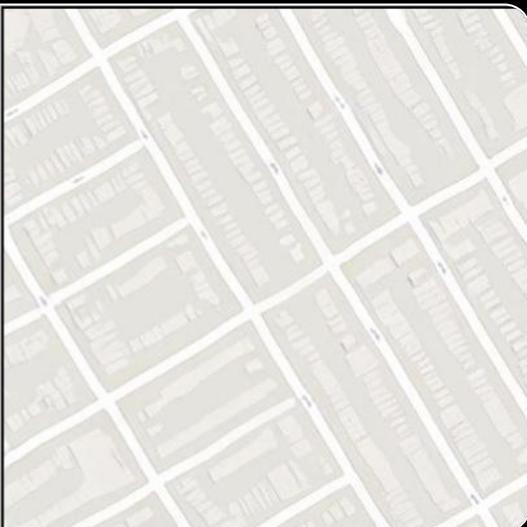
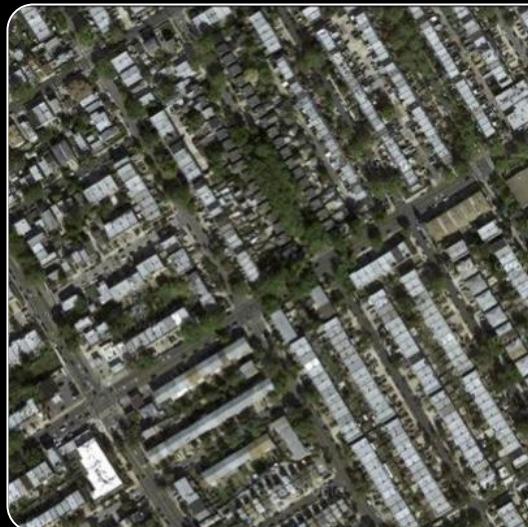
Pix2Pix - Transfer tasks



contour → photography



contour → building



photography → maps



black & white → colors

GANs in action - Style transfer



painting → photography



zebra → horses



summer → winter



photography → painting



zebra → horses



winter → summer



input picture



monet



van gogh



cezanne



ukiyo-e

Progressing growing of GANs

Looking for a way to

- Stabilize the training of GANs
- Increase the output quality
- Output images of 1024x1024



Key observation

GANs struggle with **finer details**

Key ideas

- Need to gradually increase image size
- Start by highly quantified images
- Use these as conditioning
- Add a heuristic for diversity

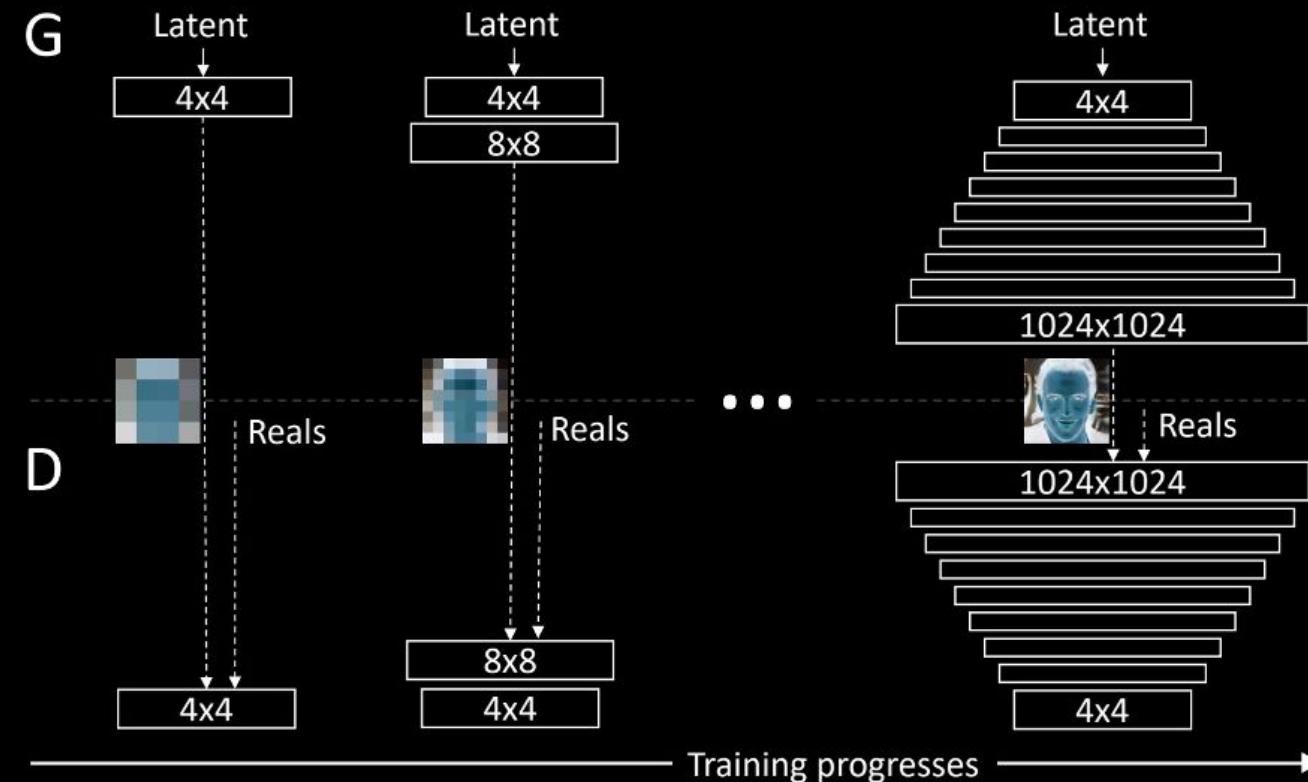


[Karras et al, *Progressive Growing of GANs for Improved Quality, Stability, and Variation*, ICLR 2018.]

Progressive growing of GANs

Major advances

- Progressively increase the size of generation
- And consequently the size of the generator
- Allows the network to discover large-scale structures
- Then refines the image details
- Faster, as it trains on smaller images



Super-resolution with GAN

16x16 input **16x16 (stretch)** **Ground truth** **GAN sample**



Evolution in a few years

