CAPACITANCE & DIELECTRICS

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CAPACITORS

Capacitors—devices that store electric charge. Capacitors are commonly used in a variety of electric circuits. For instance, they are used to

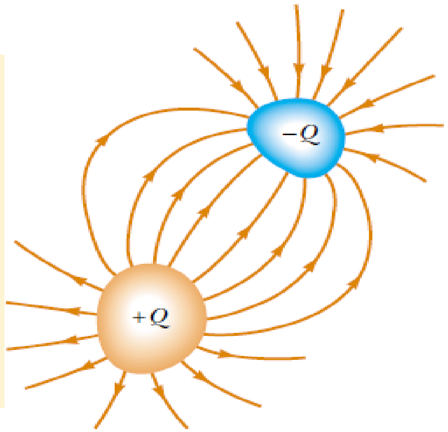
- tune the frequency of radio receivers,
- as filters in power supplies,
- to eliminate sparking in automobile ignition systems, and
- as energy-storing devices in electronic flash units.

Consider two conductors carrying charges of equal magnitude but of opposite sign, as shown in Figure. Such a combination of two conductors is called a capacitor. The conductors are called *plates.*

A ***potential difference*** *V exists between* the conductors due to the presence of the charges.

The **capacitance** C of a capacitor is defined as the ratio of the magnitude of the charge on either conductor to the magnitude of the potential difference between the conductors:

$$C \equiv \frac{Q}{\Delta V}$$



CALCULATING CAPACITANCE

Parallel-Plate Capacitors

Two parallel metallic plates of equal area *A are separated by a distance d,*

as shown in Fig.

One plate carries a charge -*Q , and the other carries a charge +Q.*

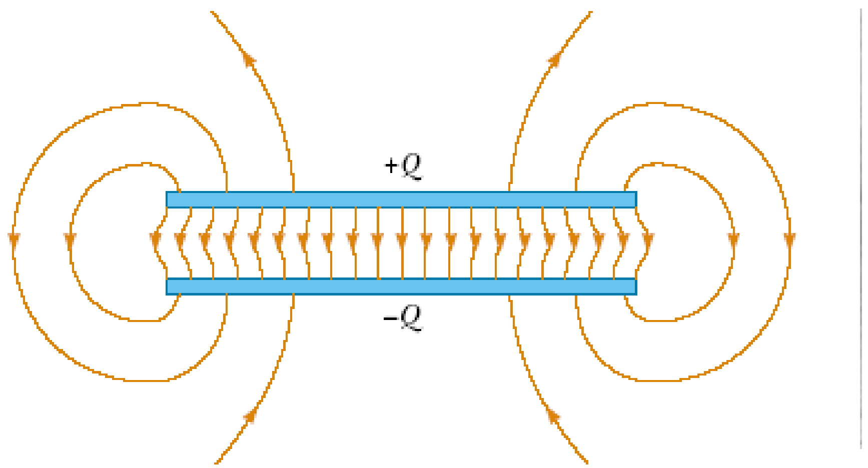
The value of the electric field between the plates is

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$$

$$\Delta V = Ed = \frac{Qd}{\epsilon_0 A}$$

$$C = \frac{Q}{\Delta V} = \frac{Q}{Qd/\epsilon_0 A}$$

$$\boxed{C = \frac{\epsilon_0 A}{d}}$$



the capacitance of a parallel-plate capacitor is proportional to the area of its plates and inversely proportional to the plate separation.

