ECE 220-203

Lab 4

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10/27/19

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Objective

The objective of this lab is to get practice with Fourier series and how to display them in MATLAB.

MATLAB Code

See Appendix A.

Results

Part a1:

1) $\omega_0 = \frac{2\pi}{2} = \pi$

$$a_0 = \frac{1}{2}\left(\int_0^1 1\,dt\right)$$

$$= \frac{1}{2}\left(t\Big|_0^1\right) = \frac{1}{2}(1-0) = \frac{1}{2}$$

$$a_k = \frac{1}{2}\left(\int_0^1 1 e^{-jk\pi t}\right)$$

$$= \frac{-1}{2jk\pi}\left(e^{-jk\pi t}\Big|_0^1\right)$$

$$= \frac{1}{2jk\pi}\left(\underbrace{e^{-jk\pi}}_{}{}^{-1} - 1\right)$$

$$= \frac{1}{2jk\pi}\left((-1)^k - 1\right)$$

Part a2:

2) $x(t) = 2 + \cos\left(\frac{2\pi}{3}t\right) + 4\sin\left(\frac{5\pi}{3}t\right)$

$\omega_0 = \pi/3$

$= 2 + 1/2\, e^{j\frac{2\pi}{3}t} + 1/2\, e^{j\frac{2\pi}{3}t} - 2j\, e^{j\frac{5\pi}{3}t} + 2j\, e^{-j\frac{5\pi}{3}t}$

$a_0 = 2 \quad a_2 = 1/2 \quad a_5 = 2j \quad a_{-2} = 1/2 \quad a_{-5} = -2j$

Part a3:

3) $\omega_0 = \frac{2\pi}{2} = \pi$

$a_k = \frac{1}{2}\int_{-1}^{1} t e^{-jk\pi t}\, dt \qquad \int u v' = uv - \int u' v$

$= \frac{1}{2}\left(\frac{t e^{-jk\pi t}}{-jk\pi}\Big|_{-1}^{1} - \int_{-1}^{1} \frac{e^{-jk\pi t}}{-jk\pi}\, dt \right)$

$= \frac{1}{2}\left(\frac{t e^{-jk\pi t}}{-jk\pi}\Big|_{-1}^{1} - \frac{e^{-jk\pi t}}{(-jk\pi)^2}\Big|_{-1}^{1} \right)$

$= \frac{1}{2}\left(\frac{e^{-jk\pi} + e^{jk\pi}}{-jk\pi} - \frac{e^{-jk\pi} - e^{jk\pi}}{(-jk\pi)^2} \right)$

$= \frac{1}{2}\left(\frac{-1^k + -1^k}{-jk\pi} - \cancel{\frac{-1^k - -1^k}{(-jk\pi)^2}}^{\,0} \right)$

$= \frac{1}{2}\left(\frac{2 \cdot -1^k}{-jk\pi} \right) \Rightarrow a_k = \frac{-1^k}{-jk\pi}$

Part a4:

4) $w_0 = \frac{2\pi}{2} = \pi$

$$a_0 = \frac{1}{2}\left(\int_0^1 \delta(t)\,dt - \int_0^1 2\delta(t-1)\,dt\right)$$

$$= \frac{1}{2}(1-2) = -\frac{1}{2}$$

$$a_k = \frac{1}{2}\left(\int_0^1 \delta(t)e^{-jk\pi t}\,dt - \int_0^1 2\delta(t-1)e^{-jk\pi t}\,dt\right)$$

$$= \frac{1}{2}\left(e^{-jk\pi t}\Big|_0^1 - 2e^{-jk\pi t}\Big|_0^1\right)$$

$$= \frac{1}{2}\left(e^{-jk\pi t} - 1 - 2\left(e^{-jk\pi t} - 1\right)\right)$$

$$= \frac{1}{2}\left(1 - \overset{-1}{\cancel{e^{-jk\pi}}}\right)$$

$$= \frac{1}{2}\left(1-(-1)^k\right)$$

Part a5:

