6th IFAC Symposium Advances in Automotive Control
Munich, Germany, July 12-14, 2010

Attitude estimation of a motorcycle in a Kalman filtering framework

Ivo Boniolo*, Stefano Corbetta*, Sergio M. Savaresi*

* *Dipartimento di Elettronica e Informazione, Politecnico di Milano, Piazza L. da Vinci, 32
20133 Milano, Italy, (e-mail: {boniolo,corbetta,savaresi}@elet.polimi.it, stefano.corbetta@mail.polimi.it).*

Abstract: **In this work the problem of attitude estimation of a motorcycle is studied proposing a Kalman Filter to identify the orientation of the vehicle. A new description of the inertial signals acquired by an IMU mounted on the motorcycle is presented to estimate the attitude of the two wheeled vehicle. The estimation performances have been tested both with simulated and experimental data to show the efficacy of the approach.**

Keywords: attitude estimation, roll angle, data fusion, Extended Kalman Filter

1. INTRODUCTION

In this paper, the theory of Kalman filtering is applied to solve the problem of attitude estimation of a motorcycle. The direct formulation of the Kalman Filter for attitude estimation is adopted (direct estimation of the attitude) proposing a new simplified model for the estimation with inertial sensors (three accelerations and three angular rates) and vehicle speed. In a model-based estimation problem such as the Kalman filter, the filtering (estimation) performance is inevitably much dependent upon how well the physics arising in an actual system are reflected in the stochastic plant model. Thus, the definition of the model of the considered process is one of the main topic of this work.

Reviewing the scientific literature, in (Gasbarro et al. 2004), an interesting approach for estimating the whole vehicle trajectory is proposed; it employs a vision system made by cameras complemented with MEMS accelerometers. The frequency separation principle has been proposed by the authors in (Boniolo, Savaresi & Tanelli 2009) and (Boniolo, Tanelli & Savaresi 2008) in which a first solution based on the frequency separation principle has been proposed and in (Boniolo et al. 2009) which is devoted to the analysis of the signals to be adopted to estimate the low frequency component of the lean angle has been conducted in a Neural Network framework.

Some contributions can be also found in the recent patent literature. For example, in (Hauser, Ohm & Rol 1995), (Schiffmann 2003), (Schubert 2005) some approaches are described, whose common purpose is to devise robust estimation methods with low cost (and small size) equipment. Specifically, (Schubert 2005) focuses on roll-over detection mainly tailored to four-wheeled vehicles while (Hauser, Ohm & Rol 1995), (Schiffmann 2003) focus on accelerometer-based estimation algorithms.

The paper is organized as follows. Section 2 is devoted to the description of the simulator that has been adopted to verify the proposed observer and of the experimental set-up that has been adopted to acquire data in racing tracks. In Section 3 the model of the process is introduced. In Section 4 the implementation of the Extended Kalman Filter is described. Section 5 is devoted to the presentation of the test results using both simulated and experimental data. In Section 6 conclusions and future works are presented.

2. SIMULATION ENVIROMENT AND EXPERIMENTAL SET-UP

The estimation algorithm proposed in this paper has been tested both with simulated and experimental data. In this Section the simulation environment and the experimental set-up are briefly described.

The simulated data are provided by the MSC BikeSim® simulator that is a full-fledged motorbike multibody simulator whose mathematical model is based on the work described in (Sharp, Evangelou & Limebeer 2004).

Fig. 1. Picture of the simulator environment.

The interesting signals that are provided by the simulator are:

- Euler angles ZXY that are the attitude angles defined commonly adopted to describe the orientation of a motorcycle (see (Shuster 1993), (Cocco 1999), (Cossalter 2002) and (Saccon 2006))
- Inertial roll angle
- Angular rates measured on the motorcycle
- Accelerations measured on the motorcycle
- Velocity of the vehicle.

The experimental data have been collected with an Aprilia TUONO1000 Factory shown in Fig. 2. On the test vehicle, the following set of sensors was employed: three 1-axis Silicon Sensing MEMS (Micro Electro-Mechanical Systems) gyroscopes (CRS-07), a 3-axis ST-Microelettronics MEMS accelerometer (LIS3L02AS4), two Hall-effect wheel encoders with 48 teeth to measure the front and rear wheel rotational speed. It is well known that the electro-optical sensors (Donati 2004) represents the unique technology to directly measure the attitude of the motorbike with respect to the asphalt as described in (Norgia et al. 2008) and (Norgia et al. 2009). Therefore, the motorcycle has been equipped with a couple of optical telemeters (one for each side) and the reference roll angle is calculated as in(1) where d_1 and d_2 are the distance measured by the left and the right sensor respectively and L is the mounting distance between them.