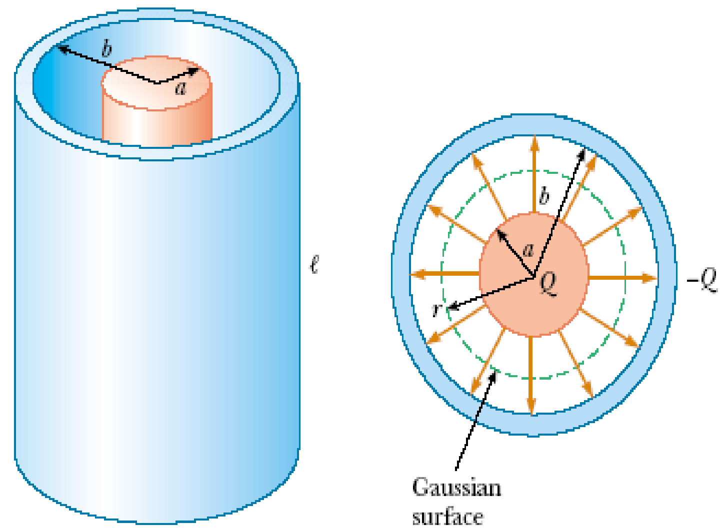
The Cylindrical Capacitor

A solid cylindrical conductor of radius a and charge Q is coaxial with a cylindrical shell of negligible thickness, radius $b > a$, and charge $-Q$

First calculate the potential difference between the two cylinders,

$$V_b - V_a = -\int_a^b \mathbf{E} \cdot d\mathbf{s} \qquad E_r = 2k_e\lambda / r$$

$$V_b - V_a = -\int_a^b E_r\, dr = -2k_e\lambda \int_a^b \frac{dr}{r} = -2k_e\lambda \ln\left(\frac{b}{a}\right)$$



$$C = \frac{Q}{\Delta V} = \frac{Q}{\dfrac{2k_e Q}{\ell} \ln\left(\dfrac{b}{a}\right)} = \boxed{\frac{\ell}{2k_e \ln\left(\dfrac{b}{a}\right)}}$$

$$\frac{C}{\ell} = \frac{1}{2k_e \ln\left(\dfrac{b}{a}\right)}$$

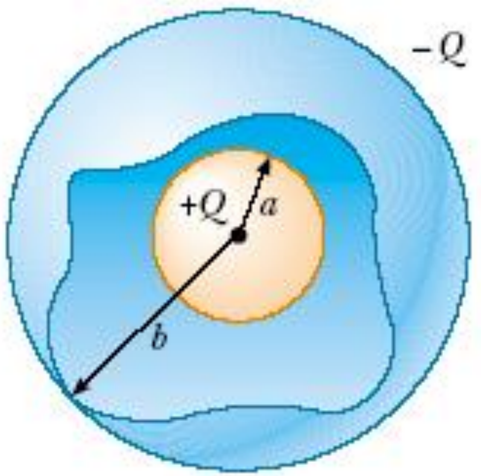
the capacitance per unit length

The Spherical Capacitor

A spherical capacitor consists of a spherical conducting shell of radius b and charge $-Q$ concentric with a smaller conducting sphere of radius a and charge Q

$$V_b - V_a = -\int_a^b E_r\,dr = -k_eQ\int_a^b \frac{dr}{r^2} = k_eQ\left[\frac{1}{r}\right]_a^b$$

$$= k_eQ\left(\frac{1}{b} - \frac{1}{a}\right)$$



$$\Delta V = |V_b - V_a| = k_eQ\frac{(b - a)}{ab}$$

$$C = \frac{Q}{\Delta V} = \boxed{\frac{ab}{k_e(b - a)}}$$

COMBINATIONS OF CAPACITORS

Parallel Combination

The individual potential differences across capacitors connected in parallel are all the same and are equal to the potential difference applied across the combination.

