How was find all formulas of maths

This book belong to:

A free owl project

Contents

Note: This book is in developing
Note: It is recommended to download the file

Chapter 1

Initial part

1.1 Introduction

1.1.1 A free owl project

Goal

To collect all the knowledge of the world in one site.

Differentiation of other projects

- This project is Open Source, to protect the books of this project of malicious changes the people that want to collaborate, they have to create a pull request that is like a proposal of a modification that have to be accept you can send it by email `lucasvarelacorrea@gmail.com` or github `https://github.com/White-Mask-230/A-free-owl`

- The norms of this project are more soft than other similar projects. There will be accept all changes except the ones are out of topic or they are malicious changes

- This books can be use as primary source.

- This project focus in demonstrated the information, not only to store it.

- This project respects the work of all people, it can' t be deleted contend of other people but it can be proved they was wrong in the error zone

- The rules of the project or of the books can receive proposals to be change with a explication of why is better than the new rules writing a email to `lucasvarelacorrea@gmail.com`

1.1.2 Book

Goal

The goal of this book is to compile how all the mathematical formulas have been discovered.

Problems solve

- To avoid the mathematics becoming a dogma, for that this book proves and display how all the formulas where discovered, instead of just thinking that they are correct.
- There is a lot of information relate to the mathematics scattered in different websites, journals and books. This make difficult to the people that want to learn by themselves all the mathematical formulas. To make it more easy, we are preparing this book which will contain all the original information that have this sources.
- Verification of information different of the classical peer review.

Book nomenclature

- **Error zone**: Is where you will find all the formulas that a person find a error.
- **Before every formula will be his name and then his id present like this** ("chapter-id"."id-chapter-formula")
- **It can have some comments that the author want to give to the reader**
  - The formula is was find in a paper cited in the reference: ["id-reference"]
  - The formula is a definition but there is any reference: [x]
  - Is a popular formula but it doesn' t find how was make: (x)
  - The formula was wrote less than a month: (*)
  - The formula was wrote more than a month: (+)
  - The formula is cited in the Error Zone: (-)
  - Use a formula that is not demonstrated: (?)
- **The editor will write clarifications like this** Note: "text-here"
- **The editor will cite a specific formula like this** "Formula" *Space of 1 cm* (id of the formula use)

  In case it have to cite more than one formula it will put first the id of the formula that was use in the further to the left of the formula

1.2 Error Zone

1.3 News

- It will use twitter to make you know the latest modifications of this project.
  Link: `https://x.com/LucasVarelaCor1`

Chapter 2

Algebra

2.1 Linear algebra

2.1.1 Summation

Definition summation (id 2.1) (+)

$$\sum_{i=j}^{n} f(i) = f(j) + f(j+1) + \cdots + f(n-1) + f(n) \qquad (2.1)$$

Change limits

Index shift (id 2.2) (+)

$$\sum_{i=j-1}^{n-1} f(i+1) = f(j-1+1) + f(j-1+2) + \cdots + f(n-2+1) + f(n-1+1) \qquad (2.1)$$

$$= f(j) + f(j+1) + \cdots + f(n-1) + f(n)$$

$$= \sum_{i=j}^{n} f(i) \qquad (2.1)$$

Subtraction superior limit (id 2.3) (+)

$$\sum_{i=j}^{n} f(i) = f(j) + f(j+1) + \cdots + f(n-2) + f(n-1) + f(n) \qquad (2.1)$$

$$= f(n) + \sum_{i=j}^{n-1} f(i) \qquad (2.1)$$

Subtraction inferior limit (id 2.4) (+)

$$\sum_{i=j}^{n} f(i) = f(j) + f(j+1) + f(j+2) + \cdots + f(n-1) + f(n) \tag{2.1}$$

$$= f(j) + \sum_{i=j+1}^{n} f(i) \tag{2.1}$$

Arithmetic Operations

Summation of the addition of two functions (id 2.5) (+)

$$\sum_{i=j}^{n} f(i)+g(i) = f(j)+g(j)+f(j+1)+g(j+1)+\cdots+f(n-1)+g(n-1)+f(n)+g(n) \tag{2.1}$$

$$= f(j) + f(j+1) + \cdots + f(n-1) + f(n) + g(j) + g(j+1) + \cdots + g(n-1) + g(n)$$

$$= \sum_{i=j}^{n} f(i) + \sum_{i=j}^{n} g(i) \tag{2.1}$$

With Constants

Summation of a constant (id 2.6) (+)

