Mooring System Analysis of an Offshore Structure: Analyzing the Effect of Changing the Mooring Parameters on the Effective Environmental Force Evaluated to Move the Anchor.

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Abstract

The environmental load on an offshore platform required to move an anchor depends on a number of parameters. The value of this had in the past been gotten through iteration and interpolation, however, this research work presents perhaps a more accurate method. The research work analyzes the effects of the variation of each of these parameters independent of the others, on the environmental load to move the anchor. A mathematical model (zero equation) is set up using the catenary line equations. The zero of this equation is found using the MATLAB fzero command. A MATLAB script Env_Load.m is developed to evaluate the values of the environmental load upon the variation of mooring parameters.

Keywords: Mooring system, catenary line equations

1.0 Introduction

A mooring system is normally required to provide sufficient restoring loads to avoid large-amplitude horizontal motions so that the floating structures can remain in an acceptable position when encountering ocean waves and current [1]. As a critical component, the mooring system guarantees the safety of the working condition of the vessel. For the water depths of up to 1000m, the most common mooring system is the catenary mooring system, which consists of a group of lines combined of chain and wire rope. For exploration and production in water depth beyond 1000m, the weight of the mooring line starts to become a limiting factor in the design of the floating system, and then the taut leg mooring system comes forth, which adopts synthetic polymeric ropes as the main section of the lines.[2]

The performance of any mooring system is dependent on some factors, which are;

- Anchor weight
- Competence of the mooring lines
- Operational water depth
- Environmental forces
- Seabed condition

These factors must be in harmonious agreement for the mooring system to harness its full potentials against environmental load. The environmental load comprises of the wind force, the current force and the wave drift force. This research shows a new technique in determining the environmental load to move the anchor using the catenary line equations. This is advantageous in cases where the environmental conditions are not easily predictable. The paper will focus on the catenary mooring systems with the following objectives:

- To form a mathematical model from the relating catenary line equations for determining the accurate value of the environmental load on the platform to move the anchor.
- To utilize computer programming (MATLAB) in the determination of the environmental load.
- To vary the mooring parameters: the length, water depth, pre-tension and wet weight of the anchor; and analyze the effects of varying these values to the environmental load on the platform needed to move the anchor.

In the past, the only way to carry out accurate estimates on ship movement and the loads acting on mooring ropes was by performing costly tests with scale models. In recent decades, numerical methods based on simplifications have become available due to increased calculation power. Moreover, it has become possible to develop mathematical models suitable for calculating moored ship motion [3]. Some past studies, for example Morrison et al. [4], have used iterative methods in the estimation of the environmental load on the platform to move the anchor, however these methods cannot produce the exact answer due to the interpolation needed between iterations. Take for instance if for the function below, one is required to find the value of x for the function to be zero.

$$f(x) = 4x^2 - 6x \qquad (1)$$

Performing iterations for $x = 2$ and $x = 1$, the values of $f(x)$ are gotten as $f(2) = 4$ and $f(1) = -2$. The change of sign indicates that the value of x would fall between 1 and 2. Interpolating to get this value,

$$\begin{array}{ccc} x & & f(x) \\ 2 & \longrightarrow & 4 \\ ? & \longrightarrow & 0 \\ 1 & \longrightarrow & -2 \end{array}$$

$$x = (0 + 2)\frac{2 - 1}{4 - (-2)} + 1 = 1.33 \qquad (2)$$

The value of x (1.33) is not as accurate as it should be (1.5). However, using the factorization method,

$$0 = 4x^2 - 6x \qquad (3)$$

The values of x are gotten as 0 and 1.5, which is the most accurate. This illustration was done to show the advantages of a direct approach to the iterative method, for the accuracy of the iterative method depends on the width of iteration. This research work introduces a direct approach for the estimation of the environmental load with the use of the MATLAB command fzero, also vary the mooring parameters to analyze the effect of variation on the environmental load. The calculations are made in accordance with the classification society, Det Norske Veritas.

