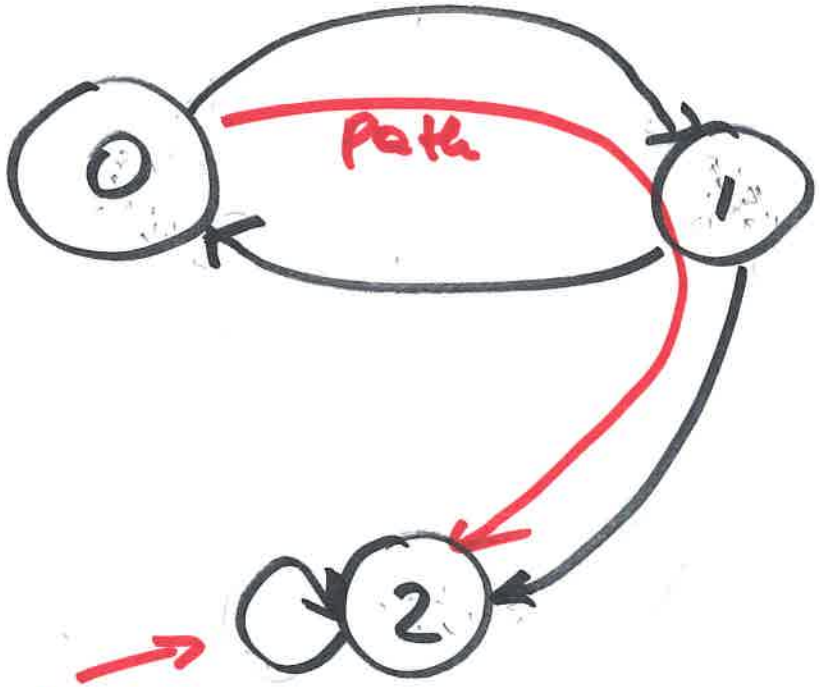
CS 2341

Chapter 9

Graphs

Graph



loop (cycle)

$G = (V, E)$

$V = \{0, 1, 2\}$

$E = \{(0,1), (1,0), (1,2), (2,2)\}$

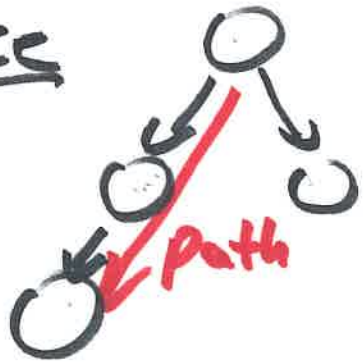
tuple representation

time: $O(|E|)$ or $O(1)$

space: $O(|E|)$

⇒ • vector of pairs
• hash table of pairs

Tree



Disconnected Graph

Adjacency Matrix

	0	1	2
0	0	1	0
1	1	0	1
2	0	0	1

Space: $O(|V|^2)$ Time: $O(1)$

Sparse matrix: space/time $O(|E|)$

⇒ • boost matrix
• boost mapped matrix

Adjacency List

vertices ↓ edges →

0: 1

1: 0, 2

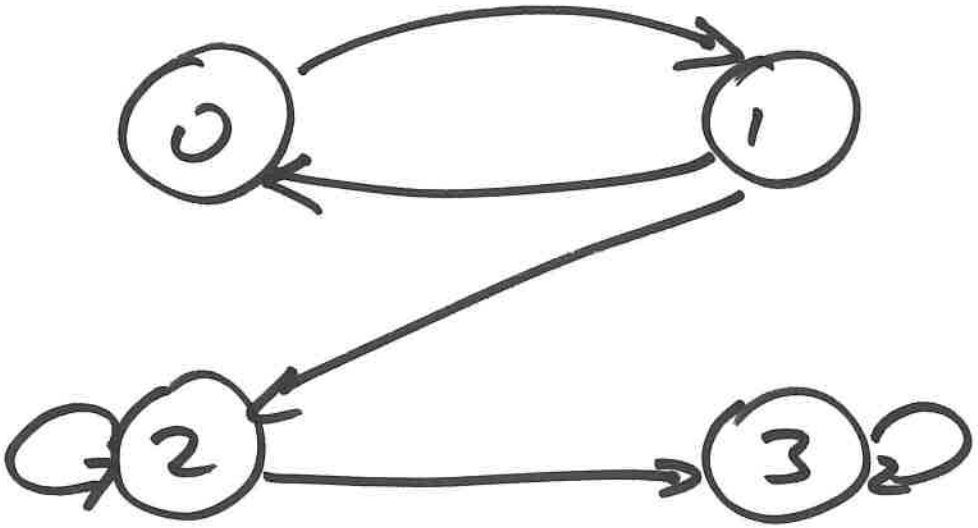
2: 2

Space: $O(|V|+|E|)$ Time: $O(|V|)$

⇒ vector of lists

Path

Where can you get from vertex 0 with a path of length 3?



You could go:

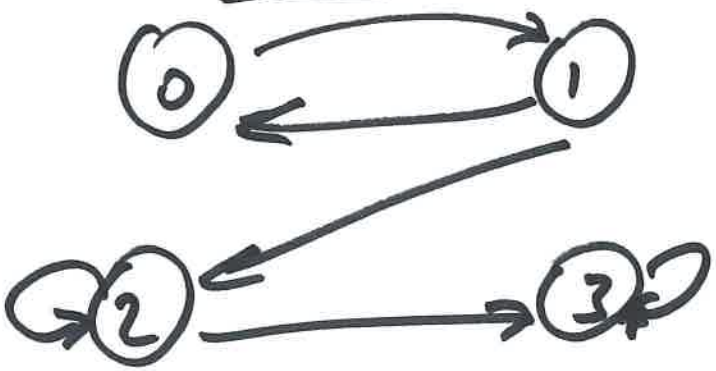
$0 \to 1 \to 2 \to 3$

$0 \to 1 \to 0 \to 1$

$0 \to 1 \to 2 \to 2$

⋮

Graph



Adjacency Matrix

H	0	1	2	3
0	0	1	0	0
1	1	0	1	0
2	0	0	1	1
3	0	0	0	1

0 → 1

= Paths of length 1

Paths of length 2 (H^2)

$$\begin{pmatrix} 0&1&0&0 \\ 1&0&1&0 \\ 0&0&1&1 \\ 0&0&0&1 \end{pmatrix} \times \begin{pmatrix} 0&1&0&0 \\ 1&0&1&0 \\ 0&0&1&1 \\ 0&0&0&1 \end{pmatrix} = \begin{pmatrix} 1&0&1&0 \\ 0&1&1&1 \\ 0&0&1&2 \\ 0&0&0&1 \end{pmatrix}$$

2 paths from 2 to 3

2 → 2 → 3

2 → 3 → 3

Paths of length 3 (H^3)

$$\begin{pmatrix} 1&0&1&0 \\ 0&1&1&1 \\ 0&0&1&2 \\ 0&0&0&1 \end{pmatrix} \times \begin{pmatrix} 0&1&0&0 \\ 1&0&1&0 \\ 0&0&1&1 \\ 0&0&0&1 \end{pmatrix} = \begin{pmatrix} 0&1&1&1 \\ 1&0&2&2 \\ 0&0&1&3 \\ 0&0&0&1 \end{pmatrix}$$

with a length of 3 you can get to 1, 2, 3