5) $\omega_0 = \frac{2\pi}{6} = \frac{\pi}{3}$

$$a_0 = \frac{1}{6}\left(\int_{-2}^{1} 1\,dt + \int_{1}^{2} -1\,dt\right)$$

$$= \frac{1}{6}\left(t\Big|_{-2}^{1} - t\Big|_{1}^{2}\right)$$

$$= \frac{1}{6}\left(-1-(-2)-(2-1)\right) = 0$$

$$a_k = \frac{1}{6}\left(\int_{-2}^{-1} 1 e^{-jk\frac{2\pi}{6}t}\,dt + \int_{1}^{2} -1 e^{-jk\frac{2\pi}{6}t}\,dt\right)$$

$$= \frac{1}{6}\left(\frac{e^{-jk\frac{\pi}{3}t}}{-jk\frac{\pi}{3}}\Bigg|_{-2}^{-1} - \frac{e^{-jk\frac{\pi}{3}t}}{-jk\frac{\pi}{3}}\Bigg|_{1}^{2}\right)$$

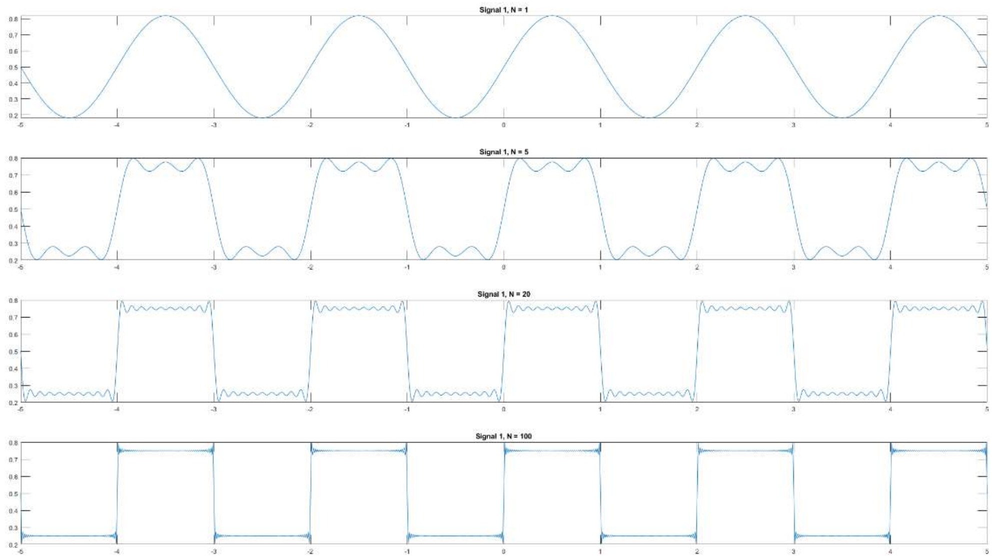
$$= \frac{1}{6}\left(\frac{e^{jk\frac{\pi}{3}} - e^{jk\frac{2\pi}{3}}}{-jk\frac{\pi}{3}} - \frac{e^{-jk\frac{2\pi}{3}} - e^{-jk\frac{\pi}{3}}}{-jk\frac{\pi}{3}}\right)$$

$$= \frac{1}{6}\,\frac{2\cos\left(\frac{2\pi k}{3}\right) - 2\cos\left(\frac{\pi k}{3}\right)}{jk\frac{\pi}{3}}$$

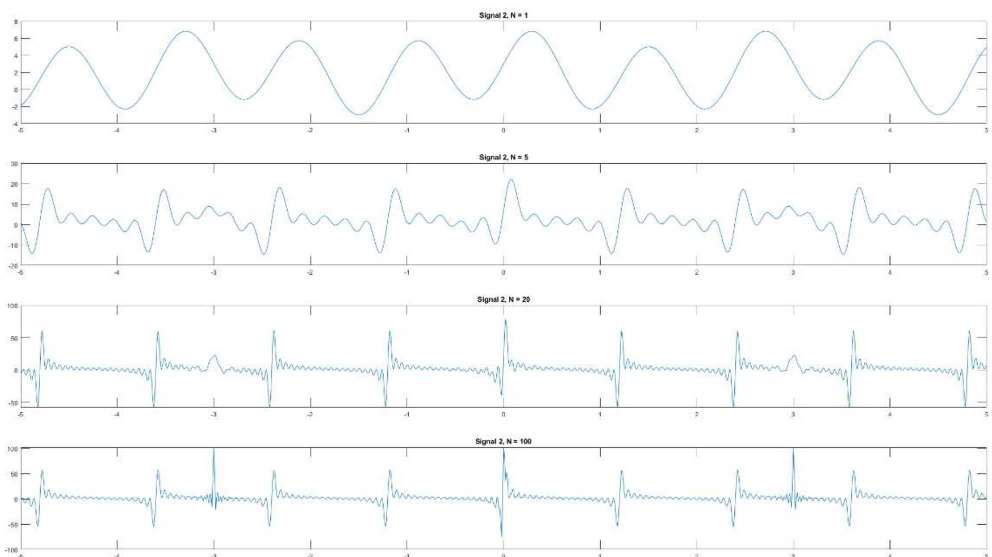
$$\Rightarrow a_k = \frac{\cos\left(\frac{2\pi k}{3}\right) - \cos\left(\frac{\pi k}{3}\right)}{jk\pi}$$