Fig. 2. Instrumented Aprilia TUONO1000 factory with inertial measuring unit and electro-optical triangulator.

$$\varphi = \arctan\left(\frac{d_1 - d_2}{L}\right) \tag{1}$$

The inertial platform has been mounted on the fuel tank of the vehicle that is not far from the COG, while the electro-optical sensors have been fixed under the pedals of the motorcycle with a distance between them of 40 cm. The signals are acquired on the Torpedo ECU of E-Shock with a sampling frequency of 100 Hz and filtered with a second order filter at a frequency of 7 Hz. The experimental test have been conducted on the circuit Santa Monica in Misano Adriatico and on the circuit Enzo Ferrari in Imola (see Fig. 3).

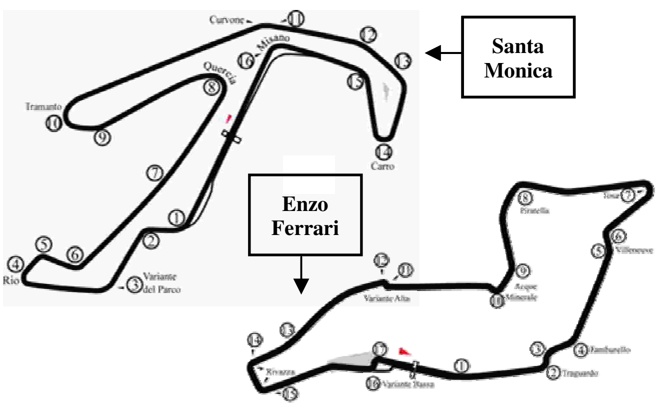

Fig. 3. Santa Monica circuit in Misano Adriatico and Enzo Ferrari circuit in Imola.

3. MODEL DEFINITION

In the attitude estimation context, the model of the process strictly depends of the available signals on the vehicle. In avionic applications (*e.g.* (van der Merwe, Wan & Julier 2004), (Lefferts, Markley & Shuster 1982), (Crassidis & Landis Markley 2003)), the vehicle dynamic can be fully measured and described combining positioning signals, angular rate signals, fixed star measurements and magnetic field measurements. In robotics just angular rates and accelerations are available and the process can be modelled considering a switching state observer (Rehbinder & Hu 2004). In four-wheeled vehicle applications different sets of sensors are used to describe just a part of the vehicle dynamic. In a motorcycle application the model definition is complicated by the noisy environment in which the measures are acquired. The first problem that needs to be solved is the definition of a model that does not describe all the contributions that influence the inertial signals, but that is sufficiently accurate for estimating the attitude parameters of the vehicle.

3.1. Measurements model

The accelerations and angular velocities are measured on the body reference frame of the motorcycle. As depicted in Fig. 4, it's considered that the measurement axes are always aligned to the axes of the body reference frame that is in accordance with the right hand rule with the x axis pointing forward and z axis pointing upward.

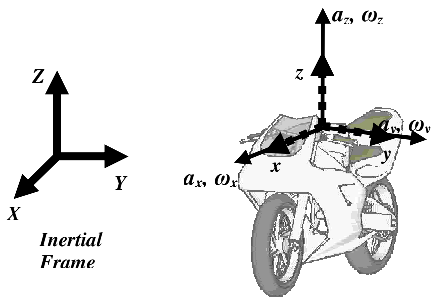

Fig. 4. Body reference frame definition and orientation of the measurement axes.

