National University of Computer & Emerging Sciences – FAST

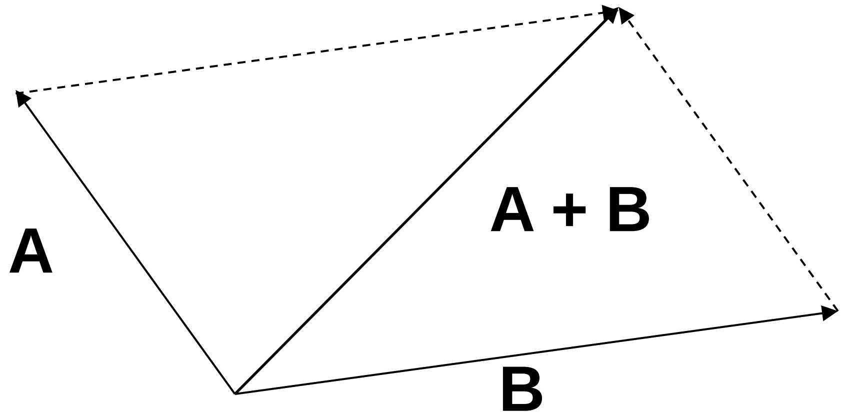
FAST SCHOOL OF COMPUTING

Course Code : NS 1001

Course Title : Applied Physics
Instructor: Dr. Muhammad Adeel
Fall Semester 2024

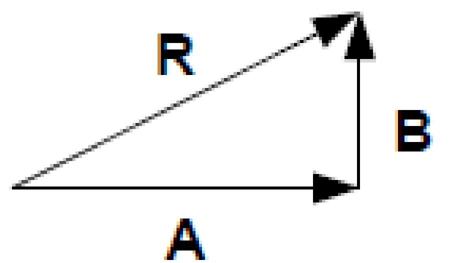
ADDITION OF VECTORS

- Graphically vectors are added by using famous head to tail rule



- Two vectors can be added using the **Parallelogram Law.**
- Considering vectors, **A** & **B,** add them by using head to tail rule

Mathematically, the vector sum is found to be **R** = **A** + **B**



The resultant vector **R** = **A** + **B**

ADDITION OF VECTORS

- Considering Fig 1(a), move **B** so that its tail coincides with the tail of **A**.
- The resultant **R** is seen to lie along the diagonal of the parallelogram formed by **A** and **B**, with the tails of all three vectors coinciding as in Fig1(b).
- Finally shift **A** such that its tail coincides with the head of the shifted **B** as shown in Fig 1(c).
- It is now obvious from Fig 1(c) that **R** =**A** + **B** = **B** + **A**,

  showing that the *commutative law* holds for vector addition

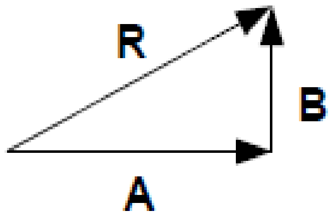


Fig 1(a)

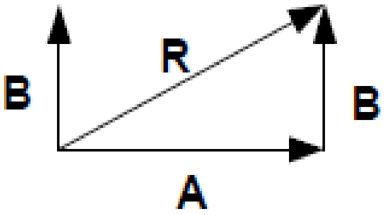


Fig 1(b)

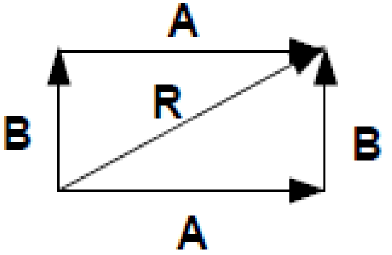


Fig 1(c)

Addition of Vectors

- The sum of more than two vectors can be found by continuing to place the tail of succeeding vectors at the head of the preceding vector, as shown in Fig. 2(c) . The resultant vector **D** = **A** + **B** + **C** is shown in Fig 2(d).