Note $C_1 = C_2 = \cdots = C_{n-1} = C_n$

$$\sum_{i=1}^{n} C = C_1 + C_2 + \cdots + C_{n-1} + C_n \tag{2.1}$$

$$= C \cdot n$$

Summation of a constant and multiplication (id 2.7) (+)

$$\sum_{i=j}^{n} C \cdot f(i) = C \cdot f(j) + C \cdot f(j+1) + \cdots + C \cdot f(n-1) + C \cdot f(n) \tag{2.1}$$

$$= C[f(j) + f(j+1) + \cdots + f(n-1) + f(n)] \tag{2.23}$$

$$= C \sum_{i=j}^{n} f(i) \tag{2.1}$$

2.1.2 Product

Definition Product (id 2.8) (*)

$$\Pi_{i=m}^{n} x_i = x_m \cdot x_{m+1} \cdots x_{n-1} \cdot x_n$$

With Constants

Product of a constant (id 2.9) (*)

$$\Pi_{i=m}^{n} C = C \cdot C \cdots C \cdot C \qquad (2.8)$$

$$C^n$$

Product of a function and a constant (id 2.10) (*)

$$\Pi_{i=m}^{n} C f(i) = \Pi_{i=m}^{n} C \Pi_{i=m}^{n} f(i) \qquad (2.10)$$

$$C^n \Pi_{i=m}^{n} f(i) \qquad (2.9)$$

Change limits

Remove one top limit (id 2.11) (*)

$$\Pi_{i=m}^{n} x_i = x_m \cdot x_{m+1} \cdots x_{n-1} \cdot x_n \qquad (2.8)$$

$$= x_n \Pi_{i=m}^{n-1} x_i \qquad (2.8)$$

Arithmetic Operations

Product of two functions (id 2.12) (*)

$$\Pi_{i=m}^{n} x^i y^i = (x^m y^m) \cdot \left(x^{(m+1)} y^{(m+1)}\right) \cdots \left(x^{(n-1)} y^{(n-1)}\right) (x^n y^n) \qquad (2.9)$$

$$= \left(x^m \cdot x^{(m+1)} \cdots x^{n-1} x^n\right) \left(y^m \cdot y^{(m+1)} \cdots y^(n-1) \cdot y^n\right)$$

$$= \Pi_{i=m}^{n} x^i \Pi_{i=m}^{n} \qquad (2.9)$$

Definition functions or numbers

Definition of a factorial (id 2.13) (*)

$$\Pi_{i=1}^{n} i = 1 \cdot 2 \cdots (n-1) \cdot n \qquad (2.9)$$

$$= n!$$

2.2 Elementary Algebra

2.2.1 Notable Identities

Square of a summation or difference (id 2.14) (+)

$$(a+b)^2 = \Pi_{i=1}^{2} a + b \qquad (2.9)$$

$$= a + b \cdot a + b \qquad (2.8)$$

$$= \sum_{i=1}^{a+b} a + b \qquad (2.2)$$

$$= \sum_{i=1}^{a+b} a + \sum_{i=1}^{a+b} b \qquad (2.5)$$

$$= a(a+b) \pm b(a+b) \qquad (2.6)$$

$$= a^2 + a \cdot b + a \cdot b + b^2 \qquad (2.23)$$

$$= a^2 + 2 \cdot a \cdot b + b^2$$

Sum by difference (id 2.15) (+)

