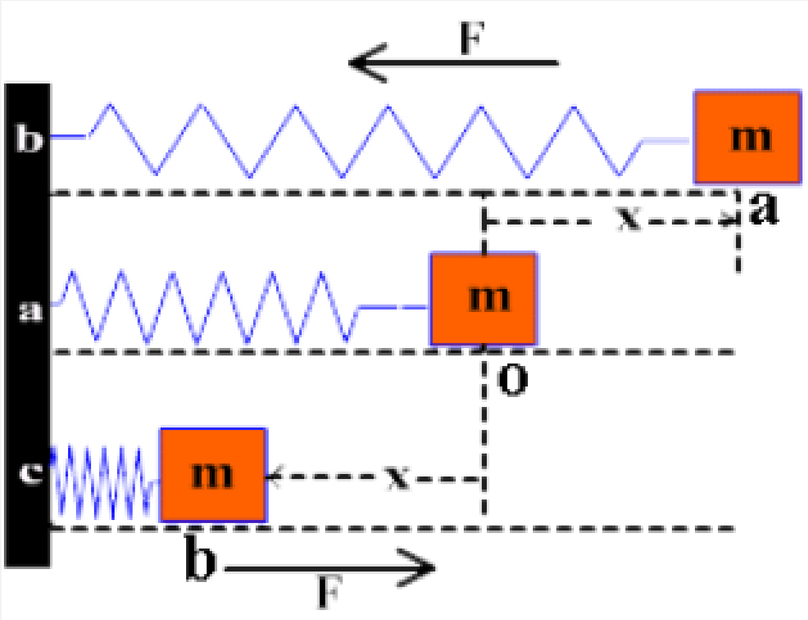
SIMPLE HARMONIC MOTION

Dr Muhammad Adeel

Definition:

- Such a motion in which acceleration is directly proportional to the displacement and is directed towards the mean position is called simple harmonic motion(SHM).



Condition FOR SHM:

- The system should haves restoring force.
- The system should have inertia.
- The system should be frictionless.

Hooke's Law

- force that is applied to spring is **directly proportional** to the displacement.
- If the spring is un stretched, there is no net force on the mass or the system is in equilibrium.
- if the mass is displaced from equilibrium, the spring will exert a restoring force, which is a force that tends to restore it to the equilibrium position.

$$F \propto x$$

where,
F → Elastic force
k → Spring constant
x → Displacement

$$\boxed{F = kx}$$

Expression for acceleration of the body executing SHM:

- Consider a mass ‘m’ attached to one end of elastic spring which can move freely on a frictionless horizontal surface.

- When the mass is released, it begins to vibrate about its mean or equilibrium position.

- But due to elasticity, spring opposes the applied force which produces the displacement. This opposing force is called restoring force.

Expression for acceleration of the body executing SHM:

The restoring force is written by;

$$F_r = -kx \text{......................} (i)$$

If 'a' is the acceleration produced by force 'F' in mass-spring system at any instant, then according to Newton's law of motion.

$$F = ma$$

$$F = m\ddot{x} \text{........................} (ii)$$

Comparing (i) and (ii)

$$m\ddot{x} = -kx \qquad \Rightarrow \ddot{x} = \left(\frac{-k}{m}\right)x$$

$$\because \frac{k}{m} = constant$$

a α -x

From above equation we can write it as

$$\ddot{x} + \left(\frac{k}{m}\right)x = 0$$

Solution of the form

$$x = A\cos(\omega t + \phi).$$

$$\therefore \omega = \sqrt{\frac{k}{m}}$$

- Velocity can found by differentiating displacement

$$v = -A\omega \sin(\omega t + \emptyset)$$

- Acceleration can found by differentiating velocity

$$a = -A\omega^2 \cos(\omega t + \emptyset)$$

Simplifying acceleration in terms of displacement:

