Obaid 24K-0793

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Q1) $C_x(x,y) = e^x - 4$ $C_y(x,y) = \cos y - 1$

$e^x - 4 = 0$ $\cos y - 1 = 0$

$e^x = 4$ $\cos y = 1$

$x \ln e = \ln 4$ $\boxed{y = 0}$

$x = \ln 4$

critical point $= (\ln 4, 2\pi n)$ for any integer n

$D = C_{xx}(x_0,y_0)\, C_{yy}(x_0,y_0) - C_{xy}^2(x_0,y_0)$

$C_{xx}(x_0,y_0) = e^x$ $C_{yy}(x_0,y_0) = -\sin y$ $C_{xy} = C(x_0,y_0) = 0$

$D = -e^x \cdot -\sin y - (0)^2$

$D = -e^x \sin y$

$D = -e^{\ln 4} \sin(0)$ 2nd order derivative test inconclusive as $D = 0$

$D = 0$

Q2) $M_a(a,b) = 100 - 4a - b$ $M_b(a,b) = 150 - 6b - a$

$100 - 4a - b = 0$ $150 - 6(100 - 4a) - a = 0$

$b = 100 - 4a$ $150 - 600 - 24a - a = 0$

$b = 100 - 4(450/23)$ $a = \frac{450}{23}$

$b = \frac{500}{23}$ Critical point $\left(\frac{450}{23}, \frac{500}{23}\right)$

$M_{aa}\left(\frac{450}{23}, \frac{500}{23}\right) = -4$ $M_{bb}\left(\frac{450}{23}, \frac{500}{23}\right) = -6$

$M_{ab}\left(\frac{450}{23}, \frac{500}{23}\right) = -1$

$D = -4 - 6 - (-1)^2 = 23$ $D > 0$ & $M_{aa} < 0$

$M(a,b)$ has a relative maxima at $\left(\frac{450}{23}, \frac{500}{23}\right)$

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Q3) $L(m,n) = 3m^2 + 2mn + 4n^2 \qquad = f(m,n)$

$g(m,n) = m^2 + n^2 + mn$

$\nabla f (m,n) = (6m + 2n)i + (8n + 2m)j$

$\nabla g (m,n) = (2m + n)i + (2n + m)j$

$\nabla f = \lambda \nabla g$

$(6m + 2n)i + (8n + 2m)j = \lambda\left((2m + n)i + (2n + m)j\right)$

$6m + 2n = \lambda(2m + n)$

$8n + 2m = \lambda(2n + m)$

$6m + 2n = 2m\lambda + n\lambda$

$n(2-\lambda) = 2m(\lambda - 3)$

$n = \dfrac{2m(\lambda-3)}{2-\lambda}$

$8n + 2m = 2n\lambda + m\lambda$

$m(2-\lambda) = 2n(\lambda-4)$

$m = \dfrac{2n(\lambda-4)}{2-\lambda}$

$\cancel{n} = \dfrac{2 \cdot \dfrac{2\cancel{n}(\lambda-4)}{2-\lambda}(\lambda-3)}{2-\lambda}$

$(2-\lambda)^2 = 4(\lambda-4)(\lambda-3)$

$4 - 4\lambda + \lambda^2 = 4(\lambda^2 - 3\lambda - 4\lambda + 12)$

$4 - 4\lambda + \lambda = 4\lambda - 28\lambda + 48$

$3\lambda - 24\lambda + 44 = 0$

$\lambda = \dfrac{12 + 2\sqrt{3}}{3} \qquad \lambda = \dfrac{12 - 2\sqrt{3}}{3}$

$n = \dfrac{2m\left(\dfrac{12 + 2\sqrt{3}}{3} - 3\right)}{2 - \left(\dfrac{12 + 2\sqrt{3}}{3}\right)}$

$n = m\left(\dfrac{6 + 4\sqrt{3}}{\cancel{3}}\right) \times \dfrac{\cancel{3}}{-6 - 2\sqrt{3}}$