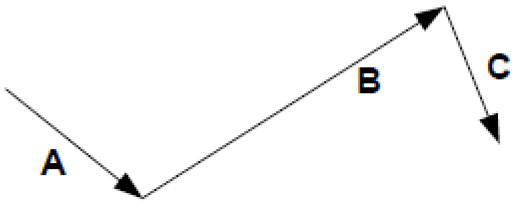


Fig 2(a)

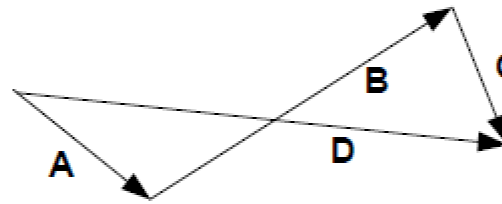


Fig 2(b)

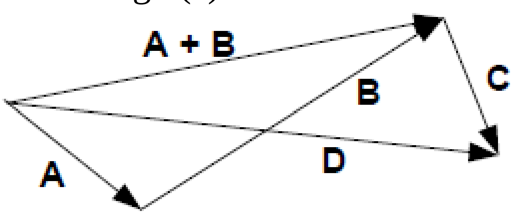


Fig 2(c)

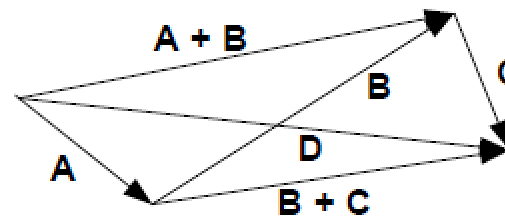


Fig 2(d)

- Draw the vector sum (**A + B**) in Fig. 2(b). The result is shown in Fig. 2(c).
- Finally, draw the vector sum (**B + C**) in Fig.2(c). The result is shown in Fig.2(d) . It is now clear from Fig. 2(d) that ,

D = A + B + C = (A + B) + C = A + (B + C), showing that

the associative law holds for vector addition.

SUBTRACTION OF A VECTOR

- Two vectors **A** and **B** are shown Fig.3(a). The vector **–A** is a vector with the same magnitude as **A** but with the opposite direction.
- Draw –**A** in Fig.3(a). The result is shown in Fig.3(b).
- Draw the resultant vector **R** = **A** + **B** in Fig. 3(b) Since **R** = **A** + **B.**
- Verify that **R** – **A** = **B** by showing that **R** + (–A) = **B** in Fig 3(b).

The result is shown in Fig. 3(c).

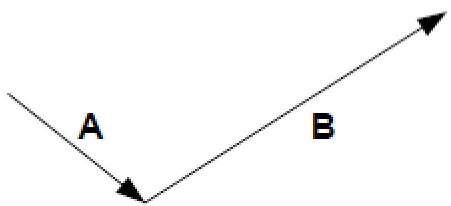


Fig 3(a)

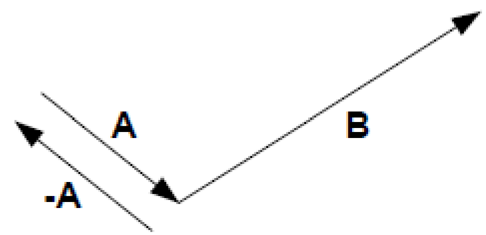


Fig 3(b)

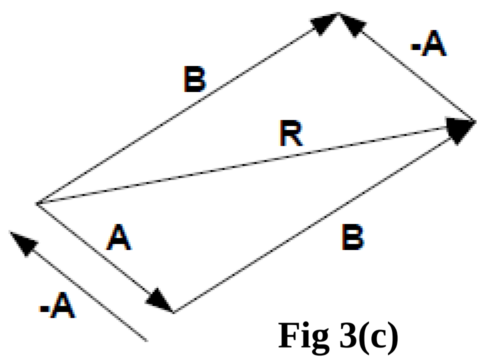


Fig 3(c)

SUBTRACTION OF A VECTOR

Class Activity:

Fig.3(d) shows two vectors **A** and **B**. Draw the resultant vector **R** = **A** + **B** and the difference vector **D** = **A - B**. Note that the difference vector **D** can be drawn by connecting the head of **A** with the head of **B** and locating the head of **D** at the head of **A** as shown in Fig. 3(e).

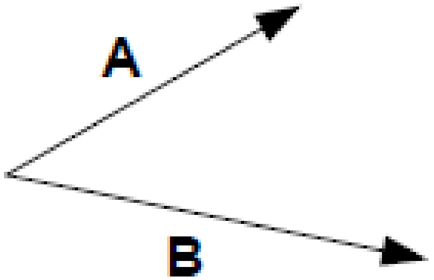


Fig 3(d)

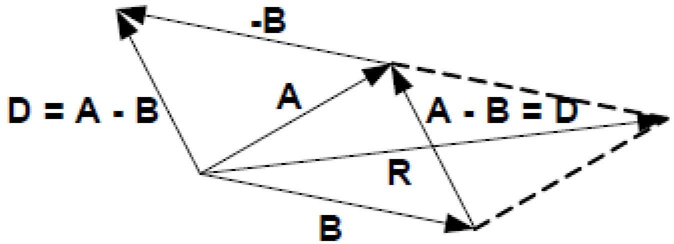


Fig 3(e)

Example:

Consider the two vectors $\mathbf{A} = 3\mathbf{i} - 2\mathbf{j}$ and $\mathbf{B} = -\mathbf{i} - 4\mathbf{j}$.

