LINEAR MOTION

Dr Muhammad Adeel

TWO-DIMENSIONAL MOTION

Example Problem 1

A rabbit runs across a parking lot on which a set of coordinate axes has, strangely enough, been drawn. The coordinates (meters) of the rabbit's position as functions of time t (seconds) are given by

$$x = -0.31t^2 + 7.2t + 28 \quad \text{———— (i)}$$

and $$y = 0.22t^2 - 9.1t + 30. \quad \text{———— (ii)}$$

(a) At $t = 15$ s, what is the rabbit's position vector $\vec{r}$ in unit-vector notation and in magnitude-angle notation?

SOLUTION:

The x and y coordinates of the rabbit's position, are the scalar components of the rabbit's position vector $\vec{r}$.

We can write

$$\vec{r}(t) = x(t)\hat{\mathrm{i}} + y(t)\hat{\mathrm{j}}.$$

(We write $\vec{r}(t)$ rather than $\vec{r}$ because the components are functions of t, and thus $\vec{r}$ is also.)

At $t = 15$ s, the scalar components are

$$x = (-0.31)(15)^2 + (7.2)(15) + 28 = 66 \text{ m}$$

and $$y = (0.22)(15)^2 - (9.1)(15) + 30 = -57 \text{ m},$$

so $$\vec{r} = (66 \text{ m})\hat{\mathrm{i}} - (57 \text{ m})\hat{\mathrm{j}},$$

SOLUTION:(CONT'D)

To get the magnitude and angle of $\vec{r}$, we use

$$r = \sqrt{x^2 + y^2} = \sqrt{(66\text{ m})^2 + (-57\text{ m})^2}$$

$$= 87\text{ m.}$$

and $\theta = \tan^{-1}\frac{y}{x} = \tan^{-1}\left(\frac{-57\text{ m}}{66\text{ m}}\right) = -41°.$

TWO-DIMENSIONAL MOTION

- Example Problem 2

For the rabbit in the preceding Sample Problem, find the velocity $\vec{v}$ at time $t = 15$ s.

- Solution:

We can find $\vec{v}$ by taking derivatives of the components of the rabbit's position vector.

$$v_x = \frac{dx}{dt} = \frac{d}{dt}(-0.31t^2 + 7.2t + 28)$$

$$= -0.62t + 7.2.$$

TWO-DIMENSIONAL MOTION

At $t = 15$ s, this gives $v_x = -2.1$ m/s.

Similarly, applying the v_y part

$$v_y = \frac{dy}{dt} = \frac{d}{dt}(0.22t^2 - 9.1t + 30)$$
$$= 0.44t - 9.1.$$

At $t = 15$ s, this gives $v_y = -2.5$ m/s.

$$\boxed{\vec{v} = (-2.1\ \text{m/s})\hat{\mathrm{i}} + (-2.5\ \text{m/s})\hat{\mathrm{j}}.}$$

TWO-DIMENSIONAL MOTION

- Example Problem 3

For the rabbit in the preceding two Sample Problems, find the acceleration $\vec{a}$ at time $t = 15$ s.

- Solution: Class Task

Graphical Integration in Motion Analysis

Integrating Acceleration. When we have a graph of an object's acceleration a versus time t, we can integrate on the graph to find the velocity at any given time. Because a is defined as $a = dv/dt$, the Fundamental Theorem of Calculus tells us that

$$v_1 - v_0 = \int_{t_0}^{t_1} a\, dt.$$

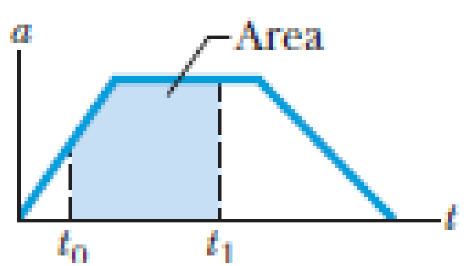
The right side of the equation is a definite integral (it gives a numerical result rather than a function), v_0 is the velocity at time t_0, and v_1 is the velocity at later time t_1. The definite integral can be evaluated from an $a(t)$ graph, such as in Fig. 2-14a. In particular,

$$\int_{t_0}^{t_1} a\, dt = \begin{pmatrix} \text{area between acceleration curve} \\ \text{and time axis, from } t_0 \text{ to } t_1 \end{pmatrix}.$$

If a unit of acceleration is 1 m/s^2 and a unit of time is 1 s, then the corresponding unit of area on the graph is

$$(1\ \text{m/s}^2)(1\ \text{s}) = 1\ \text{m/s},$$

which is (properly) a unit of velocity. When the acceleration curve is above the time axis, the area is positive; when the curve is below the time axis, the area is negative.



