ECE 220-203

Lab 6

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12/2/19

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Objective

The objective of this lab is to learn more about CT filters by designing and applying them in MATLAB as well as understand the relationship between the Laplace and Fourier domains in CT systems.

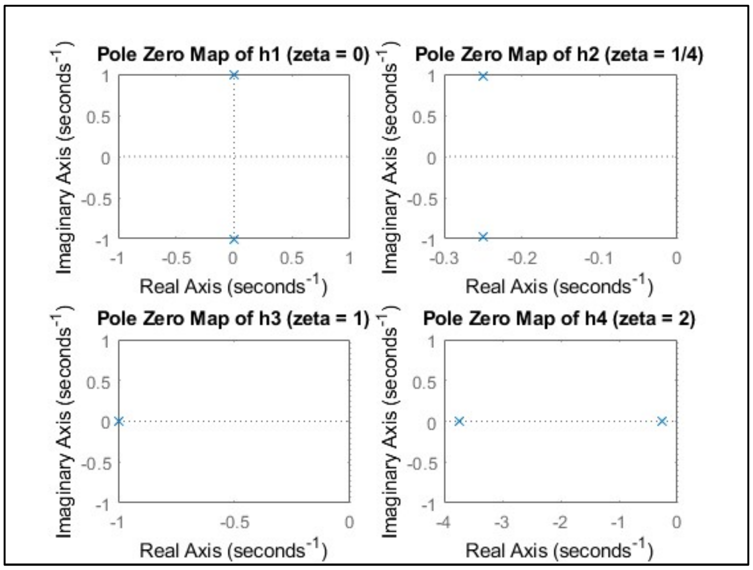
MATLAB Code

See Appendix A.

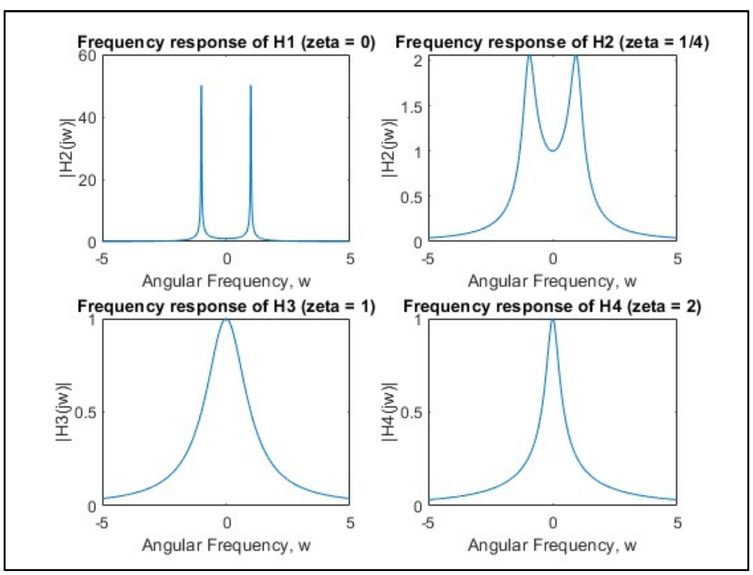
Results

Part I:

1.



2.



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The frequency response at $\omega = 0$ is the same for all systems because it is the same system, just damped with different dampening ratios.

Part II:

1.

$$1 + \left(\frac{s}{j\omega_c}\right)^{2N} = 0$$

$$-1 = \left(\frac{s}{j\omega_c}\right)^{2N}$$

$$e^{j(\pi + 2\pi k)} = \left(\frac{s}{j\omega_c}\right)^{2N}$$

$$\frac{s}{j\omega_c} = \left(e^{j(\pi + 2\pi k)}\right)^{\frac{1}{2N}}$$

$$s = j\omega_c \left(e^{j(\pi + 2\pi k)}\right)^{\frac{1}{2N}}$$

$$s = j\omega_c e^{\frac{j(\pi + 2\pi k)}{2N}}$$

2.