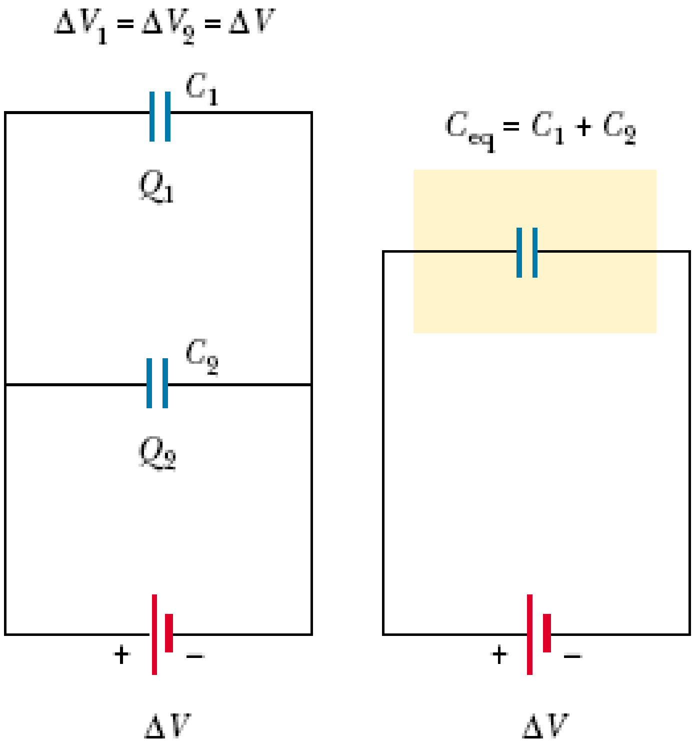
The *total charge* Q stored by the two capacitors is

$$Q = Q_1 + Q_2$$

$$Q_1 = C_1\,\Delta V \qquad Q_2 = C_2\,\Delta V \qquad Q = C_{eq}\,\Delta V$$

$$C_{eq}\,\Delta V = C_1\,\Delta V + C_2\,\Delta V$$

$$C_{eq} = C_1 + C_2 \qquad \left(\begin{array}{l}\text{parallel} \\ \text{combination}\end{array}\right)$$

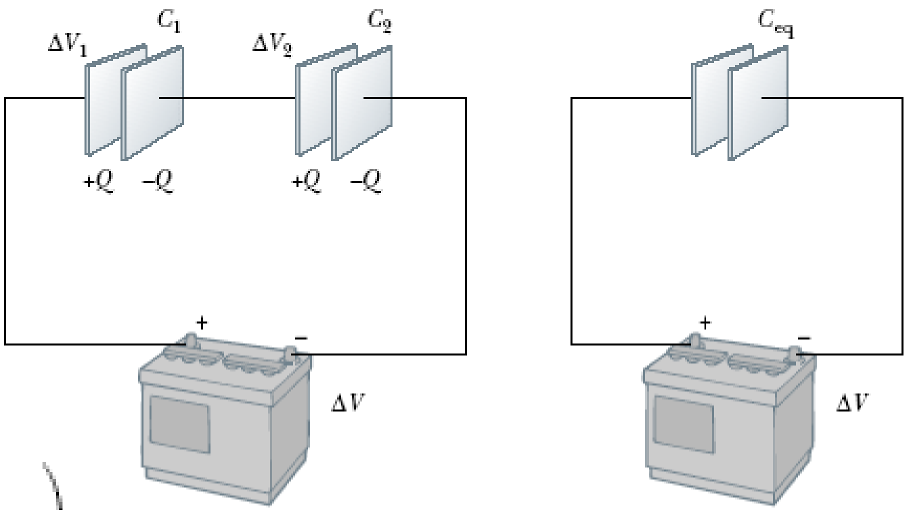


$$C_{eq} = C_1 + C_2 + C_3 + \cdots \qquad \text{(parallel combination)}$$

COMBINATIONS OF CAPACITORS

Series Combination

$$\Delta V = \Delta V_1 + \Delta V_2$$



$$\Delta V_1 = \frac{Q}{C_1} \qquad \Delta V_2 = \frac{Q}{C_2} \qquad \Delta V = \frac{Q}{C_{eq}}$$

$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \qquad \left(\begin{array}{l}\text{series}\\ \text{combination}\end{array}\right)$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots \qquad \left(\begin{array}{l}\text{series}\\ \text{combination}\end{array}\right)$$

This demonstrates that **the equivalent capacitance of a series combination is always less than any individual capacitance in the combination.**

Example:

Figure, shows a system of four capacitors, where the potential difference across ab is 50.0 V. (C_1=13.0 μF, C_2=3.0 μF, C_3=8.5 μ, and C_4=3.0 μF)

(a) Find the equivalent capacitance of this system between a and b.