This area gives the change in velocity.

Graphical Integration in Motion Analysis

Integrating Velocity. Similarly, because velocity v is defined in terms of the position x as $v = dx/dt$, then

$$x_1 - x_0 = \int_{t_0}^{t_1} v \, dt,$$

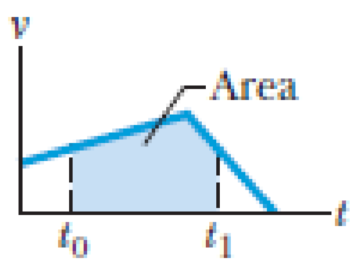
where x_0 is the position at time t_0 and x_1 is the position at time t_1. The definite integral on the right side of Eq. 2-29 can be evaluated from a $v(t)$ graph, like that shown in Fig. 2-14*b*. In particular,

$$\int_{t_0}^{t_1} v \, dt = \begin{pmatrix} \text{area between velocity curve} \\ \text{and time axis, from } t_0 \text{ to } t_1 \end{pmatrix}.$$

If the unit of velocity is 1 m/s and the unit of time is 1 s, then the corresponding unit of area on the graph is

$$(1 \text{ m/s})(1 \text{ s}) = 1 \text{ m},$$

which is (properly) a unit of position and displacement.



This area gives the change in position.

Tasks for Assignment

EXERCISE#4

Here are four descriptions of the position (in meters) of a puck as it moves in an xy plane:

(1) $x = -3t^2 + 4t - 2$ and $y = 6t^2 - 4t$ (3) $\vec{r} = 2t^2\hat{i} - (4t + 3)\hat{j}$

(2) $x = -3t^3 - 4t$ and $y = -5t^2 + 6$ (4) $\vec{r} = (4t^3 - 2t)\hat{i} + 3\hat{j}$

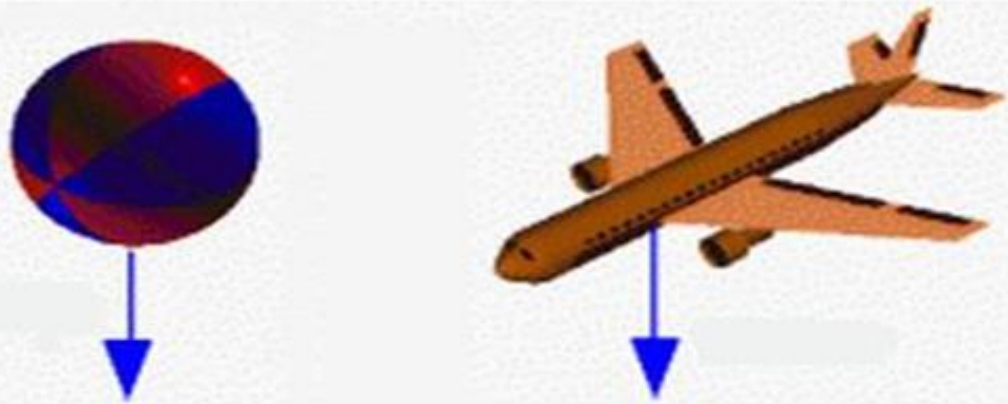
Are the x and y acceleration components constant? Is acceleration $\vec{a}$ constant?

FREE FALL MOTION/ACCELERATION

- There is one important thing to be noted here. In the solution of motion problems, we must assign vector directions.
- If you tossed an object either up or down and could somehow eliminate the effects of air on its flight, you would find that the object accelerates downward at a certain constant rate.
- That rate is called the **free-fall acceleration, and its magnitude** is represented by **g**.
- The value of g varies slightly with latitude and with elevation.
- At sea level in Earth's mid latitudes the value is 9.8 m/s^2

CONCEPTUAL VIEW

Motion of Free Falling Object
(no air resistance)



Mass and shape of object does not affect the motion.

