План лекций

Списки

Введение

```
Компилятор ghc, ghci, Haskell Platform.
   Haskell – чисто функциональный, типизированный язык программирования.
    Чистые функции.
   Типы Int, Integrer, Float, Double, Bool = True | False, Char.
    Арифметические операции.
+, -, *, /, \mathbf{div}, \mathbf{mod}
    Тип функции:
not :: Bool -> Bool
not False = True
not True = False
plus :: Int \rightarrow Int \rightarrow Int
plus x y = x + y
plus3 :: Int \rightarrow Int
plus3 = plus 3
   Кортежи (a,b).
\mathbf{fst}(x,y) = x
\mathbf{snd}(\mathbf{x}, \mathbf{y}) = \mathbf{y}
( 'a ', True)
    Списки
[a] = [] | a : [a]
1:2:[]
[1, 2]
[1..3] = [1,2,3]
 \begin{bmatrix} 0, 2...8 \end{bmatrix} = \begin{bmatrix} 0, 2, 4, 6, 8 \end{bmatrix} 
 \begin{bmatrix} 1, 1.5...3 \end{bmatrix} = \begin{bmatrix} 1.0, 1.5, 2.0, 2.5, 3.0 \end{bmatrix} 
   Конструктор списков (list comprehensions)
[x \mid x < -[1..3]] = [1,2,3]
[(x,y) \mid x \leftarrow [1,2], y \leftarrow [1,2]] = [(1,1), (1,2), (2,1), (2,2)]
[(x,y) \mid x \leftarrow [1..3], y \leftarrow [1..4], x = y] = [(1,1), (2,2), (3,3)]
```

Базовые функции со списками

```
head :: [a] \rightarrow a
head (x:xs) = x
tail :: [a] -> [a]
tail (x:xs) = xs
\begin{array}{lll} \textbf{length} & & :: & [\, a\,] \; -\!\!> \; \textbf{Int} \\ \textbf{length} & [\,] & & = & 0 \end{array}
\mathbf{length} \ (\mathbf{x} : \mathbf{xs}) = 1 + \mathbf{length} \ \mathbf{xs}
(++) :: [a] -> [a] -> [a]
(++) [] ys = ys
(++) (x:xs) ys = x : (xs ++ ys)
(!!) :: [a] \rightarrow Int \rightarrow a
(x:_) !! \hat{0} = x
(_:xs) !! n = xs !! (n-1)
\mathbf{reverse} :: [a] \rightarrow [a]
reverse [] = []
reverse (x:xs) = reverse xs ++ [x]
reverse l = rev l [] where
      rev [] 	 a = a
      rev (x:xs) a = rev xs (x:a)
[1,2,3]
[2,3] 1:[]
[3] 2:1:[]
[] 3:2:1:[]
[] [3,2,1]
\mathbf{take} \ :: \ \mathbf{Int} \ -\!\!\!> \ [\, \mathbf{a} \,] \ -\!\!\!> \ [\, \mathbf{a} \,]
take _ [] = [] take n (x:xs) | n <= 0 = []
                     otherwise = x: take (n-1) xs
```

drop

Бесконечные списки

```
[1..]

[2,4..]

take 5 [1..]

[1,2,3,4,5]

repeat :: a -> [a]

repeat x = x : repeat x

take 2 (repeat 3)

[3,3]
```

```
take 2 (3 : repeat 3)
3 : take 1 (repeat 3)
3: take 1 (3: repeat 3)
3 : 3 : take 0 (repeat 3)
3 : 3 : take 0 (3 : repeat 3)
3 : 3 : [] = [3,3]
(\$) :: (a -> b) -> a -> b
replicate :: Int \rightarrow a \rightarrow [a]
replicate n x = take n  repeat x
cycle :: [a] -> [a]
cycle xs = xs ++ cycle xs
take 5 $ cycle [1,2]
[1,2,1,2,1]
iterate :: (a \rightarrow a) \rightarrow a \rightarrow [a]
iterate f x = x : iterate f (f x)
  Линейный генератор
f x = mod (5*x + 3) 11
take 5 $ iterate f 1
[1,8,10,9,4]
Функции высших порядков
takeWhile :: (a \rightarrow Bool) \rightarrow [a] \rightarrow [a]
takeWhile _ [] = []
takeWhile p (x:xs) | p x
                                 = x : takeWhile p xs
                       | otherwise = []
dropWhile
\mathbf{filter} \ :: \ (\mathtt{a} \ -\!\!\!> \ \mathbf{Bool}) \ -\!\!\!> \ [\mathtt{a}] \ -\!\!\!> \ [\mathtt{a}]
filter_{[]} = []
filter p(x:xs) = if p x then x : filter xs else filter xs
filter even [1..5]
[2, 4]
filter (not . even) [1..5]
[1,3,5]
(.) :: (b \rightarrow c) \rightarrow (a \rightarrow b) \rightarrow a \rightarrow c
f \cdot g = \langle x - \rangle f \cdot (g \cdot x)
  Решето Эратосфена
sieve :: [Integrer] -> [Integrer]
sieve (x:xs) = x : sieve (filter (y -> y 'mod' x /= 0) xs)
primes = sieve [2..]
```