2.0 Materials and methods

Any method for defining design requisites has to envisage how the mooring system is arranged. In other words, it has to take into account the elasticity of the mooring lines and how the moored ship is subject to the action of the wind, currents and forces of the waves. In this way, the designer can choose and position mooring equipment and fittings on board and along the quay [3]. In catenary mooring design concept, load characteristics for a single line and a spread mooring are established. Ignoring fluid forces on the lines, the loading mechanism is presented in the equation below,

$$F_{environ} = F_{wind} + F_{wave} + F_{current} \qquad (4)$$

Rather than to estimate the individual constituents of the environmental load, this research work will use the catenary line equations to determine the minimum environmental load to move the anchor.

2.1 Catenary line equations

A sketch of a catenary mooring line model in two dimensions is presented in the figure below for better illustration. Detailed discussion on the catenary model can be found in Faltinsen [5].

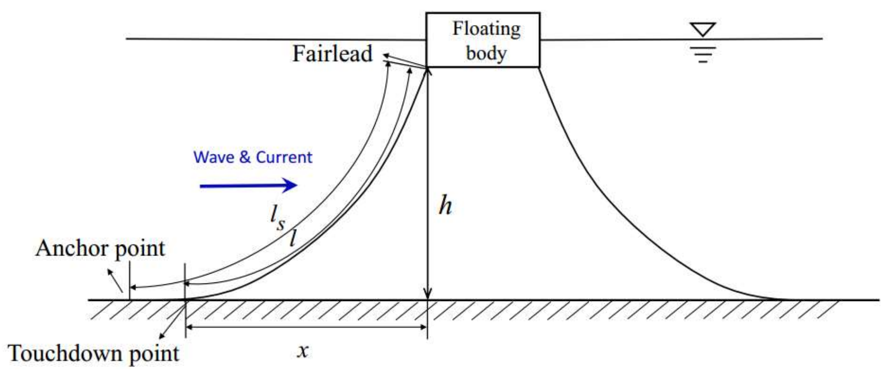

Figure 1: Sketch of catenary mooring line model in the 2D plane. [1]

Where L and Ls are the length of the mooring line from the fairlead to the anchor point and from the fairlead to the touchdown point, respectively. x is the horizontal distance between the fairlead and the touchdown point.

From the catenary line equations:

Suspended line length,

$$L_s = a \sinh\left(\frac{x}{a}\right) \qquad (5)$$

Vertical dimension (depth)

$$h = a\left[\cosh\left(\frac{x}{a}\right) - 1\right] \qquad (6)$$

combining equation (5) & (6)

$$L_s{}^2 = h^2 + 2ah \qquad (7)$$

$$L_s = \sqrt{h^2 + 2ah} \qquad (8)$$

Where,

T_H=Horizontal pre-tension (Horizontal restoring force applied by the mooring lines).

