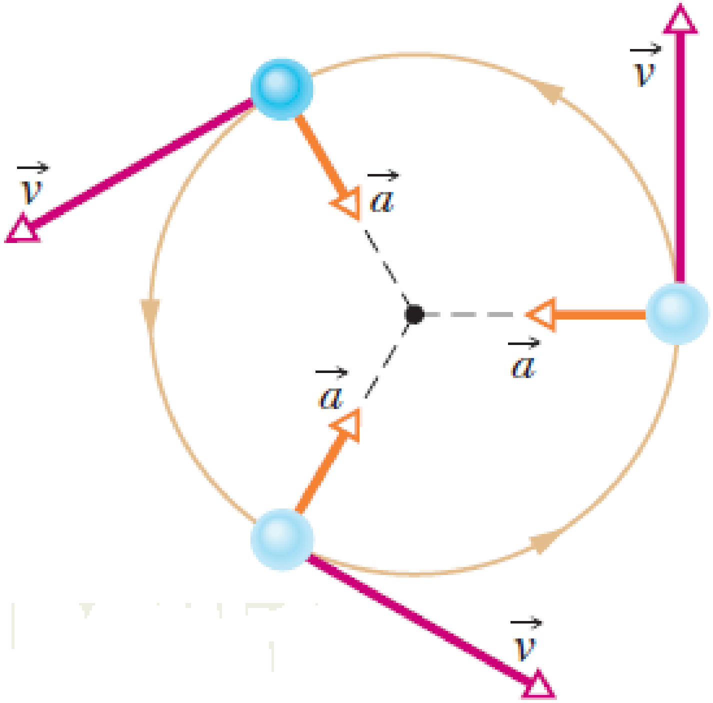
UNIFORM CIRCULAR MOTION

1

$$a = \frac{v^2}{r} \quad \text{(centripetal acceleration),}$$

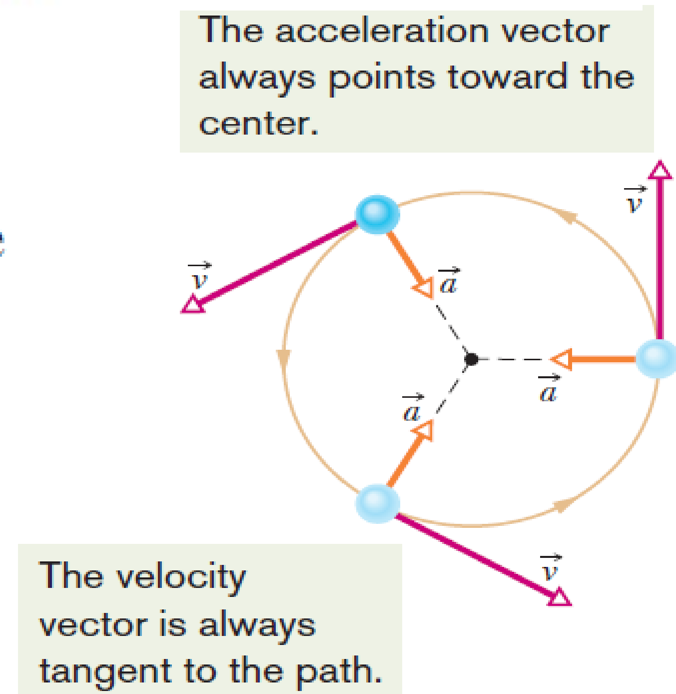
$$T = \frac{2\pi r}{v} \quad \text{(period).}$$



UNIFORM CIRCULAR MOTION

A particle is in **uniform circular motion** if it travels around a circle or a circular arc at constant (*uniform*) speed. Although the speed does not vary, *the particle is accelerating* because the velocity changes in direction.

Figure shows the relationship between the velocity and acceleration vectors at various stages during uniform circular motion.



Velocity and acceleration vectors for uniform circular motion.

