**Maximum-height human jumping simulation**

1. **Introduction**

The scope of the project is to develop a graphical simulation of the jumping motion of one human leg in two dimensions. In order to achieve it a leg model is proposed using a group of six muscles and three segments activated by neuron signals. Each muscle of the leg is modeled using a musculotendon model based on real muscle properties (e.g. pennation angle, serial and parallel elements for simulating muscle fibers). The project also focuses on how to use two types of models to simulate the interaction between the muscles, joints and body segments. The difference between those two approaches is whether they use a lumped muscle model or not. After defining the system equation, a numerical method is used to obtain the angular displacements, velocities and accelerations of the joints as a function of the muscle activation signals and the kinematics of the leg. Finally, an optimum muscle activation level is searched for in order to produce a maximum height jumping using the proposed leg model. The model described in Figure 1 governs the general model adopted in this study. In the following sections, we will explain the details of this model’s components.

u(t)

θ

Neural signals optimization

Musculotendon Dynamics

Skeletal Dynamics

a(t)

PT(t)

θ)

**Figure 1.** Block diagram showing the flow of the maximum-height jump model dynamics. u(t): neural signal, a(t): activation signal, PT(t): tendon force, θ): angular displacement, velocity and acceleration of the joints, respectively

1. **Musculoskeletal model**

The model is designed for simulating the jumping motion of a human leg. Considering that the motion will be initially simulated in two dimensions; the anatomy of the leg muscles can be simplified into a model that uses six muscles. This group of muscles interacts with three rigid body segments for the foot (1), shank (2), thigh(3) to move three joints: foot (with respect to the floor), ankle and knee.

The musculoskeletal model can be further analyzed within 3 main parts, namely Skeletal Dynamics, Musculotendon Dynamics, and Excitation-Contraction Dynamics.

* 1. **Skeletal Dynamics**

The model consists of three segments defined on the reference system defined in Figure 1. The first joint represents the foot extreme and it is attached to a rigid body segment. By using Gauss’ Principle of Least Constraint we derived the dynamical equations of motion for our model (Eq. 1).

(Eq. 1)

|  |  |  |  |
| --- | --- | --- | --- |
| , : | 3x1 vector of the angular displacement, velocity and acceleration of the joints | : | 6x1 musculotendon actuator forces – T is for tendon, not transpose. |
| : | 3x1 vector of externally applied spring to the foot | : | 3x1 vector containing gravitational terms |
| : | 3x3 system mass matrix | : | 3x6 moment arm matrix |
| : | 3x3Coriolis and centrifugal effects |  |  |

The following methodology was followed in order to define the dynamical equations for the motion of the model. The representations and the values given in Figure 2 and Table 1 are used for the following calculations.

1. Define the transformation matrix of the system based on the kinematics model presented in figure 2-a.

(Eq. 2)

1. Calculate the Jacobian matrix by taking the derivative of the transformation matrix with respect to the joint angles.

(Eq. 3)

1. Obtain the system’s inertia matrix (A), quadratic velocity (B), and gravitational (C) terms using Gauss’s principle of least constraint.

In order to determine each of the previous segments the following computations were done.

(Eq. 4)

(Eq. 5)

(Eq. 6)

is the transpose of the jacobian

is the first derivative of the jacobian

g is the gravitational acceleration acting on the –y axis.

1. Calculate the external forces acting on the system generated by the muscles () by using the moment arm applied in the direction of the muscle contraction.

(Eq. 7)

Ankle joint: Sum of external forces generated by contraction of all muscles affecting the ankle (see Table 2).

(Eq. 8)

Knee joint: Sum of external forces generated by contraction of all muscles affecting the knee (see Table 2).

Total force applied by the six muscles

(Eq. 9)

1. Place a spring-damper system under the first segment in order to simulate the contact of the foot with the floor.

Foot joint: The spring affecting the foot used to keep it on the ground till a given threshold value (θ1 > pi-34°).

