Q9) $\int_c (e^x + y^2)\,dx + (e^y + x^2)\,dy$

$y = x^2$
$y = x$

$\frac{df}{dy} = \frac{d}{dy}(e^x + y^2) = 2y$

$y \to x^2 \to x$

$x \to 0 \to 1$

$\frac{dg}{dx} = \frac{d}{dx}(e^y + x^2) = 2x$

$\iint (2x - 2y)\,dA$

$\int_0^1 \int_{x^2}^{x} (2x - 2y)\,dy\,dx$

$\int_0^1 \left| 2xy - \frac{\cancel{2}y^2}{\cancel{2}} \right|_{x^2}^{x} dx$

$\int_0^1 \left((2x(x) - x^2) - (2(x)x^2 - (x^2)^2) \right) dx$

$\int_0^1 (x^2 - 2x^3 + x^4)\,dx$

$\left| \frac{x^3}{3} - \frac{2x^4}{4} + \frac{x^5}{5} \right|_0^1$

$\frac{1}{3} - \frac{1}{2} + \frac{1}{5} \Rightarrow \boxed{\frac{1}{30}}$ Ans

Assignment 3

Obaid
24k-0793

Q1) $\frac{y^2}{2} - 3 = x$

$x = y + 1$

$\frac{y^2}{2} - 3 = y + 1$

$\frac{y^2}{2} = y + 4$

$y^2 - 2y - 8 = 0$

$y = 4 \quad , \quad y = -2$

$x = 5 \quad , \quad x = -1$

$$\int_{y=-2}^{y=4} \int_{x=\frac{y^2}{2}-3}^{x=y+1} xy \, dx \, dy$$

$$\int_{-2}^{4} \left| \frac{yx^2}{2} \right|_{\frac{y^2}{2}-3}^{y+1} dy$$

$$\int_{-2}^{4} \left(\frac{y(y+1)^2}{2} - \frac{y\left(\frac{y^2}{2}-3\right)^2}{2} \right) dy$$

$$\int_{-2}^{4} \frac{y(y^2+2y+1) - y\left(\frac{y^4}{4} - 3y^2 + 9\right)}{2} dy$$

$$\int_{-2}^{4} \frac{4y^3 - \frac{1}{4}y^5 + 2y^2 - 8y}{2} dy$$

$$\int_{-2}^{4} 2y^3 - \frac{1}{8}y^5 + y^2 - 4y \, dy$$

$$\left| \frac{y^4}{4} - \frac{y^6}{48} + \frac{y^3}{3} - 2y^2 \right|_{-2}^{4}$$

$32 - (-4) = \boxed{36}$

Q2) $$\iint_D 2x+y^2 \, dA$$

$$y^2 = y^3$$
$$y^3 - y^2 = 0$$
$$y^2(y-1) = 0$$
$$y=1 \quad y=0$$
$$x=1 \quad x=0$$

$$\int_0^1 \int_{x=y^3}^{x=y^2} 2x+y^2 \, dx \, dy$$

$$\int_0^1 \left| x^2 + y^2 x \right|_{y^3}^{y^2} dy$$

$$\int_0^1 y^4 + y^2(y^2) - \{y^6 - y^2(y^3)\} \, dy$$

$$\int_0^1 2y^4 - y^5 - y^6 \, dy$$

$$\left| \frac{2y^5}{5} - \frac{y^6}{6} - \frac{y^7}{7} \right|_0^1$$

$$\frac{2}{5} - \frac{1}{6} - \frac{1}{7} = \boxed{\frac{19}{210}}$$

Q3) $\displaystyle\int_{x=0}^{x=\pi/2}\int_{y=0}^{y=\sin x} \cos x\sqrt{1+\cos^2 x}\, dy\, dx$

$y=0$

$y=1$

$x = \arcsin y \Rightarrow x = \sin^{-1}(y)$

$x = \frac{\pi}{2}$ $\quad y = \sin x$

$$\int_0^{\pi/2} \left| y\cos x\sqrt{1+\cos^2 x} \right|_{y=0}^{y=\sin x} dx$$

$$-\frac{1}{2}\int_0^{\pi/2} -2\sin x\cos x\sqrt{1+\cos^2 x}\, dx$$

$$\left| -\frac{1}{\cancel{2}} \frac{(1+\cos^2 x)^{3/2}}{\times \frac{3}{\cancel{2}}} \right|_{x=0}^{x=\pi/2}$$

$$\left| -\frac{(1+\cos^2 x)^{3/2}}{3} \right|_0^{\pi/2}$$

$$-\frac{1}{3} - \left(\frac{-2^{3/2}}{3}\right) \Rightarrow \boxed{\frac{-1+2^{3/2}}{3}}$$ Ans

Q5) $\displaystyle\int_{\theta=0}^{\theta=\pi/4}\int_{r=0}^{r=2\cos\theta} 2r\sin\theta\, r\,dr\,d\theta$