UNIFORM CIRCULAR MOTION(CONT'D)

- Both "velocity & acceleration vectors" have constant magnitude, but their directions change continuously.
- The velocity is always directed **tangent to the circle** in the direction of motion.
- The acceleration is always directed **radially inward.** Because of this, the acceleration associated with uniform circular motion is called a **centripetal acceleration.**
- Centripetal means centre seeking.
- Mathematically**,** the magnitude of acceleration is,

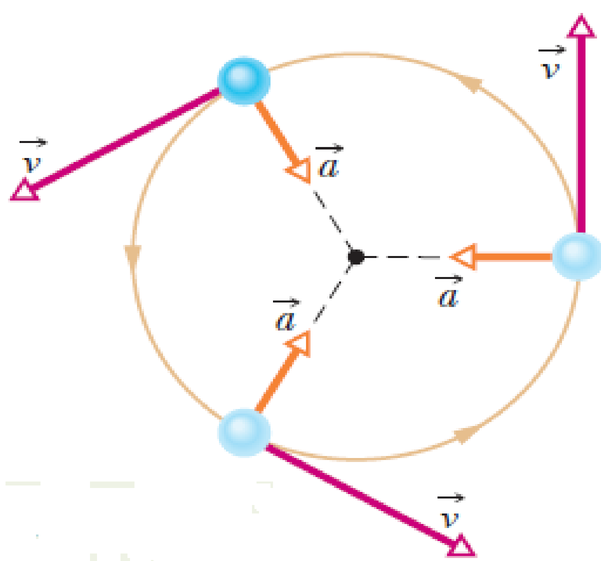
$$a_c = \frac{v^2}{r}$$

UNIFORM CIRCULAR MOTION(CONT'D)

- During this acceleration at constant speed, the particle travels the circumference of the circle (a distance of $2\pi r$ in time t.

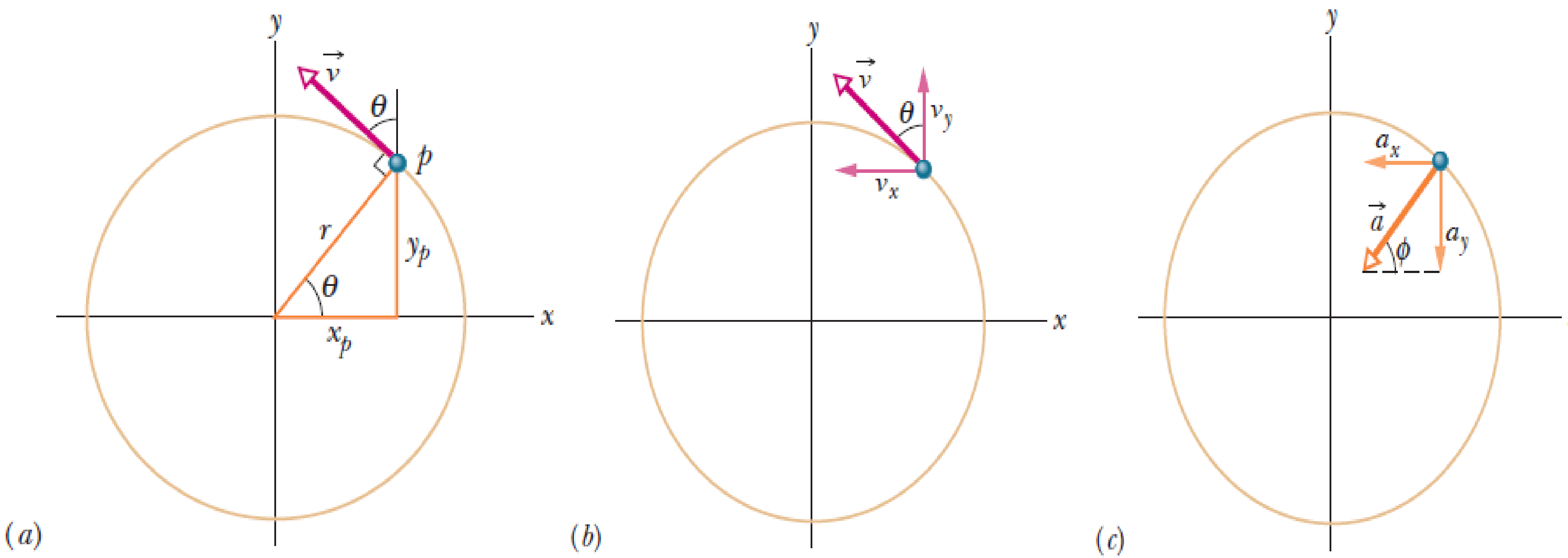
(the time for a particle to go around a closed path exactly once)

