TD 7 - Calculus

(1)

(a) Hyperbolic function

$$\cosh(x) = \frac{e^x + e^{-x}}{2}$$ We know the taylor series of e^x and e^{-x}:

$$\frac{\sum_{K=0}^{\infty} \frac{x^K}{K!} + \sum_{K=0}^{\infty} \frac{(-1)^K \cdot x^K}{K}}{2} = \sum_{K=0}^{\infty} \left(\frac{x^K}{K!} + \frac{(-1)^K \cdot x^K}{K} \right) \frac{1}{2}$$

(b) $$\cos^2(x) = \frac{1 + \cos(2x)}{2}$$

$$\left(\frac{1 + \cos(2x)}{2} \right)' = \frac{-\sin(2x) \cdot 2}{4} = \frac{\sin(-2x)}{2}$$

$$\left(\quad '' \quad \right)'' = \frac{\cos(-2x) \cdot -2}{4} = \frac{-\cos(2x)}{2}$$

$$\left(\quad '' \quad \right)''' = \frac{-\sin(2x)}{2} = \frac{\sin(-2x)}{2}$$

$$\left(\quad '' \quad \right)^{IV} = \frac{-\cos(2x)}{2}$$

General formula:

$$\frac{(-1)^{n-1} \cdot ?}{2}$$

(c)

$\sin(x) \cdot \cos(x)$

we know that $\cos(x) = \sin\left(\frac{\pi}{2} - x\right)$

$$\left(\sin(x) \cdot \sin\left(\frac{\pi}{2} - x\right)\right)' = \cancel{\sin(x)}$$

$$= \sin(x) \cdot \cos\left(\frac{\pi}{2} - x\right) \cdot \left(\frac{\pi}{2} - x\right)' + \cos(x) \cdot \sin\left(\frac{\pi}{2} - x\right)$$

$$= \sin(x) \cdot \cos\left(\frac{\pi}{2} - x\right) \cdot -1 + \cos(x) \cdot \sin\left(\frac{\pi}{2} - x\right)$$

$$= \sin(x) \cdot \sin(x) \cdot -1 + \cos(x) \cdot \cos(x)$$

$$= -\sin^2(x) + \cos^2(x) = -1$$

$\left(\quad '' \quad \right)'' = 0$, I don't have enough derivative to find a formula, and the only conversion from $\cos(x)$ to $\sin(x)$ that I found was $\sin\left(\frac{\pi}{2} - x\right)$

(2)

$$\int_{-1}^{3} x^3 + 1 \, dx$$

$$x_i = 3 + (x^3+1) \cdot \frac{3+1}{n}$$

$$= \frac{3 + 4x^3 + 4}{n} = \frac{7 + 4x^3}{n}$$

$$\sum_{i=0}^{n-1} (x_{i+1} - x_i) f(x_i) \Leftrightarrow \sum_{i=0}^{n-1} \left(\frac{7+4(x+1)^3}{n} - \frac{7+4x^3}{n} \right)$$

$$\cdot (x^3+1) = \sum_{i=0}^{n-1} \left(\frac{7+4x^3+1-7+4x^3}{n} \right) \cdot (x^3+1) =$$

$$= \sum_{i=0}^{n-1} \left(\frac{8x^3+1}{n} \right) \cdot (x^3+1) = \sum_{i=0}^{n-1} \frac{(8x^3+1)(x^3+1)}{n} =$$

$$\sum_{i=0}^{n-1} \frac{8x^9 + 8x^3 + x^3 + 1}{n} = \sum_{i=0}^{n-1} \frac{8x^9 + 11x^3 + 1}{n}$$

$$\int_{-1}^{3} x^3 + 1 \, dx = \lim_{n \to \infty} \frac{\frac{8x^{9^{\to 0}}}{n} + \frac{11x^{3^{\to 0}}}{n} + \frac{1}{n}^{\to 0}}{\frac{n}{n}} = \frac{0}{1} = 0$$