w=weight per unit length of chain cable in water

a= horizontal dimension

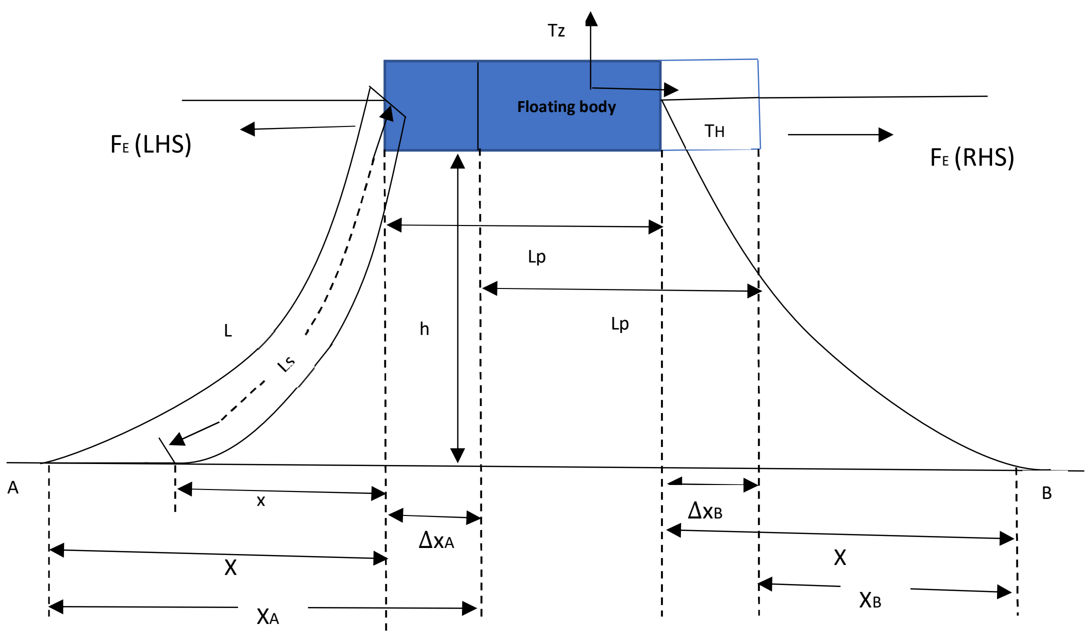

Figure 2: Environmental force on work barge to move anchor A.

$$a = \frac{T_H}{w} \qquad (9)$$

Therefore,

$$L_s = h\sqrt{1 + \frac{2T_H}{wh}} \qquad (10)$$

Maximum tension at the top,

$$T_{max} = T_H + wh \qquad (11)$$

The requirement is that $T_{max} \leq T_{br}$ (breaking strength/tension in mooring lines)

$$x = a\,cosh^{-1}\left(1 + \frac{h}{a}\right) \qquad (12)$$

Therefore,

$$x = \frac{T_H}{w} cosh^{-1}\left(1 + \frac{wh}{T_H}\right) \quad (13)$$

Considering the horizontal distance X between anchor point A and the point where the lines are connected to the vessel.

$$X = L - L_s + x \quad (14)$$

Combining the above equations,

$$X = L - h\sqrt{1 + \frac{2T_H}{wh}} + \frac{T_H}{w} cosh^{-1}\left(1 + \frac{wh}{T_H}\right) \quad (15)$$

From Fig 2, Δxa represents the increase in X_A due to movement as a result of the environmental force (Right Hand Side). While Δxb represents the decrease in X_B due to movement of platform in response to RHS environmental force to pull anchor A. The environmental force on the platform to move anchor A is a right-hand side pulling force, $F_E(RHS)$; movement of the platform stretches the touchdown point until the length of the catenary is fully extended at anchor A. As the touchdown length approximates to the full length of the catenary, the platform is equally displaced forward by an amount Δxa which is equal to a reduction Δxb towards anchor B (i.e., $\Delta xa = \Delta xb$). [4]

The minimum environmental force, F_E, required to pull the anchor is obtained at $L_{sA} \cong L$. Thus, the values of the new properties of the platform can be evaluated; considering anchor A,

$$2a_A h = {L_s}^2 - h^2 \quad (15)$$

$$a_A = \frac{{L_s}^2 - h^2}{2h} \quad (16)$$

$$T_{HA} = wa_A \quad (17)$$

$$x_A = \frac{T_{HA}}{w} cosh^{-1}\left(1 + \frac{wh}{T_{HA}}\right) \quad (18)$$

$$X_A = L - L_{sA} + x_A \quad (19)$$

Therefore,

$$X_A = \frac{T_{HA}}{w} cosh^{-1}\left(1 + \frac{wh}{T_{HA}}\right) \quad (20)$$

$$\Delta xa = X_A - X \quad (21)$$

$$X_B = X - \Delta xb = X - \Delta xa \quad (22)$$

Considering the anchor B,

$$X_B = L - L_{sB} + x_B \quad (23)$$

$$X_B = L - h\sqrt{1 + \frac{2T_{HB}}{wh}} + \frac{T_{HB}}{w} cosh^{-1}\left(1 + \frac{wh}{T_{HB}}\right) \quad (24)$$