$$T = \frac{2\pi r}{v} \text{ (period)}$$



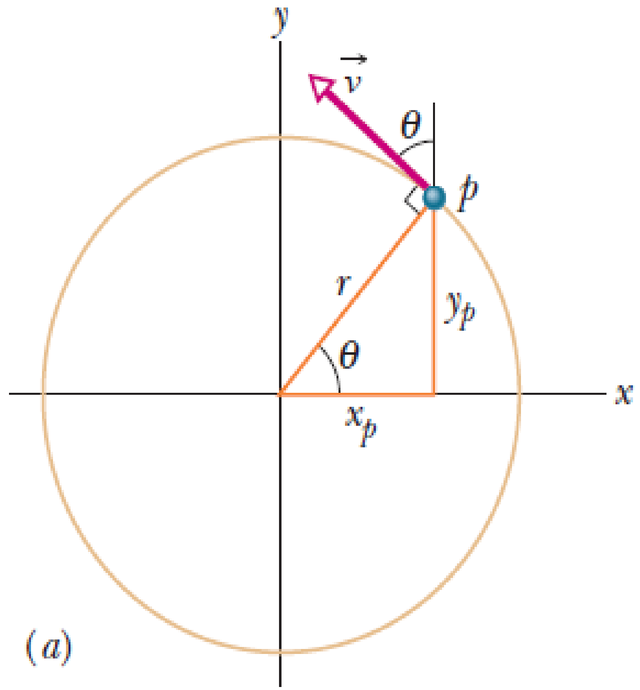
PROOF : $a_c = \dfrac{v^2}{r}$

- To find the magnitude and direction of the acceleration for uniform circular motion, we consider the following figures;

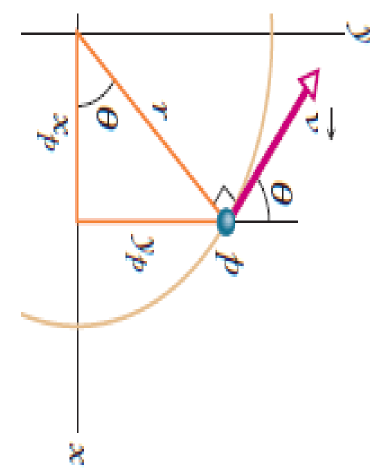


PROOF (CONT'D)

- In Fig. a, particle ***P*** moves at constant speed v around a circle of radius r.
- At the instant shown, ***P*** ha coordinates x_p and y_p
- v is tangent to the path, hence perpendicular to a radius r drawn to the particle's position. Then the angle (theta) that v makes with a vertical at P equals the angle (theta) that radius r makes with the x axis.



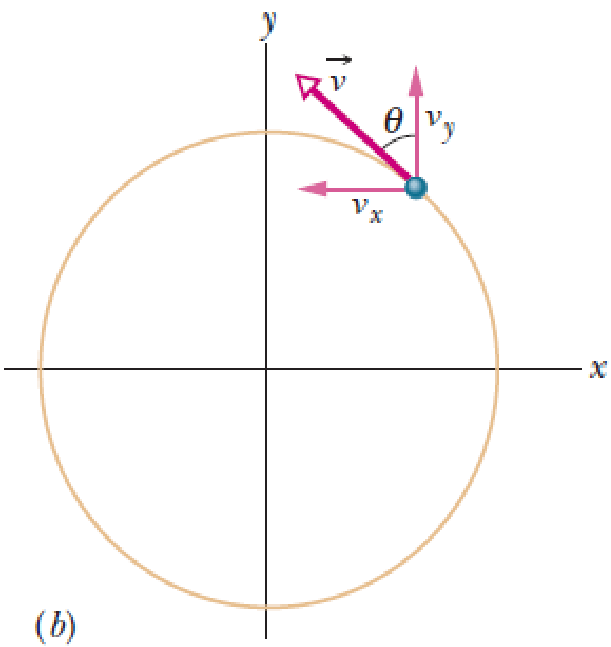
(a)



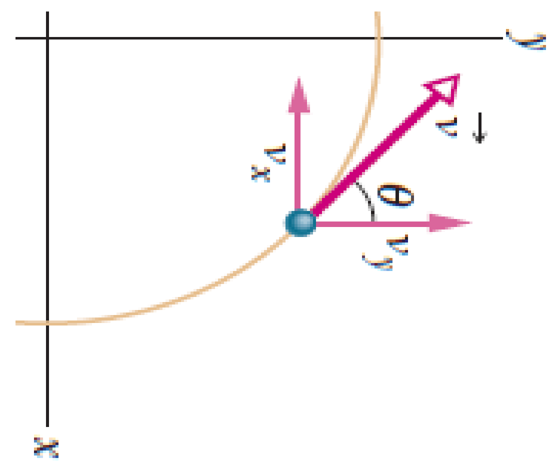
Proof (Cont'd)