Here we see that the capacitor $C_2 = 3.0\ \mu F\ and\ \ C_3 = 8.5\ \mu F$ are in parallel.

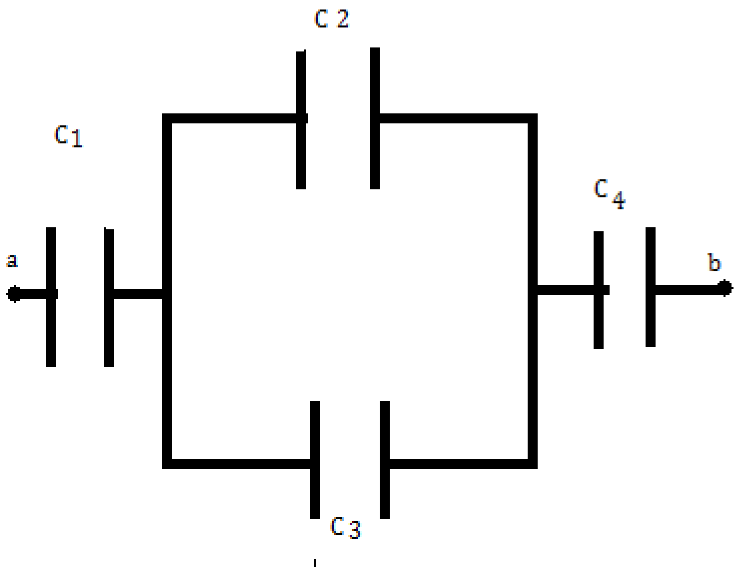
Thus we have the equivalent capacitance of them equal to $C_{eq1} = (3 + 8.5)\ \mu F = 11.5\ \mu F$.

Now we have the capacitor $C_1 = 13\ \mu F$, $C_{eq1} = 11.5\ \mu F$ and $C_3 = 3.0\ \mu F$ are in series.

Thus, we get the equivalent capacitance of the overall circuit as

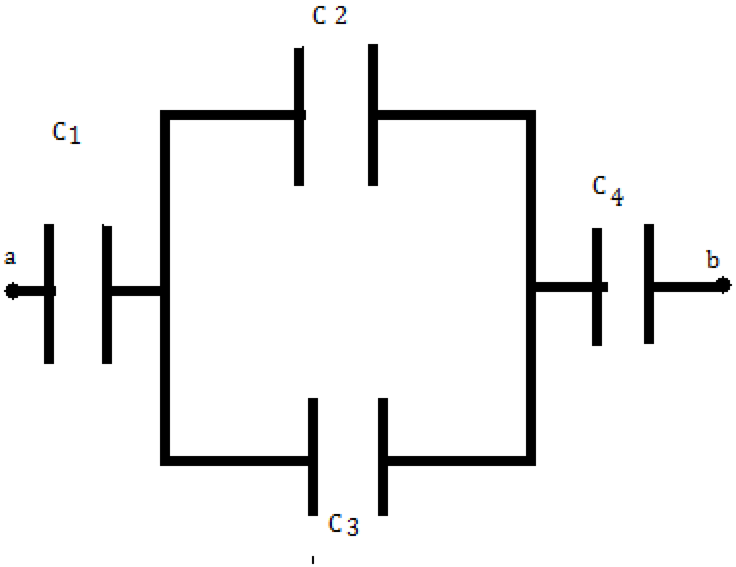
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_{eq1}} + \frac{1}{C_3}$$
$$= \frac{1}{13} + \frac{1}{11.5} + \frac{1}{3}$$

$$C_{eq} = 2\ \mu F \Rightarrow (Answer)$$



(b) How much charge is stored by this combination of capacitors?