Hence the Zero equation,

$$L - X_B - h\sqrt{1 + \frac{2T_{HB}}{wh}} + \frac{T_{HB}}{w}\, cosh^{-1}\left(1 + \frac{wh}{T_{HB}}\right) = 0 \qquad (25)$$

Then, the effective environmental force on the platform to move anchor A will be the difference between the initial pretension at A (T_{HA}); and the tension at B (T_{HB}) due to the reduced length in X_B. To achieve this, we shall use the following equation.

$$F_E = T_{HA} - T_{HB} \qquad (26)$$

2.2 Utilizing MATLAB for the Evaluation of the Zero equation

The MATLAB fzero is used to find the root of non-linear functions. The statement fzero (fun, x0) tries to find a point x where fun = 0 beside point x0 [6]. In view of equation (25), the function to find the horizontal tension at B becomes,

```
fun_TH_B=@(TH_B) L-h*sqrt(1+((2*TH_B)/(w*h)))+(TH_B/w)*...
                    acosh(1+((w*h)/TH_B))-X_B
```

2.3 Variation of Mooring Parameters

To determine the effect of variation of mooring parameters on the environmental load, increments of 10% are added to individual parameters while holding others constant. This might seem to be an arduous task, but with the MATLAB code Env_Load.m (Appendix II) developed for this research, it can be done in split seconds. The parameters to be varied are the length, wet weight of anchor, horizontal tension and water depth.

3.0 Analysis and Discussion of Results

Design Parameter and environmental wave data:

Length of Anchors Chain (L) = 1896m

Number of Anchors used = 4 Anchors

Horizontal pretension TH = 900KN

Average Weight of Anchor in water (w) = 1100N/m

Height of Water depth (h) = 1000m

Length of Work Barge Lp = 80.6m

Breadth of Work Barge Bp = 16m

Height of Work Barge Hp = 7.5m

Using the given data on the MATLAB code Env_Load.m (Appendix II), the results in Appendix III were obtained for Length_vs_EnvLoad, weight_vs_EnvLoad, Pretension_vs_EnvLoad and Depth_vs_EnvLoad. Plotting these values for analysis:

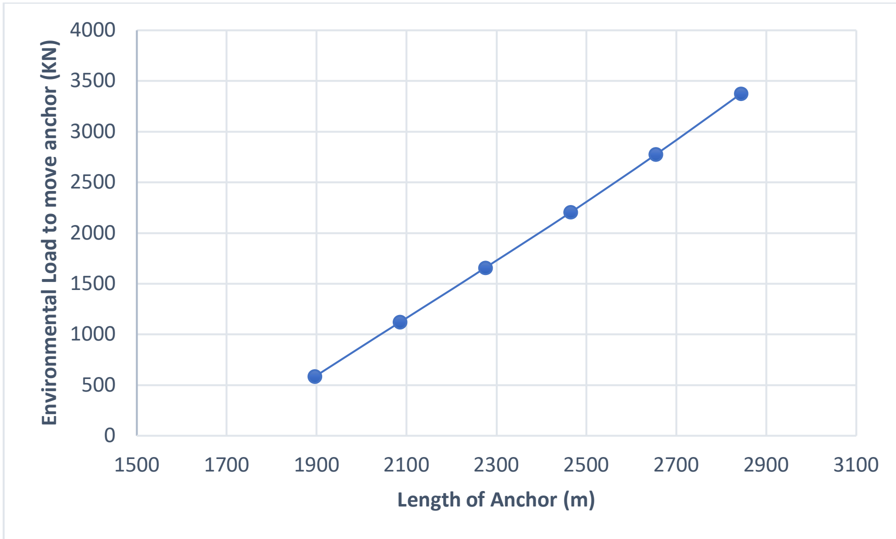

Figure 3: Plot showing the effect of increasing the length of the anchor (while holding other parameters constant) on the environmental force on the platform to move the anchor A.

