MOTION IN TWO DIMENSIONS (PROJECTILE MOTION)

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CONTENTS

- Projectile Motion
- Trajectory of Projectile
- Uniform Circular Motion
- Problems related to the topics discussed

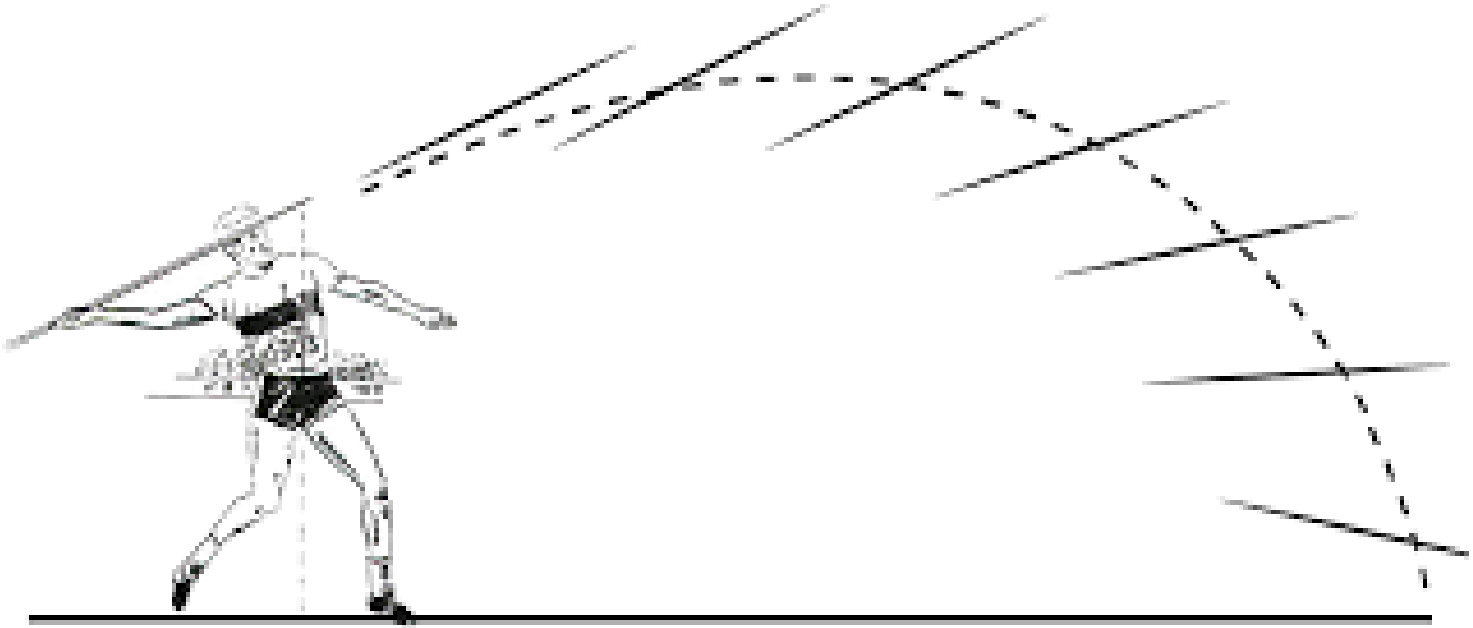
PROJECTILE & PROJECTILE MOTION

- A particle moves in a vertical plane with some initial velocity but its acceleration is always the freefall acceleration , which is downward.
- Such a particle is called a **projectile (meaning** that it is projected or launched), and its motion is called **projectile motion.**
- In projectile motion, the horizontal motion and the vertical motion are independent of each other; that is, neither motion affects the other.