$$
\begin{aligned}
Q &= C_{eq} V \\
&= 2 \times 50\ \mu C \\
&= 100\ \mu C \Rightarrow (Answer)
\end{aligned}
$$



(c) How much charge is stored in each of the 13 μF and the 3 μF capacitors?

- Thus, we have the charge on the $C_1 = 13\ \mu F$ capacitor as $100\ \mu C \Rightarrow (Answer)$.

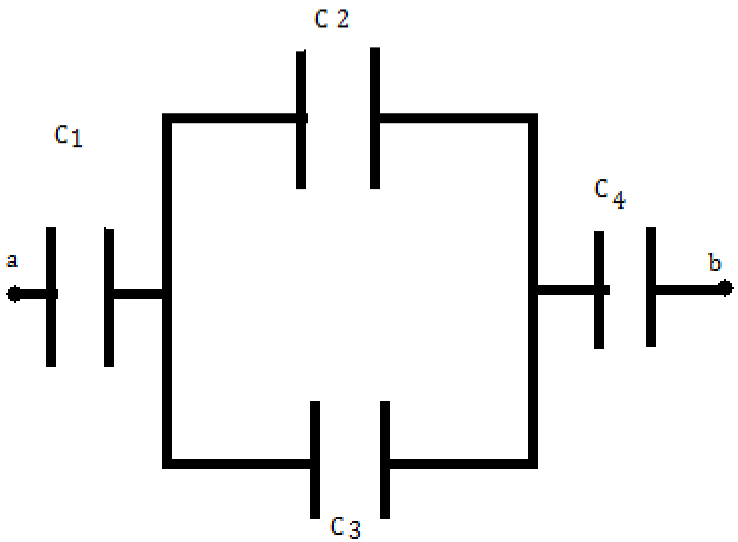
The voltage drop across the $C_{eq1} = 11.5\ \mu F$. capacitor is given by

$$Q = C_{eq1} V_2$$
$$100\mu = 11.5\mu \times V_2$$
$$V_2 = 8.7\ V$$

This is the potential across the capacitor C_2 as they are in parallel.

Thus we have the charge in C_2 given by

$$\begin{aligned} Q_2 &= C_2 V_2 \\ &= 3\mu \times 8.7\ C \\ &= 26.1\ \mu C \Rightarrow (Answer). \end{aligned}$$



Example:

For the circuit in Figure, find:

(a) the equivalent capacitance

(b) the charge and potential difference for each capacitor. (V = 9V, C1 = C2 = 30 µF , C3=C4 = 15 µF)

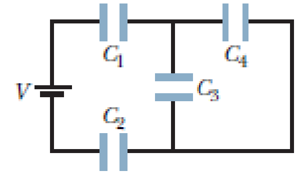


Figure depicts a system of capacitors. The pair C_3 and C_4 are in parallel.

Since C_3 and C_4 are in parallel, we replace them with an equivalent capacitance $C_{34} = C_3 + C_4 = 30\ \mu\text{F}$. Now, C_1, C_2, and C_{34} are in series, and all are numerically 30 µF, we observe that each has one-third the battery voltage across it. Hence, 3.0 V is across C_4.

The charge on capacitor 4 is $q_4 = C_4V_4 = (15\ \mu\text{F})(3.0\ \text{V}) = 45\ \mu\text{C}$.

