DAMPED AND FORCED OSCILLATIONS

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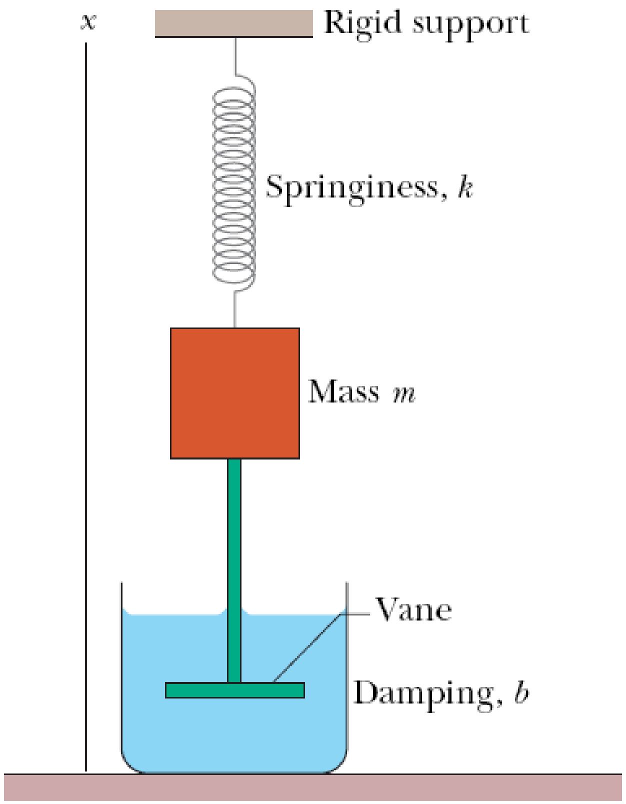
Damped Oscillations

In a damped oscillation, the motion of the oscillator is reduced by an external force.

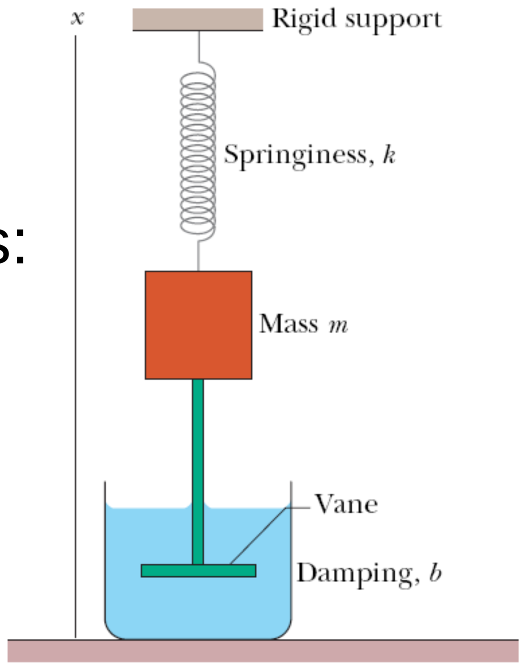
Example: A block of mass m oscillates vertically on a spring with spring constant k.
From the block a rod extends to a vane which is submerged in a liquid. The liquid provides the external damping force, F_d.



Damped SHM

Often the damping force, F_d, is proportional to the 1st power of the velocity v. That is,

$$F_d = -bv$$



From Newton's 2nd law, the following DE results:

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$

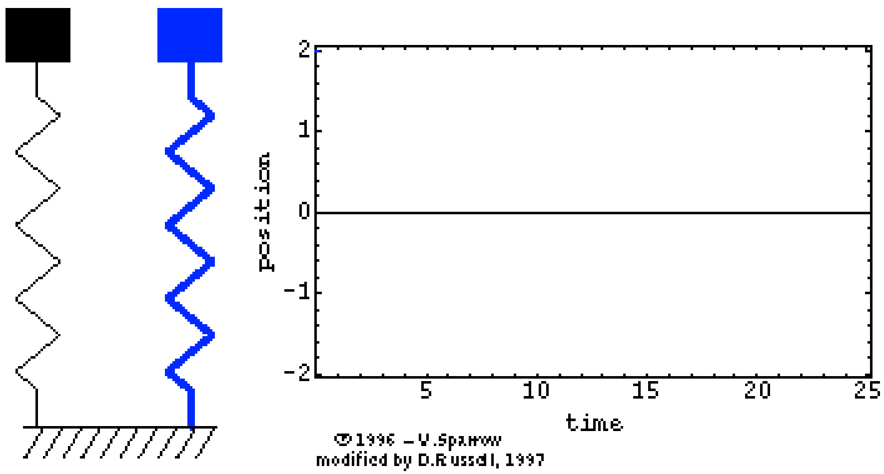
The solution is:

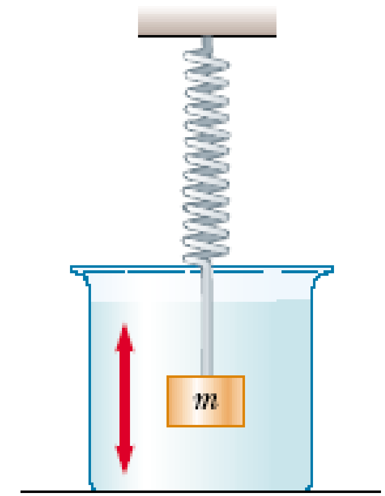
$$x(t) = x_m e^{\frac{-bt}{2m}} \cos(\omega' t + \phi)$$

Here ω' is the angular frequency, and is given by:

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

DAMPED OSCILLATIONS

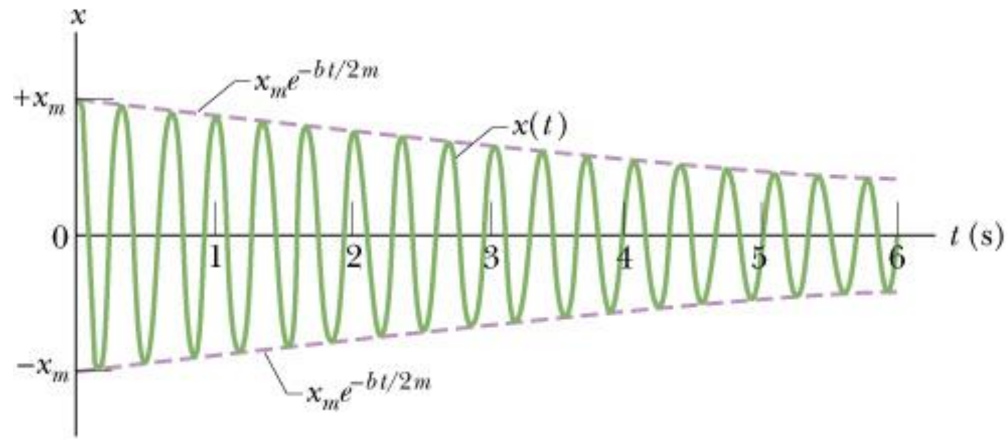




http://www.lon-capa.org/~mmp/applist/damped/d.htm

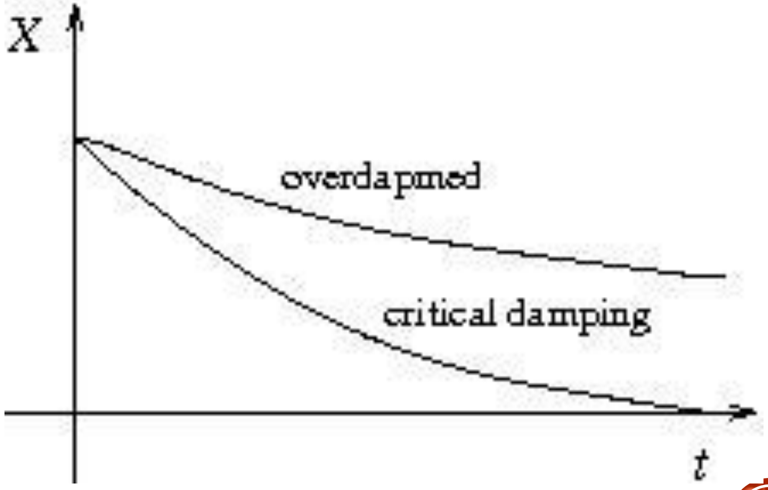
Damped Oscillations

$$x(t) = x_m e^{\frac{-bt}{2m}} \cos(\omega' t + \phi)$$



The above figure shows the displacement function *x(t)* for the damped oscillator described before.
The amplitude decreases as x_m exp (- $bt / 2m$) with time.

The above is for **b** $< 2m\omega_0$ (underdamped).



For **b** $> 2m\omega_0$ (overdamped) and **b** $= 2m\omega_0$ (critical damping), the oscillation goes like the right figure.