$$a + b \cdot a - b = \sum_{i=1}^{a+b} a - b \qquad (2.6)$$

$$= \sum_{i=1}^{a+b} a - \sum_{i=1}^{a+b} b \qquad (2.5)$$

$$= a(a+b) - b(a+b) \qquad (2.6)$$

$$= a^2 + b \cdot a - b \cdot a - b^2 \qquad (2.23)$$

$$= a^2 - b^2$$

2.2.2 Property of Fractions

Multiply a Fraction (id 2.16) (*)

$$\frac{1}{b} \cdot a = \sum_{i=1}^{a} \frac{1}{b} \qquad (2.6)$$

$$= \frac{1}{b} + \sum_{i=1}^{a-1} \frac{1}{b} \qquad (2.2)$$

$$= \frac{2}{b} + \sum_{i=1}^{a-2} \frac{1}{b} \qquad (2.2)$$

$$\vdots$$

$$= \frac{a}{b}$$

Addition of two fractions (id 2.17) (+)

$$\frac{a}{b} + \frac{c}{d} = \frac{a \cdot d}{b \cdot d} + \frac{c \cdot b}{d \cdot b} \qquad (2.21)$$

$$= \frac{a \cdot d + c \cdot b}{b \cdot d} \qquad (2.19)$$

Fraction of a fraction (id 2.18) (+)

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{\frac{a}{b} \cdot \frac{d}{c}}{\frac{c}{d} \cdot \frac{d}{c}} \qquad (2.21)$$

$$= \frac{\frac{a}{b} \cdot \frac{c}{d}}{\frac{c}{d} \cdot d \cdot \frac{1}{c}} \qquad (2.16)$$

$$= \frac{\frac{a}{b} \cdot \frac{c}{d}}{\frac{c \cdot d}{d \cdot c}} \qquad (2.20)(2.19)$$

$$= \frac{a}{b} \cdot \frac{c}{d} \qquad (2.21)$$

Summation of two fractions with same divisor (id 2.19) (*)

$$\frac{a}{b} + \frac{c}{b} = \frac{1}{b} \cdot a + c \cdot \frac{1}{b} \qquad (2.16)$$

$$= \frac{1}{b}(a + c) \qquad (2.23)$$

$$= \frac{a + c}{b} \qquad (2.16)$$

Separate numerator (id 2.20) (*)

$$\frac{a}{b \cdot c} = \frac{a \cdot \frac{1}{c}}{b \cdot c \cdot \frac{1}{c}} \qquad (2.21)$$

$$= \frac{a \cdot \frac{1}{c}}{b} \qquad (2.24)$$

$$= \frac{a}{b} \cdot \frac{1}{c} \qquad (2.16)$$

2.2.3 Property of multiplications

Definition of multiplication (id 2.22) (*)

Note $b = b_1 = b_2 = \cdots = b_{n-1} = b_n$

$$a \cdot b = \sum_{i=1}^{a} b \qquad (2.6)$$

$$= b_1 + b_2 + \cdots + b_{n-1} + b_n \qquad (2.1)$$

Distributive property (id 2.23) (+)

$$a(b + c) = \sum_{i=1}^{a} b + c \qquad (2.6)$$

$$= \sum_{i=1}^{a} b + \sum_{i=1}^{a} c \qquad (2.5)$$

$$= a \cdot b + a \cdot c \qquad (2.6)$$

Relation Other Functions

Multiplication and division of the same number (id 2.24) (+)

$$a \cdot \frac{1}{a} = \sum_{i=1}^{a} \frac{1}{a} \qquad (2.6)$$

$$= \frac{1}{a} + \sum_{i=1}^{a-1} \frac{1}{a} \qquad (2.3)$$

$$= \frac{2}{a} + \sum_{i=1}^{a-2} \frac{1}{a} \qquad (2.16)(2.3)$$

$$\vdots$$

$$= \frac{a}{a}$$

$$= 1 \qquad (2.21)$$

2.2.4 Property of raised to

Definition of raised to (id 2.25) (*)