$$s = j\omega_c e^{\frac{j(\pi + 2\pi k)}{2}} \quad ; N = 1$$

$$s_1 = j\omega_c e^{\frac{j(\pi + 2\pi)}{2}} \quad ; k = 1$$

$$= j\omega_c e^{j\frac{3\pi}{2}} = \omega_c$$

$$s_2 = j\omega_c e^{\frac{j(\pi + 4\pi)}{2}} \quad ; k = 2$$

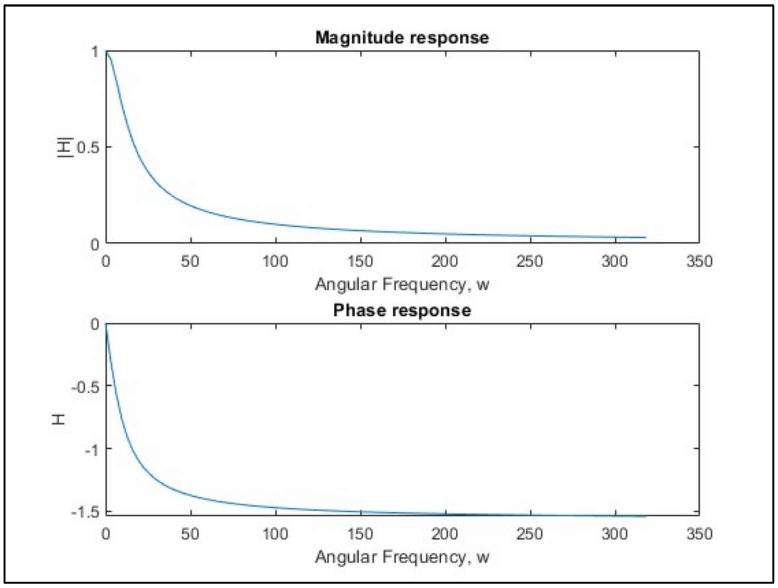
$$= j\omega_c e^{j\frac{5\pi}{2}} = -\omega_c$$

$\boxed{s = -\omega_c}$ because the pole for a causal and stable must be to the left of the $j\omega$ axis.

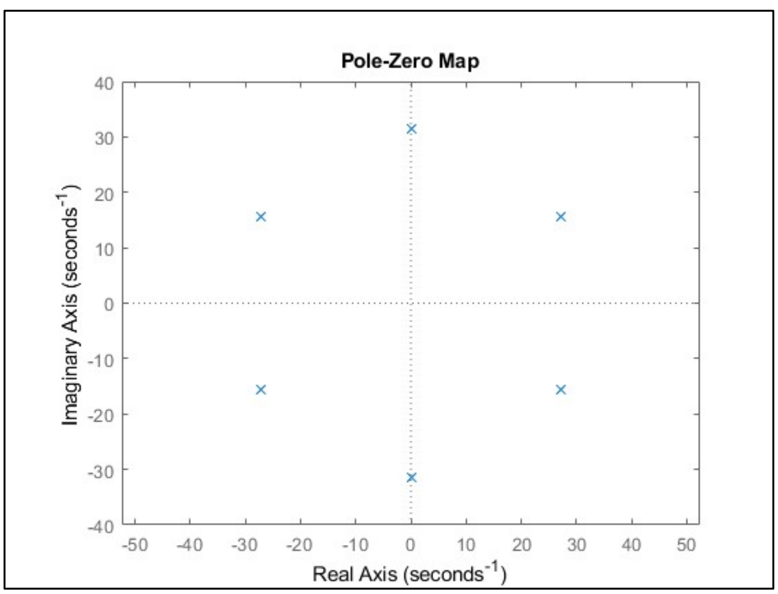
$$B(s) = \frac{k}{s+\omega_c} \qquad B(0) = 1 = \frac{k}{\omega_c}$$

$$= \boxed{\frac{\omega_c}{s+\omega_c}} \qquad k = \omega_c$$

3.



4.



