TD3, Pedro Gomes

(1)

$$x + y + z = 0$$

$$y + z = 2x \iff -2x + y + z = 0$$

Converting it into a matrix we get:

$$\begin{matrix} 1^{st}\text{ row} \\ 2^{nd}\text{ row} \end{matrix}\begin{pmatrix} 1 & 1 & 1 & 0 \\ -2 & 1 & 1 & 0 \end{pmatrix} \Rightarrow 2 \cdot 1^{st}\text{ row} + 2^{nd}\text{ row}$$

$$\Downarrow$$

$$\begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 3 & 3 & 0 \end{pmatrix}$$

We have then:

$$\begin{cases} x + y + z = 0 \\ 0 + 3y + 3z = 0 \end{cases} \iff \begin{cases} y = -z - x \\ 3(-z - x) + 3z = 0 \end{cases} \iff$$

$$\begin{cases} // \\ -3z - 3x + 3z = 0 \end{cases} \iff \begin{cases} y = -z - 0 \\ x = 0 \end{cases} \iff \begin{cases} y = -z \\ x = 0 \end{cases}$$

$$\begin{matrix} x = 0 \\ y = -z \\ z \end{matrix} \Rightarrow \text{basis found as: } \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$$

(2)

$$\begin{matrix}1^{st}\text{ row}\\ 2^{nd}\text{ row}\\ 3^{th}\text{ row}\end{matrix}\left(\begin{array}{ccc|c}1 & 2 & 2 & 2\\ 3 & -2 & -1 & 5\\ 2 & -5 & 3 & -4\\ 1 & 4 & 6 & 0\end{array}\right)$$

We have 3 columns, hence 3 variables, but 4 rows, we can drop a row, and verify at the end if it checks. (Can we?)

$$\left(\begin{array}{ccc|c}1 & 2 & 2 & 2\\ 3 & -2 & -1 & 5\\ 2 & -5 & 3 & -4\end{array}\right) \Rightarrow \left(\begin{array}{ccc|c}1 & 2 & 2 & 2\\ 0 & -8 & -7 & -1\\ 0 & -9 & -1 & -8\end{array}\right)$$

$$\left(\begin{array}{ccc|c}1 & 2 & 2 & 2\\ 0 & -8 & -7 & -1\\ 0 & 0 & -55 & 55\end{array}\right) \Leftarrow \left(\begin{array}{ccc|c}1 & 2 & 2 & 2\\ 0 & -8 & -7 & -1\\ 0 & 0 & 1 & -1\end{array}\right)$$

Because:

$2^{nd}\text{ row} = 9 \cdot 2^{nd}\text{ row} + (-8) \cdot 3^{th}\text{ row}$

Finally we get:

$$\begin{cases} x + 2y + 2z = 2 \\ 0 + (-8y) + (-7z) = -1 \\ z = -1 \end{cases}$$

$$\Downarrow$$

$$\begin{cases} \quad // \\ -8y - 7 \cdot 1 = -1 \\ \quad // \end{cases} \quad \begin{cases} // \\ -8y = -8 \\ // \end{cases} \quad \begin{cases} // \\ y = -1 \\ // \end{cases}$$

$x + 2z = 0 \Leftrightarrow x = 2z$

$x = -2$

$$\Downarrow$$

$$\begin{cases} x + 2(-1) + 2z = 2 \\ // \\ // \end{cases} \quad \begin{cases} x - 2 + 2z = 2 \\ // \\ // \end{cases} \quad \begin{cases} x = -2 \\ y = -1 \\ z = -1 \end{cases}$$

Response: There is a solution.

(2)