TD 2 – CALCULUS

(1) Solve in $\mathbb{R}$:

   (a) $|x - 2| \leq 4$,

   (b) $|x^2 - x - 1| = 1$.

(2) Are the following sequences convergent or divergent for n going to infinity (give an argument for your answer)? For convergent sequences, also determine their limit:

   (a) $a_n = \frac{n}{n^2+1}$

   (b) $b_n = \frac{3n^2+1/n}{2n^2+n}$

   (c) $c_n = 2^{-n}$

(3) **(Recurrent sequences)** The general term of a recurrent sequences is defined by a function of one or more previous terms. Consider for $n \geq 0$ the sequence

$$u_{n+1} = \frac{1}{2}\left(u_n + \frac{a}{u_n}\right) \qquad \text{where } a \in \mathbb{R}^+ \text{ and } u_0 = a \ .$$

Prove that $\lim_{n\to\infty} u_n = \sqrt{a}$. Note: finding the limit under the assumption that the sequence is known to be convergent is relatively easy (note that in this situation we have $\lim_{n\to\infty} u_n = \lim_{n\to\infty} u_{n+1}$). Proving convergence is more challenging.