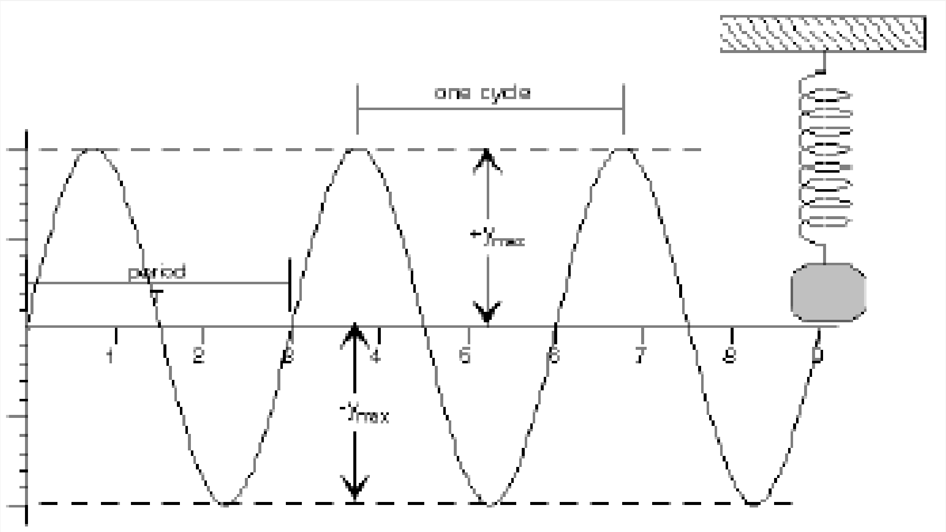
$$a = \frac{d^2x}{dt^2} = -\omega^2 x,$$

Acceleration can also be expressed as:

$$a(t) = -(2\pi f)^2 x(t)$$

Characteristic of Mass-spring system executing SHM:

When a body is vibrating, its displacement from the mean position changes with time. The value of its distance from the mean position at any time is known as its instantaneous displacement.



- The maximum value of displacement is known as its amplitude.

- A vibration means one complete round trip of the body in motion.

- The time required to complete one vibration is called time period

$$T = 2\pi\sqrt{\frac{m}{k}}$$

- The number of cycles per second. A cycle is a complete round trip is called Frequency

$$f = \frac{1}{T}$$

$$f = \frac{1}{2\pi\sqrt{\frac{m}{k}}}$$

$$f = 2\pi\sqrt{\frac{k}{m}}$$

- If T is time period of a body executing SHM, its angular frequency can be written as;

$$\omega = \frac{2\pi}{T} = 2\pi f$$

Phase angle:

The angle $\theta = \omega t$ which specifies the displacement as well as the direction of the point executing SHM is known as phase angle.