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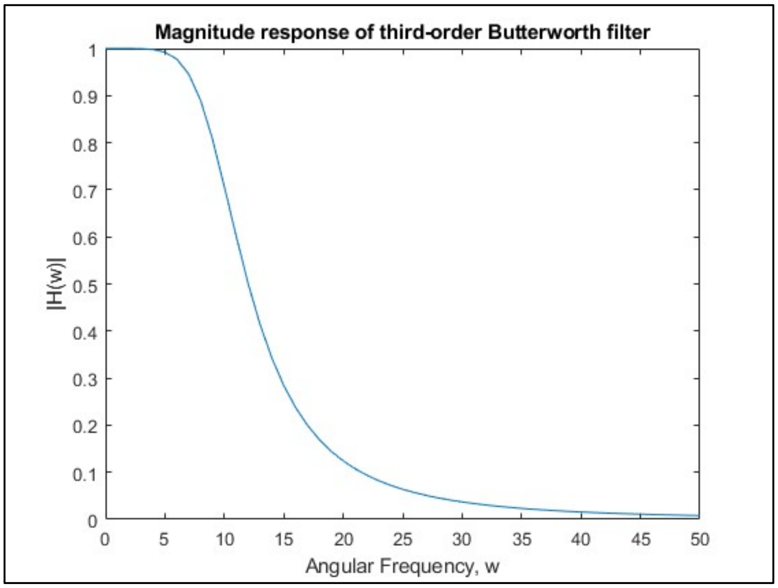
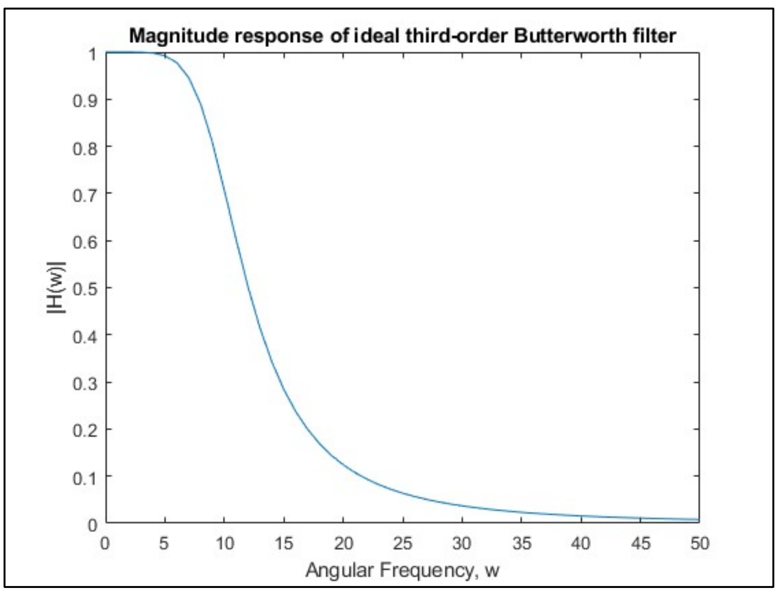
5. $b = 31007$
6.



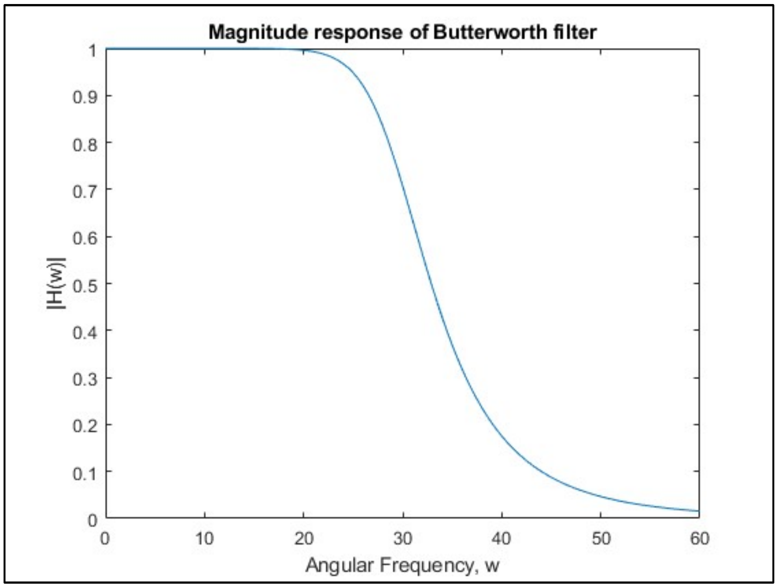
7.



8.

$$N = \frac{\dfrac{log_{10}\left(\dfrac{\frac{1}{0.001^2} - 1}{\frac{1}{0.99^2} - 1}\right)}{\log_{10}\left(\frac{50}{10}\right)}}{2} \approx 6$$

10.