In this work, the Euler angles parameterization is used even if it brings to a non-linear description of the process. The Roll-Pitch-Yaw rotational matrix R_{ZXY} in (2) defines the geometrical relation between the body reference frame and the inertial reference where φ is the roll angle, ϑ is the pitch angle and ψ is the yaw angle of the vehicle.

$$R_{ZXY}(\varphi,\vartheta,\psi) = \begin{bmatrix} c_\vartheta c_\psi - s_\varphi s_\vartheta s_\psi & c_\vartheta s_\psi + s_\varphi s_\vartheta c_\psi & -c_\varphi s_\vartheta \\ -c_\varphi s_\psi & c_\varphi c_\psi & s_\varphi \\ s_\vartheta c_\psi + s_\varphi c_\vartheta s_\psi & s_\vartheta s_\psi - s_\varphi c_\vartheta c_\psi & c_\varphi c_\vartheta \end{bmatrix} \tag{2}$$

The expression of the angular rates acquired by set of gyroscopes along the vehicle axes depends on the adopted representation of the motorcycle orientation. As a consequence, the gyroscope signals are defined as in (3) where $\dot{\varphi}$ is the roll angular rate, $\dot{\vartheta}$ is the pitch angular rate and $\dot{\psi}$ is the yaw angular rate (see (Shuster 1993) and (Saccon 2006)).

$$\begin{bmatrix}\omega_x\\ \omega_y\\ \omega_z\end{bmatrix}=\begin{bmatrix}c_\vartheta\dot{\varphi}-s_\vartheta c_\varphi\dot{\psi}\\ \dot{\vartheta}+s_\varphi\dot{\psi}\\ s_\vartheta\dot{\varphi}+c_\varphi c_\vartheta\dot{\psi}\end{bmatrix}\tag{3}$$

A simplified formulation of the kinematic accelerations can be deduced observing that the principal terms that affect the measured accelerations in a two-wheeled vehicle are the gravitational acceleration (g), the longitudinal acceleration ($\dot{V}_x$) and the centrifugal acceleration ($\dot{\psi}V_x$). The gravitational acceleration is expressed in the inertial reference frame, while the longitudinal and centrifugal acceleration can be expressed in a reference frame that is rotated by an angle ψ around the absolute Z axis with respect to the inertial frame, then (4) is yield.

$$\begin{bmatrix}a_x\\ a_y\\ a_z\end{bmatrix}=\begin{bmatrix}-c_\varphi s_\vartheta g+c_\vartheta\dot{V}_x+s_\varphi s_\vartheta\dot{\psi}V_x\\ s_\varphi g+c_\varphi\dot{\psi}V_x\\ c_\varphi c_\vartheta g+s_\vartheta\dot{V}_x-s_\varphi c_\vartheta\dot{\psi}V_x\end{bmatrix}\tag{4}$$

It can be easily observed that all the terms that appear in (4) are measurable. In fact, the roll rate $\dot{\varphi}$, the pitch rate $\dot{\vartheta}$ and the yaw rate $\dot{\psi}$ can be expressed as a function of the measured angular rates in the body coordinate system:

$$\begin{bmatrix}\dot{\varphi}\\ \dot{\vartheta}\\ \dot{\psi}\end{bmatrix}=\begin{bmatrix}c_\vartheta & 0 & -s_\vartheta\\ t_\varphi s_\vartheta & 1 & -t_\varphi c_\vartheta\\ -s_\vartheta/c_\varphi & 0 & c_\vartheta/c_\varphi\end{bmatrix}\begin{bmatrix}\omega_x\\ \omega_y\\ \omega_z\end{bmatrix},\tag{5}$$

and substituting in (4), the measured accelerations can be expressed as a function of known signals:

$$\begin{bmatrix}a_x\\ a_y\\ a_z\end{bmatrix}=\begin{bmatrix}-c_\varphi s_\vartheta g+c_\vartheta\dot{V}_x+s_\varphi s_\vartheta\left(-s_\vartheta/c_\varphi\,\omega_x+c_\vartheta/c_\varphi\,\omega_z\right)V_x\\ s_\varphi g+c_\varphi\left(-s_\vartheta/c_\varphi\,\omega_x+c_\vartheta/c_\varphi\,\omega_z\right)V_x\\ c_\varphi c_\vartheta g+s_\vartheta\dot{V}_x-s_\varphi c_\vartheta\left(-s_\vartheta/c_\varphi\,\omega_x+c_\vartheta/c_\varphi\,\omega_z\right)V_x\end{bmatrix}.\tag{6}$$

The model in (6) represents the acceleration of the vehicle in the hypothesis that the centre of rotation of the roll and pitch dynamic is the mounting position of the inertial unit, thus, the contribution of accelerations that are neglected in (6) are:

- *Translational lateral acceleration*: effect of the sideslip of the vehicle, (Pacejka 2002), (Corno 2008), (Cossalter, Doria & Lot 1999);
- *Translational vertical acceleration*: contribution of the heave dynamic of the vehicle and COG elevation;
- *Angular accelerations*: centrifugal and tangential contribution of the accelerations due to roll rate $\dot{\varphi}$ and pitch rate $\dot{\vartheta}$;
- *Displacement*: the difference between the position of the COG of the vehicle and the mounting position of the accelerometers is neglected.