General Equation

$$x(t) = A\cos(2\pi f t + \phi)$$

where,

x $\rightarrow$ Displacement

A $\rightarrow$ Amplitude of the oscillation

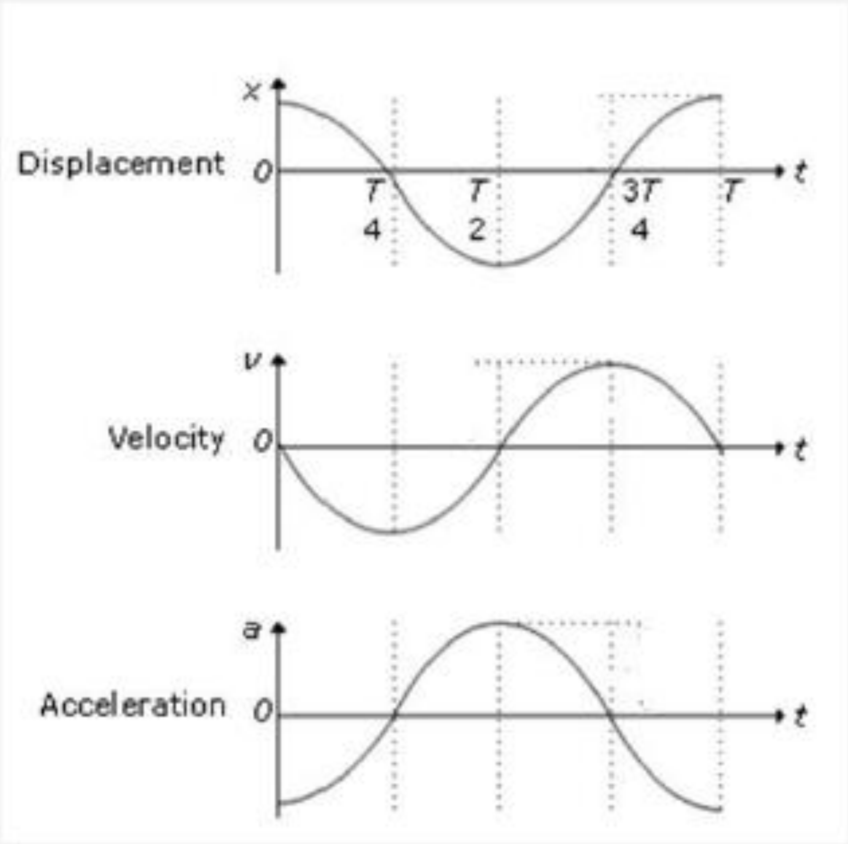
f $\rightarrow$ Frequency

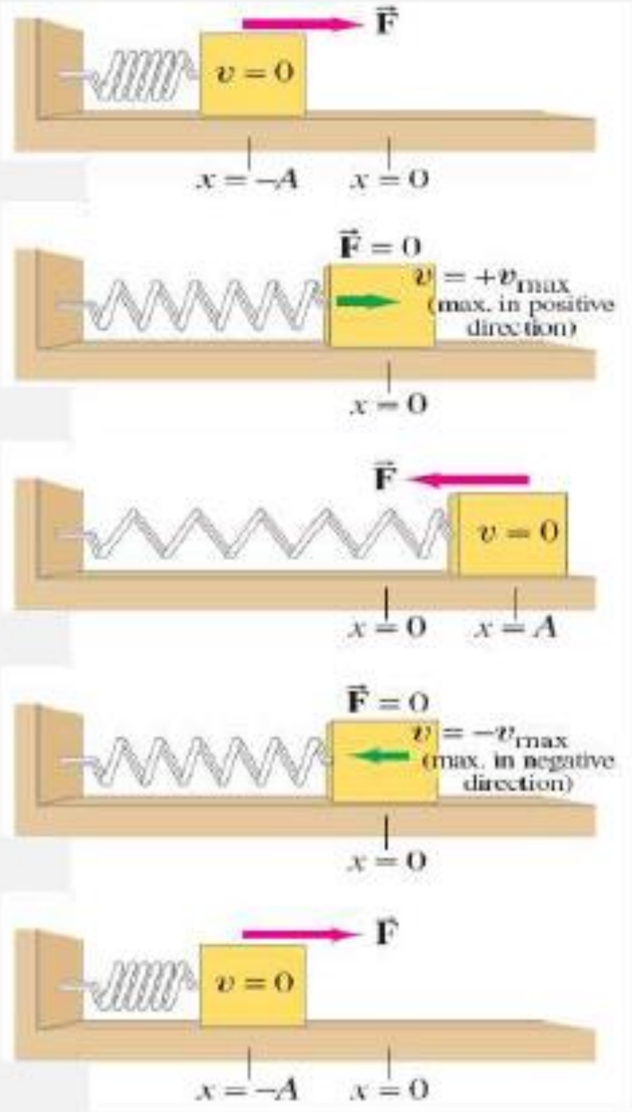
t $\rightarrow$ time

ϕ $\rightarrow$ Phase of oscillation

If there is no displacement at time t = 0, the phase is $\phi = \pi/2$

Graphical representation





A block whose mass m is 680 g is fastened to a spring whose spring constant k is 65 N/m. The block is pulled a distance $x = 11$ cm from its equilibrium position at $x = 0$ on a frictionless surface and released from rest at $t = 0$.