(Eq. 10)

|  |  |  |
| --- | --- | --- |
|  |  |  |
| **a** | **b** | **c** |
| **Figure 2.** a) Representation of the three segment model (foot, shank and thigh). are the angular positions of the foot, ankle, and knee with respect to the floor, foot and ankle, respectively.  b) Representation of the musculoskeletal model of the leg using six muscles: soleus (SOL), gastrocnemius (GAS), other plantarflexors (OPF), tibialis anterior (TA), vasti (VAS) and hamstrings (HAMS)  c) Physical representation of the musculotendon model: Tendon is in series with the muscle, which has a pennation angle, α, and consists of passive and active elements. | | |

|  |  |  |
| --- | --- | --- |
| ***Segments properties*** | ***M (kg)*** | ***li (m)*** |
| **Foot** | 2.2 | 0.175 |
| **Shank** | 7.5 | 0.435 |
| **Thigh** | 15.15 | 0.400 |

**Table 1**: Properties of the body segments used for the jump model.

* 1. **Musculotendon Dynamics**

Musculotendon actuator was modeled as described in Pandy *et* *al.* (1990) with a three element, lumped muscle, in series with the tendon (Figure 2c). The mechanics of the muscle was described by a contractile element which demonstrates Hill-type behaviour, a series-elastic element, and a parallel-elastic element. The behavior of a single muscle can be seen in Figure 6. The equation for the dynamics of musculotendon component is based on the model adopted in Pandy *et al.* (1990) (Eq. 11).

Besides the Hill-type model used by Pandy *et al.*, a spring-damper based muscle model was also implemented. All the parameters (proportional and derivative gains, Kp and Kd) are tuned in order to represent the physical properties of the relevant muscles. Using spring-damper muscle model, we managed to obtain similar effects with the original model. However, since the latter model is assumed to be a more accurate representation, we used the Hill-type model for our optimization calculations.

(Eq. 11)

|  |  |
| --- | --- |
|  |  |
| **a** | **b** |
| **Figure 3:** a)Force-Velocity and b) Force-Length Relations of a single muscle, i.e. soleus. | |

* 1. **Excitation-Contraction Dynamics**

In order to define the relation between the input neural signal and the muscle activation, a first-order differential equation is constructed.

(Eq. 12)

where u(t): muscle excitation; a(t): muscle activation; : rise and decay time constants for muscle activation, respectively; a designated lower bound on muscle activation.

* 1. **Musculotendon Properties and Musculoskeletal Geometry**

We initially adopted the parameters used by Pandy *et al.* (1990) who used the data reported by Wickiewicz *et al.* (1983), Brand *et al.* (1986), Alexander and Vernon (1975), Woo *et al.* (1982), and Butler *et al.* (1984). Muscle and tendon parameters used in our study are presented in Table 2.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | Muscle | | | Tendon | |
| Actuator | α0  (deg) | (m) | (N) | (m) | (%) |
| SOL | 20.0 | 0.034 | 4235 | 0.360 | 2.5 |
| OPF | 10.0 | 0.036 | 3590 | 0.405 | 2.6 |
| TA | 5.0 | 0.070 | 1400 | 0.265 | 2.7 |
| GAS | 12.0 | 0.062 | 2370 | 0.411 | 3.9 |
| VAS | 10.0 | 0.090 | 5400 | 0.206 | 3.0 |
| HAMS | 9.0 | 0.106 | 2350 | 0.390 | 2.6 |

**Table 2.** Muscle and tendon properties used in the model.

The musculoskeletal geometry of the model, which defines the origin and insertion sites, was based on the data reported by Scott *et al* (1989).

For defining the global muscle model, the joints that each muscle manipulates are obtained following an analysis on the leg physiology. Once the joints are obtained as a function of the system variables (angular displacements), the muscle origins and insertions must be calculated in terms of the joints coordinates.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Number** | **Muscle properties** | **Joint affected** | **Muscle contraction** | **Model** |
| 1 | **TA** | Ankle | Flexion | T1 |
| 2 | **SOL** | Ankle | Extension | T1 |
| 3 | **GAS** | Ankle | Extension | T2 |
| 4 | **OPF** | Ankle | Extension | T1 |
| 5 | **VAS** | Knee | Extension | T3 |
| 6 | **HAMS** | Knee | Flexion | T2 |