- The scalar components of v are shown in Fig b. With them, we can write the velocity "v" as

$$\vec{v} = v_x\hat{\mathrm{i}} + v_y\hat{\mathrm{j}}$$
$$\vec{v} = (-v\sin\theta)\hat{\mathrm{i}} + (v\cos\theta)\hat{\mathrm{j}}$$



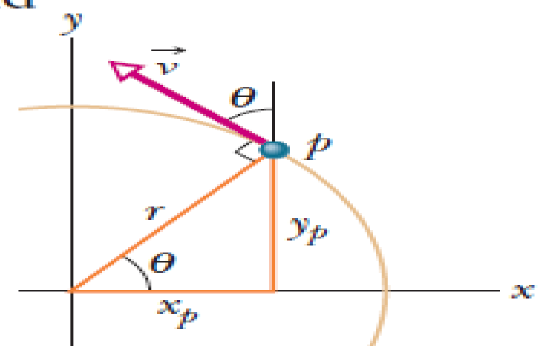
(b)



Proof (Cont'd)

Now, using the right triangle in Fig. *a*, we can replace sin θ with y_p/r and cos θ with x_p/r to write

$$\vec{v} = \left(-\frac{v y_p}{r}\right)\hat{\mathrm{i}} + \left(\frac{v x_p}{r}\right)\hat{\mathrm{j}}$$



To find the acceleration $\vec{a}$ of particle p, we must take the time derivative of this equation. Noting that speed v and radius r do not change with time, we obtain

$$\vec{a} = \frac{d\vec{v}}{dt}$$

$$\vec{a} = \frac{d}{dt}\left(-\frac{v y_p}{r}\right)\hat{\imath} + \frac{d}{dt}\left(\frac{v x_p}{r}\right)\hat{\jmath}$$

$$= \left(-\frac{v}{r}\frac{dy_p}{dt}\right)\hat{\mathrm{i}} + \left(\frac{v}{r}\frac{dx_p}{dt}\right)\hat{\mathrm{j}}. \longrightarrow \textbf{Eq (i)}$$

PROOF (CONT'D)

Now note that the rate dy_p/dt at which y_p changes is equal to the velocity component v_y. Similarly, $dx_p/dt = v_x$, and, from figure (b) we have the further substitutions

Making these substitutions in Eq (i) i.e.,

$$\vec{a} = \frac{d\vec{v}}{dt} = \left(-\frac{v}{r}\frac{dy_p}{dt}\right)\hat{\mathrm{i}} + \left(\frac{v}{r}\frac{dx_p}{dt}\right)\hat{\mathrm{j}}$$

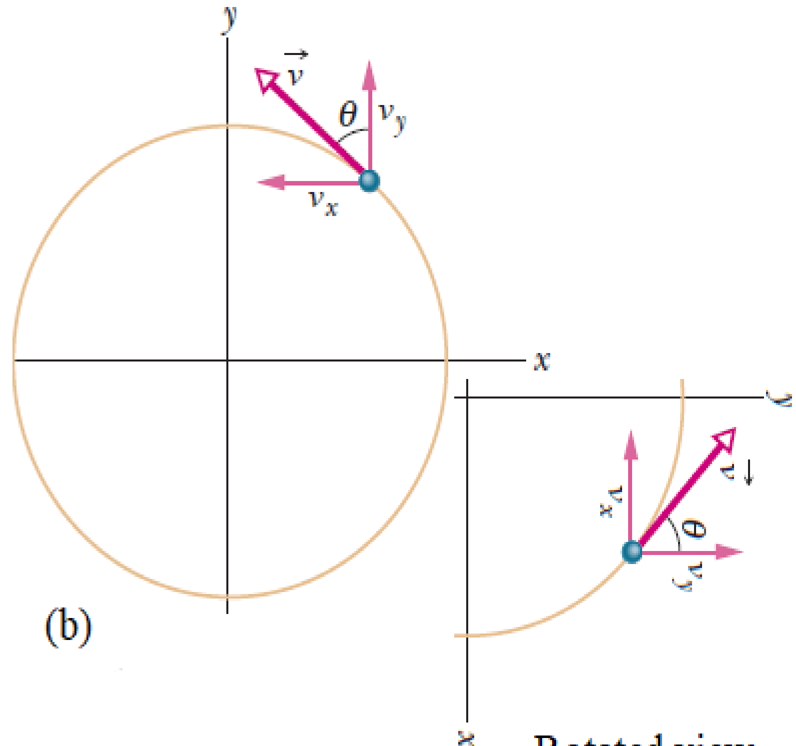
$$\vec{a} = \frac{d}{dt}\left(-\frac{vv_y}{r}\right)\hat{\imath} + \frac{d}{dt}\left(\frac{vv_x}{r}\right)\hat{\jmath}$$