(a) What are the angular frequency, the frequency, and the period of the resulting motion?

KEY IDEA The block–spring system forms a linear simple harmonic oscillator, with the block undergoing SHM.

Calculations: The angular frequency is given by Eq. 15-12:

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{65\ \mathrm{N/m}}{0.68\ \mathrm{kg}}} = 9.78\ \mathrm{rad/s}$$

$$\approx 9.8\ \mathrm{rad/s}. \qquad \text{(Answer)}$$

The frequency follows from Eq. 15-5, which yields

$$f = \frac{\omega}{2\pi} = \frac{9.78\ \mathrm{rad/s}}{2\pi\ \mathrm{rad}} = 1.56\ \mathrm{Hz} \approx 1.6\ \mathrm{Hz}. \qquad \text{(Answer)}$$

The period follows from Eq. 15-2, which yields

$$T = \frac{1}{f} = \frac{1}{1.56\ \mathrm{Hz}} = 0.64\ \mathrm{s} = 640\ \mathrm{ms}. \qquad \text{(Answer)}$$

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(b) What is the amplitude of the oscillation?

KEY IDEA With no friction involved, the mechanical energy of the spring–block system is conserved.

Reasoning: The block is released from rest 11 cm from its equilibrium position, with zero kinetic energy and the elastic potential energy of the system at a maximum. Thus, the block will have zero kinetic energy whenever it is again 11 cm from its equilibrium position, which means it will never be farther than 11 cm from that position. Its maximum displacement is 11 cm:

$$x_m = 11 \text{ cm}. \qquad \text{(Answer)}$$

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(c) What is the maximum speed v_m of the oscillating block, and where is the block when it has this speed?

KEY IDEA The maximum speed v_m is the velocity amplitude ωx_m in Eq. 15-6.

Calculation: Thus, we have

$$v_m = \omega x_m = (9.78 \text{ rad/s})(0.11 \text{ m}) = 1.1 \text{ m/s}. \quad \text{(Answer)}$$

This maximum speed occurs when the oscillating block is rushing through the origin; compare Figs. 15-4*a* and 15-4*b*, where you can see that the speed is a maximum whenever $x = 0$.

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(d) What is the magnitude a_m of the maximum acceleration of the block?

KEY IDEA The magnitude a_m of the maximum acceleration is the acceleration amplitude $\omega^2 x_m$ in Eq. 15-7.

Calculation: So, we have

$$a_m = \omega^2 x_m = (9.78 \text{ rad/s})^2(0.11 \text{ m})$$
$$= 11 \text{ m/s}^2. \qquad \text{(Answer)}$$

This maximum acceleration occurs when the block is at the ends of its path. At those points, the force acting on the block has its maximum magnitude; compare Figs. 15-4*a* and 15-4*c*, where you can see that the magnitudes of the displacement and acceleration are maximum at the same times.

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(e) What is the phase constant ϕ for the motion?

Calculations: Equation 15-3 gives the displacement of the block as a function of time. We know that at time $t = 0$, the block is located at $x = x_m$. Substituting these *initial conditions*, as they are called, into Eq. 15-3 and canceling x_m give us

$$1 = \cos \phi. \qquad (15\text{-}14)$$

Taking the inverse cosine then yields

$$\phi = 0 \text{ rad}. \qquad \text{(Answer)}$$

(Any angle that is an integer multiple of 2π rad also satisfies Eq. 15-14; we chose the smallest angle.)

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(f) What is the displacement function $x(t)$ for the spring–block system?

Calculation: The function $x(t)$ is given in general form by Eq. 15-3. Substituting known quantities into that equation gives us

$$\begin{aligned} x(t) &= x_m \cos(\omega t + \phi) \\ &= (0.11 \text{ m}) \cos[(9.8 \text{ rad/s})t + 0] \\ &= 0.11 \cos(9.8t), \qquad \text{(Answer)} \end{aligned}$$

where x is in meters and t is in seconds.