$$x^a = \Pi_{i=1}^{a} x$$

Value x raised to 0 (id 2.26) (*)

$$x^0 = \Pi_{i=1}^{0} x \qquad (2.25)$$

$$= \frac{x}{x} \Pi_{i=1}^{0} x$$

$$= \frac{1}{x} \Pi_{i=1}^{1} x \qquad (2.11)$$

$$= \frac{1}{x} \cdot x \qquad (2.8)$$

$$= 1 \qquad (2.24)$$

Value x raised to a + 1 (id 2.27) (*)

$$x^{a+1} = \Pi_{i=1}^{a+1} x \qquad (2.9)$$

$$= x \Pi_{i=1}^{a} x \qquad (2.11)$$

$$= xx^a \qquad (2.9)$$

Value x raised to -a (id 2.28) (*)

$$x^{-a} = \Pi_{i=1}^{-a} x \qquad (2.8)$$

$$= \frac{x^a}{x^a} \Pi_{i=1}^{-a} x$$

$$= \frac{1}{x^a} \Pi_{i=1}^{a} x \Pi_{i=1}^{-a} x \qquad (2.9)$$

$$= \frac{1}{x^a} \Pi_{i=1}^{a-a} x \qquad (2.11)$$

Note: The above explanation formula. Using the formula of removing one top limit, reversing the operation, we can add 1 to the upper limit of the production of living $\frac{1}{x}$. This process can be repeated a times because we have x multiplicate a times. So we can write it as $a \cdot 1$ which is equal to a

$$= \frac{1}{x^a} \qquad (2.26)$$

Chapter 3

Mathematical analysis

3.1 Calculus

3.1.1 Limits

Properties

If only constant (id 3.1) [1] (+)

$$\lim_{x \to c} k = k$$

If only the term that approximates to (id 3.2) [1] (+)

$$\lim_{x \to c} x = c$$

If there is a constant and a function (id 3.3) [1](+)

$$\lim_{x \to c} k \cdot f(x) = k \lim_{x \to c} f(x)$$

If there are two functions that are ± (id 3.4) [1] (+)

$$\lim_{x \to c} [f(x) \pm g(x)] = \lim_{x \to c} f(x) \pm \lim_{x \to c} g(x)$$

If there are two functions that multiply (id 3.5) [1] (+)

$$\lim_{x \to c} [f(x) \cdot g(x)] = \lim_{x \to c} f(x) \cdot \lim_{x \to c} g(x)$$

If there are two functions that divide (id 3.6) [1] (+)

$$\lim_{x \to c} \left[\frac{f(x)}{g(x)} \right] = \frac{\lim_{x \to c} f(x)}{\lim_{x \to c} g(x)}$$

If one function composite the other function (id 3.7) [1] (+)

$$\lim_{x \to c} f(g(x)) = f(\lim_{x \to c} g(x))$$

Theorem

Squeeze Theorem $f(x) \leq g(x) \leq h(x)$ [1] (id 3.8) (x)

$$\lim_{x \to c} f(x) = \lim_{x \to c} h(x) = L \to \lim_{x \to c} g(x) = L$$

Definition functions or numbers

Definition of a derivative (id 3.9) (+)

$$\frac{d}{dx} f(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Definition of a integral (id 3.10) (+)

$$\int_a^b f(x)dx = \lim_{n \to \infty} \frac{b-a}{n} \sum_{i=1}^{n} f\left(a + i\frac{b-a}{n}\right)$$