Мар и zipWith

```
map :: (a -> b) -> a -> b
\mathbf{map} \ \mathbf{f} \ [] = []
map f(x:xs) = fx : map fxs
map (^2) [1..5]
[1,4,9,16,25]
map (2^{\hat{}}) [1..5]
[2,4,8,16,32]
zipWith :: (a -> b -> c) -> [a] -> [b] -> [c]
zipWith f (x:xs) (y:ys) = f x y : zipWith f xs ys
zipWith _ _ _
zipWith3 :: (a -> b -> c -> d) -> [a] -> [b] -> [c] -> [d]
fibs = 0:1:zipWith (+) fibs (tail fibs)
Вычисление в стратегии сверху вниз:
fib n = fibs !! n
fib 3
2
fibs !! 3
(0:1:zipWith (+) fibs (tail fibs)) !! 3
(1:zipWith (+) fibs (tail fibs)) !! 2
(zipWith (+) fibs (tail fibs)) !! 1
(0 + 1 : zipWith (+)
     (1:zipWith (+) fibs (tail fibs))
     (zipWith (+) fibs (tail fibs))) !! 1
(zipWith (+)
     (1:zipWith (+) fibs (tail fibs))
     (zipWith (+) fibs (tail fibs))) !! 0
(zipWith (+)
     (1:\mathbf{zipWith} (+) \text{ fibs } (\mathbf{tail} \text{ fibs}))
     (zipWith (+)
         (0:1:\mathbf{zipWith}\ (+)\ \mathrm{fibs}\ (\mathbf{tail}\ \mathrm{fibs}))
         (1:zipWith (+) fibs (tail fibs)))) !! 0
(zipWith (+)
     (1:\mathbf{zipWith} (+) \text{ fibs } (\mathbf{tail} \text{ fibs}))
     (0 + 1 : \mathbf{zipWith} (+))
         (1:zipWith (+) fibs (tail fibs))
         (zipWith (+) fibs (tail fibs)))) !! 0
(1 + 1 : zipWith (+)
    (zipWith (+) fibs (tail fibs))
     (zipWith (+)
         (1:zipWith (+) fibs (tail fibs))
         (zipWith (+) fibs (tail fibs)))) !! 0
```

В Haskell применяется измененная версия этой стратегии. Создается ссылка на fibs и fibs будет вычисляться только один раз, на каждом шаге добавляя новые элементы.

Свёртка

```
\mathbf{sum} \quad [ \ ] \quad = 0
sum (x:xs) = x + sum xs
concat [] = []
concat (xs:xss) = xs ++ concat xss
foldr :: (a -> b -> b) -> b -> [a] -> b
foldr f e = 0
foldr f e (x:xs) = f x foldr f e xs
\mathbf{sum} = \mathbf{foldr} \ (+) \ 0
concat = foldr (++) []
{\bf foldl} \ :: \ (b \ -\!\!> \ a \ -\!\!> \ b) \ -\!\!> \ b \ -\!\!> \ [a] \ -\!\!> \ b
foldl f e = 0
foldl f e (x:xs) = foldl f (f e x) xs
reverse = foldl (flip (:)) []
\mathbf{flip} \ :: \ (a \rightarrow b \rightarrow c) \rightarrow b \rightarrow a \rightarrow c
flip f x y = f y x
foldr1 :: (a -> a -> a) -> [a] -> a
foldr1 f [x] = x
foldr1 f (x:xs) = f x foldr1 f xs
maximum = foldr1 max
filter p = foldr (\xs -> if p x then x:xs else xs) []
\mathbf{map} \ \mathbf{f} = \mathbf{foldr} \ ((:) \ . \ \mathbf{f}) \ []
\mathbf{length} = \mathbf{foldr} \ ( \setminus \ \mathbf{n} \rightarrow 1 + \mathbf{n} ) \ 0
Data.List и сортировки
transpose []
                               = []
transpose ([] : xss) = transpose xss
transpose ((x:xs) : xss) =
     (x : [h | (h: ) < -xss]) : transpose (xs : [t | (:t) < -xss])
qsort :: Ord a \Rightarrow [a] \rightarrow [a]
qsort [] = []
qsort(x:xs) = qsort(filter(=x) xs) ++ [x] ++ qsort(filter(>x) xs)
isort :: Ord a \Rightarrow [a] \rightarrow [a]
isort [] = []
isort (x:xs) = insert x (isort xs) where
  insert x [] = [x]
  \mathbf{insert} \ \ x \ \ ys@(y\!:\!ys\ ') \ \ | \ \ x>y \qquad = y \ : \ \mathbf{insert} \ \ x \ \ ys\ '
                            | otherwise = x : ys
```

```
msort :: Ord a \Rightarrow [a] \rightarrow [a]
msort = mergeAll . sequences
 where
    sequences (a:b:xs)
     | a > b = descending b [a] xs
     | otherwise = ascending b (a:) xs
    sequences xs = [xs]
    descending a as bs@(b:bs')
     |a>b = descending b (a:as) bs'
    descending a as bs = (a:as): sequences bs
    ascending a as bs@(b:bs')
    | a \ll b = ascending b (\ys -> as (a:ys)) bs'
    ascending a as bs = as [a]: sequences bs
    mergeAll [x] = x
    mergeAll xs = mergeAll (mergePairs xs)
    mergePairs (a:b:xs) = merge a b: mergePairs xs
   mergePairs xs = xs
   merge~as@(a:as')~bs@(b:bs')
     | a > b = b : merge as bs'
      | otherwise = a:merge as' bs
   merge [] bs = bs
   merge as [] = as
```