$$v_x = -v \sin\theta$$

and

$$v_y = v\cos\theta$$

$$\vec{a} = \left(-\frac{v^2}{r}\cos\theta\right)\hat{\mathrm{i}} + \left(-\frac{v^2}{r}\sin\theta\right)\hat{\mathrm{j}}$$



(b)

Rotated view

This vector and its components are shown in Fig. *c*.

PROOF (CONT'D)

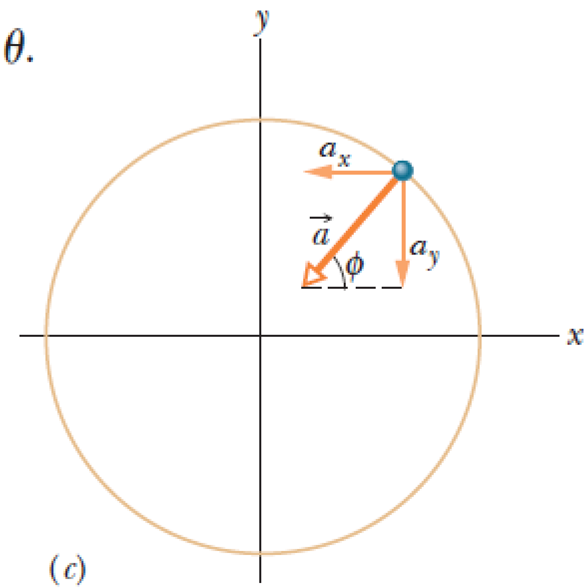
$$\vec{a} = \left(-\frac{v^2}{r}\cos\theta\right)\hat{\mathrm{i}} + \left(-\frac{v^2}{r}\sin\theta\right)\hat{\mathrm{j}}.$$

$$a = \sqrt{a_x^2 + a_y^2} = \frac{v^2}{r}\sqrt{(\cos\theta)^2 + (\sin\theta)^2} = \frac{v^2}{r}\sqrt{1} = \frac{v^2}{r},$$

as we wanted to prove. To orient $\vec{a}$, we find the angle ϕ shown in Fig. c

$$\tan\phi = \frac{a_y}{a_x} = \frac{-(v^2/r)\sin\theta}{-(v^2/r)\cos\theta} = \tan\theta.$$

Thus, $\phi = \theta$, which means that $\vec{a}$ is directed along the radius r of Fig. a, toward the circle's center, as we wanted to prove.



(c)

Conceptual Question:

Describe circumstances (physical examples) in which velocity and acceleration vectors are:

(i) parallel

When the car is moving with increasing velocity then the direction of acceleration will be parallel to the velocity.

(ii) Anti parallel

When the car is moving with decreasing velocity then the direction of acceleration will be anti-parallel to the velocity.

(iii) perpendicular

When the car will move in a circle then the direction of velocity and acceleration will be perpendicular to each other.

In this case the direction of velocity will be along the tangent to the circle and the direction of acceleration depends upon the change in velocity so if we take the vector sum then the resultant will always point toward the center of the circle so we can say that velocity and acceleration will be perpendicular to each other in this case.

(iv) **v** is zero, but **a** is not zero.
When the brakes are applied on the moving car, it slows down and comes to rest due to negative acceleration in the opposite direction. Thus, velocity v is zero but a is not zero.

(v) **a** is zero but **v** is not zero.
When the car is moving with constant velocity then there will be no change in velocity so acceleration will be zero, but velocity will not be zero.