Calculate

(a) $\mathbf{A} + \mathbf{B}$,

(b) $\mathbf{A} - \mathbf{B}$,

(c) $|\mathbf{A} + \mathbf{B}|$,

(d) $|\mathbf{A} - \mathbf{B}|$

(e) the directions of $\mathbf{A} + \mathbf{B}$ and (vi) $\mathbf{A} - \mathbf{B}$.

$$\vec{A} = 3\hat{i} - 2\hat{j} \quad \text{and} \quad \vec{B} = -\hat{i} - 4\hat{j}$$

$$\text{(a)} \ \vec{A} + \vec{B} = \left(3\hat{i} - 2\hat{j}\right) + \left(-\hat{i} - 4\hat{j}\right) = 2\hat{i} - 6\hat{j}$$

$$\text{(b)} \ \vec{A} - \vec{B} = \left(3\hat{i} - 2\hat{j}\right) - \left(-\hat{i} - 4\hat{j}\right) = 4\hat{i} + 2\hat{j}$$

$$\text{(c)} \ \left|\vec{A} + \vec{B}\right| = \sqrt{2^2 + (-6)^2} = \sqrt{40}$$

$$\text{(d)} \ \left|\vec{A} - \vec{B}\right| = \sqrt{4^2 + 2^2} = \sqrt{20}$$

$$\text{Direction of } \left(\vec{A} + \vec{B}\right) \text{ is } = \frac{\vec{A} + \vec{B}}{\left|\vec{A} + \vec{B}\right|} = \frac{2\hat{i} - 6\hat{j}}{\sqrt{40}}$$

$$\text{Direction of } \left(\vec{A} - \vec{B}\right) \text{ is } = \frac{\vec{A} - \vec{B}}{\left|\vec{A} - \vec{B}\right|} = \frac{4\hat{i} + 2\hat{j}}{\sqrt{20}}$$

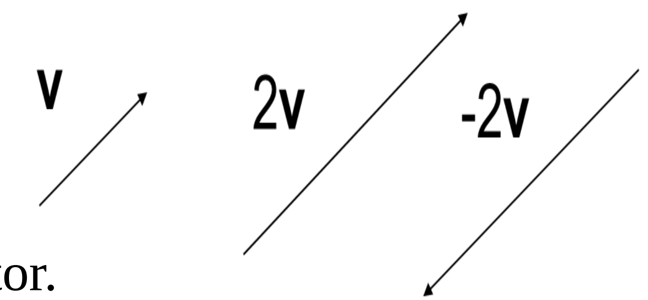
Scalar Multiplication of a Vector

We can multiply a vector by a real number c

That means multiplying its magnitude by c:

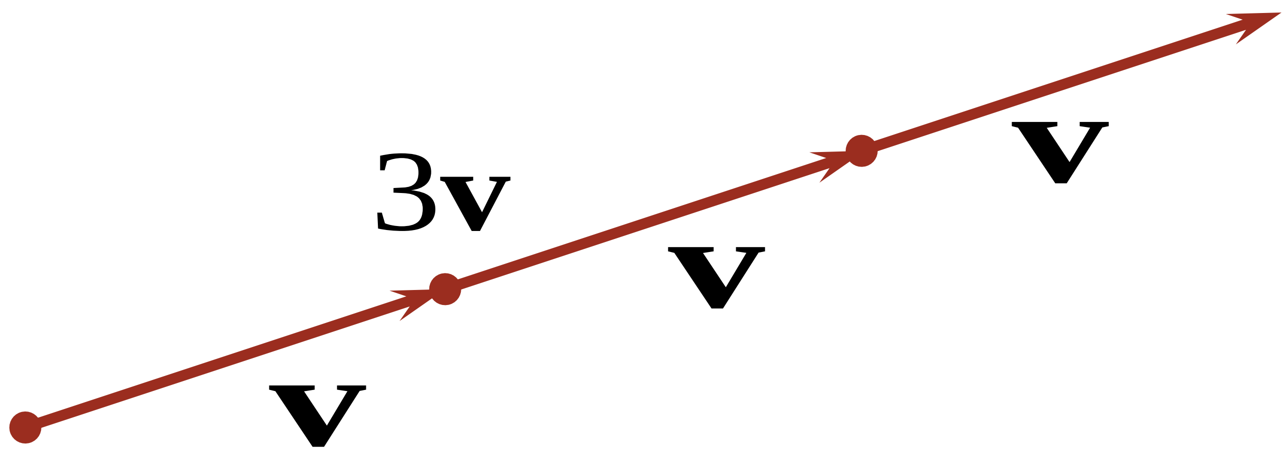
Notice that multiplying a vector by a

negative real number **reverses the direction** of vector.