**Table 3**: Properties of the body segments used for the jump model

**Monoarticular model (T1)**

The first type defines muscles that have an origin in some part of the leg segment directly attached to the joint and can be modeled as shown in Figure 4. The length of the muscle is computed as the Euclidian distance between the origin and the insertion

|  |  |  |
| --- | --- | --- |
|  |  |  |
| **Figure 4**: The monoarticular muscle model and the calculation of the moment arm. | | |

**Biarticular model (T2) and Circular Model for knee (T3)**

The second type defines muscles where the origin is not part of the body segment directly attached to the joint affected by the muscle. The origin is modeled in a virtual segment of a “grounded bone” as shown in the following figure.

The length of the muscle is obtained using the Euclidian distance between the origin and the insertion.

The moment arm for the biarticular model is obtained using a pulley system, depicted in Figure 5, with the given equation 13.

(Eq. 13)

|  |  |  |
| --- | --- | --- |
|  |  | : Length of the shank  : Distance from the ankle joint to the insertion  : Distance from the knee joint to the origin  : Inner angle of the ankle  : 90 degrees for the ankle moment arm. |
|  | |

**Figure 5:** Biarticular muscle model (T2), where the origin is not attached to the segment.

Finally, the VAS muscle on the knee is modeled using a pulley centered at the knee joint, where the moment arm is the radius and the length is obtained using the arc-length.

(Eq. 14)

1. **Optimal Control Problem**

For maximum height jumping, the height reached by the center of mass of the body is chosen for the measure of performance (Eq.).

(Eq. 15)

In order to find the optimal control signals for maximizing the performance, we applied a greedy search on the combinations of six input neural signals. The time for simulating the jump is divided into 6 equidistant intervals. The neural signals can only be changed when switching from one interval to another. After discarding some redundant cases, we searched for the whole space of neural signal combinations, and found out the lift-off time. The constraints, which define the problem, are the dynamical equations (Eq. 1, 11, and 12), and a set of inequality constraints which bound the magnitude of each neural signal. The equality constraint specifies the instant of lift-off, i.e. a zero vertical ground reaction force. Those constraints are:

(Eq. 16)

(Eq. 17)

Where is the mass of the *i*th segment, is the vertical acceleration of the center of mass of the *i*th segment, is the magnitude of the vertical ground reaction force, and indicates that each quantity is evaluated at the final lift-off time.

1. **Results**

**4.1 Moment arm**

For different representations of moment arm models, we tested our calculations on two-link arm kinematic model. For each model, we provided the simulation videos of muscle contraction under gravity constraints without using inertia of the muscles.

1. Monoarticular model

(Supplementary.zip/2LinkMonoarticular.avi)

1. Biarticular model

(Supplementary.zip/2LinkBiarticular.avi)

1. Circular model

(Supplementary.zip/2LinkCircular.avi)

**4.2 Inertia of the muscles**

We tested our model with and without integrating the inertia of the muscles into the kinematics equations while the force generated by the muscle was also taken into account in addition to the gravity. As suggested by Pai (2010), simulation results show that the two approaches affect the motion of the two-link model (See Supplementary.zip/2LinkWithIm.avi and 2LinkWithoutIm.avi). Here, mass of the muscles are chosen as 4.20 kg.

**4.3. Jump**

We have implemented a Matlab simulation for the forward calculations of a single leg motion subject to arbitrary muscle activations. Using the ODE solver (i.e. ode45) of the Matlab, we solve for the skeletal dynamics (Equation 1) together with the musculotendon dynamics (Equation 2) by feeding the system with muscle activation levels (i.e. a(t), see Figure 1) that can approximate a simulation implemented by Pandy *et al.*. Activation of the muscles follow a proximal-distal sequence as suggested by Pandy *et al.*. First, at second of six equidistant time intervals, HAMS is activated, then for the last four time interval, VAS is activated. SOL and OPF are activated for the last half of the simulation. Finally, GAS is activated.

Our simulation result can be seen in the provided video (Supplementary.zip/Jump.avi). The jump height calculated by the Eq. 15, and the result is ~17 cm, which shows us that the neural signals we provided are not actually the optimum ones.

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