$\omega(0) = 1$

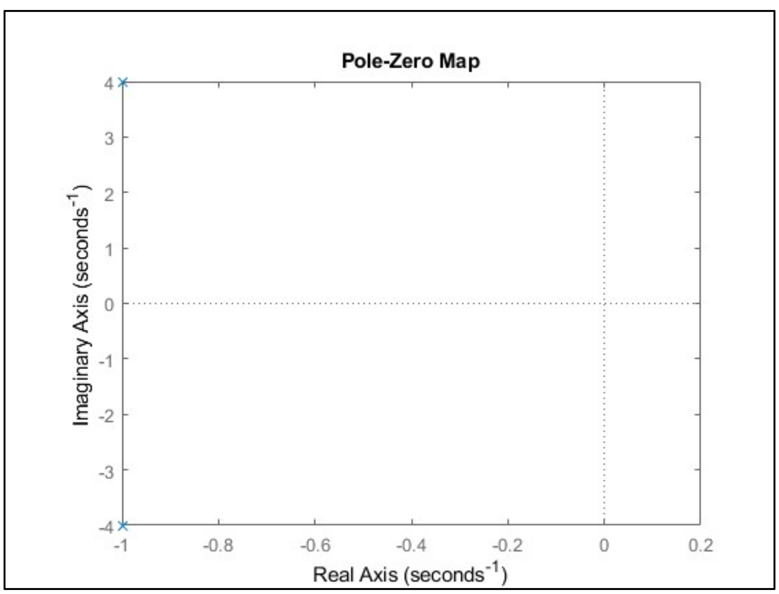
$\omega(10) = 1$

$\omega(50) = 0.0466$

My filter meets the specifications given.

Part III:

1.



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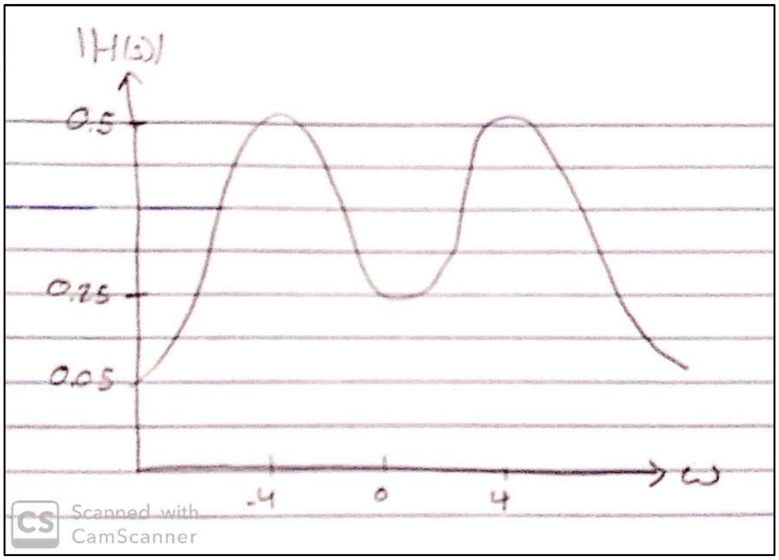
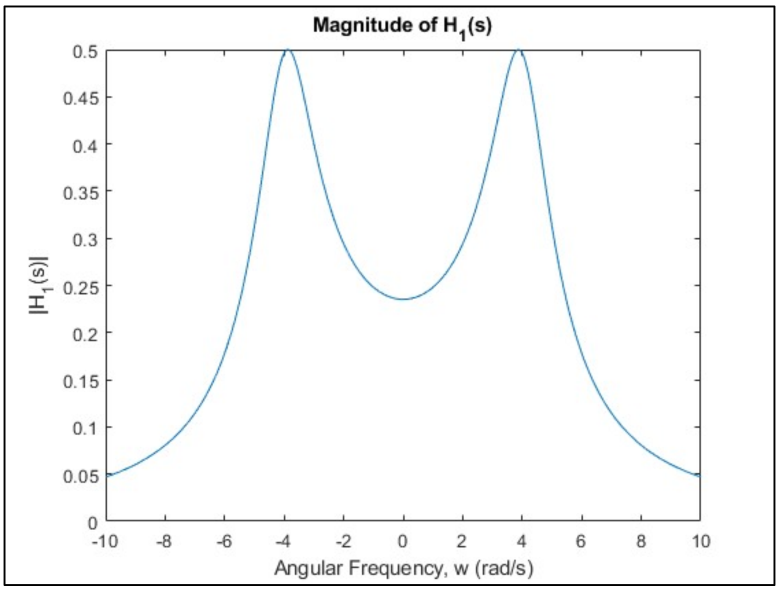
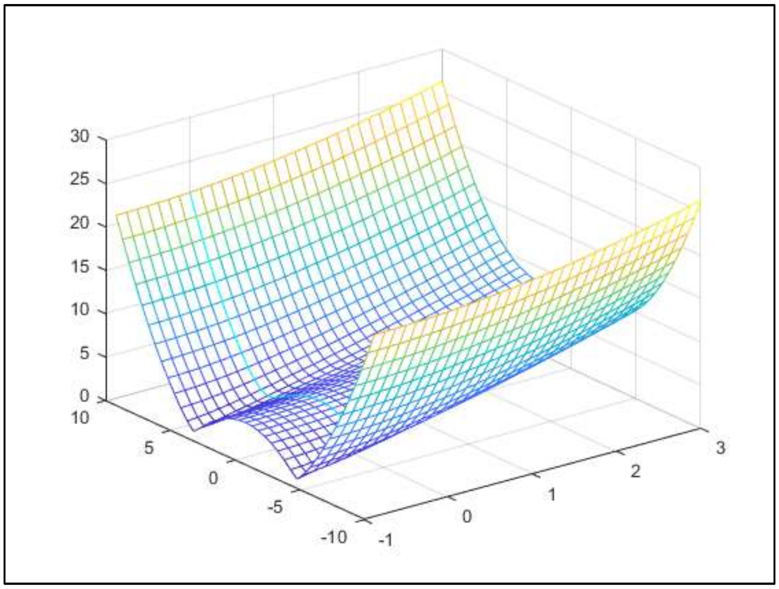
2.