$n = \dfrac{m(6 + 4\sqrt{3})}{-6 - 2\sqrt{3}}$

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$m^2+n^2+mn=400$

$m^2+\left(\frac{-m(6+4\sqrt{3})}{6+2\sqrt{3}}\right)^2+m\left(\frac{-m(6+4\sqrt{3})}{6+2\sqrt{3}}\right)=400$

$m^2+\frac{1}{2}m^2=400$

$\frac{3}{2}m^2=400$

$m=\frac{20\sqrt{6}}{3}$ rej → cannot be negative

$n=\frac{\frac{20\sqrt{6}}{3}(6+4\sqrt{3})}{-6-2\sqrt{3}}$

$n=\frac{-10\sqrt{6}-30\sqrt{2}}{3}$ rej → cannot be negative

$\lambda=\frac{12-2\sqrt{3}}{3}$

$n=2m\left(\frac{12-2\sqrt{3}}{3}\right)$

$\frac{2-12-2\sqrt{3}}{3}$

$n=m\left(\frac{-1+\sqrt{3}}{2}\right)$

$m^2+m^2\left(\frac{2-\sqrt{3}}{2}\right)+m^2\left(\frac{-1+\sqrt{3}}{2}\right)=400$

$\frac{3}{2}m^2=400$

$m=\frac{20\sqrt{6}}{3}$ $\quad m=\frac{-20\sqrt{6}}{3}$ X

$n=\frac{-10\sqrt{6}+30\sqrt{2}}{3}$

$L.P=\left(\frac{20\sqrt{6}}{3}, \frac{-10\sqrt{6}+30\sqrt{2}}{3}\right)$ (m, n)

Q4) $E(p,q,r)=f(p,q,r)=p^2+4q^2+2r^2$

$g(p,q,r)=p^2+q^2+r^2$

$\nabla f(p,q,r)=2pi+8qj+4rk$

$\nabla g(p,q,r)=2pi+2qj+2rk$

$\nabla f=\lambda\nabla g$

$2pi+8qj+4rk=\lambda(2pi+2qj+2rk)$

$2p=\lambda 2p$ $\quad$ $8q=2\lambda q$ $\quad$ $4r=\lambda 2r$

$\lambda=$ $\quad$ $\lambda=4$ $\quad$ $\lambda=2$

$p^2+q^2+r^2=250{,}000$

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Q5) $J_{\theta_1}(\theta_1, \theta_2) = \cos(\theta_1) + \theta_2$

$J_{\theta_2}(\theta_1, \theta_2) = 2\theta_2 + \theta_1$

Iterations	θ_1	θ_2	J_{θ_1}	J_{θ_2}	J
0	1	1	1.54030	3	2.84147
1	0.84597	0.7	1.363005	2.24597	1.83079
2	0.70967	0.4754031	1.23398	1.66048	1.27497

$$x_{n+1} = x_n - \alpha \frac{df}{dx}$$

$J_f^{\theta_1} = 1 - 0.1(1.54030) = 0.84597$

$J_1^{\theta_2} = 1 - 0.1(3) = 0.7$

$J_2^{\theta_1} = 0.84597 - 0.1(1.363008) = 0.70967$

$J_2^{\theta_2} = 0.7 - 0.1(2.24597) = 0.475403$

Q6) $R(f,g,h) = f^2 + 2g^2 + 3h^2 + fg - 5f - 7g - 9h + 30$

$R_f(f,g,h) = 2f + g - 5$

$R_g(f,g,h) = 4g + f - 7$

$R_h(f,g,h) = 6h - 9$

Iterations	f	g	h	R_f	R_g	R_h	R
0	1	1	1	−2	−2	−3	16
1	1.1	1.1	1.15	−1.7	−1.5	−2.1	15.2575

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$$x_{n+1} = x_n - \alpha\left(\frac{df}{dx}\right)$$