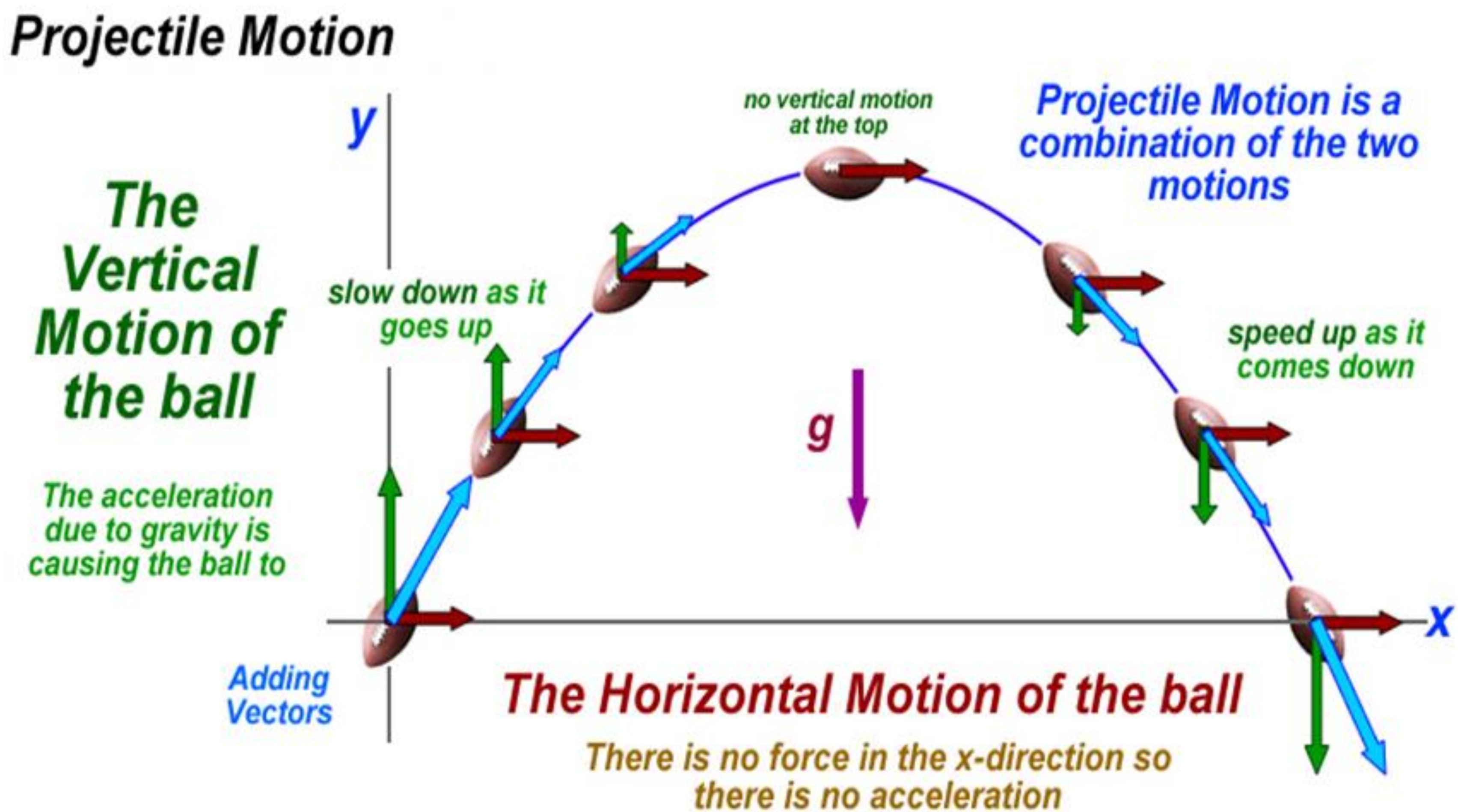
PICTORIAL VIEW

A stroboscopic photograph of a yellow tennis ball bouncing off a hard surface. Between impacts, the ball has projectile motion

Javelin Throw



VECTOR PERSPECTIVE



WHAT MAY BE THE PROJECTILES?

Projectile might be ;

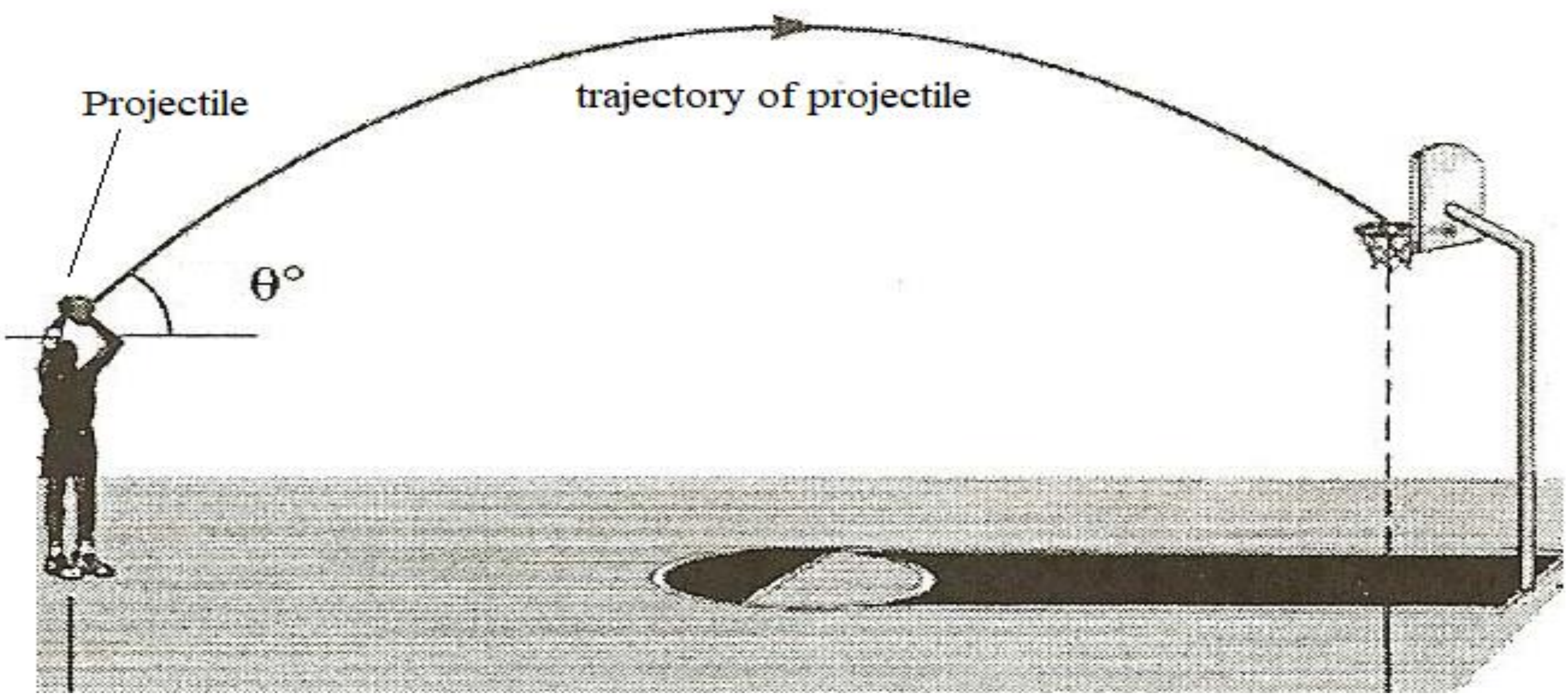
- a tennis ball
- a baseball in flight
- Cannon ball shot from a cannon
- A basketball thrown in a basket
- A kicked football

Many sports (from golf and football to lacrosse and Racquetball)involve the projectile motion of a ball, and much effort is spent in trying to control that motion for an advantage.