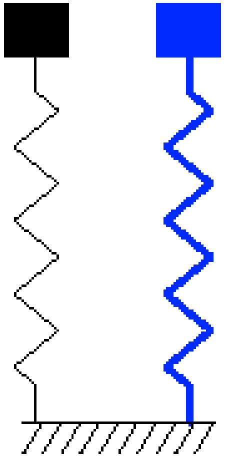
DAMPED OSCILLATIONS

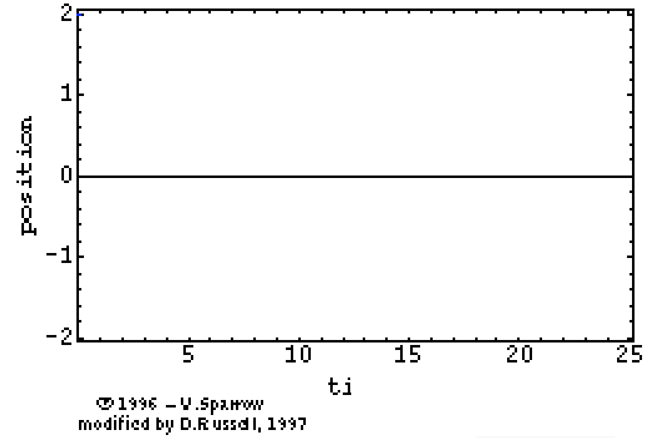
In many real systems, dissipative forces, such as friction, retard the motion.

Consequently, the mechanical energy of the system diminishes in time, and the motion is said to be Damped.

Retarding force

$$\mathbf{R} = -b\mathbf{v}$$





; we can write Newton's second law as

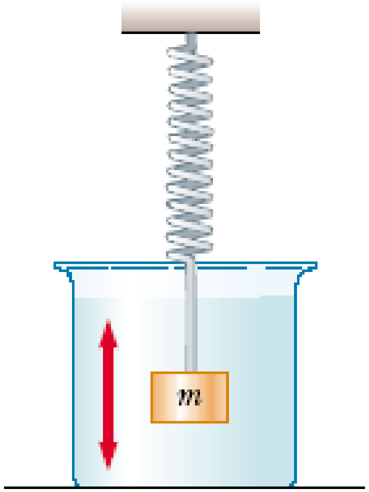
$$\sum F_x = -kx - bv = ma_x$$

$$-kx - b\frac{dx}{dt} = m\frac{d^2x}{dt^2}$$

The solution of this equation $x = Ae^{-\frac{b}{2m}t}\cos(\omega t + \phi)$

where the angular frequency of oscillation is

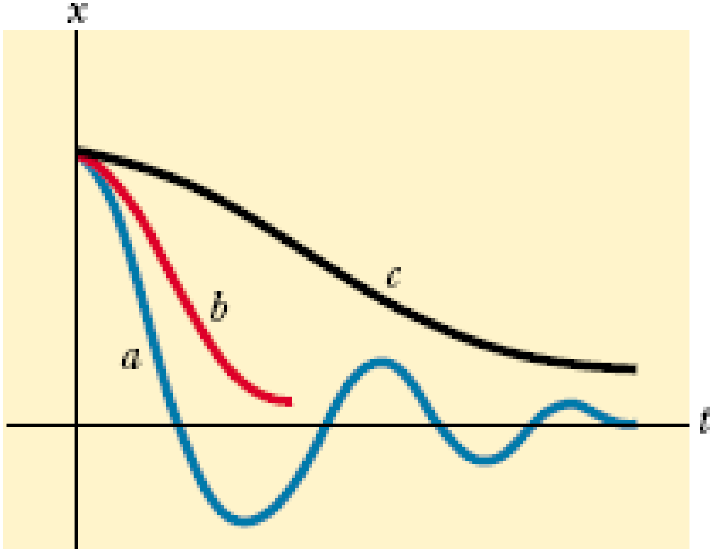
$$\omega = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$$



DAMPED OSCILLATIONS

$$\omega' = \sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2}$$

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}.$$



Graphs of displacement versus time for (a) an underdamped oscillator, (b) a critically damped oscillator, and (c) an overdamped oscillator.

where $\omega_0 = \sqrt{k/m}$ represents the angular frequency in the absence of a retarding force (the undamped oscillator) and is called the **natural frequency**

Forced Oscillations and Resonance

When the oscillator is subjected to an external force that is periodic, the oscillator will exhibit forced/driven oscillations.

