TD 4

14/10/2017
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(1)

We know that $n > 0$, and that the sequence converges, which means:

$$\lim_{n \to \infty} U_n = U$$

We have

$$\lim_{n \to \infty} U_n = \lim_{n \to \infty} U_n - 1 + \frac{1}{\lim_{n \to \infty} n - 1} \Leftrightarrow$$

$$\Leftrightarrow U = U - 1 + \frac{1}{U-1} \Leftrightarrow U - U + 1 = \frac{1}{U-1} \Leftrightarrow$$

$$\Leftrightarrow 1 = \frac{1}{U-1} \Leftrightarrow 1 \cdot 1 = U - 1 \Leftrightarrow 1 = U - 1 \Leftrightarrow$$

$\Leftrightarrow U = 2$, The sequence converges to a finite limit 2.

(I tried to follow the instructions of a similar exercice : TD2 (3))

(2)

(a)

let's say $n=1$

$$\sum_{n=1} \frac{(n+2)^2}{2^{n+2}} \quad \text{quotient criterion} \quad \frac{a_{n+1}}{a_n}$$

$$\frac{\frac{(n+1+2)^2}{2^{n+2+1}}}{\frac{(n+2)^2}{2^{n+2}}} = \frac{\frac{(n+3)^2}{2^{n+3}}}{\frac{n^2+4n+4}{2^{n+2}}} = \frac{\frac{n^2+6n+9}{2^n \cdot 2^3}}{\frac{n^2+4n+4}{2^n \cdot 2^2}} =$$

$$= \frac{\frac{n^2+6n+9}{8}}{\frac{n^2+4n+4}{4}} = \frac{n^2+6n+9}{8} \cdot \frac{4}{n^2+4n+4} =$$

$$= \frac{4(n^2+6n+9)}{8(n^2+4n+4)} = \frac{(n^2+6n+9)}{2(n^2+4n+4)} = \frac{n^2(1+\frac{6\cancel{n}}{\cancel{n}}+\cancel{\frac{9}{n^2}})}{2n^2+8n+8}$$

$$= \frac{\cancel{n^2}(1+6)}{\cancel{n^2}(2+\frac{8n}{n}+\frac{8}{\cancel{n^2}})} = \frac{7}{2+8} = \boxed{\frac{7}{10}}$$

The serie converges to $\frac{7}{10}$

(b)

$$\sum_{n\geq 0} 0,9999^n$$

$$\sum_{n=0} 0,9999^n$$

$0,9999 < 1$

$$\frac{1}{1-x} = \frac{1}{1-0,9999}$$

$$= 10000$$

The limit is 10000.

(c)

$$\sum_{n\geq 2} (-1)^{n-1} \frac{n^2-n}{n^2+n}$$

$\sum_{n=2}$ correct the "move" $\rightarrow$ $\sum_{n=0} (-1)^{n-1+2} \frac{(n+2)^2 - n+2}{(n+2)^2 + n+2}$

$$\sum_{n=0} (-1)^n \underbrace{\frac{(n+2)^2 - n+2}{(n+2)^2 + n+2}}_{(1)}$$

(1)

$$\frac{n^2+4n+4-n+2}{n^2+4n+4+n+2} = \frac{\cancel{n^2}\left(1+\frac{4n}{n}+\cancel{\frac{4}{n^2}}-\cancel{\frac{n}{n^2}}+\cancel{\frac{2}{n^2}}\right)}{\cancel{n^2}\left(1+\frac{4n}{n}+\cancel{\frac{4}{n^2}}+\cancel{\frac{n}{n^2}}+\cancel{\frac{2}{n^2}}\right)}$$

$$= \frac{1+\frac{4n}{n}}{1+\frac{4n}{1}} = 1$$, This means:

$$\sum_{n=0} (-1)^n \cdot 1 = \sum_{n=0} (-1)^n$$

$$\downarrow$$

$$\frac{1}{1-x} = \frac{1}{1-(-1)} = \boxed{\frac{1}{2}}$$

The limit of the serie is $\frac{1}{2}$

(Not sure if this made any sense)