Relation with other functions

Relation summation and limit (id 3.11) (*)

$$\lim_{h \to a} \sum_{i=j}^{n} f(h,i) = \lim_{h \to a} [f(h,j) + f(h,j+1) + \cdots + f(h,n-1) + f(h,n)] \qquad (2.1)$$

$$= \lim_{h \to a} f(h,j) + \lim_{h \to a} f(h,j+1) + \cdots + \lim_{h \to a} f(h,n-1) + \lim_{h \to a} f(h,n) \qquad (3.4)$$

$$= \sum_{i=j}^{n} \lim_{h \to a} f(h,j) \qquad (2.1)$$

3.1.2 Derivative

With Constants

Derivative of constant multiply and function (id 3.12) (+)

$$\begin{aligned} \frac{d}{dx}[Cf(x)] &= \lim_{h \to 0} \frac{C \cdot f(x+h) - C \cdot f(x)}{h} && (3.9) \\ &= \lim_{h \to 0} C \frac{f(x+h) - f(x)}{h} && (2.23) \\ &= C \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} && (3.3) \\ &= C \frac{d}{dx} f(x) && (3.9) \end{aligned}$$

Derivative of a constant (id 3.13) (+)

$$\begin{aligned} \frac{d}{dx}[C] &= \lim_{h \to 0} \frac{C - C}{h} && (3.9) \\ &= 0 \end{aligned}$$

Arithmetic Operations

Derivative of the addition or subtraction of two functions (id 3.14) (+)

$$\frac{d}{dx}[f(x) \pm g(x)] = \lim_{h\to 0} \frac{f(x+h) \pm g(x+h) - f(x) - g(x)}{h} \qquad (3.9)$$

$$= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \pm \lim_{h\to 0} \frac{g(x+h) - g(x)}{h} \qquad (3.4)$$

$$= \frac{d}{dx} f(x) \pm \frac{d}{dx} g(x) \qquad (3.9)$$

Derivative of a multiply of two functions (id 3.15) (+)

$$\frac{d}{dx}[f(x) \cdot g(x)] = \lim_{h\to 0} \frac{f(x+h) \cdot g(x+h) - f(x) \cdot g(x)}{h} \qquad (3.9)$$

Note: addition and subtraction of $f(x) \cdot g(x+h)$

$$= \lim_{h\to 0} \frac{f(x+h) \cdot g(x+h) - f(x) \cdot g(x) + f(x) \cdot g(x+h) - f(x)g(x+h)}{h}$$

$$= \lim_{h\to 0} \frac{[f(x+h) \cdot g(x+h) - f(x) \cdot g(x+h)] + [f(x) \cdot g(x+h) - f(x) \cdot g(x)]}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h) \cdot g(x+h) - f(x) \cdot g(x+h)}{h} + \lim_{h\to 0} \frac{f(x) \cdot g(x+h) - f(x) \cdot g(x)}{h} \qquad (3.4)$$

$$= \lim_{h\to 0} \frac{g(x+h) \cdot [f(x+h) - f(x)]}{h} + \lim_{h\to 0} \frac{f(x) \cdot [g(x+h) - g(x)]}{h} \qquad (2.23)$$

$$= \lim_{h\to 0} g(x+h) \cdot \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} + \lim_{h\to 0} f(x) \cdot \lim_{h\to 0} f(x) \cdot \lim_{h\to 0} \frac{g(x+h) - g(x)}{h} \qquad (3.3)$$

$$= g(x) \cdot \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} + f(x) \cdot \lim_{h\to 0} \frac{g(x+h) - g(x)}{h} \qquad (3.2)(3.1)$$

$$= g(x) \cdot \frac{d}{dx}[f(x)] + f(x) \cdot \frac{d}{dx}[g(x)] \qquad (3.9)$$

Derivative of a function elevate by a constant (id 3.16) (?)