Part d:

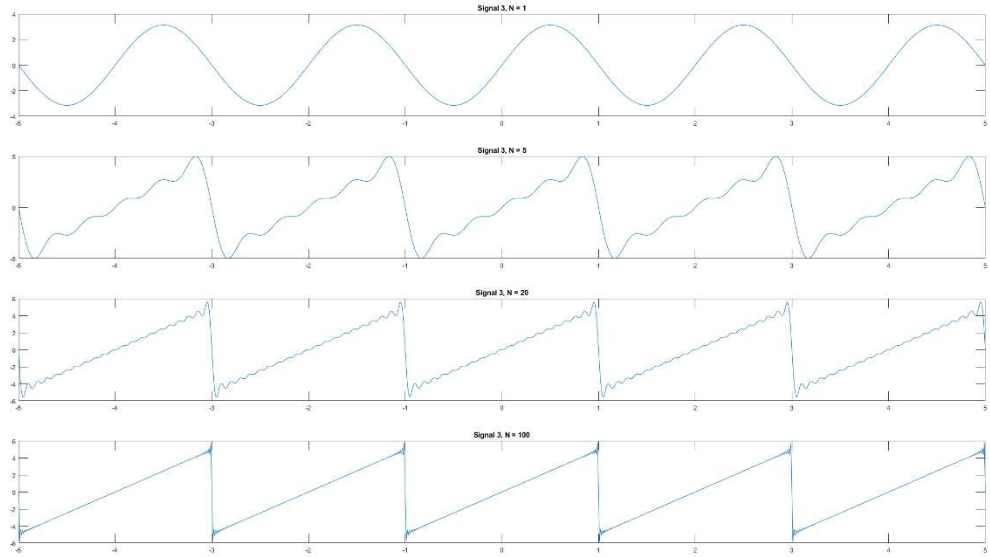
Signal 1:



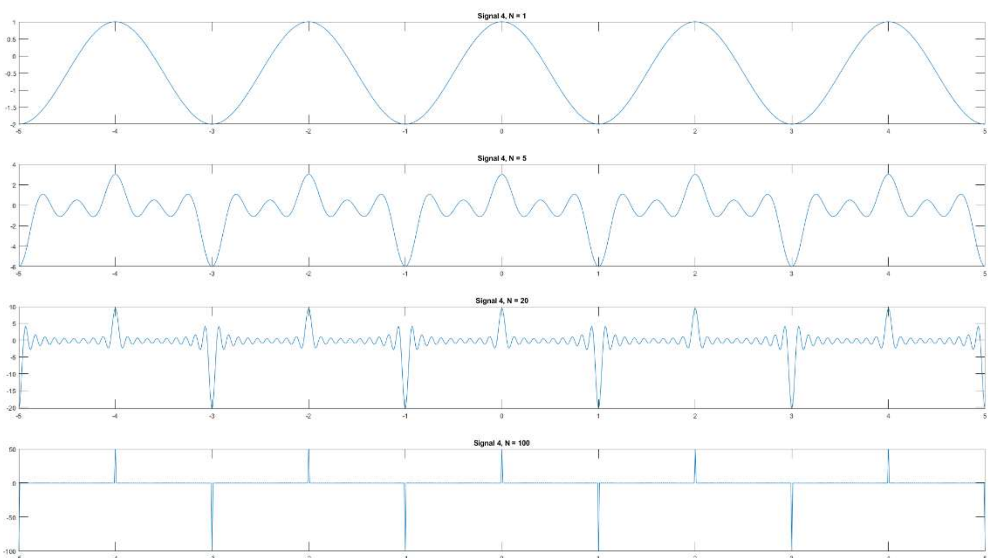
Signal 2:



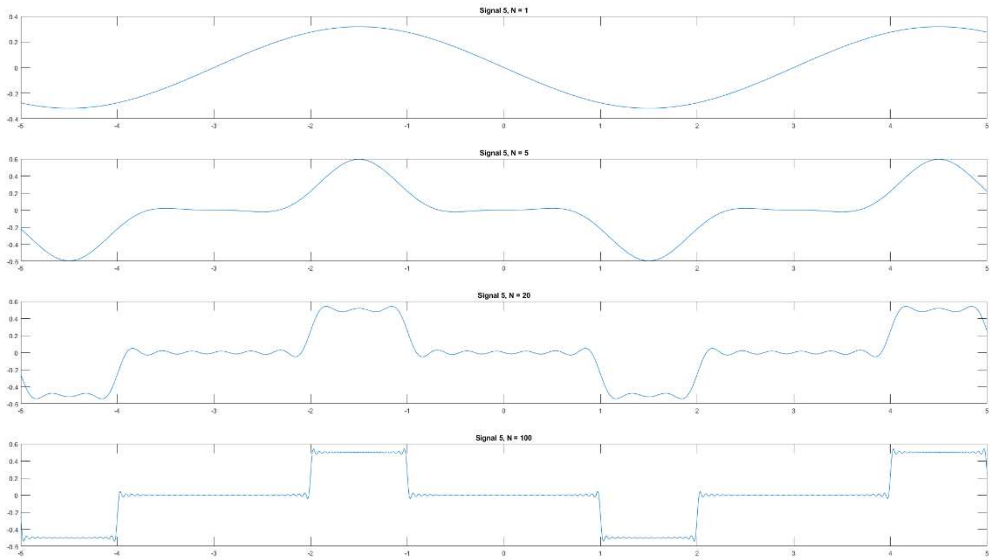
Signal 3:



Signal 4:



Signal 5:



Judging from the graphs, as N increases, the quality of representation of the original signal increases. As N increases, the MATLAB output becomes closer and closer to the original signal, but still has some fringes which you can see very easily on the square wave samples. This is known as Gibb's Phenomenon.

Part e:

You can see Gibb's Phenomenon in signals 1 (Square), 3 (Sawtooth), and 5 (Square). The reason it is possible to see the phenomenon during these signals is because there exists an immediate jump, or state change, like going immediately from amplitude 1 to amplitude 0 as is common in square and sawtooth waves. During these jumps, the summation of the Fourier series overshoots the immediate change in slope and causes more oscillation than what was intended.

Discussion/Conclusion

This lab went smoothly and was completed with few issues. The most difficult part of this lab was finding the a_k function in part a because of the decent possibility of human error. Past that, writing the MATLAB code was straightforward since we were given the function $x_{(N)}t$. It was interesting to see how Fourier series, with conjunction with harmonics, can (semi)-accurately portray the original signal. Although I knew about Gibb's phenomenon from lecture, it was cool to see how it slowly becomes more pronounced as the number of harmonics increase. I believe my results to be accurate and found no errors when writing my MATLAB script.