There are two frequencies involved in a forced oscillator:

I. w_0, the natural angular frequency of the oscillator, without the presence of any external force, and

II. w_e, the angular frequency of the applied external force.

The equation of motion is like the following:

$$m\frac{d^2x}{dt^2}+g\frac{dx}{dt}+kx=F_0\cos(w_e t)$$

Forced Oscillations and Resonance

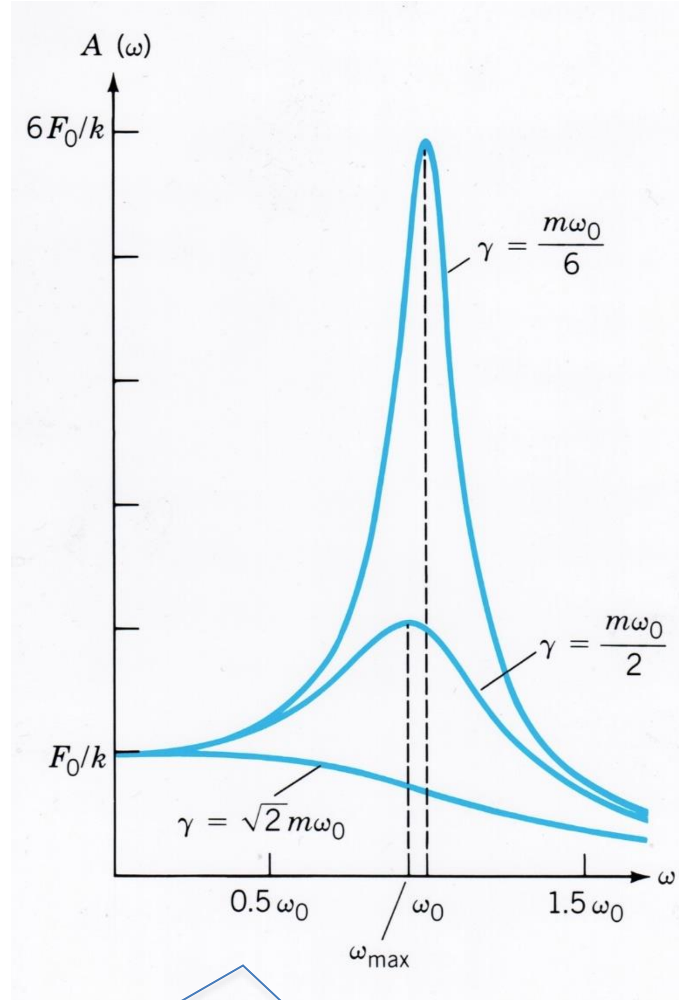
$$m\frac{d^2x}{dt^2}+\gamma\frac{dx}{dt}+kx=F_0\cos(\omega_e t)$$

The *steady state* solution is

$$x(t)=A\cos(\omega_e t+\delta)$$

$$A=\frac{F_0/m}{\sqrt{(\omega_0^2-\omega_e^2)^2+\left(\frac{\gamma}{m}\omega_e\right)^2}}$$

$$\tan\delta=\frac{\gamma}{m}\frac{\omega_e}{\omega_0^2-\omega_e^2} \qquad \omega_0=\sqrt{\frac{k}{m}}$$