(3)

(a)

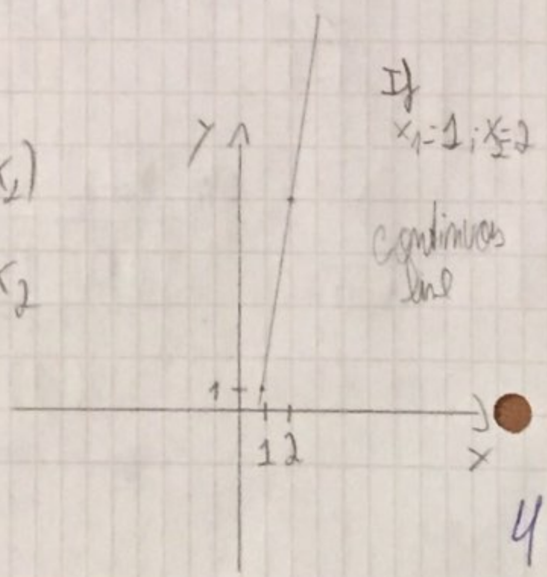
$$x_1 \neq x_2 = f(x_1) \neq f(x_2)$$

$$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$$

$$(x_1)^3 = (x_2)^3$$

$$\pm\sqrt[3]{x_1} = \pm\sqrt[3]{x_2}$$

$$+\sqrt[3]{x_1} \,!= -\sqrt[3]{x_2}$$



The function $f(x) = x^3$, is injective and continuous

(b)

$x_1 \neq x_2 = g(x_1) \neq g(x_2)$

$g(x_1) = g(x_2) \Rightarrow x_1 = x_2$

$(x_1)^3 - x = (x_2)^3 - x$

$(x_1)^3 - x + x = (x_2)^3$

$(x_1)^3 = (x_2)^3$ (Same situation as the previous function? Or, am I wrong?)

(c)

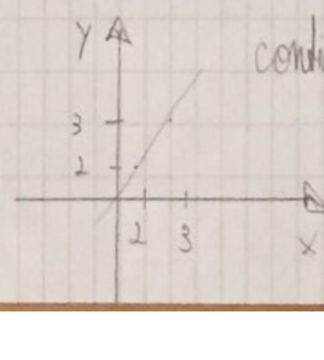
$x_1 \neq x_2 = h(x_1) \neq h(x_2)$

$h(x_1) = h(x_2) \Rightarrow x_1 = x_2$

$\lfloor x_1 \rfloor = \lfloor x_2 \rfloor$; for $\forall x \in \mathbb{Z}$ it's true, for example

$x_1 = 2$, $x_2 = 3$

$\lfloor 2 \rfloor = 2$, $\lfloor 3 \rfloor = 3$

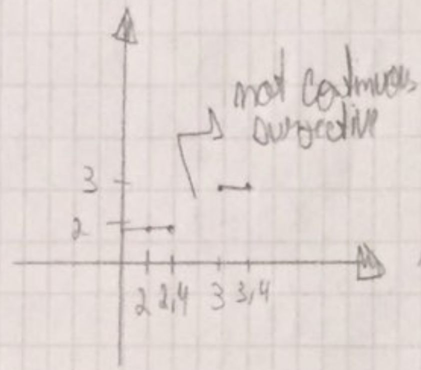


continuous injective

BUT Not true for $\forall x \in \mathbb{R}$

E.g: $x_1 = 2$; $x_2 = 2{,}4$

$\lfloor 2 \rfloor = 2$; $\lfloor 2{,}4 \rfloor = 2$,



same for $x_1 = 3$, $x_2 = 3{,}4$

(d)

$(|3|$ and $|-3|) = 3$, which means two values of x have the same y : Not injective, continuous surjective.

(e)

Injective continuous function

$x_1 = 1$; $x_2 = 2$ $\frac{1}{1} = 1$, $\frac{1}{2} = 0{,}5$