

TD6

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(1)  
(a)

$$\sin\left(x + \frac{\pi}{2}\right) = \sin(x) \cdot \cos\left(\frac{\pi}{2}\right) + \sin\left(\frac{\pi}{2}\right) \cos(x) =$$

$$= \sin(x) \cdot 0 + 1 \cdot \cos(x) = \cos(x)$$

(b)

$$\cos\left(x + \frac{\pi}{2}\right) = \cos(x) \cdot \cos\left(\frac{\pi}{2}\right) - \sin(x) \cdot \sin\left(\frac{\pi}{2}\right) =$$
$$= \cos(x) \cdot 0 - \sin(x) \cdot 1 = -\sin(x)$$

(c)

$$\sin(\pi - x) = -\sin(\pi + x) \quad \left| \begin{array}{l} \text{or do we must change the} \\ \text{sign of } \pi \text{ as well?; and get:} \\ -\sin(-\pi + x) \end{array} \right.$$
$$= -(\sin(\pi) \cos(x) + \sin(x) \cos(\pi)) =$$
$$= -(0 \cdot \cos(x) + \sin(x) \cdot (-1)) =$$
$$= -(-\sin(x)) = \sin(x)$$

(2)

(a)

$$f(x) = \cos(2x+1)$$

$$f(x)' = -[\sin(2x+1)] \cdot 2 =$$

$$= [-(\sin(2x)\cos(1) + \sin(1)\cos(2x))] \cdot 2$$

(b)

$$g(x) = \ln(\sin(2x+1))$$

$$(\sin(2x+1))' = \cos(2x+1) \cdot 2$$

$$g(x)' = \frac{\cos(2x+1) \cdot 2}{\sin(2x+1)} =$$

$$= \frac{(\cos(2x)\cos(1)) \cdot 2 - (\sin(2x)\sin(1))}{\sin(2x)\cos(1) - \sin(1)\cos(2x)} =$$

$$= \frac{\cos(1)}{\cos(1)} \left( \frac{\cos(2x) \cdot 2}{\sin(2x)} \right) - \frac{\sin(1)}{\sin(1)} \left( \frac{\sin(2x)}{\cos(2x)} \right)$$

$$= \frac{\cos(2x) \cdot 2}{\sin(2x)} - \frac{\sin(2x)}{\cos(2x)}$$

2

(c)

$$\begin{aligned} h(x) &= \sin(x)^{\sin(x)} = \exp(\ln(\sin(x)^{\sin(x)})) = \\ &= (\exp(\sin(x) \ln(\sin(x))))' = \\ &= \sin(x) \cdot \ln(\sin(x)) \cdot \triangle (\text{see resolution}) \end{aligned}$$

$$\triangle \text{ Resolution: } [\sin(x) \cdot \ln(\sin(x))]'$$

$$= \sin(x) \left( \frac{\cos(x)}{\sin(x)} \right) + \cos(x) \cdot \ln(\sin(x)) =$$

$$= \frac{\sin(x) \cdot \cos(x)}{\sin(x)} + \cos(x) \cdot \ln(\sin(x)) =$$

$$= \cos(x) + \cos(x) \cdot \ln(\sin(x)) = 2 \cos(x) \cdot \ln(\sin(x))$$

Final expression:

$$\begin{aligned} &\sin(x) \cdot \ln(\sin(x)) \cdot 2 \cos(x) \cdot \ln(\sin(x)) = \\ &= \underbrace{[\ln(\sin(x))]^2}_{\circ} \cdot \sin(x) \cdot 2 \cos(x) \end{aligned}$$

Here we do not have " $\sin(x)^2$ ", but is this true? :

$$(\ln(\sin(x)))^2 \Leftrightarrow 2 \ln(\sin(x))$$

(3)