TD 2 Calculus

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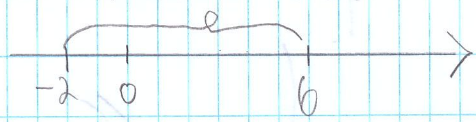
(1) Solve in $\mathbb{R}$

(a) $|x-2| \leq 4$ $\begin{cases} (x-2) & \text{for } x \geq 2 \\ -(x-2) & \text{for } x \leq 2 \end{cases}$

Scratch calculus
$(x-2) = (2-x) = 0$
Racine carrée!

$$|x-2| \leq 4 \begin{cases} (x-2) \leq 4 \\ \text{or} \\ (x-2) \geq -4 \end{cases} \Leftrightarrow \begin{cases} x \leq 4+2 \\ \\ x \geq -4+2 \end{cases} \Leftrightarrow \begin{cases} x \leq 6 \\ \\ x \geq -2 \end{cases}$$

!result's here!



(b) $|x^2 - x - 1| = 1$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Leftrightarrow \frac{+1 \pm \sqrt{1^2 - 4.1.1}}{-2}$$

$$\Leftrightarrow \frac{+1 + \sqrt{1^2 + 4}}{-2} \quad \text{ou} \quad \frac{+1 + \sqrt{1^2 + 4}}{-2}$$

$$\Leftrightarrow \frac{+1 + \sqrt{5}}{-2} \quad \text{ou} \quad \frac{+1 - \sqrt{5}}{-2}$$

(b) continuation

$$|x^2-x-1| \begin{cases} (x^2-x-1) \text{ for } \frac{-1-\sqrt{5}}{2} \leqslant x \leqslant \frac{-1+\sqrt{5}}{2} \\ -(x^2-x-1) \text{ for } \frac{-1-\sqrt{5}}{2} \leqslant x \leqslant \frac{-1+\sqrt{5}}{2} \end{cases}$$

$$|x^2-x-1|=1 \begin{cases} x^2-x-1-1=0 \\ -(x^2-x-1)-1=0 \end{cases} \Leftrightarrow \begin{cases} 2^2-2-1-1=0 \\ -x^2+x+1-1=0 \end{cases}$$

$$\Leftrightarrow \begin{cases} 0=0 \\ -x^2+x=0 \end{cases} \Leftrightarrow \begin{cases} \text{"} \\ -1^2+1=0 \end{cases} \Leftrightarrow \begin{cases} 0=0;\ x=2 \\ 0=0 \quad x=1 \end{cases}$$

$\cancel{(2)\ \tan\, z\, \frac{n}{2}}$

(2)

(a) $a_n = \dfrac{n \rightarrow 1^{st} \text{ degree}(P)}{n^2+1 \rightarrow 2^{nd} \text{ degree}(Q)}$ $P < Q$

Because Q is a 2^{nd} degree equation, and P a 1^{st} degree equation the limit of $a_n == 0$, and it converges.

(Would this be enough?) if not:

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n}{n^2+1} = \lim_{n\to\infty} \frac{\cancel{n}}{\cancel{n}\cdot n+1} = \lim_{n\to\infty} \frac{1}{n+1}$$

$$= \lim_{n\to\infty} \frac{\frac{1}{n}}{\frac{n}{n}+\frac{1}{n}} = \lim_{n\to\infty} \frac{0}{1+0} = \lim_{n\to\infty} 0 = 0$$

(b) $b_n = \dfrac{3n^2+1/n}{2n^2+n}$ n^2 is the ~~most~~ largest power

$$\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{\frac{3\cancel{n^2}}{\cancel{n^2}} + \frac{\frac{1}{n}}{n^2}}{\frac{2\cancel{n^2}}{\cancel{n^2}} + \frac{n}{n^2}} = \lim_{n\to\infty} \frac{3+\frac{1/n}{n^2}}{2+\frac{\cancel{n}}{\cancel{n}\cdot n}} =$$

$$\lim_{n\to\infty} \frac{3+\frac{1/n}{n^2}}{2+1/n} = \lim_{n\to\infty} \frac{3+\frac{1/\cancel{n}}{n\cdot\cancel{n}}}{2+1/n} = \lim_{n\to\infty} \frac{3+1/n}{2+1/n}$$

(Continue in next page)

the numerator $(3+\frac{1}{n})$ is always going to be bigger than the denominator $(2+\frac{1}{n})$, the sequence diverges.

(c)

$$c_n = 2^{-n}$$

n_1	n_2	n_3	n
$2^{-1}=0,5$	$2^{-2}=0,25$	$2^{-3}=0,125$	...

We see that 2^{-n} is going to get closer and closer to 0. Eventually, $\lim_{n\to\infty} 2^{-n}$ will equal 0, which means the sequence converges.

(3)

Not able to solve it, tried on scratch paper