Among the described contributions, the translational lateral acceleration is the main relevant and it particularly influence the description of the inertial signal a_y . Consequently, the model in (6) commits the greatest error in the description of the acceleration acquired along the *y* axis of the vehicle.

3.2. Description of the process

Defining

$$\begin{aligned}&\mathbf{x}=[\varphi\ \vartheta]^T\\ &\mathbf{u}=\left[\omega_x\ \omega_y\ \omega_z\ g\ \dot{V}_x\ V_x\right]^T,\\ &\mathbf{y}=\left[a_x\ a_y\ a_z\right]^T\end{aligned}\tag{7}$$

the system can be described as

$$\begin{aligned}\dot{\mathbf{x}}&=\mathbf{f}(\mathbf{x},\mathbf{u})+\eta_\mathbf{x}\\ \mathbf{y}&=\mathbf{g}(\mathbf{x},\mathbf{u})+\eta_\mathbf{y}\end{aligned}\tag{8}$$

where the functions $\mathbf{f}(\mathbf{x},\mathbf{u})$ and $\mathbf{g}(\mathbf{x},\mathbf{u})$ are defined as in (9) in which x_i, *i=1,2*, and u_j, *j=1,…,6*, are the *i-th* and *j-th* component of the vectors $\mathbf{x}$ and $\mathbf{u}$ respectively and the noises $\boldsymbol{\eta}_\mathbf{x}$ and $\boldsymbol{\eta}_\mathbf{y}$ are assumed to be uncorrelated and to have a normal distribution with zero mean and covariance Q and R respectively as defined in (10).

$$\begin{aligned}&\mathbf{f}(\mathbf{x},\mathbf{u})=\begin{bmatrix}c_{x_2} & 0 & -s_{x_2} & 0\,0\,0\\ t_{x_1}s_{x_2} & 1 & -t_{x_1}c_{x_2} & 0\,0\,0\end{bmatrix}\mathbf{u}\\ &\mathbf{g}(\mathbf{x},\mathbf{u})=\begin{bmatrix}-c_{x_1}s_{x_2} & c_{x_2} & s_{x_1}s_{x_2}\left(-s_{x_2}/c_{x_1}\,u_1+c_{x_2}/c_{x_1}\,u_3\right)\\ s_{x_1} & 0 & c_{x_1}\left(-s_{x_2}/c_{x_1}\,u_1+c_{x_2}/c_{x_1}\,u_3\right)\\ c_{x_1}c_{x_2} & s_{x_2} & -s_{x_1}c_{x_2}\left(-s_{x_2}/c_{x_1}\,u_1+c_{x_2}/c_{x_1}\,u_3\right)\end{bmatrix}\begin{bmatrix}u_4\\ u_5\\ u_6\end{bmatrix}\\ &\boldsymbol{\eta}_\mathbf{x}=[\eta_\varphi\ \eta_\vartheta]^T\quad \boldsymbol{\eta}_\mathbf{y}=\left[\eta_{a_x}\ \eta_{a_y}\ \eta_{a_z}\right]^T\end{aligned}\tag{9}$$

$$E\left[\boldsymbol{\eta}_\mathbf{x}\boldsymbol{\eta}_\mathbf{x}^T\right]=Q\quad E\left[\boldsymbol{\eta}_\mathbf{y}\boldsymbol{\eta}_\mathbf{y}^T\right]=R\quad E\left[\boldsymbol{\eta}_\mathbf{x}\boldsymbol{\eta}_\mathbf{y}^T\right]=0\,.\tag{10}$$

Some comments can be done:

- The model defined in (9) is proper for the estimation of the orientation but not for inertial navigation;
- The yaw angle ψ is not a state variable, because it cannot be observed in the output signals; this is not a limit for the motorcycle application;
- The process description has just two state variables;
- The system in (9) does not have singularity conditions in the range of variation of the attitude angles for the considered application.