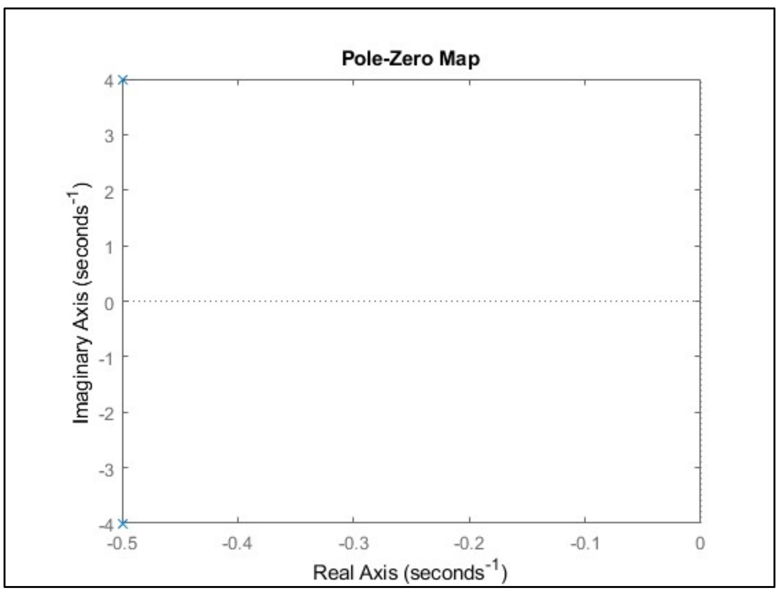


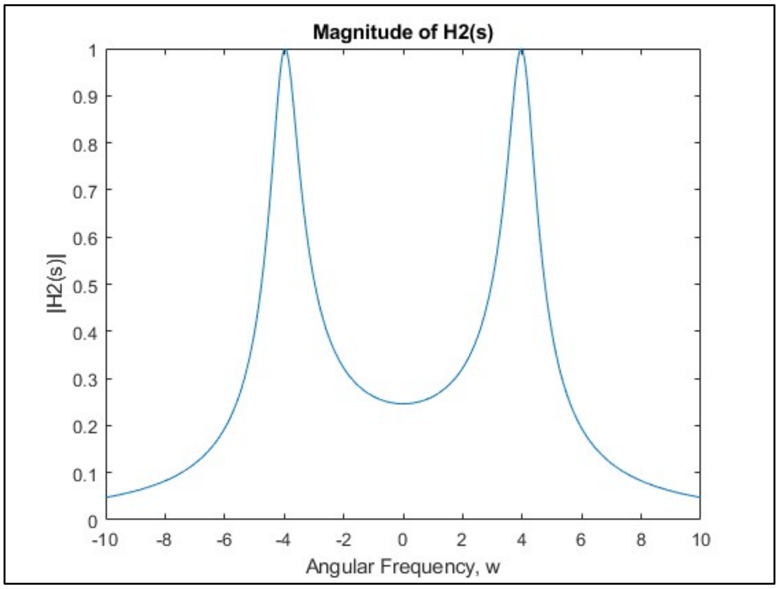
The plots of the analytical and MATLAB magnitude responses match.