Монады

Maybe

```
data Maybe a = Nothing | Just a
safeHead :: [a] \rightarrow Maybe a
safeHead [] = Nothing
safeHead (x:_) = Just x
                               :: (\mathbf{Eq} \ \mathbf{a}) \Rightarrow \mathbf{a} \rightarrow [(\mathbf{a}, \mathbf{b})] \rightarrow \mathbf{Maybe} \ \mathbf{b}
lookup
         _ []
                               = Nothing
lookup
lookup key ((k,v):ps)
   | key == k
                             = Just v
     otherwise
                             = lookup key ps
lookup 2 [(1,"one"), (2,"two"), (3,"three")]
Just "two"
lookup 4 [(1,"one"), (2,"two"), (3,"three")]
Nothing
class Monad m where
     \mathbf{return} \ :: \ \mathbf{a} \ -\!\!> \mathbf{m} \ \mathbf{a}
     (>>=) :: m a -> (a -> m b) -> m b
```

```
instance Monad Maybe where
     \mathbf{return} \ \ \mathbf{x} \ = \ \mathbf{Just} \ \ \mathbf{x}
     {\bf Nothing} \ \ >>= \ f \ = \ {\bf Nothing}
     (Just x) \gg f = f x
   Последовательный поиск в нескольких списках (аналогичный подход при поиске в БД).
                      = [("Mike", "It"), ("Jan", "Sales")]
depCountry = [("It", "Japan"), ("Sales", "USA")]
countryCurrency = [("Japan", "JPY"), ("USA", "USD")]
currencyRate = [("JPY", 112), ("USD", 1)]
f :: String -> Maybe Int
f = case lookup emp empDep of
              {\bf Nothing} \quad -\!\!> \, {\bf Nothing}
              Just dep -> case lookup dep depCountry of
                                  Nothing -> Nothing
                                   Just country -> case lookup country country Currency of
                                                           Nothing -> Nothing
                                                            Just curr -> lookup curr currencyRate
\mathrm{fB}\ \mathrm{emp}\ =\ \mathbf{lookup}\ '\ \mathrm{empDep}\ \mathrm{emp}\ >\!>=\ \mathbf{lookup}\ '\ \mathrm{depCountry}\ >\!\!>=\ \mathbf{lookup}\ '\ \mathrm{countryCurrency}
            >>= lookup' currencyRate where
               lookup' ps k = lookup k ps
\mathrm{fD} \ \mathrm{emp} \, = \, \mathbf{do} \ \mathrm{dep} \, < \!\!\! - \, \, \mathbf{lookup} \ \mathrm{emp} \, \, \mathrm{empDep}
                country <- lookup dep depCountry</pre>
                currency <- lookup country country Currency
                lookup currency currencyRate
List
instance Monad [a] where
     return x = [x]
     xs \gg f = concat (map f xs)
   Построение грамматик.
f'' a' = "ca"
f'', b'' = "bb"
f'', c'' = "a"
take 3 $ iterate (>>= f) "abc"
["abc", "cabba", "acabbbbca"]
IO
class Monad m where
     \mathbf{return} \ :: \ \mathbf{a} \ -\!\!> \mathbf{m} \ \mathbf{a}
     (>>=) :: m a -> (a -> m b) -> m b
     (>>) :: m a -> m b -> m b
     m>>>\ k\ =\ m>>=\ \backslash x\ -\!\!\!> \ k
putChar :: Char -> IO ()
\mathbf{putStrLn} \ :: \ [\mathbf{Char}] \ -\!\!\!> \mathbf{IO} \ ()
putStrLn xs = foldr (>>) (return ()) (map putChar xs) >> putChar '\n'
```

```
\label{eq:getChar} \begin{split} \text{getLine} &:: \text{IO Char} \\ \text{getLine} &:: \text{IO [Char]} \\ \text{getLine} &= \text{do } x <- \text{ getChar} \\ &\quad \text{if } x == \ \ \ \ \ \ \ \ \ \\ &\quad \text{then return ""} \\ &\quad \text{else do } xs <- \text{ getLine} \\ &\quad \text{return } (x : xs) \end{split}
```

Структура программы.

Создайте файл hello.hs, со следующим содержимым и скомпелируйте: ghc hello.hs.

module Main where

```
\begin{array}{ll} \text{main} & :: & \textbf{IO} & (\,) \\ \text{main} & = & \textbf{putStrLn} & \text{"Hello} \;, \text{\_World!"} \end{array}
```