A number multiplied in front of a vector is called a **SCALAR**.

It means to take the vector and add together that many times.



PROPERTIES OF MULTIPLICATION OF A VECTOR BY A NUMBER

- Commutative Law:

$$m\vec{A}=\vec{A}m$$

- Associative Law:

$$m(\mathrm{n}\,\vec{A})=\vec{A}(m\,\mathrm{n})$$

- Distributive Law:

$$(m+\mathrm{n})\,\vec{A}=m\vec{A}+\mathrm{n}\vec{A}$$

$$m(\vec{A}+\vec{B})=m\vec{A}+m\vec{B}$$

UNIT VECTORS

- A **unit vector** is a vector with magnitude 1.
- Given a vector **v**, we can form a unit vector by multiplying the vector by 1 / |**v**|.
- A vector such as <3,4> can be written as

3<1,0> + 4<0,1>.

For this reason, these vectors are given special names:

i = <1,0> and **j** = <0,1>.

A vector in component form **v** = <a,b> can be written as a**i** + b**j**.

UNIT VECTORS AND COORDINATE SYSTEMS

- If a vector $\mathbf{A}$ is multiplied by a scalar m, the resulting product $m\mathbf{A}$ is a vector whose magnitude is equal to $|m|$ times the magnitude of $\mathbf{A}$. The direction of $m\mathbf{A}$ is the same as that of $\mathbf{A}$ if m is positive and opposite to that of $\mathbf{A}$ if m is negative. If λ is a vector having a magnitude of unity, then $m\lambda$ is a vector whose magnitude is $|m|$.
- The magnitude of the vector $\mathbf{A}$ is written as $|\mathbf{A}| = A$.
- In Fig. 4(a) the unit vector $A\,\boldsymbol{\lambda}$, which has a magnitude of unity, is in the same direction as $\mathbf{A}$. We can therefore write the vector $\mathbf{A}$ as the magnitude of $\mathbf{A}$ multiplied by the unit vector $A\,\boldsymbol{\lambda}$.

  That is, $A\,\mathbf{A} = A\boldsymbol{\lambda}$.
- The unit vector $A\,\boldsymbol{\lambda}$ in the direction of $\mathbf{A}$

  can then be written as $\boldsymbol{\lambda} = \frac{A}{A}$

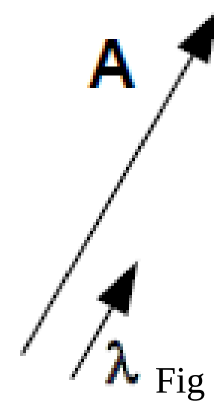


Fig 4(a)

UNIT VECTORS AND COORDINATE SYSTEMS

- Fig. 4(b) shows **A** to be the vector sum of **A***x* and **A***y*. That is,

$$\mathbf{A} = \mathbf{A}x + \mathbf{A}y\ .$$

- The vectors **A***x* and **A***y* lie along the *x* and *y* axes; therefore, we say that the vector **A** has been resolved into its *x* and *y* components.
- The unit vectors **i** and **j** are directed along the *x* and *y* axes as shown in Fig. 4(b).
- Using the technique of Fig. 4(a), we can therefore write

$\mathbf{A}x = Ax\mathbf{i}$ and $\mathbf{A}y = Ay\mathbf{j}$

- We can then write **A** in terms of the unit vectors as the vector sum $\mathbf{A} = Ax\mathbf{i} + Ay\mathbf{j}$.