3.

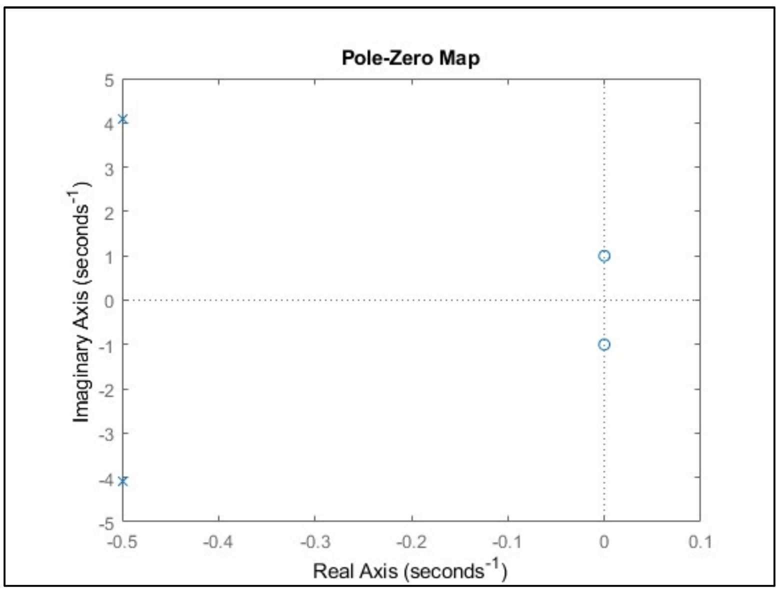

30
25
20
15
10
5
0
10
5
0
-5
-10
-1
0
1
2
3

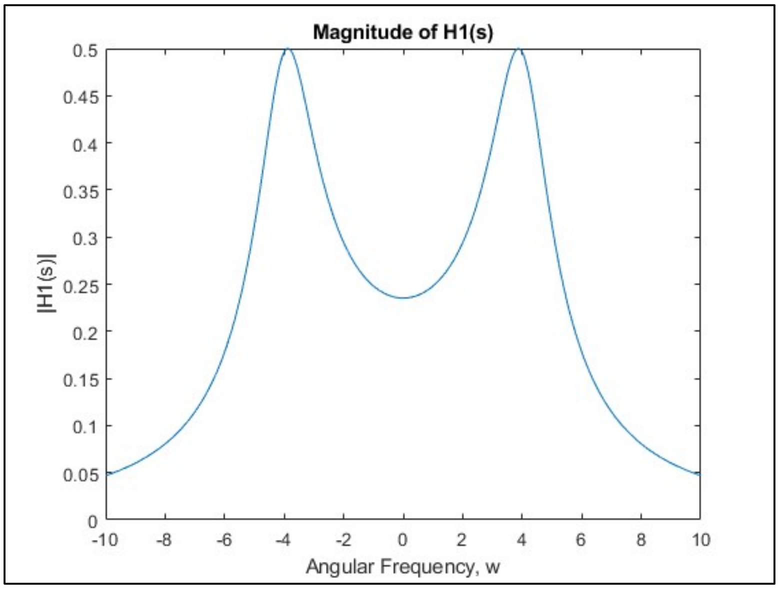
4.