4. EXTENDED KALMAN FILTER

The Extended Kalman Filter (EKF) is implemented in a discrete framework as depicted in Fig. 5.

The non-linear functions in (9) are discretized with a Euler forward method as in (11) where *Ts* is the sampling time of the acquired signals and the subscript *k-1* and *k* are indicating the sample at *(k-1)-th* and *k-th* instant respectively.

$$\begin{aligned} \mathbf{x}_k &= \mathbf{x}_{k-1} + \mathbf{f}(\mathbf{x}_{k-1}, \mathbf{u}_{k-1})Ts + \boldsymbol{\eta}_{x,k} \\ \mathbf{y}_k &= \mathbf{g}(\mathbf{x}_k, \mathbf{u}_k) + \boldsymbol{\eta}_{y,k} \end{aligned} \tag{11}$$

The formulation of the EKF (see (Grewal & Andrews 1993) and (Haykin 2001)) is applied to (11). The a priori estimation can be computed with a first order Euler update as

$$\begin{aligned} \hat{\mathbf{x}}_k(-) &= \hat{\mathbf{x}}_{k-1}(+) + \mathbf{f}(\hat{\mathbf{x}}_{k-1}(+), \mathbf{u}_{k-1})Ts \\ \hat{\mathbf{y}}_k &= \mathbf{g}(\hat{\mathbf{x}}_k(-), \mathbf{u}_k) \end{aligned} . \tag{12}$$

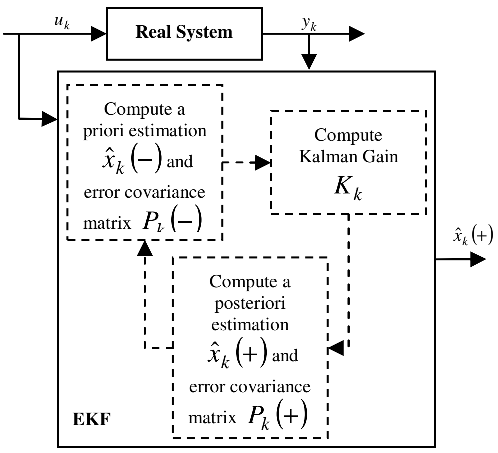

Fig. 5. Block diagram of Kalman Filtering.

The linearization of the model evaluated around the most recent estimation can be computed as

$$\begin{aligned} A_c(\hat{\mathbf{x}}, \mathbf{u}, k-1) &= \left.\frac{\partial \mathbf{f}(\mathbf{x}, \mathbf{u})}{\partial \mathbf{x}}\right|_{\mathbf{x}=\hat{\mathbf{x}}_{k-1}(+), \mathbf{u}=\mathbf{u}_{k-1}} \\ A(\hat{\mathbf{x}}, \mathbf{u}, k-1) &= e^{A_c(\hat{\mathbf{x}}, \mathbf{u}, k-1)Ts} \\ C(\hat{\mathbf{x}}, \mathbf{u}, k) &= \left.\frac{\partial \mathbf{g}(\mathbf{x}, \mathbf{u})}{\partial \mathbf{x}}\right|_{\mathbf{x}=\hat{\mathbf{x}}_k(-), \mathbf{u}=\mathbf{u}_k} \end{aligned} \tag{13}$$

and $A(\hat{\mathbf{x}}, \mathbf{u}, k-1)$ can be approximated with the second order truncation of the Taylor series of exponential matrix $e^{A_c(\hat{\mathbf{x}}, \mathbf{u}, k-1)Ts}$ (Van Loan 1978) as in (14).

$$A(\hat{\mathbf{x}}, \mathbf{u}, k-1) \cong I + A_c(\hat{\mathbf{x}}, \mathbf{u}, k-1)Ts + \frac{1}{2}(A_c(\hat{\mathbf{x}}, \mathbf{u}, k-1)Ts)^2 \tag{14}$$