$f_1 = 1 - 0.05(-2) = 1.1$

$g_1 = 1 - 0.05(-2) = 1.1$

$h_1 = 1 - 0.05(-3) = 1.15$

$f_2 = 1.1 - 0.05(-1.7) = 1.185$

$g_2 = 1.1 - 0.05(-1.5) = 1.175$

$h_2 = 1.15 - 0.05(-2.1) = 1.255$

$f_3 = 1.185 - 0.05(-1.455) = 1.25775$

$g_3 = 1.175 - 0.05(-1.115) = 1.23075$

$h_3 = 1.255 - 0.05(-1.47) = 1.3285$

$f_4 = 1.25775 - 0.05(-1.25375) = 1.3204375$

$g_4 = 1.23075 - 0.05(-0.81925) = 1.2717125$

$h_4 = 1.3285 - 0.05(-1.029) = 1.37995$

Q7) $J(\theta_1, \theta_2, \theta_3) = \theta_1^2 + \theta_2^2 + 4\theta_3^2 + \theta_1\theta_2$

$$\begin{bmatrix} J\theta_1\theta_1 & J\theta_2\theta_1 & J\theta_3\theta_1 \\ J\theta_1\theta_2 & J\theta_2\theta_2 & J\theta_3\theta_2 \\ J\theta_1\theta_3 & J\theta_2\theta_3 & J\theta_3\theta_3 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 8 \end{bmatrix}$$

$J\theta_1 = 2\theta_1 + \theta_2 \longrightarrow 2\theta_1 + \theta_2 = 0$

$J\theta_2 = 2\theta_2 + \theta_1 \longrightarrow 2\theta_2 + \theta_1 = 0$

$J\theta_3 = 8\theta_3 \rightarrow 8\theta_3 = 0$

$\theta_2 = -2\theta_1$

$J\theta_1\theta_1 = 2$

$2(-2\theta_1) + \theta_1 = 0$

$J\theta_2\theta_2 = 2$

$-4\theta_1 + \theta_1 = 0$

$J\theta_3\theta_3 = 8$

$-3\theta_1 = 0$

$J\theta_1\theta_2 = 1$

$\theta_1 = 0$

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Q8) $E = \int_1^3 \int_0^1 xy - x^2 \, dx \, dy$

$E = \int_1^3 \left[\frac{x^2 y}{2} - \frac{x^3}{3} \right]_0^1 dy$

$E = \int_1^3 \left(\frac{y}{2} - \frac{1}{3} \right) dy$

$E = \left[\frac{y^2}{4} - \frac{1}{3} y \right]_1^3$

$E = \left[\frac{3^2}{4} - \frac{3}{3} \right] - \left[\frac{(1)^2}{4} - \frac{1}{3} \right]$

$E = \frac{9}{4} - 1 - \frac{1}{4} + \frac{1}{3}$

$\boxed{E = \frac{4}{3}}$

Q9) $V = \iint_D 10 - x^2 - y^2 \, dA$

$(0,0)$ $(2,0)$ $(2,3)$

$V = \int_{y=0}^{3} \int_{x=\frac{2}{3}y}^{2} 10 - x^2 - y^2 \, dx \, dy$

$V = \int_0^3 \left[10x - \frac{x^3}{3} - xy^2 \right]_{\frac{2}{3}y}^{2} dy$

$V = \int_0^3 20 - \frac{8}{3} - 2y^2 - \frac{20}{3} y + \frac{8}{81} y^3 + \frac{2}{3} y^3 \, dy$

$V = \int_0^3 \frac{52}{3} - 2y^2 - \frac{20}{3} y + \frac{62}{81} y^3 \, dy$

$V = \left[\frac{52}{3} y - \frac{2y^3}{3} - \frac{10y^2}{3} + \frac{31}{162} y^4 \right]_0^3$

$V = \left(\frac{39}{2} \right) - 0$

$V = \frac{39}{2} \text{ unit}^3$