The plot in Figure 3 depicts a positive slope, meaning that an increase in length will bring about an increase in the environmental load on the platform required to move the anchor. However, increasing the length of the anchor may not be most appropriate for a particular mooring system design as it may come with some disadvantages, for example in cost, durability etc. Consideration has to be made when taking this step as examinations are to be carried out on its effects on the general mooring system design.

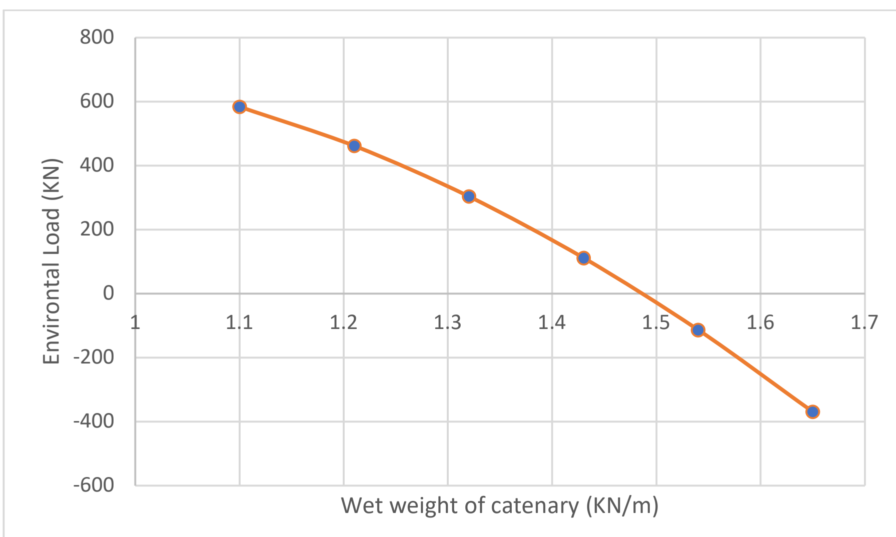

Figure 4: Plot showing the effect of increasing the wet weight of the catenary (while holding other parameters constant) on the environmental force on the platform to move the anchor A.

The plot in Figure 4 depicts a negative slope, meaning that an increase in the weight of the catenary in water will bring about a decrease in the environmental load on the platform required to move the anchor A. Hence, the more the weight of the catenary in water, the lower environmental force required to pull out the anchor; that is to say, increasing the wet weight of the catenary will reduce the capacity of the mooring system. It can be noted from figure 4 that the environmental load turned negative at some values of the catenary weight, this signifies an unsafe system at those values (while all other parameters constant).

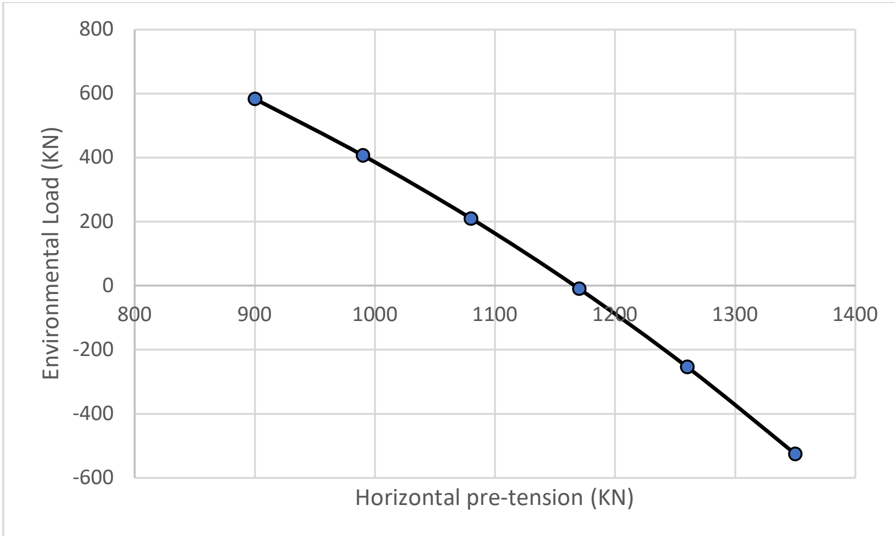

Figure 5: Plot showing the effect of increasing the horizontal tension (while holding other parameters constant) on the environmental force on the platform to move the anchor A.