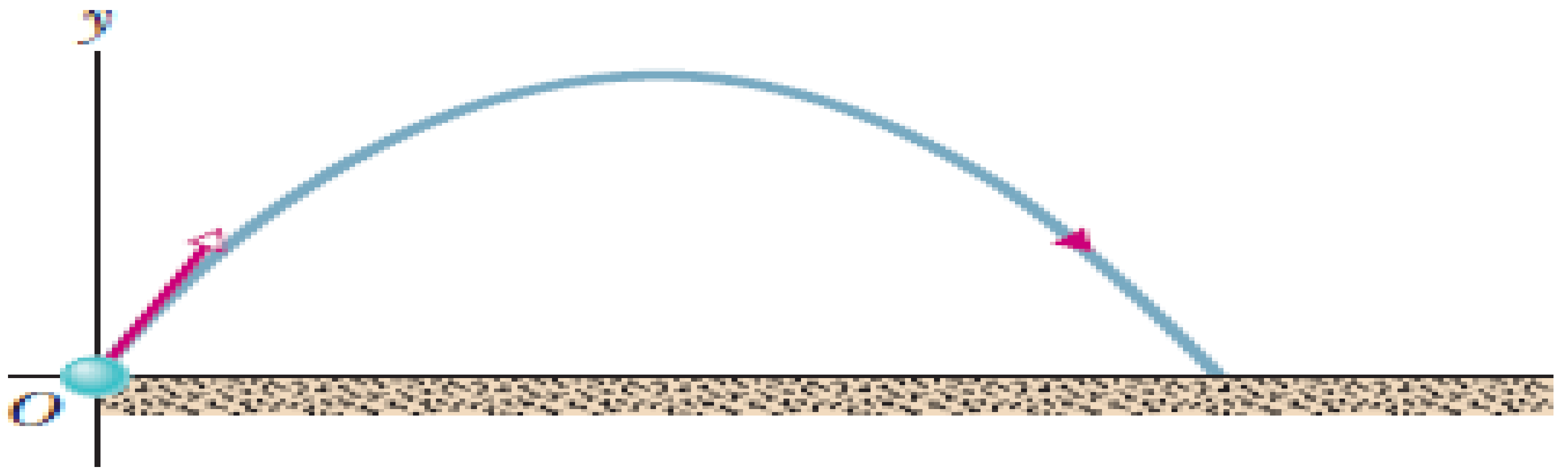
Note: Projectile is not an airplane or a duck in flight

TRAJECTORY OF PROJECTILE

- Path followed by the projectile is called as **trajectory of that projectile.**



PICTORIAL REPRESENTATION



The *projectile motion* of an object launched into the air at the origin of a coordinate system and with launch velocity $\vec{v}_0$ at angle θ_0. The motion is a combination of vertical motion (constant acceleration) and horizontal motion (constant velocity), as shown by the velocity components.

PICTORIAL REPRESENTATION

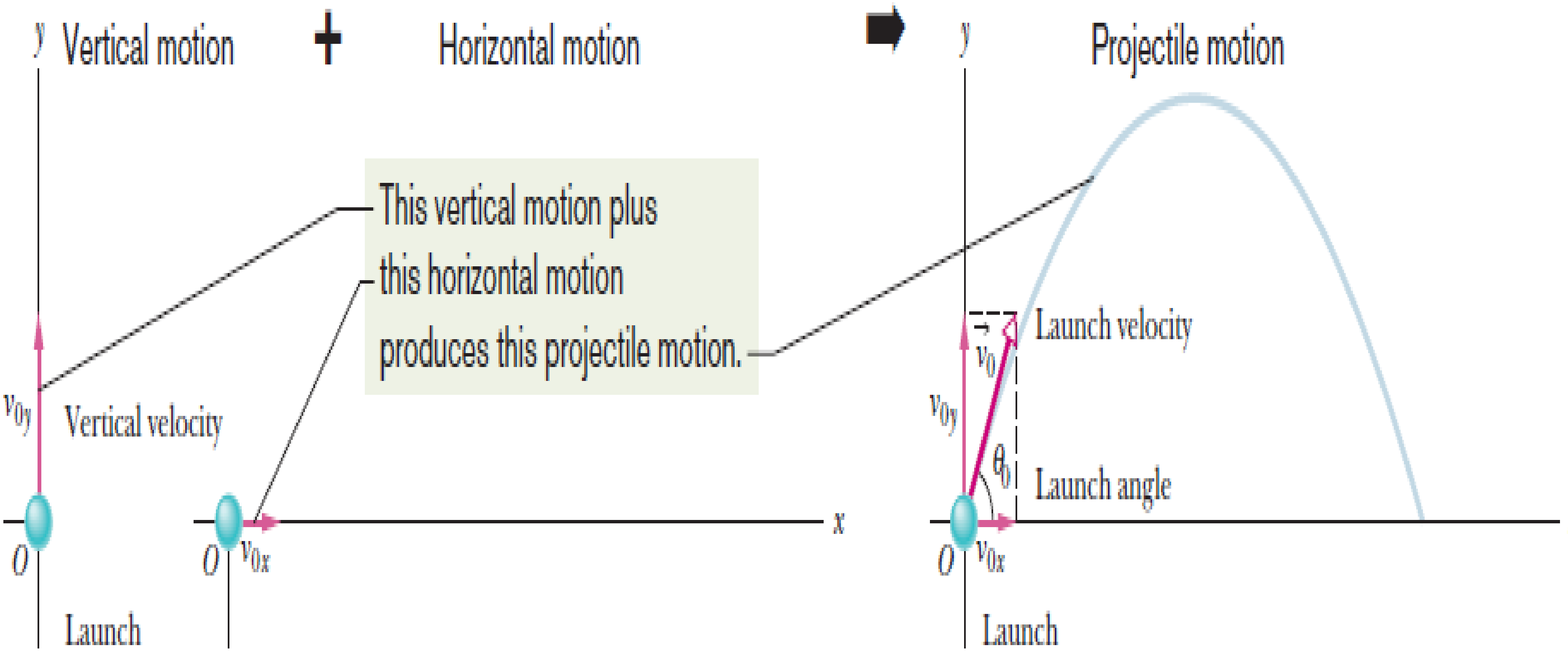


Fig 1(a)

