Assignment 1

24k-0793

problem 1.

1. $f(x,y,z) = xy \ln z$

sol

domain $z > 0$

$= \{(x,y,z) \in R^3, z > 0\}$

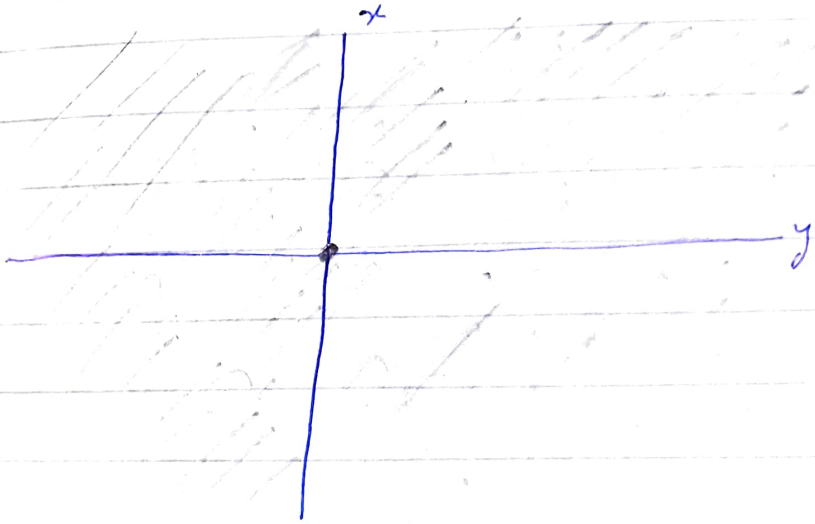
- As discussed with Sir, done in software

2. $f(x,y) = \ln(x^2+y^2)$

Domain : $x^2+y^2 > 0$

↳ circle only undefined at $(0,0)$

$D = \{(x,y) \in R \mid x,y \neq (0,0)\}$



key =

• = unshaded only at (0,0)

3) $f(x,y) = \sin^{-1}(y-x)$

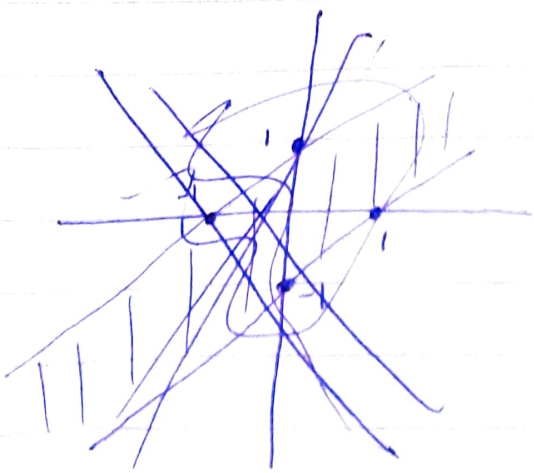
$\rightarrow \sin^{-1}(y-x)$

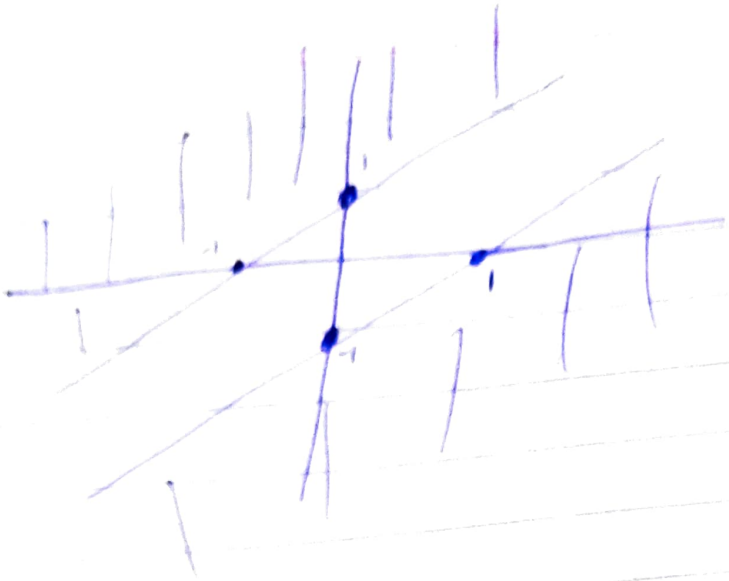
$1 < y-x$ $\quad -1 > y-x$

$1+x < y$ $\quad y < x-1$

$y = x+1$

$y = x-1$





4. $f(x,y,z) = \frac{1}{x+1} + \frac{1}{y-1} + \frac{1}{x+y+z}$

$\rightarrow x+1 \neq 0 \quad \rightarrow y-1 \neq 0$

$x \neq -1 \quad \hookrightarrow y \neq 1$

$\rightarrow x+y \neq z$

done in software

Problem 2:

1) $f(x,y) = 4x^2 + y^2 + 1$

$c = 4x^2 + y^2 + 1$

$c-1 = 4x^2 + y^2$

$1 = \frac{x^2}{\frac{c-1}{4}} + \frac{y^2}{c-1}$

$a^2 = \frac{c-1}{4}$, $a = \frac{\sqrt{c-1}}{2}$

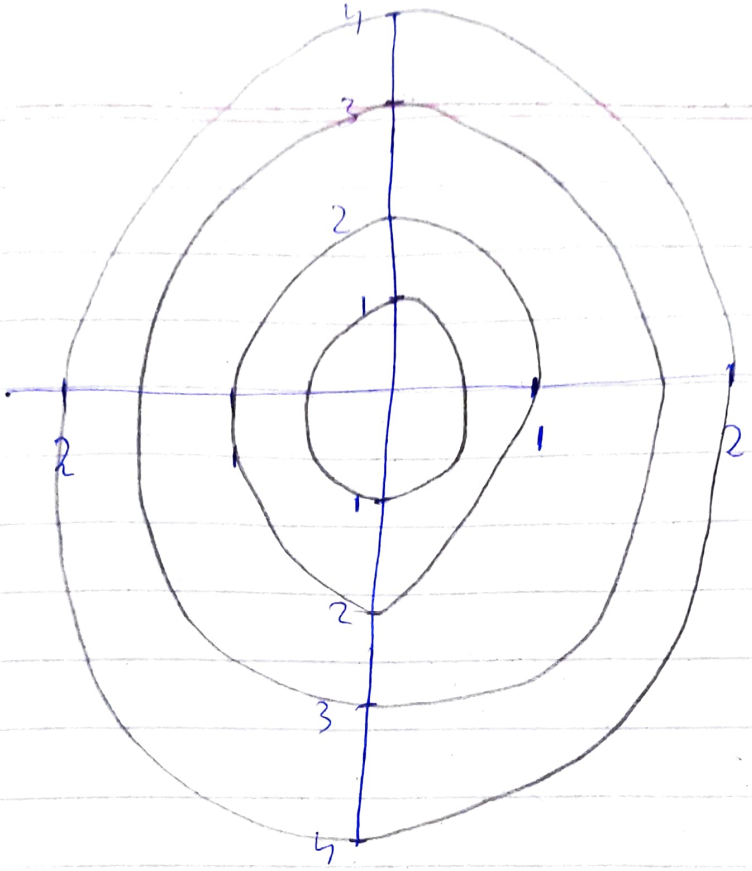
$b^2 = c-1$, $b = \sqrt{c-1}$

$c=1$, single point so

$c=2$, $a=\frac{1}{2}$, $b=1$

$c=5$, $a=1$, $b=2$

$c=10$, $a=3/2$, $b=3$



ii) $-2 = 2x - 6y$

∴ As level of curve so $2 = k = \text{cons}$

$-2 = 2x - 6y + k$

$-2 = 2x - 6y + k$

$k = 6y - 2x - 2$

$k + 2 = 2(3y - x)$

$\frac{k+2}{2} = 3y - x \rightarrow$ plane

$k = 0$

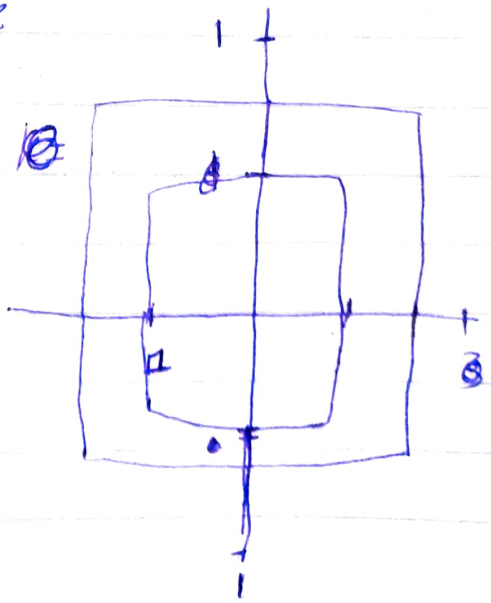
$1 = 3y - x$

x	0	-1
y	-1/3	0

$k = 1$

$\frac{3}{2} = 3y - 1$

x	-3/2	0
y	0	1/2



Problem 5 :-

$\frac{dh}{dt} = 3\ mm/s \rightarrow 0.3\ cm/s$

$\frac{dr}{dt} = -2\ mm/s \rightarrow -0.2\ cm/s$

$h = 3.2\ cm$

$r = 1.5\ cm$

$\frac{dv}{dt} = ??$

Since $V = \frac{\pi r^2 h}{3}$

$dv = \frac{\partial v}{\partial r} dr + \frac{\partial v}{\partial h} dh$

$\frac{dv}{dt} = \frac{\partial v}{\partial r} \cdot \frac{dr}{dt} + \frac{\partial v}{\partial h} \cdot \frac{dh}{dt}$

$\frac{dv}{dt} = \left(\frac{2\pi rh}{3}\right)(-0.2) + \left(\frac{\pi r^2}{3}\right)(0.3)$

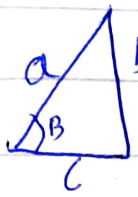
$= \frac{2\pi (1.5)(3.2)(-0.2)\pi}{3} + \frac{(0.3)(1.5)^2 \pi}{3}$

$\frac{dv}{dt} \quad -0.64\pi + 0.225\pi$

$\frac{dv}{dt} = -0.415\pi$

$\frac{dv}{dt} = -1.31\ cm^3/s$

Q6)



$$A = \frac{1}{2} \times ac \times \sin B$$

$$\frac{da}{dt} = 0.4 \text{ units/s} \qquad \frac{dc}{dt} = -0.8 \qquad \frac{dB}{dt} = 0.2$$

$$a = 3, \quad c = 4, \quad B = \pi/6$$

$$\frac{dA}{dt} = \frac{1}{2}\left(c \sin B \frac{da}{dt} + ac \cos B \frac{dB}{dt} + ac \sin B \frac{dc}{dt}\right)$$

$$\frac{1}{2}\left(4 \sin \pi/6 \times 0.4 + 12 \cos \pi/6 \times 0.2 + 3 \sin \pi/6 \times -(0.8)\right)$$

$$\frac{1}{2}\left(0.8 + 2.08 - 1.2\right)$$

$$\frac{dA}{dt} = 0.84 \text{ units}^2/\text{s} \quad \text{increasing}$$