Note: Credits to

- Url: `https://www.youtube.com/watch?v=dZnc3PtNaN4&t=304s`
- Title: Proof: `d/dx(x^n)` | Taking derivatives | Differential Calculus | Khan Academy
- Make by: Khan Academy

$$\frac{d}{dx}f^n(x) = \lim_{h\to 0}\frac{f^n(x+h) - f^n(x)}{h} \qquad (3.9)$$

$$= \lim_{h\to 0}\frac{-f^n(x) + \sum_{k=0}^{n}\binom{n}{k}f^{n-k}(x+h)\cdot h^k}{h} \qquad (?)$$

$$= \lim_{h\to 0}\sum_{k=1}^{n}\frac{\binom{n}{k}f^{n-k}(x+h)\cdot h^k}{h} \qquad (2.4)$$

$$= \lim_{h\to 0}\sum_{k=1}^{n}\binom{n}{k}f^{n-k}(x+h)\cdot h^{k-1} \qquad (2.27)$$

$$= \sum_{k=1}^{n}\lim_{h\to 0}\binom{n}{k}f^{n-k}(x+h)\cdot h^{k-1} \qquad (3.11)$$

$$= \lim_{h\to 0}\binom{n}{1}f^{n-1}(x+h)h^{1-1} + \lim_{h\to 0}\binom{n}{2}f^{n-2}(x+h)h^{2-1} + \cdots + \lim_{h\to 0}\binom{n}{k}f^{n-k}(x+h)h^{k-1} \qquad (2.1)$$

$$= \lim_{h\to 0}\binom{n}{1}f^{n-1}(x+h)h^{0} + \lim_{h\to 0}\binom{n}{2}f^{n-2}(x+h)h^{1} + \cdots + \lim_{h\to 0}\binom{n}{k}f^{n-k}(x+h)h^{k-1}$$

$$= \lim_{h\to 0}\binom{n}{1}f^{n-1}(x+h) + \lim_{h\to 0}\binom{n}{2}f^{n-2}(x+h)h^{1} + \cdots + \lim_{h\to 0}\binom{n}{k}f^{n-k}(x+h)h^{k-1} \qquad (2.26)$$

$$= \binom{n}{1}n\cdot f^{n-1}(x) \qquad (3.1)$$

$$= n\cdot f^{n-1}(x) \qquad (?)$$

Derivative of a division of two functions (id 3.17) (+)

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{d}{dx}[f(x)\cdot g^{-1}(x)]$$

$$= g^{-1}(x)\cdot\frac{d}{dx}f(x) + f(x)\cdot\frac{d}{dx}g^{-1}(x) \qquad (3.15)$$

$$= g^{-1}(x)\frac{d}{dx}f(x) - f(x)\cdot g^{-2}(x)\cdot\frac{d}{dx}g(x) \qquad (3.16)$$

$$= \frac{g(x)\frac{d}{dx}f(x) - f(x)\cdot\frac{d}{dx}g(x)}{g^2(x)} \qquad (2.28)$$

Relation with other functions

Relation with e^x (id 3.18) (*)

$$\frac{d}{dx}[e^x] = \lim_{h\to 0} \frac{e^{x+h} - e^x}{h} \qquad (3.9)$$

$$= \lim_{h\to 0} \frac{e^x \cdot e^h - e^x}{h} \qquad (2.27)$$

$$= \lim_{h\to 0} \frac{e^x(e^h - 1)}{h} \qquad (2.23)$$

$$= \lim_{h\to 0} \frac{e^x(1 - 1)}{h} \qquad (2.26)$$

$$= e^x \lim_{h\to 0} \frac{1 - 1}{h} \qquad (3.3)$$

$$= e^x \lim_{h\to 0} \frac{h}{h} \qquad (3.2)$$

$$= e^x \qquad (2.21)$$

Relation with $\int$ (id 3.19) (+)

$$\frac{d}{dx}\left[\int_a^b dx f(x)\right] = \lim_{h\to 0} \frac{1}{h}\left(\int_a^b dx f(x+h) - \int_a^b dx f(x)\right) \qquad (3.14)$$

$$= \lim_{h\to 0} \frac{1}{h} \int_a^b dx f(x+h) - f(x) \qquad (3.24)$$

$$= \lim_{h\to 0} \int_a^b dx \frac{f(x+h) - f(x)}{h} \qquad (3.22)(2.16)$$

$$= \int_a^b dx \frac{d}{dx} f(x) \qquad (3.9)$$

Relation with $\sum$ (id 3.20) (+)