PICTORIAL REPRESENTATION(CONT'D)

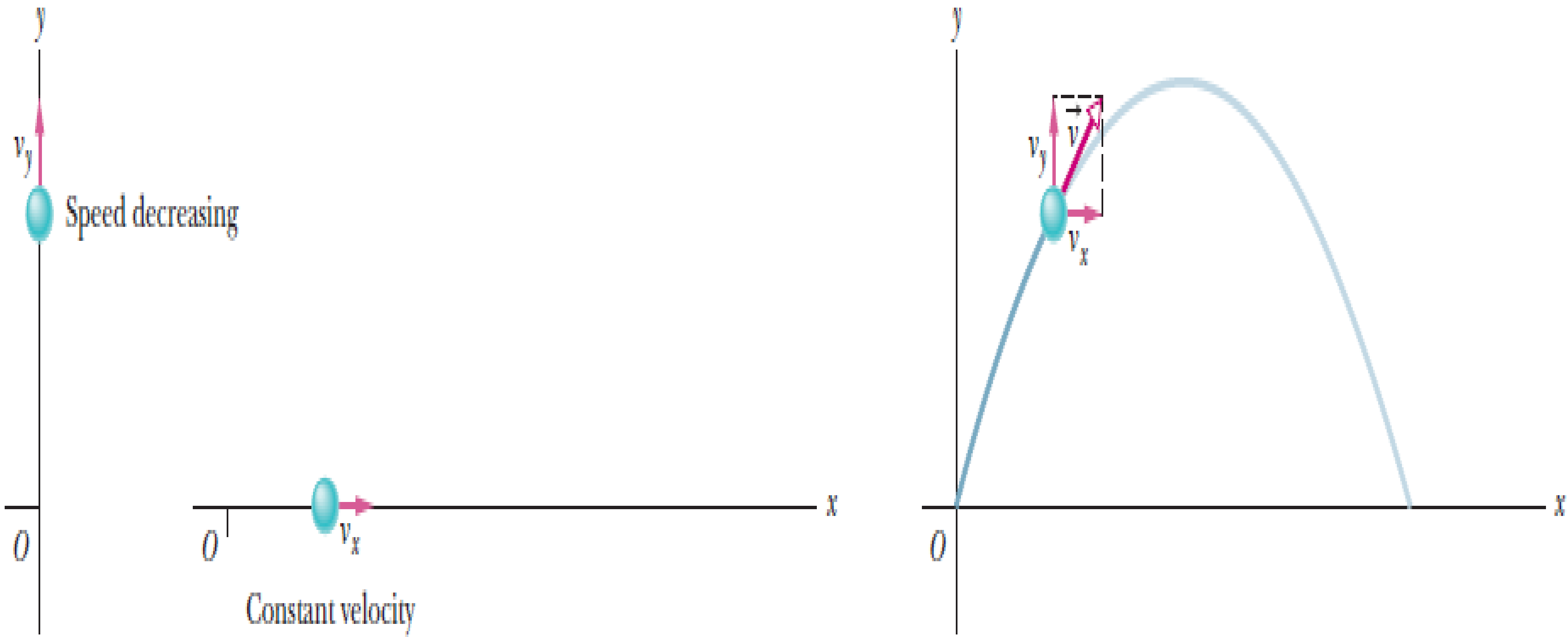


Fig 1(b)

PICTORIAL REPRESENTATION(CONT'D)

