

TD 3 – CALCULUS

- (1) Prove the formula for a geometric series:

$$\forall n \geq 0, \forall x \neq 1 : \quad 1 + x + x^2 + \dots + x^n = \frac{x^{n+1} - 1}{x - 1} .$$

- (2) Find a general formula for $\sum_{i=n_0}^{\infty} x^i$ and $\sum_{i=n_0}^m x^i$ for $n_0, m \in \mathbb{N}$ and $|x| < 1$.

- (3) Compute the values of the following series:

(a) $10 + 20 + 40 + 80 + \dots + 10240$

(b) $12 + 4 + \frac{4}{3} + \frac{4}{9} + \frac{4}{27} + \dots$

(c) $2 - 1 + \frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} + \dots$

(d) $\sum_{n=1}^{\infty} \frac{(-1)^n \cdot 2^n}{3^{n-1}}$

- (4) Assume a series whose general term a_i is given by $a_i = 2 - \frac{1}{3^{2i}}$. What is the value of the partial sum $\sum_{i=0}^n a_i$? Does the series converge for $n \rightarrow \infty$?