The plot depicts a negative slope, meaning that an increase in the horizontal pre-tension will bring about an increase in the environmental load on the platform required to move the anchor A. Hence, the more the horizontal pre-tension, the lower the environmental force required to pull out the anchor. Which implies that increasing the horizontal tension on the catenary mooring line will reduce the capacity of the mooring system. It can be noted from figure 5 that the environmental load turned negative for some values of the horizontal pre-tension, this signifies an unsafe system at those values (while holding all other parameters constant).

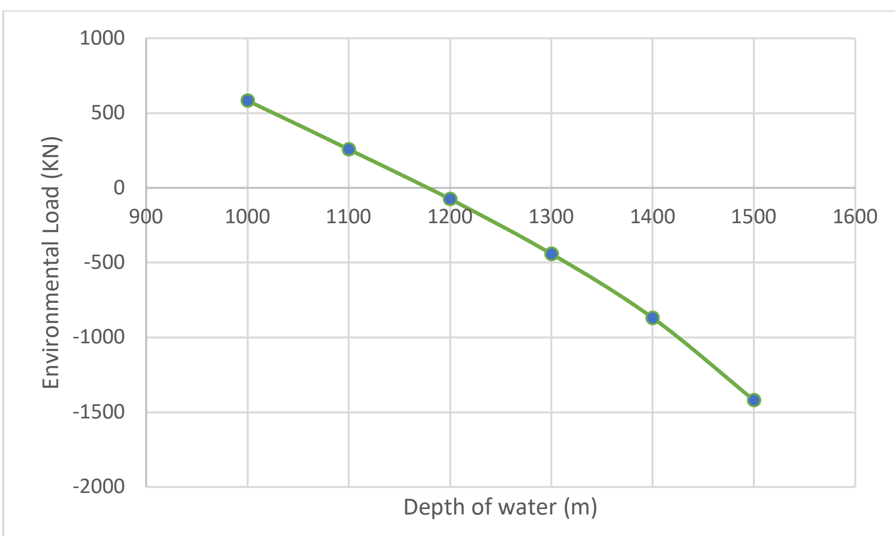

Figure 6: Plot showing the effect of increasing the depth of water (while holding other parameters constant) on the environmental force on the platform to move the anchor.

The plot depicts a negative slope, meaning that an increase in the depth of water will bring about a decrease in the environmental load on the platform required to move the anchor. Hence, using the same mooring system design for another system with a lower depth (i.e., higher distance from vessel to sea bed) will reduce the capacity of the system. It can be noted from figure 6 that the environmental load turned negative at some depth, this signifies an unsafe system at those values (while holding all other parameters constant).

4.0 Conclusion

This research work has been able to form a mathematical model from the catenary line equations, utilizing the model along with MATLAB in the determination of the environmental loads at varied cases. The variation of mooring parameters enabled the analysis of the effects of increasing a certain parameter on the environmental load. However, the variation done in this research was for only a single variable per analysis, that is to say, changing one parameter while holding the others constant. Hence, the results give no information on the right mixture of values of parameters in a mooring system design, but paves a way for the next phase of this design which is the optimization of the mooring system design through non-linear programming. In this phase, all parameters would be varied (except for the design water depth) within a region bounded by constraints e.g., minimum length, maximum tension etc.; to maximize the environmental load on the platform to pull the anchor A.