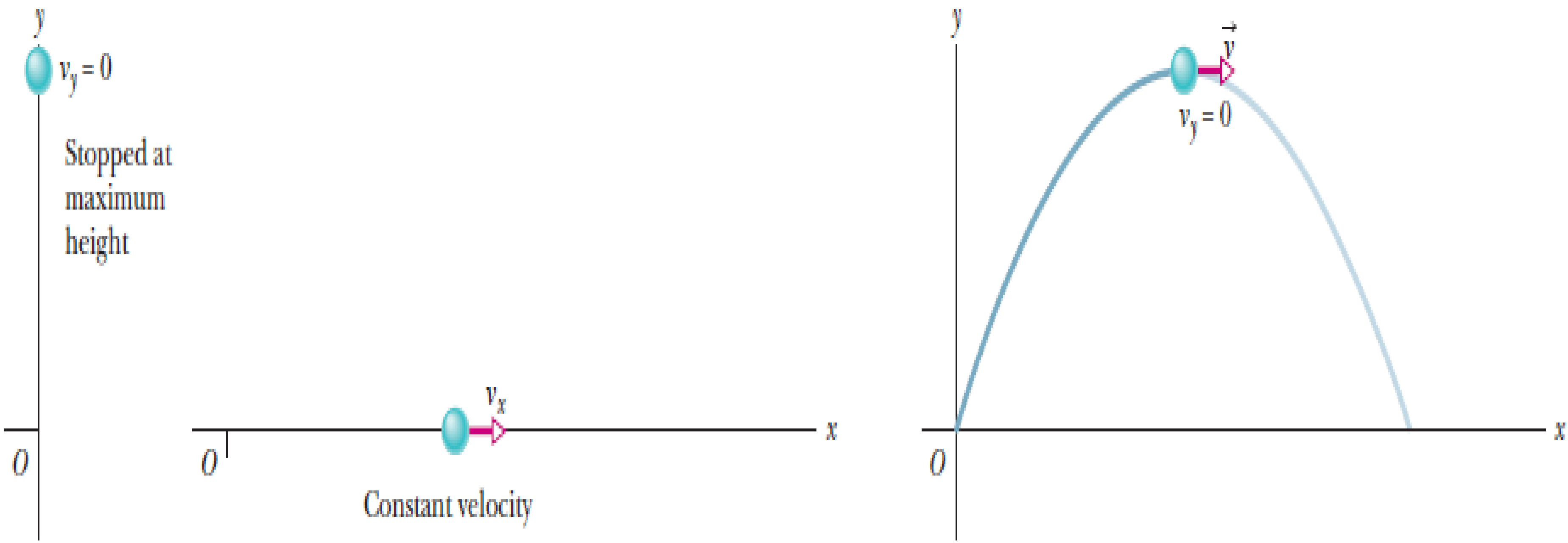


Fig 1(c)

PICTORIAL REPRESENTATION(CONT'D)

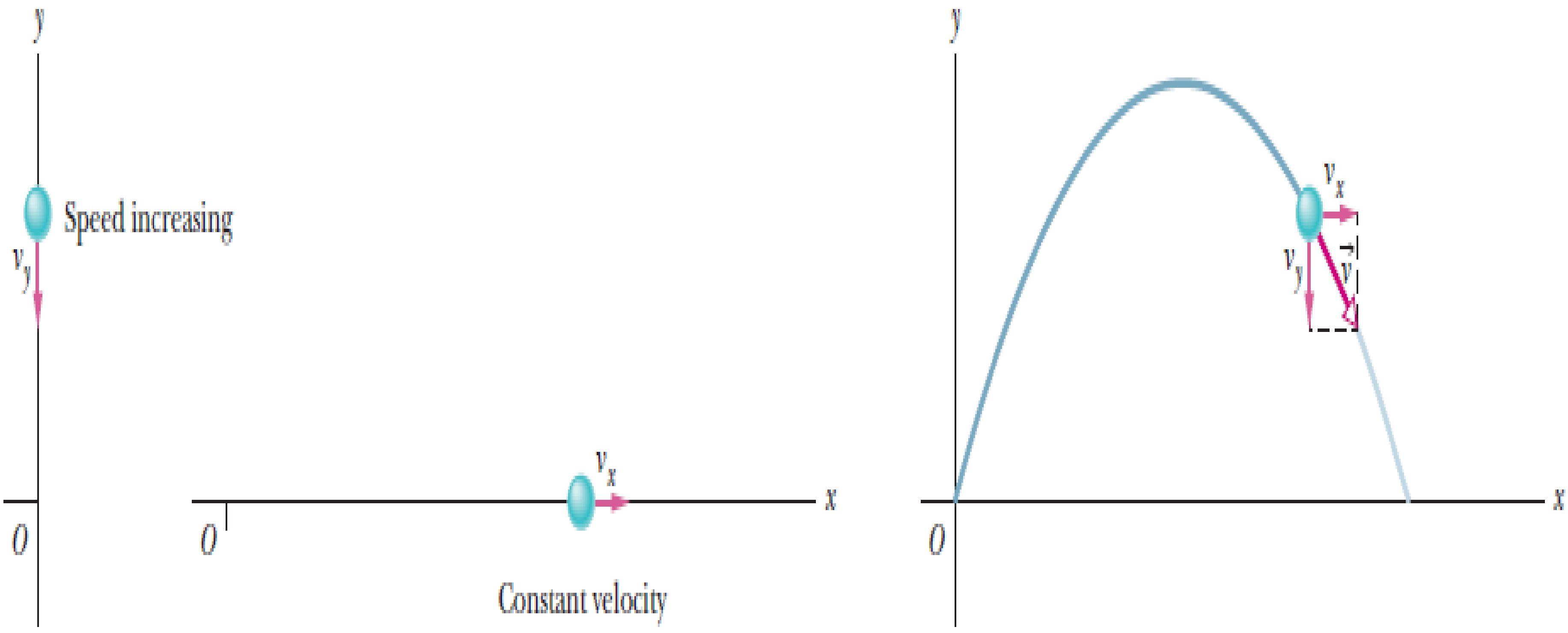


Fig 1(d)

PICTORIAL REPRESENTATION(CONT'D)