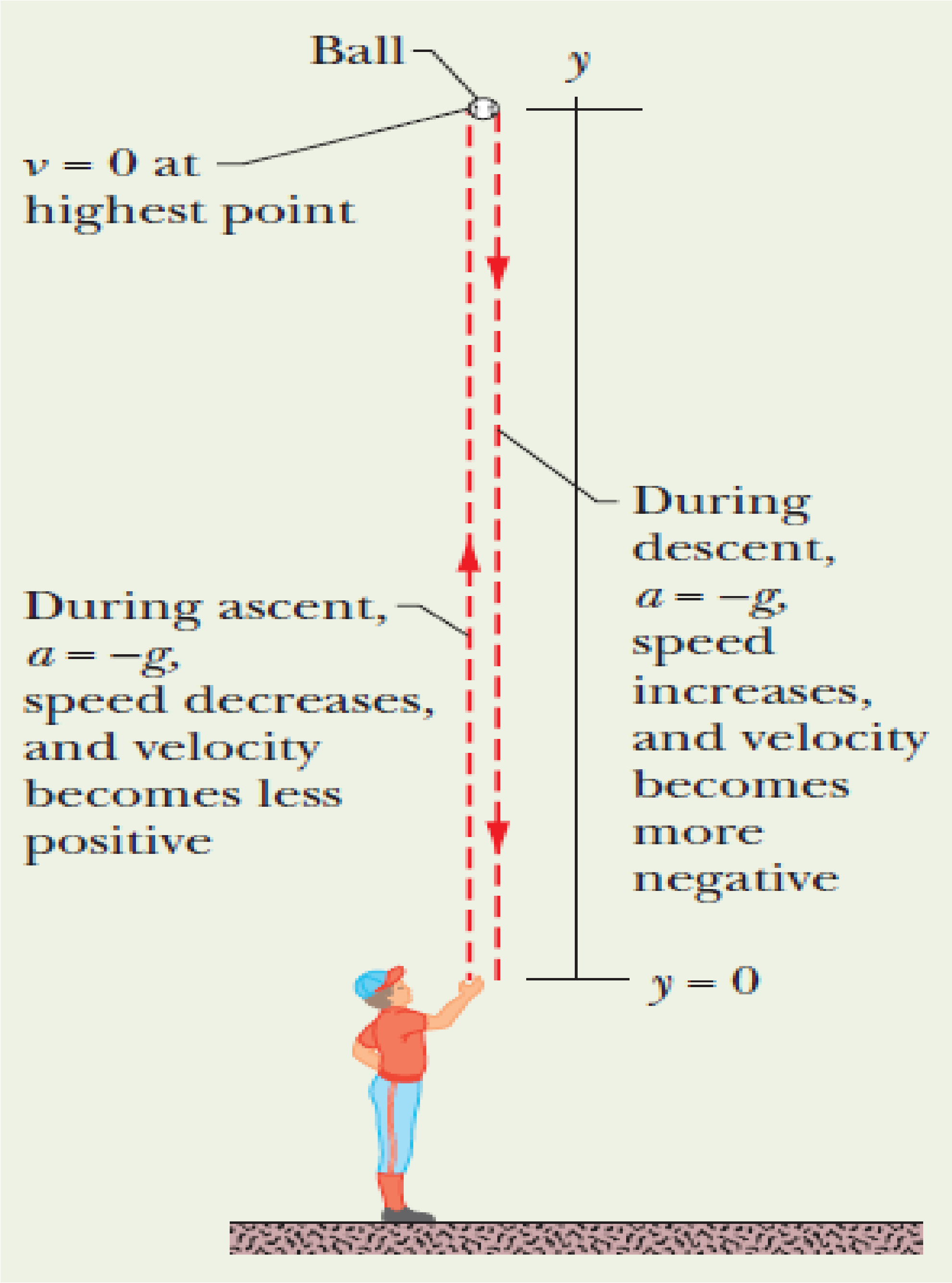
All objects fall at the same rate in a vacuum. -- Galileo.