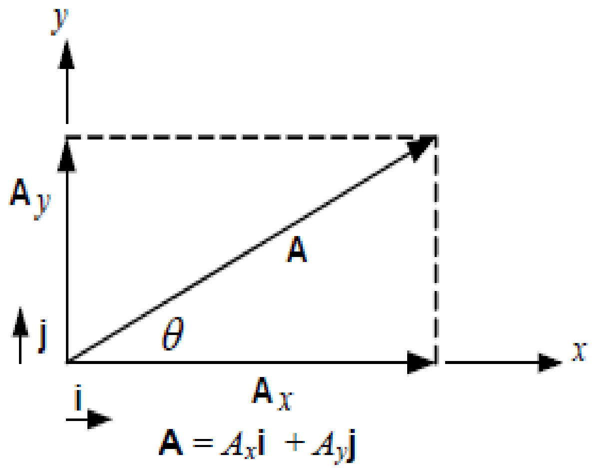


Fig 4(b)

UNIT VECTORS AND COORDINATE SYSTEMS

- In the previous frame we saw that a vector **A** lying in the the *x*-*y* plane can be written as $\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j}$.
- From the figure we see that the magnitudes are related by

$$A_x = A\cos\theta \qquad A_y = A\sin\theta$$

$$A_x^2 + A_y^2 = A^2\left(\cos^2\theta + \sin^2\theta\right) = A^2$$

from which the ratio
$$\frac{A_y}{A_x} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

Square *Ax* and *Ay* and add the results to obtain

$$A_x^2 + A_y^2 = A^2\left(\cos^2\theta + \sin^2\theta\right) = A^2$$

from which
$$A = \sqrt{A_x^2 + A_y^2}\,.$$

ADDITION OF VECTORS BY COMPONENTS

To illustrate the addition of vectors by components, consider the vector sum $\mathbf{R} = \mathbf{A} + \mathbf{B}$ shown in Fig 5(a) By resolving **A** and **B** into x and y components, we can write

$$\mathbf{R} = \mathbf{A} + \mathbf{B}$$
$$= A_x\mathbf{i} + A_y\mathbf{j} + B_x\mathbf{i} + B_y\mathbf{j}$$

from which

$$\mathbf{R} = (A_x + B_x)\mathbf{i} + (A_y + B_y)\mathbf{j}$$

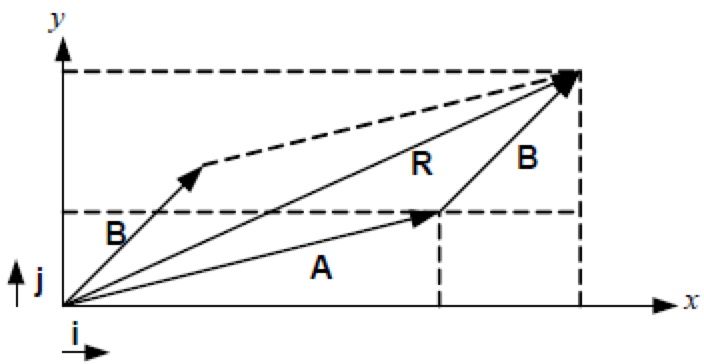


Fig 5(a) $\mathbf{R} = (A_x + B_x)\mathbf{i} + (A_y + B_y)\mathbf{j}$

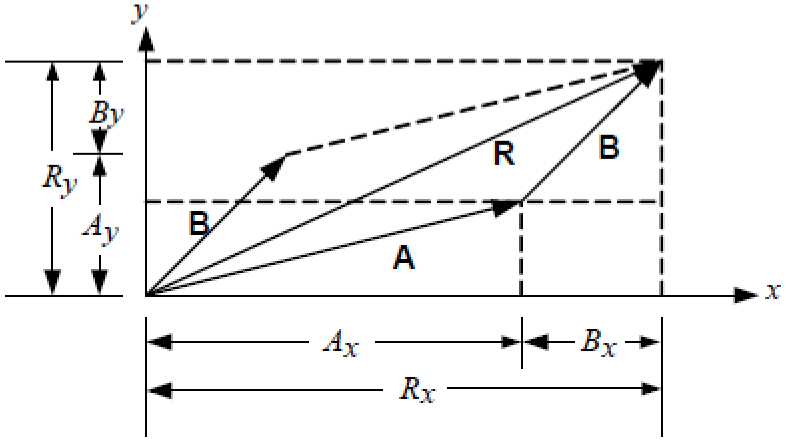
ADDITION OF VECTORS BY COMPONENTS

But in **Fig 5(a)** **R** can be written as $\mathbf{R} = R_x\mathbf{i} + R_y\mathbf{j}$. Therefore