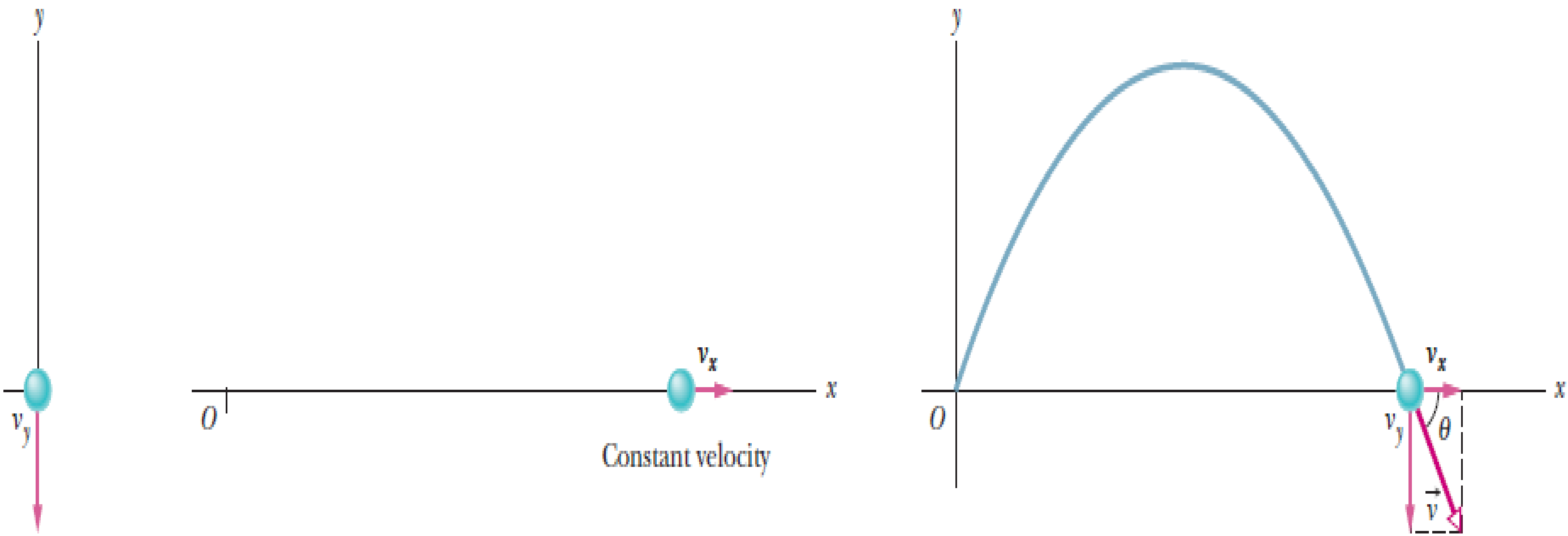


Fig 1(e)

THE HORIZONTAL MOTION

Because there is *no acceleration* in the horizontal direction, the horizontal component v_x of the projectile's velocity remains unchanged from its initial value v_{0x} throughout the motion, as demonstrated in Fig. 1(a) to 1(e)

At any time t, the projectile's horizontal displacement $x - x_0$ from an initial position x_0 is given by

$$x - x_0 = v_0 t + \frac{1}{2} a t^2.$$

where a=0

Therefore, the above equation will become,

$$x - x_0 = v_{0x} t.$$

Because $v_{0x} = v_0 \cos \theta_0$, this becomes

$$\boxed{x - x_0 = (v_0 \cos \theta_0) t.}$$

THE VERTICAL MOTION

- The vertical motion is a motion of a particle in a free fall.
- The acceleration in free fall remains constant
- We substitute "negative acceleration" because the direction of motion of the body is against the gravity.
- As the motion is vertical or in y-axis we switch to the y-component notation, therefore

$$y - y_0 = v_{0y}t - \frac{1}{2}gt^2$$

$$y - y_0 = (v_0 \sin\theta_0)t - \frac{1}{2}gt^2 \longrightarrow \textbf{Eq (i)}$$

THE VERTICAL MOTION(CONT'D)

where the initial vertical velocity component v_{0y} is replaced with the equivalent $v_0 \sin \theta_0$.

By using first equation of motion in y-direction

$$v_y = v_{oy} + (-gt)$$

$$v_y = v_{oy} - gt$$

$$v_y = v_o \sin \theta_o - gt \longrightarrow \textbf{Eq (ii)}$$

THE VERTICAL MOTION(CONT'D)

$$v^2{}_y = (v_o \sin\theta_o - gt)^2$$

$$v^2{}_y = (v_o \sin\theta_o)^2 - 2(v_o \sin\theta_o)(gt) + (gt)^2$$

$$v^2{}_y = (v_o \sin\theta_o)^2 - 2g(v_o \sin\theta_o\, t + \frac{1}{2} g t^2)$$

$$\boxed{v^2{}_y = (v_o \sin\theta_o)^2 - 2g(y - y_o)}$$

THE VERTICAL MOTION(CONT'D)

$$\boxed{v_y = v_0 \sin\theta_0 - gt}$$

$$\boxed{v^2{}_y = (v_o \sin\theta_o)^2 - 2g(y - y_o)}$$

Conclusion

the vertical velocity component behaves just as for a ball thrown vertically upward. It is directed upward initially, and its magnitude steadily decreases to zero, *which marks the maximum height of the path.*

The vertical velocity component then reverses direction, and its magnitude becomes larger with time.

EQUATION OF TRAJECTORY OF PROJECTILE

We can find the equation of the projectile's path (its **trajectory**) by eliminating time t