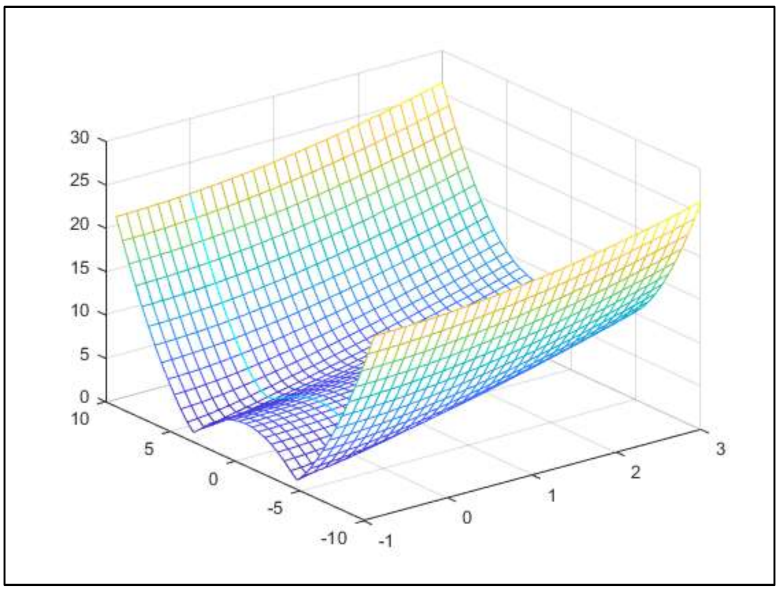

Pole-Zero Map
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})
4
3
2
1
0
-1
-2
-3
-4
-0.5
-0.4
-0.3
-0.2
-0.1
0


Magnitude of H2(s)
|H2(s)|
Angular Frequency, w
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-10
-8
-6
-4
-2
0
2
4
6
8
10

5.


Pole-Zero Map
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})
5
4
3
2
1
0
-1
-2
-3
-4
-5
-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1





Adding the extra zeros did not noticeably affect the Laplace transform.

Conclusion

This lab contained many difficulties, most of them in Part I and some in Part II. As for Part I, I know my analytical and MATLAB work was correct, but I could not figure out questions 3 and onward. Part II was completed without difficulty and it was interesting to see how a Butterworth filter could be created theoretically before made into a circuit. Part III contained more difficulties such that I could not finish question 6. The previous questions in Part III were completed without issue and I'm confident that my answers for them are correct. Besides the aforementioned issues, I believe my results to be accurate and found no errors when executing my MATLAB script.

