TD 6 CALCULUS – SOLUTIONS

(1) **Properties of trigonometric functions. Using the addition formulae for sin and cos, deduce other properties by computing:**

(a) $\sin(x + \frac{\pi}{2})$,

We use the addition formula for the sine function and get

$$\sin(x + \tfrac{\pi}{2}) \;=\; \sin(x)\cos(\tfrac{\pi}{2}) + \cos(x)\sin(\tfrac{\pi}{2}) \;=\; \sin(x)\cdot 0 + \cos(x)\cdot 1 \;=\; \cos(x)\ .$$

(b) $\cos(x + \frac{\pi}{2})$,

Similarly we get:

$$\cos(x + \tfrac{\pi}{2}) \;=\; \cos(x)\cos(\tfrac{\pi}{2}) - \sin(x)\sin(\tfrac{\pi}{2}) \;=\; \cos(x)\cdot 0 - \sin(x)\cdot 1 \;=\; -\sin(x)\ .$$

(c) $\sin(\pi - x)$,

Same idea:

$$\sin(\pi - x) \;=\; \sin(\pi)\cos(x) - \cos(\pi)\sin(x) \;=\; \cos(x)\cdot 0 - \sin(x)\cdot(-1) \;=\; \sin(x)\ .$$

(d) $\tan(x + \pi)$.

We use the definition of the tan function together with the addition formulae for sin and cos and get

$$\tan(x+\pi) \;=\; \frac{\sin(x+\pi)}{\cos(x+\pi)} \;=\; \frac{-\sin(x)}{-\cos(x)} \;=\; \tan(x)\ .$$

(2) **Compute the derivatives of the following functions:**

(a) $f(x) = \cos(2x + 1)$,

The derivative of $f(x)$ can be determined with the help of the derivative of the cos function in combination with the composition rule. Therefore, we get

$$f'(x) \; = \; -\sin(2x+1)\cdot 2 \; = \; -2\cdot\sin(2x+1)\ .$$

(b) $g(x) = \ln(\sin(2x+1))$,

Again we use the two basic derivatives for ln and sin together the composition rule, and we conclude

$$g'(x) \; = \; \frac{1}{\sin(2x+1)}\cdot(\sin(2x+1))' \; = \; \frac{2\cdot\cos(2x+1)}{\sin(2x+1)} \; = \; 2\,\cot(2x+1)\ .$$

(c) $h(x) = \sin(x)^{\sin(x)}$.

We use the standard trick explained in the class for functions with variable x in the exponent and write the given function as an exp function:

$$h(x) \; = \; \exp\Big(\ln\Big(\sin(x)^{\sin(x)}\Big)\Big) \; = \; \exp\Big(\sin(x)\cdot\ln\left(\sin(x)\right)\Big)\ .$$

To determine the derivative of $h(x)$ with the composition rule, it is important to find the derivative of the inner function in this composed function. We calculate it with the help of the product rule as

$$\Big(\sin(x)\cdot\ln(\sin(x))\Big)' \; = \; \cos(x)\cdot\ln(\sin(x)) \; + \; \sin(x)\cdot\frac{\cos(x)}{\sin(x)}\ .$$

Combing these partial results, we get the final result as

$$h'(x) \; = \; \sin(x)^{\sin(x)}\cdot\Big(\ln(\sin(x))+1\Big)\cdot\cos(x)\ .$$