TD 4 – Linear ALGEBRA; Pedro Gomes

version 1

(1)

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 6 & 5 & 4 \\ 13 & 10 & 8 \end{pmatrix}$$

<u>Determine:</u>

(a)

$$\begin{pmatrix} 1 & 1 & 1 \\ 6 & 5 & 4 \\ 13 & 10 & 8 \end{pmatrix} \xrightarrow{-6 \cdot 1^{st}\text{ row} + 2^{nd}\text{ row}} \begin{pmatrix} 1 & 1 & 1 \\ 0 & -1 & -2 \\ 13 & 10 & 8 \end{pmatrix}$$

$$\xrightarrow{-13 \cdot 1^{st}\text{ row} + 3^{th}\text{ row}} \begin{pmatrix} 1 & 1 & 1 \\ 0 & -1 & -2 \\ 0 & -3 & -5 \end{pmatrix} \xrightarrow{-3 \cdot 2^{nd}\text{ row} + 3^{th}\text{ row}} \begin{pmatrix} 1 & 1 & 1 \\ 0 & -1 & -2 \\ 0 & 0 & 1 \end{pmatrix}$$

~~$x \cdot 0 + y \cdot 0 + 1 z = 0$~~

~~$\Rightarrow 0 + 0 + z = 0$~~

~~$\Rightarrow z = 0$ thus $1 \cdot 0 = 0$~~

~~the final row is 0 0 0 | 0~~

We have no rows that are none zero, thus the rank of the matrix is 3.

(b) the determinant of A.

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 1 & 1 \\ 6 & 5 & 4 & 6 & 5 & 4 \\ 13 & 10 & 8 & 13 & 10 & 8 \end{array}\right)$$

we get:

$$(1 \cdot 5 \cdot 8) + (1 \cdot 4 \cdot 13) + (1 \cdot 6 \cdot 10) - (13 \cdot 5 \cdot 1) - (10 \cdot 4 \cdot 1) - (8 \cdot 6 \cdot 1) = \boxed{-1} \text{ : determinant}$$

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(c) ~~The inverse of A if it exists~~

↳ determinant $\neq 0$, thus

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 6 & 5 & 4 & 0 & 1 & 0 \\ 13 & 10 & 8 & 0 & 0 & 1 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & -2 & -6 & 1 & 0 \\ 13 & 10 & 8 & 0 & 0 & 1 \end{array}\right) \to$$

$$\to \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & -2 & -6 & 1 & 0 \\ 0 & -3 & -5 & -13 & 0 & 1 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & -2 & -6 & 1 & 0 \\ 0 & 0 & 1 & 5 & -3 & 1 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 6 & -1 & 0 \\ 0 & 0 & 1 & 5 & -3 & 1 \end{array}\right) \to$$

$$\to \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 16 & 5 & -2 \\ 0 & 0 & 1 & 5 & -3 & 1 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & -4 & 3 & -1 \\ 0 & 1 & 0 & 16 & 5 & -2 \\ 0 & 0 & 1 & 5 & -3 & 1 \end{array}\right) \to$$

$$\to \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -20 & -2 & 1 \\ 0 & 1 & 0 & 16 & 5 & -2 \\ 0 & 0 & 1 & 5 & -3 & 1 \end{array}\right)$$

(c) The inverse of A if it exists:
↳ determinant $\neq 0$, thus

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 6 & 5 & 4 & 0 & 1 & 0 \\ 13 & 10 & 8 & 0 & 0 & 1 \end{array}\right) \rightarrow \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & -2 & -6 & 1 & 0 \\ 13 & 10 & 8 & 0 & 0 & 1 \end{array}\right) \rightarrow$$

$$\rightarrow \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & -2 & -6 & 1 & 0 \\ 0 & -3 & -5 & -13 & 0 & 1 \end{array}\right) \rightarrow \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & -2 & -6 & 1 & 0 \\ 0 & 0 & 1 & 5 & -3 & 1 \end{array}\right) \rightarrow$$

$$\rightarrow \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 4 & -5 & 2 \\ 0 & 0 & 1 & 5 & -3 & 1 \end{array}\right) \rightarrow \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & -4 & 3 & -1 \\ 0 & -1 & 0 & 4 & -5 & 2 \\ 0 & 0 & 1 & 5 & -3 & 1 \end{array}\right) \rightarrow$$

$$\rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & -2 & 1 \\ 0 & -1 & 0 & 4 & 5 & 2 \\ 0 & 0 & 1 & 5 & -3 & 1 \end{array}\right) \rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & -2 & 1 \\ 0 & 1 & 0 & -4 & 5 & -2 \\ 0 & 0 & 1 & 5 & -3 & 1 \end{array}\right)$$

thus:

$$\begin{pmatrix} 1 & 1 & 1 \\ 6 & 5 & 4 \\ 13 & 10 & 8 \end{pmatrix}^{-1} = \begin{pmatrix} 0 & -2 & 1 \\ -4 & 5 & -2 \\ 5 & -3 & 1 \end{pmatrix}$$

(2)

$$\left(\begin{array}{ccc|ccc} 3 & -1 & 2 & 3 & -1 & 2 \\ 4 & -3 & 3 & 4 & -3 & 3 \\ 1 & 3 & 0 & 1 & 3 & 0 \end{array}\right)$$

We have then:

$$(3 \cdot -3 \cdot 0) + (-1 \cdot 3 \cdot 1) + (2 \cdot 4 \cdot 3)$$
$$-(1 \cdot -3 \cdot 2) - (3 \cdot 3 \cdot 3) - (0 \cdot 4 \cdot -1) =$$

$$= 0 - 3 + 24 + 6 - 27 - 0 = \boxed{3 - 3 = 0}$$

$A^t \cdot X$ has no solution, because $\det(A^t) = 0$, thus $A^t \cdot X - B = 0$ cannot have a solution.