# TD 8 - Calculus

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(1)

(a)

$$\underbrace{(2x+1)}_{f(x)}\underbrace{\exp(x)}_{g'(x)} =$$

$$= (2x+1).\exp(x) - \int \overset{f'(x)}{2}\,\overset{g'(x)}{\exp(x)} =$$

$$= (2x+1).\exp(x) - (2.\exp(x) - \int 0.(\dots))$$

$$= (2x+1).\exp(x) - 2.\exp(x) =$$

$$= \exp(x)(2x+1-2) = \exp(x)(2x-1)$$

(b)

$$\underbrace{(5x^2+1)}_{f(x)}\underbrace{\cos(x)}_{g'(x)} =$$

$$= (5x^2+1)\sin(x) - \int \overset{f'(x)}{10x}.\overset{g'(x)}{\sin(x)} =$$

$$= (5x^2+1).\sin(x) - (10x.(-\cos(x)) - \int 10.(-\cos(x))) =$$

$$= (5x^2+1)\sin(x) - (10x.(-\cos(x)) - (10.(-\sin(x))) =$$

$$= (5x^2+1)\sin(x) + 10x.\cos(x) + 10.\sin(x) =$$

Final result:

$$= \sin(x)(5x^2+1+10) + 10x.\cos(x) = \underline{\underline{\sin(x)(5x^2+11) + 10x.\cos(x)}}$$

(c) $\underbrace{(3x^2+2x+1)}_{f(x)} \cdot \underbrace{\exp(ax)}_{g'(x)} =$

$= (3x^2+2x+1) \cdot \exp(ax) - \int \overbrace{(6x+2)}^{f(x)} \overbrace{\exp(ax)}^{g'(x)} =$

$= (3x^2+2x+1) \cdot \exp(ax) - ((6x+2) \cdot \exp(ax) - \int 6 \cdot \exp(ax) =$

$= (3x^2+2x+1) \exp(ax) - [(6x+2) \cdot \exp(ax) - (6 \cdot \exp(ax))] =$

$= (3x^2+2x+1) \cdot \exp(ax) - (6x+2) \exp(ax) - 6 \cdot \exp(ax) =$

Final result:

$= \exp(ax)(3x^2+2x+1-6x+2-6) = \exp(ax)(3x^2-4x-3)$

$\boxed{(2)}$

$\underbrace{x^2}_{f(x)} \cdot \underbrace{\exp(x)}_{g'(x)} = x^2 \cdot \exp(x) - \int 2x \cdot \exp(x)\, dx =$

$= x^2 \cdot \exp(x) - (2x \cdot \exp(x)) - \int 2 \cdot \exp(x) =$

$= x^2 \cdot \exp(x) - (2x \cdot \exp(x)) - (2 \cdot \exp(x)) =$

$= \cancel{x} \exp(x)(x^2-2x-2) + c$, where $c=0$

$= \exp(x)(x^2-2x-2)$

(3)

(a)

$$\int_0^{\frac{\pi}{2}} (-x^2+x+1) \cdot \sin(x)\, dx =$$

$= (-x^2+x+1) \cdot (-\cos(x)) - \int (-2x+1)\,(-\cos(x)) =$

$= (-x^2+x+1) \cdot (-\cos(x)) - \left[ (-2x+1)\,(-\sin(x)) - \int -2 \cdot (-\sin(x)) \right] =$

$= (-x^2+x+1)\ (-\cos(x)) - (-2x+1)(-\sin(x)) - (-2 \cdot (\cos(x))) =$

$= (-x^2+x+1)(-\cos(x)) - (-2x+1)(-\sin(x)) + 2 \cdot (-\cos(x)) =$

$= -\cos(x)\,(-x^2+x+1+2) - (-2x+1)(-\sin(x)) =$

$= -\cos(x)\,(-x^2+x+3) - (2x\sin(x) - \sin(x)) =$

$= x^2\cos(x) - x\cos(x) - 3\cos(x) - 2x\sin(x) + \sin(x) + c$, where $c$ is a constant.

<u>Boundaries</u>

$$\lim_{x \to 0} \left( x^2\cos(x) - x\cos(x) - 3\cos(x) - 2x\sin(x) + \sin(x) \right) =$$

$$= x^{2\nearrow 0} - x^{\nearrow 0} - 3 - 2x^{\nearrow 0} = \boxed{3}$$

$$\lim_{x \to \frac{\pi}{2}} \left( x^2\cos(x) - x\cos(x) - 3\cos(x) - 2x\sin(x) + \sin(x) \right) =$$

$= 1 - \pi$

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We simplify and get:

$$(1-\pi)-(-3) = 1-\pi+3 = \boxed{4-\pi}$$

(b)

$$\int_0^x t.e^t dt$$

$$t.\exp(t) - \int 1.\exp(t) =$$

$$= t.\exp(t) - (1 \exp(t)) = t.\exp(t) - \exp(t) + c$$

$\boxed{\text{Boundaries}}$

$$\lim_{t \to 0} = -1 \quad ; \quad \lim_{t \to x} = x \exp(x) - \exp(x) =$$

$$= \exp(x)(x-1)$$

We subtract UP - DOWN

$$\exp(x)(x-1) - (-1) = \exp(x)(x-1)+1$$