Appendix A.

```
%% Part I
%1
figure(1)
wn = 1;
zeta = [0, 1/4, 1, 2];

subplot(2,2,1);
x = 1;
h(x) = tf(wn^2,[1, 2*zeta(x)*wn, wn^2]);
pzmap(h(x)), title('Pole Zero Map of h1 (zeta = 0)');

subplot(2,2,2);
x = 2;
h(x) = tf(wn^2,[1, 2*zeta(x)*wn, wn^2]);
pzmap(h(x)), title('Pole Zero Map of h2 (zeta = 1/4)');

subplot(2,2,3);
x = 3;
h(x) = tf(wn^2,[1, 2*zeta(x)*wn, wn^2]);
pzmap(h(x)), title('Pole Zero Map of h3 (zeta = 1)');

subplot(2,2,4);
x = 4;  %switches between h1-h4
h(x) = tf(wn^2,[1, 2*zeta(x)*wn, wn^2]);
pzmap(h(x)), title('Pole Zero Map of h4 (zeta = 2)');

%2
figure(2)
omega = -5:0.01:5;

subplot(2,2,1);
x = 1;
[H, omega] = freqs(wn^2,[1, 2*zeta(x)*wn, wn^2], omega);
plot(omega, abs(H)), title('Frequency response of H1 (zeta = 0)');
xlabel('Angular Frequency, w');
ylabel('|H2(jw)|');

subplot(2,2,2);
x = 2;
[H, omega] = freqs(wn^2,[1, 2*zeta(x)*wn, wn^2], omega);
plot(omega, abs(H)), title('Frequency response of H2 (zeta = 1/4)');
xlabel('Angular Frequency, w');
ylabel('|H2(jw)|');

subplot(2,2,3);
x = 3;
[H, omega] = freqs(wn^2,[1, 2*zeta(x)*wn, wn^2], omega);
plot(omega, abs(H)), title('Frequency response of H3 (zeta = 1)');
xlabel('Angular Frequency, w');
ylabel('|H3(jw)|');

subplot(2,2,4);
x = 4;
[H, omega] = freqs(wn^2,[1, 2*zeta(x)*wn, wn^2], omega);
plot(omega, abs(H)), title('Frequency response of H4 (zeta = 2)');
```

```
xlabel('Angular Frequency, w');
ylabel('|H4(jw)|');
%% Part II
%3
figure(1)
wc = 10*pi;
N = 1;
w = linspace(0,1000);
H = freqs(wc,[1 wc],w);
subplot(211)
plot(w/pi,abs(H));
xlabel('Angular Frequency, w')
ylabel('|H|')
title('Magnitude response')

subplot(212)
plot(w/pi,angle(H));
xlabel('Angular Frequency, w')
ylabel('H')
title('Phase response')

%4
figure(2)
s = tf('s');
N = 3;
H =  1/(1+(s/(wc))^(2*N));
p = pole(H);
pzmap(H);
axis('equal');

%5
csp = [ -1.000 + 0.000i, -0.500 - 0.866i, -0.500 + 0.866i].*wc;
a = poly(csp);
b = -csp(1)*csp(2)*csp(3)

%6
figure(3)
w = (0:1:50)*pi;
B3 = freqs(b,a,w);
plot(w/pi,abs(B3));
xlabel('Angular Frequency, w')
ylabel('|H(w)|')
title('Magnitude response of third-order Butterworth filter')

%7
figure(4)
[b,a] = butter(3,wc,'s');
plot(w/pi,abs(freqs(b,a,w)));
xlabel('Angular Frequency, w')
ylabel('|H(w)|')
title('Magnitude response of ideal third-order Butterworth filter')

%10
figure(5)
A1 = 0.99;
A2 = 0.001;
Rp = -20*log(1-0.02);
```

```
Rs = -20*log(0.001);
wp = 10;
ws = 50;
w  = 0:0.5:60;

N = round(0.5*(log10(((1/(A2^2))-1)/((1/(A1^2))-1)))/(log10(ws/wp)));
wc = (wp+ws)/2;
[b,a] = butter(N,wc,'s');
B4 = freqs(b,a,w);

plot(w,abs(B4))
xlabel('Angular Frequency, w');
ylabel('|H(w)|')
title('Magnitude response of Butterworth filter')
%% Part 3
%1
figure(1)
a1 = 4;
b1 = [1, 2, 17];

r1 = roots(b1);
sys1 = tf(a1,b1);
h = pzplot(sys1);

%2
figure(2)
omega = (-10:0.5:10);
[B4, omega] = freqs(a1, b1, omega);
plot(omega, abs(B4));
title('Magnitude of H1(s)');
xlabel('Angular Frequency, w');
ylabel('|H1(s)|');

%3
figure(3)
sigma=-1+(1/8)*(1:32);
[sigmagrid,omegagrid] = meshgrid(sigma,omega);
sgrid = sigmagrid+1i*omegagrid;

B4 = polyval(b1,sgrid)./polyval(a1,sgrid);
mesh(sigma,omega,abs(B4));

hold on;
plot3(zeros(1,41),omega,abs(B4(: ,8))+0.05,'c')
hold off;

%4
figure(4)
a2 = 4;
b2 = [1, 1, 16.25];

r2 = roots(b1);
sys2 = tf(a2,b2);
h = pzplot(sys2);

figure(5)
omega = (-10:0.01:10);
```

```
[B4, omega] = freqs(a2, b2, omega);
plot(omega, abs(B4));
title('Magnitude of H2(s)');
xlabel('Angular Frequency, w');
ylabel('|H2(s)|');

%5
figure(6)
a3 = [1/4, 0, 1/4];
b3 = [1, 1, 17];

r2 = roots(b3);
sys3 = tf(a3,b3);
h = pzplot(sys3);

figure(7)
omega = (-10:0.5:10);
[B4, omega] = freqs(a1, b1, omega);
plot(omega, abs(B4));
title('Magnitude of H1(s)');
xlabel('Angular Frequency, w');
ylabel('|H1(s)|');

figure(8)
sigma=-1+(1/8)*(1:32);
[sigmagrid,omegagrid] = meshgrid(sigma,omega);
sgrid = sigmagrid+1i*omegagrid;

B4 = polyval(b1,sgrid)./polyval(a1,sgrid);
mesh(sigma,omega,abs(B4));

hold on;
plot3(zeros(1,41),omega,abs(B4(: ,8))+0.05,'c')
hold off;
```