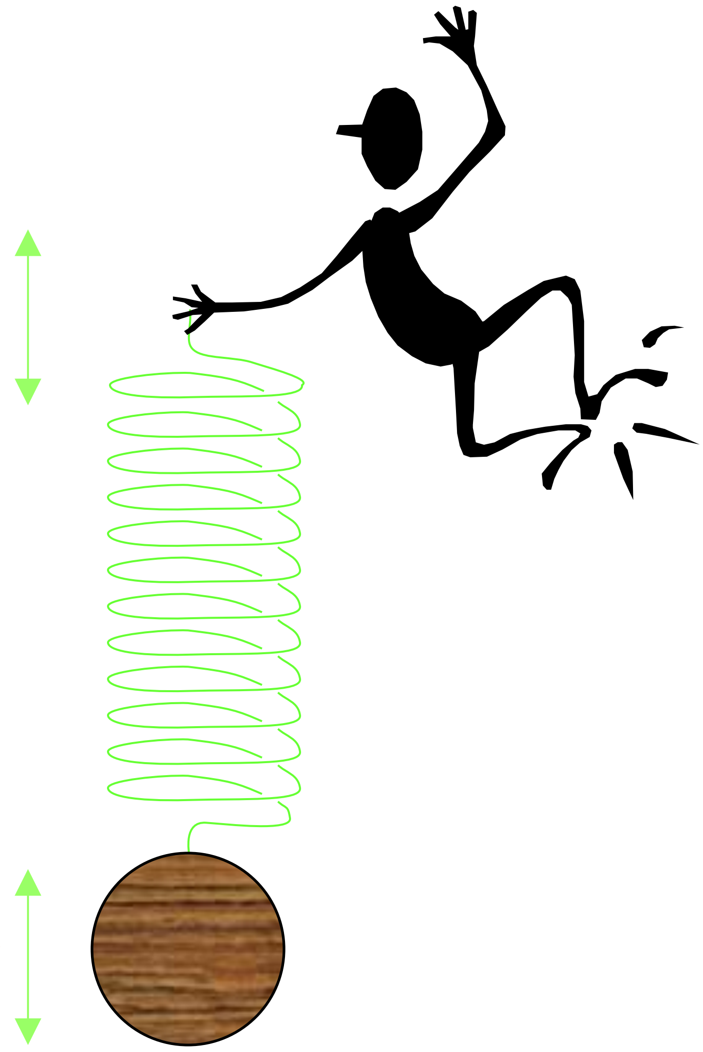


Resonance occurs at $\omega_e \sim \omega_{max} < \omega_0$, for $\gamma<\sqrt{2}m\omega_0$

EXAMPLE (MASS-SPRING SYSTEM)

Periodic driving force of freq. f

Oscillating with natural freq. f_o



RESONANCE

When a system is disturbed by a periodic driving force which frequency is ***equal to the natural frequency (fo)*** of the system, the system will oscillate with ***LARGE amplitude***.

Resonance is said to occur.

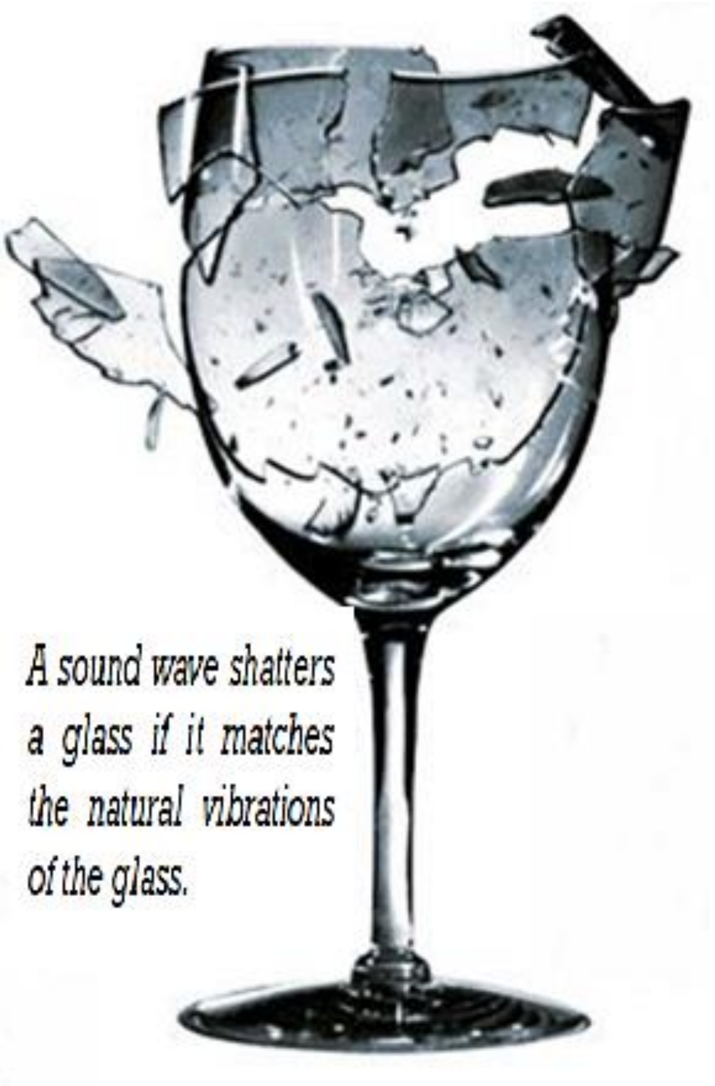
http://www.acoustics.salford.ac.uk/feschools/waves/shm3.htm

EXAMPLE 1

Breaking Glass

System : ***glass***

Driving Force : ***sound wave***



EXAMPLE 2

Collapse of the Tacoma Narrows suspension bridge in America in 1940

System : ***bridge***

Driving Force : ***strong wind***

FORCED OSCILLATIONS

When a system is disturbed by a ***periodic driving force*** and then oscillate, this is called ***forced oscillation***.
The system will oscillate with ***<u>its</u> natural frequency*** ***(f_o)*** which is ***independent of*** the frequency of the driving force

$$x = A\cos(\omega t + \phi)$$

$$F_{ext}\cos\omega t - kx - b\frac{dx}{dt} = m\frac{d^2x}{dt^2}$$

Where,

$$A = \frac{F_{ext}/m}{\sqrt{(\omega^2 - {\omega_0}^2)^2 + \left(\frac{b\omega}{m}\right)^2}}$$