$$R_x = A_x + B_x$$

$$R_y = A_y + B_y$$

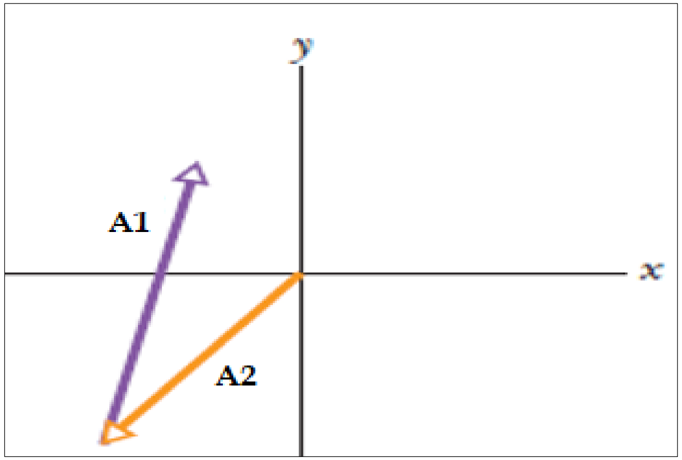
These results are summarized in **figure below.**



$$\mathbf{R} = R_x\mathbf{i} + R_y\mathbf{j} = (A_x + B_x)\mathbf{i} + (A_y + B_y)\mathbf{j}$$

Example:

The two vectors shown in **Figure** lie in an xy plane. What are the signs of the x and y components, respectively, of

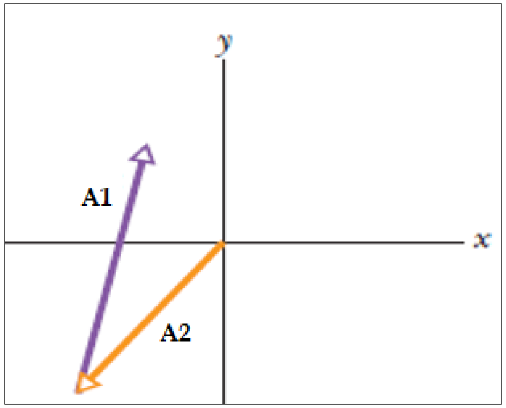


(a) $\vec{A}_1 + \vec{A}_2$

(b) $\vec{A}_1 - \vec{A}_2$,

(c) $\vec{A}_2 - \vec{A}_1$?

(a) As the resultant of (A_1 + A_2) lies in the second quadrant in which the X-component is negative and Y-component is positive, **so the signs of X and Y component of (A_1 + A_2) are negative (-) and positive (+) respectively.**



(b) The direction of (A_1 - A_2) is in the third quadrant where X and Y are negative (-), so **the signs of both X and Y component of (A_1 - A_2) are positive (-).**

(c) The direction of (A_2 -A_1) is in the first quadrant where X and Y both are positive (+), so **the signs of both X and Y component of (A_2 - A_1) are negative (-).(a**

Example:

Find the magnitude of vector **C** that satisfy the equation:

$2\mathbf{A} - 6\mathbf{B} + 3\mathbf{C} = 2\boldsymbol{j},$ where $\mathbf{A} = \boldsymbol{i} - 2\boldsymbol{k}$ and $\mathbf{B} = -\boldsymbol{j} + \boldsymbol{k}/2$

$$
\begin{aligned}
2\vec{A} - 6\vec{B} + 3\vec{C} &= 2\hat{j} \\
3\vec{C} &= 2\hat{j} - 2\vec{A} + 6\vec{B} \\
\vec{C} &= \tfrac{2}{3}\hat{j} - \tfrac{2}{3}\vec{A} + 2\vec{B} \\
&= \tfrac{2}{3}\hat{j} - \tfrac{2}{3}(\hat{i} - 2\hat{k}) + 2(-\hat{j} + \tfrac{\hat{k}}{2}) = \tfrac{2}{3}\hat{j} - \tfrac{2}{3}\hat{i} + \tfrac{4}{3}\hat{k} - 2\hat{j} + \hat{k} \\
&= -\tfrac{2}{3}\hat{i} + (\tfrac{2}{3} - 2)\hat{j} + (\tfrac{4}{3} + 1)\hat{k} \\
&= -\tfrac{2}{3}\hat{i} - \tfrac{4}{3}\hat{j} + \tfrac{7}{3}\hat{k}.
\end{aligned}
$$

The components are $C_x = -2/3$, $C_y = -4/3$, and $C_z = 7/3$, and substituting into (Figure) gives

$$C = \sqrt{C_x^2 + C_y^2 + C_z^2} = \sqrt{(-2/3)^2 + (-4/3)^2 + (7/3)^2} = \sqrt{23/3}.$$