$$x - x_0 = (v_0 \cos \theta_0)t. \qquad \text{........(Eq iii)}$$

$$y - y_0 = v_{0y}t - \tfrac{1}{2}gt^2$$

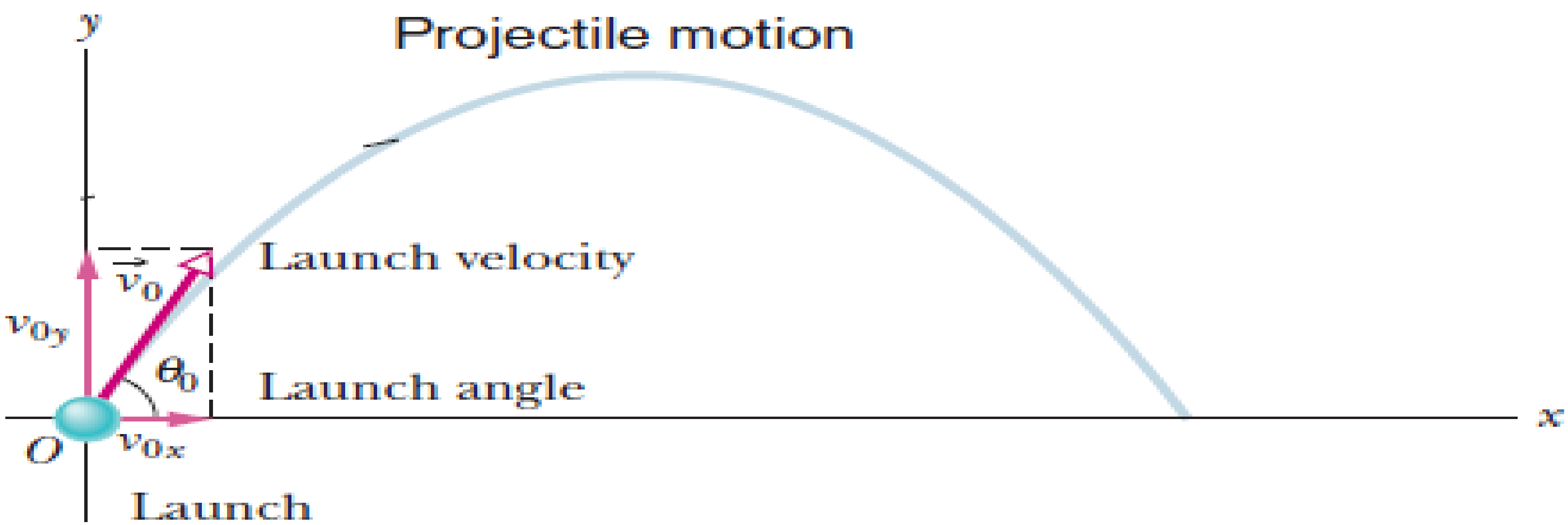
$$= (v_0 \sin \theta_0)t - \tfrac{1}{2}gt^2 \quad \text{........(Eq iv)}$$

Solving Eq. iii for t and substituting into

Eq. iv we obtain, after a little rearrangement.

$$y = (\tan \theta_0)x - \frac{gx^2}{2(v_0 \cos \theta_0)^2} \qquad \text{(trajectory)} \longrightarrow \text{(Eq v)}$$

EQUATION OF TRAJECTORY OF PROJECTILE



This is the equation of the path shown in Figure In deriving it, for simplicity we let $x_0 = 0$ and $y_0 = 0$ in Eqs. iii and iv respectively. Because g, θ_0, and v_0 are constants, Eq. v is of the form $y = ax + bx^2$, in which a and b are constants. This is the equation of a parabola, so the path is *parabolic*.

THE HORIZONTAL RANGE

The *horizontal range* R of the projectile is the *horizontal* distance the projectile has traveled when it returns to its initial height (the height at which it is launched). To find range R, let us put

$$x - x_0 = R$$

in Eq. $x - x_0 = (v_0 \cos \theta_0)t.$

$$R = (v_0 \cos \theta_0)t \quad \text{......(Eq vi)}$$

and $y - y_0 = 0$ in Eq. $y - y_0 = (v_0 \sin \theta_0)t - \frac{1}{2}gt^2.$

obtaining $$0 = (v_0 \sin \theta_0)t - \frac{1}{2}gt^2. \text{ ....(Eq vii)}$$

Eliminating t between these two equations yields

$$R = \frac{2v_0^2}{g} \sin \theta_0 \cos \theta_0.$$

Using the identity $\sin 2\theta_0 = 2 \sin \theta_0 \cos \theta_0$

we obtain

$$R = \frac{v_0^2}{g} \sin 2\theta_0. \quad \text{......(Eq viii)}$$

THE HORIZONTAL RANGE(CONT'D)

Caution: This equation does *not* give the horizontal distance traveled by a projectile when the final height is not the launch height.

Note that R in Eq viii has its maximum value when $\sin 2\theta_0 = 1$, which corresponds to $2\theta_0 = 90^\circ$ or $\theta_0 = 45^\circ$.

The horizontal range R is maximum for a launch angle of 45°.

However, when the launch and landing heights differ, as in shot put, hammer throw, and basketball, a launch angle of 45° does not yield the maximum horizontal distance.

Example:

Find the angle of the projectile if the maximum height and the range of the projectile are equal.

$$R = \frac{u^2 \sin 2\theta}{g}$$

$$H = \frac{u^2 \sin^2\theta}{2g}$$

$$\frac{u^2 \sin 2\theta}{g} = \frac{u^2 \sin^2\theta}{2g}$$

$$2 \sin\theta \cos\theta = \frac{\sin^2\theta}{2}$$

$$4 \cos\theta = \sin\theta$$

$$\tan\theta = 4$$

$$\theta = \tan^{-1} 4$$

= 76 degrees

Example:

Show that for a projectile motion the maximum range covered is equal to four times of its maximum height attained. (R_{max}=4H).

The range of the projectile $R = \frac{u^2 \sin 2\theta}{g}$,R will be the maximum, if $\sin 2\theta = 1 \Rightarrow 2\theta = 90^0 \Rightarrow \theta = 45^0$

Then,

$$R_{max} = \frac{u^2}{g}$$

So the maximum height attained by the projectile is:

$$H = \frac{u^2 \sin^2 45^0}{2g} = \frac{u^2}{g} \times \frac{1}{2} = \frac{R_{max}}{4} \Rightarrow R_{max} = 4H$$

EFFECTS OF AIR ON A PROJECTILE

- We have assumed that the air through which the projectile moves has no effect its motion.
- However, in many situations, the disagreement between our calculations and the actual motion of the projectile can be large because the air resists the motion.
- As an example figure below shows two paths for a fly ball that leaves the bat at an angle of 60° with the horizontal and an initial speed of 44.7 m/s.

GRAPHICAL VIEW

- Path I (the baseball player's fly ball) is a calculated path that
- approximates normal conditions of play, in air. Path II (the physics professor's fly ball) is the path the ball would follow in a vacuum.

