TD2: Linear Algebra, Pedro Gomes

(1)

$$x_1\begin{pmatrix}1\\2\\3\end{pmatrix} + x_2\begin{pmatrix}-1\\2\\-3\end{pmatrix} + x_3\begin{pmatrix}-1\\6\\-3\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}$$

$A: x_1 - x_2 - x_3 = 0$

$B: 2x_1 + 2x_2 + 6x_3 = 0 \implies 4x_2 + 8x_3$

$C: 3x_1 + (-3x_2) - 3x_3 = 0$

$B = -2 \cdot A + B$

$= -2x_1 + 2x_2 + 2x_3 + B = 4x_2 + 8x_3$

$C = -3 \cdot A + C$

$= -3x_1 + 3x_2 + 3x_3 + C = x_3 = 0$

Knowing that $x_3 = 0$

$$4x_2 + 8 \cdot 0 = 0 \iff 4x_2 + 0 = 0$$
$$\iff x_2 = 0$$

Finally:

$$x_1 - 0 - 0 = 0 \iff x_1 = 0$$

$\left.\begin{array}{l} x_1 = 0 \\ x_2 = 0 \\ x_3 = 0 \end{array}\right\}$ linear independant

(2)

$A: 3x + 2y = 1$

$B: x - y = 2$

$C: 4x + 2y = 2$

$C = -4 \cdot B + C$

$= (-4x + 4y = -8) + C$

$= (0 + 6y = -6)$

$= (y = -1)$

Knowing this:

$3x + 2(-1) = 1 \iff 3x - 2 = 1$

$\iff 3x = 3 \iff \boxed{x = 1}$

? All the solutions? (don't know how)

↳ Tried to use extended euclide algorithme on: $3x + 2y = 1$, but not enough calculations to apply recursion afterwards:

$3 \,|\, 2 \qquad 3 = 1 \cdot 2 + 1$

$1 \quad 1 \qquad 1 = 3 - 1 \cdot 2$

...?

(3)

number: $x_1 x_2 x_3$

we know:

$x_2 + x_3 \leq 5$

$x_3 x_2 x_1 - x_1 x_2 x_3 = 792$

$$\begin{pmatrix} x_3 \\ x_2 \\ x_1 \end{pmatrix} - \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{matrix} 7 \\ 9 \\ 2 \end{matrix}$$

$$\left(x_1 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + x_2 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right) - \left(x_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + x_2 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right) = \begin{pmatrix} 7 \\ 9 \\ 2 \end{pmatrix}$$

$A: x_3 - x_1 = 7$

$B: x_2 - x_2 = 9$

$C: x_1 - x_3 = 2$

We know that we're working in $\mathbb{Z}/9\mathbb{Z}$, because:

$\begin{pmatrix} 7 \\ 9 \\ 2 \end{pmatrix} \to$ biggest, thus after some tries if $x_3 = 5$ (which is the maximum it can be, because of the $x_2 + x_3 \leq 5$, where $x_2 = 0$) we have:

A: $5 - x_1 = 7 \Leftrightarrow -x_1 = 2 \Leftrightarrow x_1 = -2$, which is $7 \bmod 9$

We have then

$$\begin{array}{ccc} 7 & 9 & 5 \\ x_1 & x_2 & x_3 \end{array}$$

if we make $597 - 795 = -198$, and -198 is $0 \bmod 9$, and so is $792 \bmod 9$

Remark. Could you please give us more exercises like this to practice? It took me ages to solve this one, a lot of tries, and is most likely wrong, it would be very hard to do such an exercise on the exam because of the time.