Appendix A

```
%%
% Signal 1
figure(1)
t = -5 : 0.01 : 5;
x1 = 1/2;
N = 1;
for k = 1 : N
    x1 = x1 + (-1/(1i*2*k*pi))*(exp(-1i*k*pi)-1)*exp(1i*k*pi*t);
end
subplot(411), plot(t, x1), title('Signal 1, N = 1');
x2 = 1/2;
N = 5;
for k = 1 : N
    x2 = x2 + (-1/(1i*2*k*pi))*(exp(-1i*k*pi)-1)*exp(1i*k*pi*t);
end
subplot(412), plot(t, x2), title('Signal 1, N = 5');
x3 = 1/2;
N = 20;
for k = 1 : N
    x3 = x3 + (-1/(1i*2*k*pi))*(exp(-1i*k*pi)-1)*exp(1i*k*pi*t);
end
subplot(413), plot(t, x3), title('Signal 1, N = 20');
x4 = 1/2;
N = 100;
for k = 1 : N
    x4 = x4 + (-1/(1i*2*k*pi))*(exp(-1i*k*pi)-1)*exp(1i*k*pi*t);
end
subplot(414), plot(t, x4), title('Signal 1, N = 100');
%%
% Signal 2
figure(2)
x1 = 2;
N = 1;
for k = 1 : N
    x1 = x1 + (1/2)*(exp(1i*(2*pi/3)*k*t) + exp(1i*(2*pi/3)*k*t)) +
(2/1i)*(exp(1i*(5*pi/3)*k*t) - exp(-1i*(5*pi/3)*k*t));
end
subplot(411), plot(t, x1), title('Signal 2, N = 1');
x2 = 2;
N = 5;
for k = 1 : N
    x2 = x2 + (1/2)*(exp(1i*(2*pi/3)*k*t) + exp(1i*(2*pi/3)*k*t)) +
(2/1i)*(exp(1i*(5*pi/3)*k*t) - exp(-1i*(5*pi/3)*k*t));
end
subplot(412), plot(t, x2), title('Signal 2, N = 5');
x3 = 2;
N = 20;
for k = 1 : N
    x3 = x3 + (1/2)*(exp(1i*(2*pi/3)*k*t) + exp(1i*(2*pi/3)*k*t)) +
(2/1i)*(exp(1i*(5*pi/3)*k*t) - exp(-1i*(5*pi/3)*k*t));
end
subplot(413), plot(t, x3), title('Signal 2, N = 20');
```

```
x4 = 2;
N = 100;
for k = 1 : N
    x4 = x4 + (1/2)*(exp(1i*(2*pi/3)*k*t) + exp(1i*(2*pi/3)*k*t)) +
(2/1i)*(exp(1i*(5*pi/3)*k*t) - exp(-1i*(5*pi/3)*k*t));
end
subplot(414), plot(t, x4), title('Signal 2, N = 100');
%%
% Signal 3
figure(3)
x1 = 0;
N = 1;
for k = 1 : N
    x1 = x1 + ((1i*(-1)^k)/k*pi)*exp(1i*k*pi*t);
end
subplot(411), plot(t, x1), title('Signal 3, N = 1');
x2 = 0;
N = 5;
for k = 1 : N
    x2 = x2 + ((1i*(-1)^k)/k*pi)*exp(1i*k*pi*t);
end
subplot(412), plot(t, x2), title('Signal 3, N = 5');
x3 = 0;
N = 20;
for k = 1 : N
    x3 = x3 + ((1i*(-1)^k)/k*pi)*exp(1i*k*pi*t);
end
subplot(413), plot(t, x3), title('Signal 3, N = 20');
x4 = 0;
N = 100;
for k = 1 : N
    x4 = x4 + ((1i*(-1)^k)/k*pi)*exp(1i*k*pi*t);
end
subplot(414), plot(t, x4), title('Signal 3, N = 100');
%%
% Signal 4
figure(4)
x1 = -1/2;
N = 1;
for k = 1 : N
    x1 = x1 + ((1/2)-(-1)^k)*exp(1i*k*pi*t);
end
subplot(411), plot(t, x1), title('Signal 4, N = 1');
x2 = -1/2;
N = 5;
for k = 1 : N
    x2 = x2 + ((1/2)-(-1)^k)*exp(1i*k*pi*t);
end
subplot(412), plot(t, x2), title('Signal 4, N = 5');
x3 = -1/2;
N = 20;
for k = 1 : N
    x3 = x3 + ((1/2)-(-1)^k)*exp(1i*k*pi*t);
end
```

```
subplot(413), plot(t, x3), title('Signal 4, N = 20');
x4 = -1/2;
N = 100;
for k = 1 : N
    x4 = x4 + ((1/2)-(-1)^k)*exp(1i*k*pi*t);
end
subplot(414), plot(t, x4), title('Signal 4, N = 100');
%%
% Signal 5
figure(5)
x1 = 0;
N = 1;
for k = 1 : N
   x1 = x1 +  (1/(1i*k*pi))*(cos(2*pi*k/3)-
cos(pi*k/3))*exp(1i*k*(pi/3)*t);
end
subplot(411), plot(t, x1), title('Signal 5, N = 1');
x2 = 0;
N = 5;
for k = 1 : N
   x2 = x2 +  (1/(1i*k*pi))*(cos(2*pi*k/3)-
cos(pi*k/3))*exp(1i*k*(pi/3)*t);
end
subplot(412), plot(t, x2), title('Signal 5, N = 5');
x3 = 0;
N = 20;
for k = 1 : N
   x3 = x3 +  (1/(1i*k*pi))*(cos(2*pi*k/3)-
cos(pi*k/3))*exp(1i*k*(pi/3)*t);
end
subplot(413), plot(t, x3), title('Signal 5, N = 20');
x4 = 0;
N = 100;
for k = 1 : N
   x4 = x4 +  (1/(1i*k*pi))*(cos(2*pi*k/3)-
cos(pi*k/3))*exp(1i*k*(pi/3)*t);
end
subplot(414), plot(t, x4), title('Signal 5, N = 100');
```