Alternatively, one may show that the equivalent capacitance of the arrangement is given by

$$\frac{1}{C_{1234}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_{34}} = \frac{1}{30\ \mu\text{F}} + \frac{1}{30\ \mu\text{F}} + \frac{1}{30\ \mu\text{F}} = \frac{1}{10\ \mu\text{F}}$$

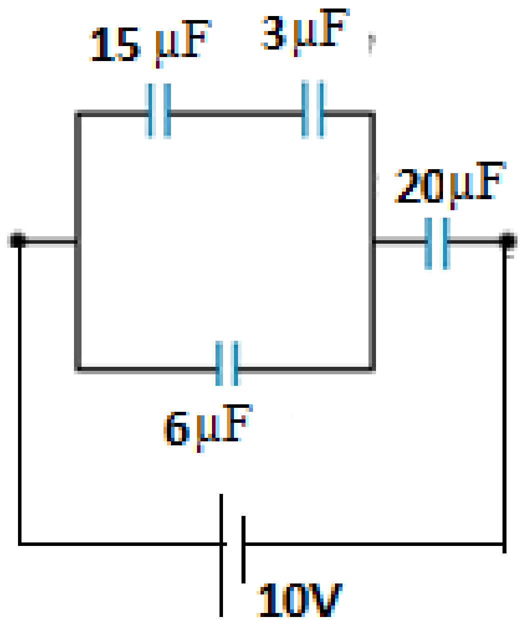
or $C_{1234} = 10\ \mu\text{F}$. Thus, the charge across C_1, C_2, and C_{34} are

$$q_1 = q_2 = q_{34} = q_{1234} = C_{1234}V = (10\ \mu\text{F})(9.0\ \text{V}) = 90\ \text{nC}.$$

Now, since C_3 and C_4 are in parallel, and $C_3 = C_4$, the charge on C_4 (as well as on C_3) is $q_3 = q_4 = q_{34}/2 = (90\ \mu\text{F})/2 = 45\ \mu\text{F}$.

Class Activity:

For the circuit in Fig-5, find the equivalent capacitance and the charge on 20μF capacitor.



Example:

From a supply of identical capacitors rated 8 μF, 250 V, the minimum number of capacitors required to form a composite 16 μF,1000 V capacitor is:

The required voltage is 1000 V and the capacitors are parallel as 250 V.

$$\frac{1}{C_{eq}} = \frac{1}{8\mu F} + \frac{1}{8\mu F} + \frac{1}{8\mu F} + \frac{1}{8\mu F} = \frac{1}{2\mu F}$$

$$\Rightarrow C_{eq} = 2\mu F$$

So, number of capacitors required will be **4** i.e**., 250×4=1000 in series**.

Now example of four capacitor in series will be equal **2μf** ,

but the equivalent capacitance required is given as **16μf**

So, **there must be 8 series of parallel arrange capacitors each of capacitor 2 micro farad hence total number of capacitor = 4×8 = 32**

ENERGY STORED IN A CHARGED CAPACITOR

Suppose that q is the charge on the capacitor at some instant during the charging process. At the same instant, the potential difference across the capacitor is $\Delta V = q/C$. The work necessary to transfer an increment of charge dq from the plate carrying charge -q to the plate carrying charge q (which is at the higher electric potential) is

$$dW = \Delta V\, dq = \frac{q}{C}\, dq$$

$$W = \int_0^Q \frac{q}{C}\, dq = \frac{1}{C}\int_0^Q q\, dq = \frac{Q^2}{2C}$$

$$U = \frac{Q^2}{2C} = \tfrac{1}{2}Q\,\Delta V = \tfrac{1}{2}C(\Delta V)^2$$

Energy stored in a parallel-plate capacitor

$$U = \frac{1}{2}\frac{\epsilon_0 A}{d}(E^2 d^2) = \frac{1}{2}(\epsilon_0 A d)E^2$$

energy per unit volume $u_E = U/V = U/Ad$,

$$u_E = \tfrac{1}{2}\epsilon_0 E^2$$

Energy density in an electric field

energy density in any electric field is proportional to the square of the magnitude of the electric field at a given point.