Time – sec.	0	1	2	3	4	5	6	7	8
Accel – m/sec^2	9.8	9.8	9.8	9.8	9.8	9.8	9.8	9.8	9.8
Velocity – m/sec	0	9.8	19.6	29.4	39.2	49.0	58.8	68.6	78.4
Dist – meters	0	4.9	19.6	44.1	78.4	122.5	176.4	240.1	313.6

DIRECTION OF VELOCITY IN FREE FALL

- In order to work with free fall problems choosing a particular coordinate system is a matter of personal convenience
- Consider a boy throwing a ball vertically upwards then we will take the motion of ball is along "y-axis" and we will take the upward motion of ball as positive.
- It is important to note that once we choose a coordinate system, all parameters have their vector direction controlled by it.

The equations of free fall apply for rising as well as for falling objects, provided any effects from the air can be neglected



DIRECTION OF VELOCITY IN FREE FALL

- If we choose the positive *y* direction as up and the boy throws the ball straight up, then the vector displacement from the ground to its highest position is positive.
- During its upward travel, because velocity is the displacement divided by the scalar time, it too is positive.
- The only motion is in the "y direction", so we therefore use in the equations previously derived

$$y, v_y, a_y$$

$$\boxed{v_y = v_{0y} + a_y t}$$

CONDITIONS WHEN VELOCITY(OF FREE FALL) IS MAX OR MIN

- Condition 1:When velocity is maximum

  We observe that in throwing the ball upward the largest value for the magnitude of the y velocity occurs as it leaves the boy's hand. For which the value of g should be positive.

- Condition 2:When velocity is minimum
- It is when the ball reaches to highest point

a_y is the acceleration caused by the force of gravity acting on the ball

SOLVED EXAMPLE 1

EXAMPLE A boy throws a ball upward with an initial velocity of 12 m/sec. How high does it go?

Solution We choose the starting point as the origin and the upward direction as positive. Because velocity is a vector displacement divided by time, upward velocity is also positive. The force of gravity is in the negative y direction, so the sign of the acceleration is therefore negative. First list what is known and what is to be found

$$v_{0y} = 12\,\text{m/sec}, \quad v_y = 0 \text{ (at its highest point)}, \quad a_y = g = -9.8\,\text{m/sec}^2$$

$$y = ?$$

We select the 3rd equation of motion because all the quantities in that equation are known except y, the quantity that we want to find

$$v_y^2 - v_{0y}^2 = 2a_y y$$

SOLVED EXAMPLE(CONT'D)

Solving for y, we write

$$y = \frac{v_y^2 - v_{0y}^2}{2a_y}$$

Substituting the numerical values for the quantities in the equation,

$$y = \frac{0 - (12\,\text{m/sec})^2}{2(-9.8\,\text{m/sec}^2)}$$

$$= 7.3\,\text{m}$$

SOLVED EXAMPLE 2

EXAMPLE A boy throws a ball upward with an initial velocity of 12 m/sec and catches it when it returns. How long was it in the air?

Solution As in the previous example, we choose the starting point as the origin and the upward direction as positive.

$$v_{0y} = 12\,\text{m/sec}, \quad a_y = -9.8\,\text{m/sec}^2, \quad y = 0 \text{ (vector displacement is zero}$$

$$\text{because it returns to his hand)}, \; t = ?$$

Select 2nd equation of motion

$$y = v_{0y}t + \frac{1}{2}a_y t^2 \qquad \text{..........eq. 1}$$

Using the fact that $y = 0$, Eq. 1 becomes

$$0 = v_{0y}t + \frac{1}{2}a_y t^2$$

We see immediately that if we divide both sides of the equation by t, we obtain

$$0 = v_{0y} + \frac{1}{2}a_y t$$

SOLVED EXAMPLE(CONT'D)

$$t = -\frac{2\,v_{0y}}{a_y}$$

$$= -\frac{2 \times 12\ \text{m/sec}}{-9.8\ \text{m/sec}^2}$$

$$= 2.45\ \text{sec}$$

Note: If the ball had landed on a roof, then the left side of Eq. 1 would not be zero and the equation to be solved would be quadratic.