Thus, the a posteriori estimation of the attitude of the motorcycle can be computed as

$$\begin{aligned} P_k(-) &= A(\hat{\mathbf{x}}, \mathbf{u}, k-1)P_{k-1}(+)A(\hat{\mathbf{x}}, \mathbf{u}, k-1)^T + Q_{k-1} \\ K_k &= P_k(-)C(\hat{\mathbf{x}}, \mathbf{u}, k)^T\left[C(\hat{\mathbf{x}}, \mathbf{u}, k)P_k(-)C(\hat{\mathbf{x}}, \mathbf{u}, k)^T + R_k\right]^{-1} \\ \hat{\mathbf{x}}_k(+) &= \hat{\mathbf{x}}_k(-) + K_k[\mathbf{y_k} - \hat{\mathbf{y}}_\mathbf{k}] \\ P_k(+) &= [I - K_k C(\hat{\mathbf{x}}, \mathbf{u}, k)]P_k(-) \end{aligned} \tag{15}$$

where Q_k and R_k are the tuning parameters of the EKF, $P_k(-)$ and $P_k(+)$ are respectively the a priori and posteriori estimation of the covariance matrix of the state error and K_k is the optimal gain of the Kalman Filter.

The problem of definition of the covariance matrices of the state equation and output transformation of an EKF is not trivial. In general it is well known that:

- The higher the values of R_k, the smaller the gain of the filter;
- The higher the values of Q_k, the higher the gain of the filter.

The output transformation introduce many approximations, as a consequence, the research of the best EKF parameters can be constrained considering that $Q_k << R_k$.

To tune the parameters of the filter, the MSE (Mean Square Error) of the performed estimation is evaluated. In particular, the easiest way of definition of the covariance matrices is

$$\begin{aligned} Q_k &= qI_2 \\ R_k &= rI_3 \end{aligned} . \tag{16}$$

Once the parameters q and r are defined so that

$$\begin{aligned} &q = q_o, \quad r = r_o \\ &q_o, r_o : \min\left\{\sqrt{mse(\varphi - \hat{\varphi})}\right\} \end{aligned} , \tag{17}$$

it can be further observed that the focus of this application is the estimation of the lean angle of the motorcycle and that the model of the acquired accelerations better fits the signals measured along the x and z axis of the body reference frame, consequently, the variance of the noise signals can be differentiated as in (18) where the parameters q_o and r_o are fixed and the parameters K_q and K_r are defined so that MSE of the roll angle estimation is minimized.

$$\begin{aligned} Q_k &= q_o\begin{bmatrix} 1 & 0 \\ 0 & K_q \end{bmatrix} \\ R_k &= r_o\begin{bmatrix} 1 & 0 & 0 \\ 0 & K_r & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{aligned} . \tag{18}$$

To define the EKF parameters in the simulation environment, different tracks have been simulated considering some possible non-idealities as banked and sloped road and variable speed. For each run the estimation error is computed and the mean of the MSE of each simulation is minimized as a function of q, r, K_q and K_r. In the experimental context, all the runs in Misano and Imola are considered, for each of them the estimation error and the corresponding MSE are calculated and the mean of the error variance is minimized.

With this procedure it is ensured that the tuning of the Kalman Filter is not biased by the considered run and that the parameters are general for different circuits.

In Fig. 6, the average MSE of the estimation error for the experimental test is depicted as a function of the parameters of the EKF; it is shown that:

- The best performance are reached considering $q \in \left[10^{-5}, 10^{-6}\right]$, and in this range the sensitivity to r is very low;

- The best performance are reached considering $K_q = 1$ and $K_r = 1$.

The results of the tuning procedure are reported in Table 4.1.

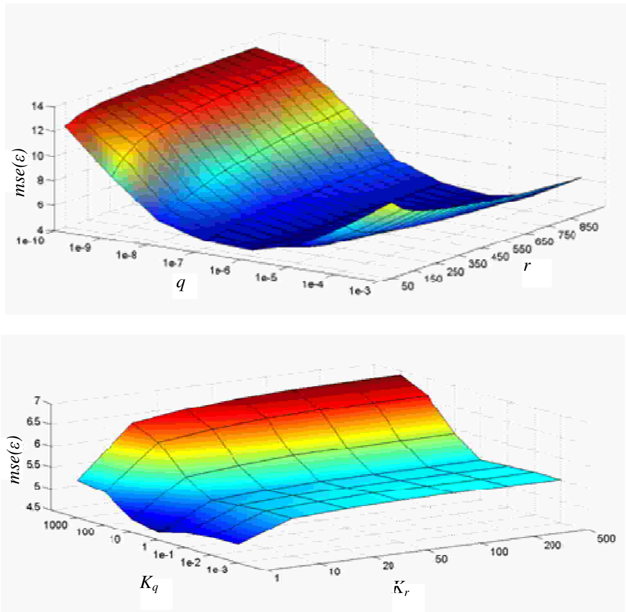

Fig. 6. Optimization of the EKF parameters for experimental data (minimization of the MSE of the estimation error ε).