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Appendix I: Main particulars of the platform [2]

Length (m)	80.6
Breadth of pontoon (m)	16
Height of pontoon (m)	7.5
Diameter of columns (m)	12.9
Spacing of columns (m) (centre to centre)	54.72
Displacement (m3)	23548
Distance between pontoons (m)	1.17
Vertical position of CG (above BL) (m)	14.9
Longitudinal coordinate of CG (forward midship) (m)	0
Radius of inertia for pitch (m)	30
Coordinate of Fairlead 1 (m)	(33.6, 29.0, 0)
Coordinate of Fairlead 2 (m)	(31.9, 32.9, 0)
Coordinate of Fairlead 3 (m)	(29.0, 33.6, 0)

CG, centre of gravity; BL, base

Appendix II: the MATLAB code Env_Load.m

```
% to evaluate the effect of change in individual mooring particulars on the
permissible environmental load
format shortG % setting the format style of the output.
L1= 1896; % initial length of anchor in metres before increment
w1= 1.1; % initial weight in water of anchor in kilonewton/metre before
increment
TH1= 900; % initial horizontal pretension in kilonewton before increment
h1= 1000; % initial height of water depth in metres before increment
L=[L1:0.1*L1:1.5*L1 L1*ones(1,6) L1*ones(1,6) L1*ones(1,6)];
w=[w1*ones(1,6) w1:0.1*w1:1.5*w1 w1*ones(1,6) w1*ones(1,6)];
TH=[TH1*ones(1,6) TH1*ones(1,6) TH1:0.1*TH1:1.5*TH1 TH1*ones(1,6)];
h=[ h1*ones(1,6) h1*ones(1,6) h1*ones(1,6) h1:0.1*h1:1.5*h1];
% initialising values for X,Ls_A,a_A,TH_A,x_A,TH_B,Env_Load etc.
X=zeros(1,24);
Ls_A=zeros(1,24);
a_A=zeros(1,24);
TH_A=zeros(1,24);
X_A=zeros(1,24);
dx_A=zeros(1,24);
X_B=zeros(1,24);
TH_B=zeros(1,24);
EnvLoad=zeros(1,24);
for n=1:24
       X(n)=L(n)-h(n)*sqrt(1+((2*TH(n))/(w(n)*h(n))))+(TH(n)/w(n))*...
           acosh(1+((w(n)*h(n))/TH(n)));
       % extension of lines: Ls equivalent to L (for maximum allowable
environmental load)
       Ls_A(n)=L(n);
       a_A(n)=((Ls_A(n))^2-(h(n))^2)/(2*h(n));
       TH_A(n)=w(n)*a_A(n);
       X_A(n)=(TH_A(n)/w(n))*acosh(1+((w(n)+h(n))/TH_A(n)));
       dx_A(n)=X_A(n)-X(n);
       X_B(n)=X(n)-dx_A(n);
```

```
        % Objective function
        fun_TH_B=@(TH_B) L(n)-
h(n)*sqrt(1+((2*TH_B)/(w(n)*h(n))))+(TH_B/w(n))*...
          acosh(1+((w(n)*h(n))/TH_B))-X_B(n);
      TH_B(n)=fzero(fun_TH_B,1400);
      EnvLoad(n)=TH_A(n)-TH_B(n);
end
Length_vs_EnvLoad=transpose([L(1:6);EnvLoad(1:6)])
weight_vs_EnvLoad=transpose([w(7:12);EnvLoad(7:12)])
Pretension_vs_EnvLoad=transpose([TH(13:18);EnvLoad(13:18)])
Depth_vs_EnvLoad=transpose([h(19:24);EnvLoad(19:24)])
```

Appendix III: Results obtained running the script file Env_Load.m

```
>> Env_Load


Length_vs_EnvLoad =


         1896       583.52

       2085.6       1122.3

       2275.2       1656.1

       2464.8       2203.6

       2654.4       2774.3

         2844       3373.4



weight_vs_EnvLoad =


          1.1       583.52

         1.21       461.48

         1.32       303.72

         1.43        111.6

         1.54      -113.14

         1.65      -368.55



Pretension_vs_EnvLoad =


          900       583.52

          990       406.81
```

1080	209.94
1170	-9.2936
1260	-253.44
1350	-525.43

Depth_vs_EnvLoad =

1000	583.52
1100	258.57
1200	-75.178
1300	-439.94
1400	-868.73
1500	-1420.1