CAPACITORS WITH DIELECTRICS

A dielectric is a non conducting material, such as rubber, glass, or waxed paper.

When a dielectric is inserted between the plates of a capacitor, the capacitance increases.

If the dielectric completely fills the space between the plates, the capacitance increases by a dimensionless factor κ, which is called the dielectric constant.

$$\Delta V = \frac{\Delta V_0}{\kappa} \qquad C = \frac{Q_0}{\Delta V} = \frac{Q_0}{\Delta V_0/\kappa} = \kappa\frac{Q_0}{\Delta V_0}$$

$$C = \kappa\frac{\epsilon_0 A}{d}$$

$$\boxed{C = \kappa C_0}$$

advantages:

- Increase in capacitance
- Increase in maximum operating voltage
- Possible mechanical support between the plates, which allows the plates to be close together without touching, thereby decreasing d and increasing C

Types of Capacitors

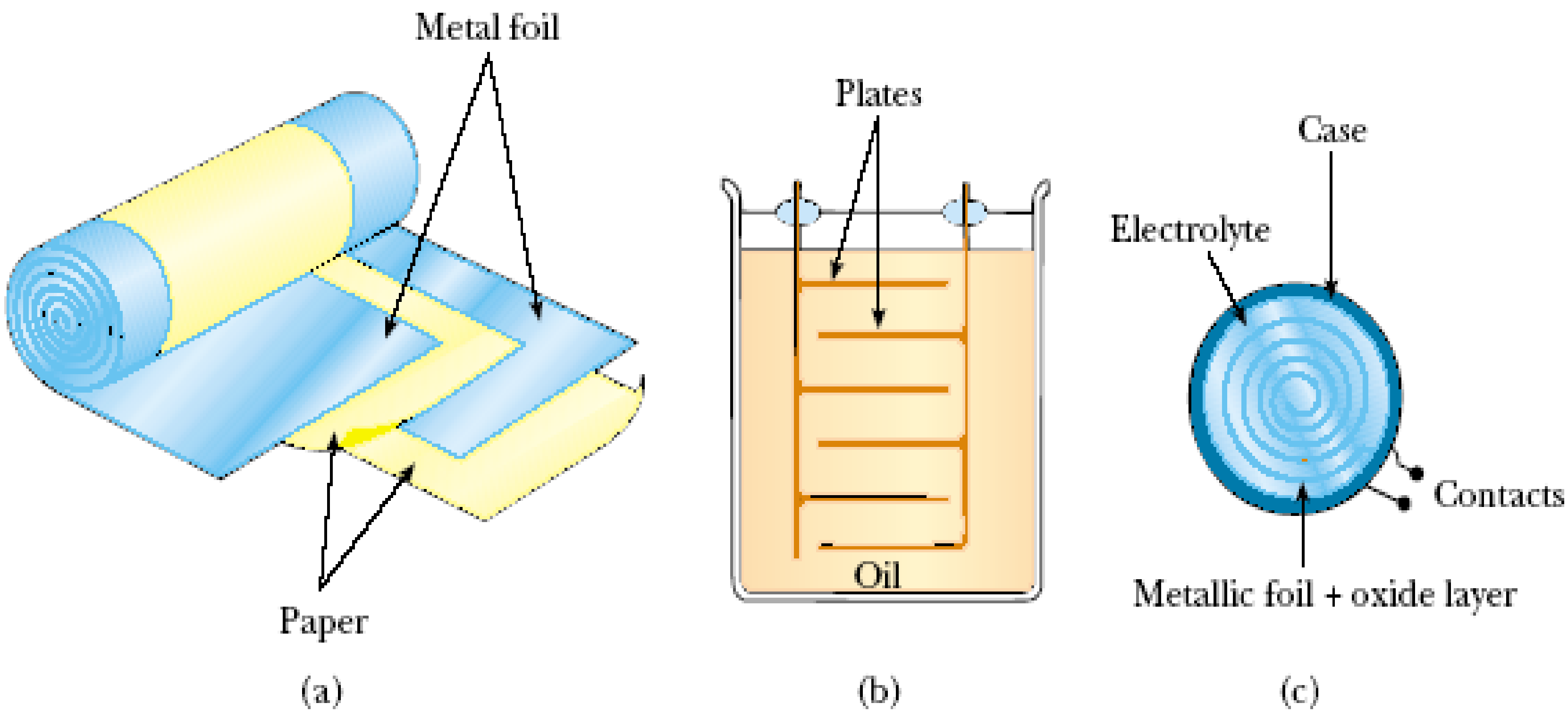


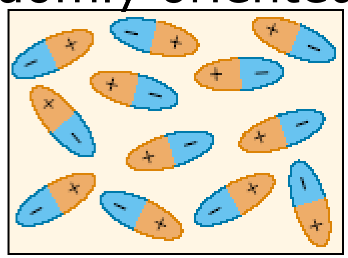
Figure 26.15 Three commercial capacitor designs. (a) A tubular capacitor, whose plates are separated by paper and then rolled into a cylinder. (b) A high-voltage capacitor consisting of many parallel plates separated by insulating oil. (c) An electrolytic capacitor.

AN ATOMIC DESCRIPTION OF DIELECTRICS

the field in the presence of a dielectric is

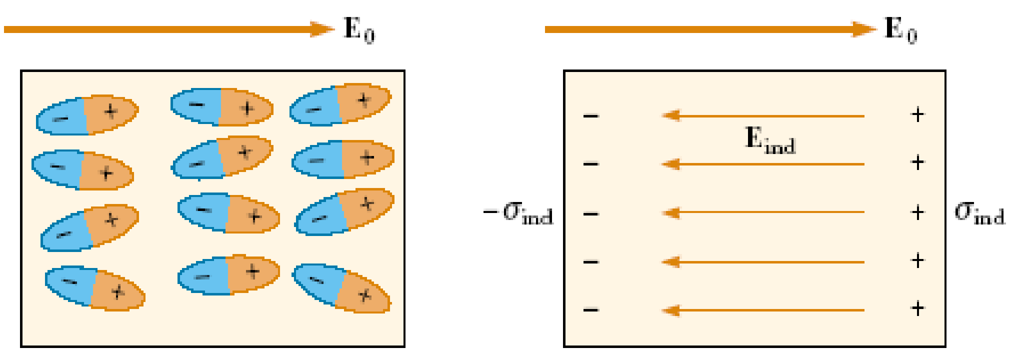
$$\mathbf{E} = \frac{\mathbf{E}_0}{\kappa}$$

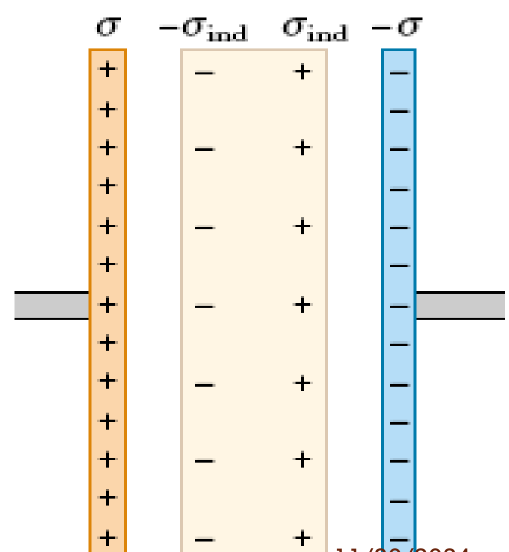
(a)Polar molecules are randomly oriented in the absence of an external electric field.



(a)

(b) When an external field is applied, the molecules partially align with the field.





$$E = E_0 - E_{\text{ind}}$$