	Test Condition	
Parameter	*Simulation*	*Experimental test*
q	10^{-3} [rad/s]^2	10^{-6} [rad/s]^2
r	100 [m/s^2]^2	375 [m/s^2]^2
K_q	1	1
K_r	1	1

Table 4.1: Extended Kalman Filter parameters for simulated.

5. SIMULATIONS AND EXPERIMENTAL RESULTS

The proposed approach has great performance both considering simulated data and experimental data.

In Fig. 7, the estimated signals $\hat{\varphi}_{EKF}$ and $\hat{\vartheta}_{EKF}$ are compared to the reference quantities. The committed error has an ESR (Error to Signal Ratio) of 0.8%.

In Fig. 8, the estimated roll angle $\hat{\varphi}_{EKF}$ is compared to the attitude parameter obtained by the application of the frequency separation based algorithm ($\hat{\varphi}_{f_{sp}}$) proposed in (Boniolo, Savaresi & Tanelli 2009) and to the reference quantity measured with the electro-optical system. It is shown that the parameter estimated by the EKF better fits the reference angle. In particular the frequency separation algorithm has an ESR of 6% while the ESR of the EKF is 2%. The improvement of the performance is mainly due to the estimation of the pitch angle of the vehicle.

6. CONCLUSIONS

In this paper the problem of estimation of the attitude of a motorcycle is tackled in a Kalman filtering framework. A new model is introduced to describe the signals acquired with an IMU on a motorbike. This model is proper for the estimation of the orientation of the vehicle. The proposed observer has been tested and tuned for estimating the roll angle of the motorcycle and it has been tested both with simulated and experimental data.

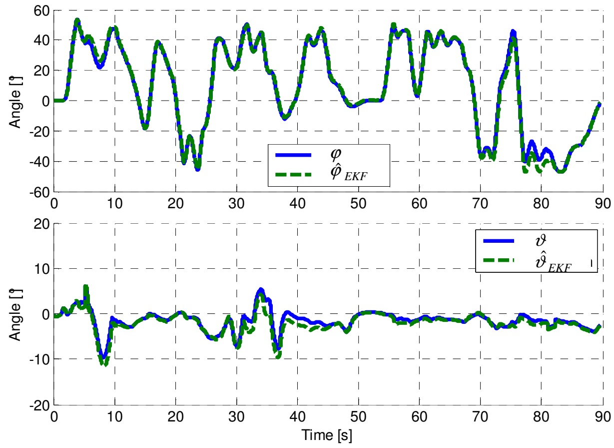

Fig. 7. Application of the observer to simulated data.

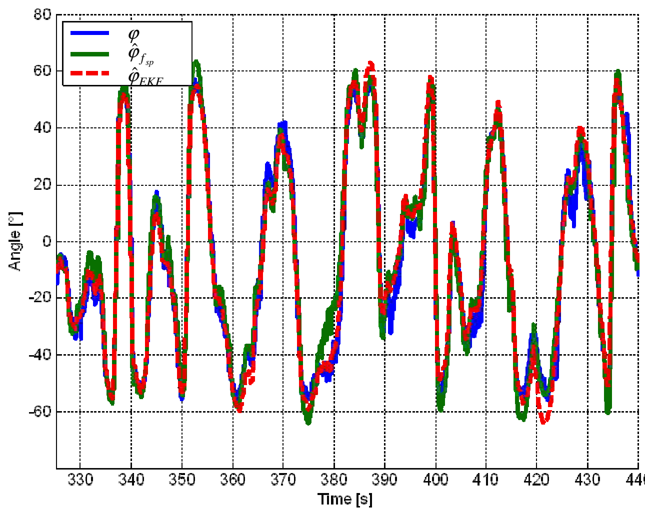

Fig. 8. Comparison of the performance of the EKF with the frequency separation based algorithm proposed in (Boniolo, Savaresi & Tanelli 2009).

7. REFERENCE

Boniolo, I, Panzani, G, Savaresi, SM, Scamozzi, A & Testa, L 2009, 'On the Roll Angle Estimate via Inertila Sensors: Analysis of the Principal Measurement Axes', *DSCC 2009*.