$$\frac{d}{dx}\left[\sum_{i=j}^{n} f(i,x)\right] = \frac{d}{dx}[f(j,x)+f(j+1,x)+\cdots+f(n-1,x)+f(n,x)] \qquad (3.9)$$

$$= \frac{d}{dx}f(j,x) + \frac{d}{dx}f(j+1,x) + \cdots + \frac{d}{dx}f(n-1,x) + \frac{d}{dx}f(n,x) \qquad (3.14)$$

$$= \sum_{i=j}^{n} \frac{d}{dx} f(i,x) \qquad (2.1)$$

Definition functions or numbers

Function digamma (id 3.21) (?)

$$\psi(x) = \frac{d}{dx}\ln[\Gamma(x)] \qquad (?)$$

$$= \frac{\frac{d}{dx}\Gamma(x)}{\Gamma(x)} \qquad (?)$$

3.1.3 Integrals

Constants

Integral of the multiply of a function and a constant (id 3.22) (+)

$$\int_a^b C\cdot f(x)dx = \lim_{n\to\infty}\frac{b-a}{n}\sum_{i=1}^{n} C\cdot f\left(a+i\frac{b-a}{n}\right) \qquad (3.10)$$

$$= C\lim_{n\to\infty}\frac{b-a}{n}\sum_{i=1}^{n} f\left(a+i\frac{b-a}{n}\right) \qquad (2.7)(3.3)$$

$$= C\int_a^b f(x)dx \qquad (3.10)$$

Integral of a constant (id 3.23) (+)

$$\int_a^b dx\cdot C = \lim_{n\to\infty}\frac{b-a}{n}\sum_{i=1}^{n} C \qquad (3.10)$$

$$= \lim_{n\to\infty}\frac{b-a}{n}\cdot C\cdot n \qquad (2.6)$$

$$= C(b-a) \qquad (2.24)$$

Arithmetic Operations

Integral of a addition and a subtraction of two functions (id 3.24) (+)

$$\int_a^b f(x)\pm g(x)dx = \lim_{n\to\infty}\frac{b-a}{n}\left[\sum_{i=1}^{n} f\left(a+i\frac{b-a}{n}\right)\pm g\left(a+i\frac{b-a}{n}\right)\right] \qquad (3.10)$$

$$= \lim_{n\to\infty}\frac{b-a}{n}\left[\sum_{i=1}^{n} f\left(a+i\frac{b-a}{n}\right)\pm\sum_{i=1}^{n} g\left(a+i\frac{b-a}{n}\right)\right] \qquad (2.5)$$

$$= \lim_{n\to\infty}\frac{b-a}{n}\sum_{i=1}^{n} f\left(a+i\frac{b-a}{n}\right)\pm\lim_{n\to\infty}\frac{b-a}{n}\sum_{i=1}^{n} g\left(a+i\frac{b-a}{n}\right) \qquad (2.23)(3.4)$$

$$= \int_a^b f(x)dx \pm \int_a^b g(x)dx \qquad (3.10)$$

Relation with other functions

Integral of a summation (id 3.25) (*)

$$\int_a^b dx \sum_{i=j}^{n} f(x,i) = \int_a^b dx[f(x,j)+f(x,j+1)+\cdots+f(x,i-1)+f(x,i)] \tag{2.1}$$

$$= \int_a^b dx f(x,j) + \int_a^b dx f(x,j+1) + \cdots + \int_a^b f(x,i-1) + \int_a^b f(x,i) \tag{3.24}$$

$$= \sum_{i=j}^{n} \int_a^b dx f(x,i) \tag{2.1}$$

Chapter 4

Final part

4.1 Reference

[1] Alan Stein. Properties of Limits: https://www2.math.uconn.edu/~stein/math115/Slides/math115-130notes.pdf

4.2 Make by

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