$x = r\cos\theta$

$y = r\sin\theta$

$(r\cos\theta - 1)^2 + (r\sin\theta)^2 = 1$

$r^2\cos^2\theta - 2r\cos\theta + \cancel{1} + r^2\sin^2\theta = \cancel{1}$

$r^2 - 2r\cos\theta = 0$

$r(r - 2\cos\theta) = 0$

$r = 0 \qquad r = 2\cos\theta$

$\displaystyle\int_{\theta=0}^{\theta=\pi/4}\int_{r=0}^{r=2\cos\theta} 2r^2\sin\theta\, dr\,d\theta$

$\displaystyle\int_0^{\pi/4} \left| \frac{2r^3}{3}\sin\theta \right|_0^{2\cos\theta} d\theta$

$\displaystyle\int_0^{\pi/4} \frac{2(8\cos^3\theta)\sin\theta}{3}\, d\theta$

$\displaystyle\frac{16}{3}\int_0^{\pi/4} \cos^3\theta\sin\theta\, d\theta$

$\displaystyle\left| \frac{16}{3} \times \frac{\cos^4\theta}{4} \right|_0^{\pi/4} = -\frac{1}{3} + \frac{4}{3} = \boxed{1}$

Q6) $$\int_0^3\int_0^1\int_0^{\sqrt{1-z^2}} ze^y\,dx\,dz\,dy$$

$$\int_0^3\int_0^1 \left| xze^y \right|_0^{\sqrt{1-z^2}} dz\,dy$$

$$\int_0^3\int_0^1 ze^y\sqrt{1-z^2}\,dz\,dy$$

$$\int_0^3 \left| -\frac{1}{2}e^y \frac{(1-z^2)^{3/2}}{\frac{3}{2}} \right|_0^1 dy$$

$$\int_0^3 0-\left(-\frac{e^y(1)}{3}\right) dy$$

$$\int_0^3 \frac{e^y}{3}\,dy$$

$$\left|\frac{1}{3}e^y\right|_0^3 \Rightarrow \frac{1}{3}e^3-\frac{1}{3} \Rightarrow \boxed{\frac{e^3-1}{3}}$$

Q7) $$\int_{\theta=\pi/6}^{\theta=5\pi/6}\int_{r=2+2\sin\theta}^{r=6\sin\theta} r\,dr\,d\theta$$

$$\int_{\pi/6}^{5\pi/6} \left|\frac{r^2}{2}\right|_{r=2+2\sin\theta}^{6\sin\theta} d\theta$$

$$\int_{\pi/6}^{5\pi/6}\left(\frac{36\sin^2\theta}{2}-\frac{(4+8\sin\theta+4\sin^2\theta)}{2}\right)d\theta$$

$$\int_{\pi/6}^{5\pi/6} 16\sin^2\theta-4\sin\theta-2\,d\theta$$

$$\cos 2\theta = 1-2\sin^2\theta$$

$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$r=6\sin\theta$

$r=2+2\sin\theta$

$6\sin\theta = 2+2\sin\theta$

$4\sin\theta = 2$

$\sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$5\pi/6$

Q7) $\int_C F \cdot d\gamma = \int_a^b F(t) \cdot \gamma'(t)\,dt$ $\quad \gamma'(t) = 2i + 3j + 2tk$

$F(x(t), y(t), z(t)) = 2ti - (-t^2)j + 3tk$

$f(\gamma(t)) = 2ti + \cancel{3}t^2 j + 3tk$

$f(\gamma(t)) \cdot \gamma'(t) = (2ti + t^2 j + 3tk) \cdot (2i + 3j - 2tk)$

$4t + 3t^2 - 6t^2 \Rightarrow 4t - 3t^2$

$\int_{-1}^{1} 4t - 3t^2\,dt$

$\left| 2t^2 - t^3 \right|_{-1}^{1} \Rightarrow 2 - 1 - (2 - (-1)) \Rightarrow \cancel{2} - 1 - \cancel{2} - 1 = \boxed{-2}$ Ans

Q8) $f(x(t), y(t), z(t)) = (-1+2t)(5+t)(4t)^2$ $\quad x(t) = -1 + 2t$

$= 16^2(-5 - t + 10t + 2t^2) \Rightarrow -80t^2 + 144t^3 + 32t^4$ $\quad y(t) = 5 + t$

$z(t) = 4t$

$\int F \cdot d\gamma = \int_0^1 (-80t^2 + 144t^3 + 32t^4) \cdot \sqrt{2^2 + 1^2 + 4^2}\,dt$ $\quad 0 \le t \le 1$

$\sqrt{21} \int_0^1 -80t^2 + 144t^3 + 32t^4$

$\sqrt{21} \left| -\frac{80t^3}{3} + 36t^4 + \frac{32t^5}{5} \right|_0^1$

$\sqrt{21} \left(-\frac{80}{3} + 36 + \frac{32}{5} \right)$

$\boxed{\frac{236\sqrt{21}}{15}}$ Ans