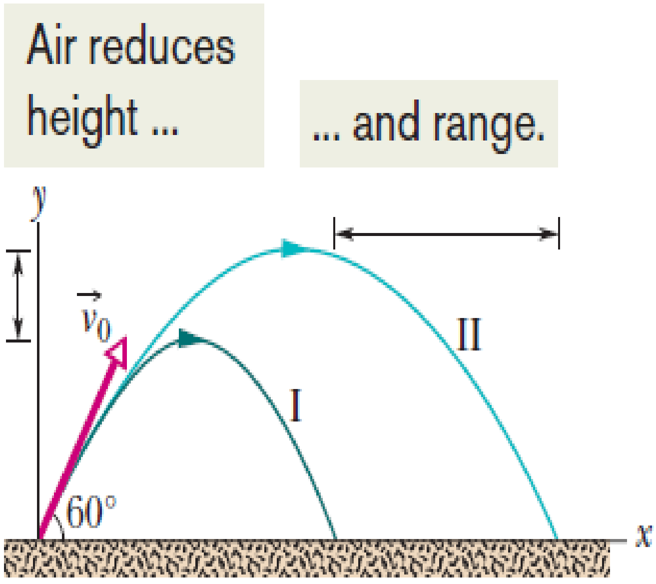


Figure (I) The path of a fly ball calculated by taking air resistance into account. (II) The path the ball would follow in a vacuum, calculated by the methods of this chapter.

CORRESPONDING DATA OF THE FLY BALLS

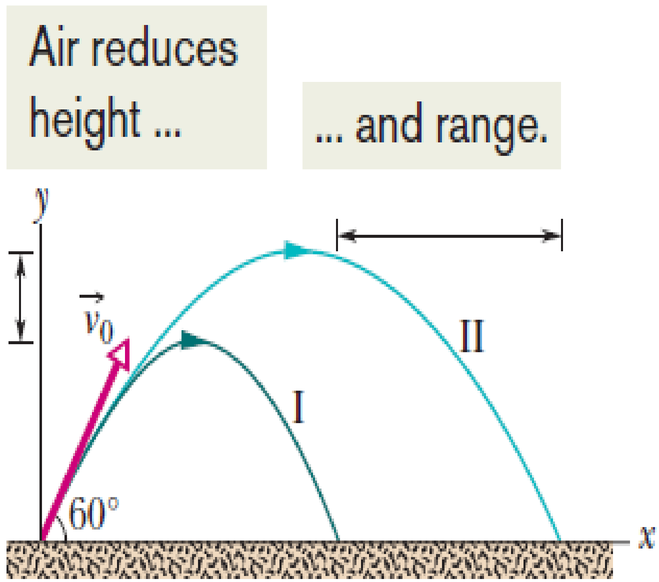


Figure (I) The path of a fly ball calculated by taking air resistance into account. (II) The path the ball would follow in a vacuum, calculated by the methods of this chapter.

Table 1 Two Fly Balls[a]

	Path I (Air)	Path II (Vacuum)
Range	98.5 m	177 m
Maximum height	53.0 m	76.8 m
Time of flight	6.6 s	7.9 s

[a]The launch angle is 60° and the launch speed is 44.7 m/s.

Class Activity:

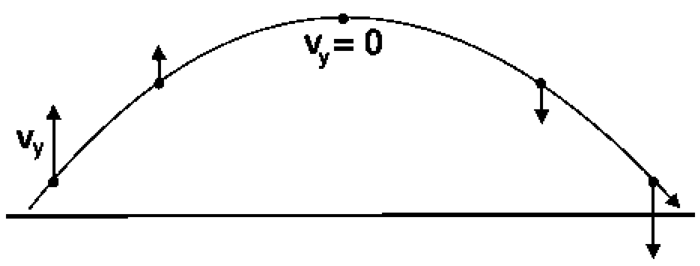
In a projectile motion:

(i) which velocity component retains its initial value throughout the flight and why?

Solution: Horizontal component, due to the absence of horizontal forces, a projectile remains in motion with a constant horizontal velocity.

(ii) At what point in the path of a projectile is the speed a minimum? Show this point by drawing a projectile path.

Solution: At the maximum height.



Example:

Given figure shows three paths for a football kicked from ground level. Ignoring the effects of air, rank the paths according to

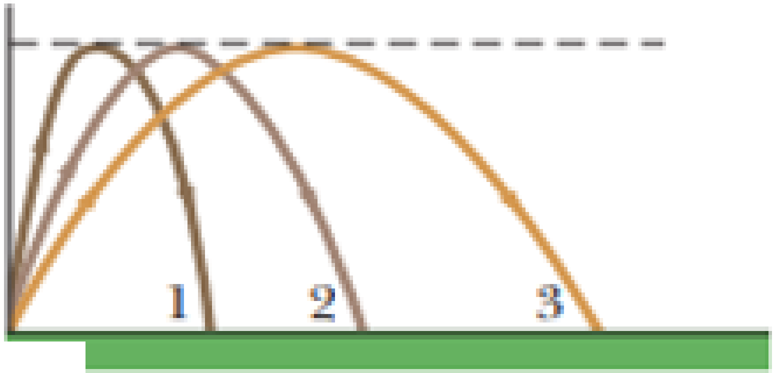
(A) time of flight

(B) initial vertical velocity component

(C) initial horizontal velocity component

(D) initial speed

greatest first.



As we know,

the same max height means the same vertical component of initial velocity,

and the same amount of time object spends in the air.

Horizontal component of initial velocity determines range.

Also, $V = \sqrt{u_x^2 + u_y^2}$ so,

(A) $1 = 2 = 3$

(B) $1 = 2 = 3$

(C) $3 > 2 > 1$

(D) $3 > 2 > 1$