

Gomes Pedro  
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## TD 1 - Calculus

(1)

$$8 \mid 9^n - 1$$

Prove that  $n \geq 1$  we have

Verify

$$9^1 - 1 = 8 \text{ TRUE}$$

Assumption

$$8 \mid m = m+1$$

$$9^{m+1} - 1 = 9 \cdot 9^m - 1$$

(Do not know what to do next)

(2) Prove that for  $n \geq 1$  we have

$$\sum_{i=1}^n 2^i = 2^{n+1} - 2$$

Verify

$$n=1$$

$$\sum_{i=1}^1 2^i$$

$$= 2^{1+1} - 2$$

$$2$$

$$= 2 \text{ TRUE}$$

Assumption for k

$$\sum_{i=1}^k 2^i = 2^{k+1} - 2$$

Prove for k+1

$$\sum_{i=1}^{k+1} 2^i = 2^{(k+1)+1} - 2$$

$$\sum_{i=1}^{k+1} 2 = 2^{k+2} - 2$$

(Do not know what to do next)

(3) Verify for n=4

$$n! > 2^n = 4! > 2^4 = 24 > 16 \text{ TRUE}$$

Assumption for k

$$k! > 2^k \text{ is TRUE}$$

Prove for k+1

$$(k+1)! > 2^{(k+1)}$$

$$1+k! > 2 \cdot 2^k$$

$$1+k! > 2^k \cdot 2$$

(Do not know what to do next)

(4) Binomial theorem (Binomial coefficient)

$$\begin{aligned}\sum_{k=0}^m \binom{m}{k} &= \sum_{k=0}^m \binom{m}{0} = \frac{m!}{0!(m-0)!} \\ &= \frac{m!}{0!m!} = \frac{1}{0!} = 1\end{aligned}$$