Boniolo, I, Savaresi, SM & Tanelli, M 2009, 'Roll angle estimation in two-wheeled vehicle', *Control Theory & Applications, IET*, vol 3, no. 1, pp. 20-32.

Boniolo, I, Tanelli, M & Savaresi, SM 2008, 'Roll Angle Estimation in Two-Wheeled Vehicles', *17th IEEE International Conference on Control Applications*, San Antonio.

Cocco, G 1999, *Motorcycle Design and Technology*, Giorgio Nada Editore, Milan.

Corno, M 2008, 'Active Stability Control Design for Road Vehcles', Ph.D Thesis, Politecnico di Milano, Milan.

Cossalter, V 2002, *Motorcycle Dynamics*, Race Dyanmics, Milwakee.

Cossalter, V, Doria, A & Lot, R 1999, 'Steady Turning of Two-wheeled Vehicles', *Vehicle System Dynamics*, vol 31, pp. 157-181.

Crassidis, JL & Landis Markley, F 2003, 'Unscented filtering for spacecraft attitude estimation', *AIAA Guidance, Navigation and COntrol Conference and Exhibit*, Austin.

Donati, S 2004, *Electro-Optical Instrumentation - Sensing and Measuring with Lasers*, Prentice Hall.

Gasbarro, L, Beghi, A, Frezza, R, Nori, F & Spagnol, C 2004, 'Motorcycle trajectory reconstruction by integration of vision and MEMS accelerometers', *Proceedings of the 43th Conference on Decision and Control*, Nassau.

Grewal, MS & Andrews, AP 1993, *Kalman filtering: theory and Practice*, Prantice-Hall.

Hauser, B, Ohm, FH & Rol, G 1995, 'Motorcycle ABS using horizontal and vertical acceleration sensors', US Patent 5,445,44.

Haykin, S 2001, *Kalman Filtering and Neural Networks*, John Wiley & Sons Inc.

Leffererts, EJ, Markley, FL & Shuster, MD 1982, 'Kalman Filtering for Spacecraft Attitude Estimation', *Journal of Guidance, Control and Dynamics*, vol 5, no. 5, pp. 417-428.

Norgia, M, Boniolo, I, Tanelli, M, Savaresi, SM & Svelto, C 2009, 'Optical Sensors for Real-Time Measurement of Motorcycle Tilt Angle', *IEEE Transactions on Instrumentation & Measurement*, vol 58, no. 5, pp. 1640-1649.

Norgia, M, Svelto, C, Boniolo, I, Tanelli, M & Savaresi, SM 2008, 'Characterization of Optical Sensors for Real-Time Measurement of Motorcycle Tilt Angles', *Instrumentation and Measurement Technology Conference Proceedings, 2008. IMTC 2008. IEEE.*

Pacejka, HB 2002, *Tyre and Vehicle Dynamics*, Buttherworth Heinemann, Oxford.

Rehbinder, H & Hu, X 2004, 'Drift-free attitude estimation for accelerated rigid bodies', *Automatica*, vol 40, no. 4, pp. 653-659.

Saccon, A 2006, 'Maneuver Regulation of Non-Linear Systems: The Challenge of Motorcycle Control', Ph.D Thesis, Università degli studi di Padova, Padova.

Schiffmann, JK 2003, 'Vehicle roll angle estimation and method', European Patent EP1,346,883.

Schubert, PJ 2005, 'Vehicle rollover sensing using angular accelerometer', US Patent US 1,139,83.

Sharp, RS, Evangelou, S & Limebeer, DJN 2004, 'Advances in the modelling of motorcycle dynamics', *Multibody System Dynamics*, vol 12, p. 251–283.

Shuster, MD 1993, 'A Survey of Attitude Representations', *The Journal of the Astronautical Sciences*, vol 41, no. 4, p. 439–517.

van der Merwe, R, Wan, EA & Julier, SI 2004, 'Sigma-point Kalman filters for nonlinear estimation and sensor-fusion: Applications to Integrated Navigation', *Proceedings of the AIAA Guidance, Navigation & Control Conference*.

Van Loan, C 1978, 'Computing integrals involving the matrix exponential', *IEEE Transactions on Automatic Control*, vol 23, no. 3, pp. 395-404.