TD3

08/10/2017
Pedro Gomes

(1)

$$1 + x + x^2 + \dots x^n = \frac{x^{n+1} - 1}{x - 1}$$

$$\sum_{i=0}^{n} x^i = \frac{x^{n+1} - 1}{x - 1}$$

Let's try some values for n, when $x = 2$ $(x \neq 0)$

$$n_0 = \frac{2^{0+1} - 1}{2 - 1} = \frac{1}{1} = 1$$

$$n_1 = \frac{2^{1+1} - 1}{2 - 1} = \frac{3}{1} = 3$$

$$n_2 = \frac{2^{2+1} - 1}{2 - 1} = 7$$

$$n_3 = \frac{2^{3+1} - 1}{2 - 1} = 15$$

$$n_4 = \frac{2^{4+1} - 1}{2 - 1} = 31$$

$$2^{n+1} - 1$$

...

$$\boxed{2^{n+1} - 1}$$

to calculate the ~~sum~~ partial sum of a n value

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$\dots$

$2^{n+1} - 1$

$$\boxed{2^{n+1} - 1}$$

to calculate the ~~sum~~ partial sum of a n value

We guess that

$$\sum_{i=0}^{n} x^i = 2^{n+1} - 1$$

$$\Rightarrow \sum_{i=0}^{\infty} x^i = \lim_{n\to\infty} \sum_{i=0}^{n} x^i = \lim_{n\to\infty} 2^{n+1} - 1 = \infty$$

(I think smth is incomplet in the last part, but I cannot spot what).

(2)

(Tried on scratch, couldn't do it)

(3)

(a)

$$\underbrace{10 + 20}_{\cdot 2} \underbrace{+ 40}_{\cdot 2} + 80 + \ldots + 10240$$

$a_1 = 10 = a$

$a_4 = 80$

ratio $= r = 2$

partial sum of a_4 :

$$\sum_{i=1}^{a_4} a_4 = a\left(\frac{1-r^n}{1-r}\right)$$

$$= 10\left(\frac{1-2^4}{1-2}\right)$$

$$= 10(15) = \underline{\underline{150}}$$

$10240 \div 2 = 5120$
$5120 \div 2 = 2560$
$2560 \div 2 = 1280$
$1280 \div 2 = 640$
$640 \div 2 = 320$
$320 \div 2 = 160$
$160 \div 2 = \underline{80}$

$4+7=11$

$a_{11} = 10240$

Sum of serie:

$$\sum_{i=1}^{n} a_{11} = a\left(\frac{1-r^n}{1-r}\right)$$

$$= 10\left(\frac{1-2^{11}}{1-2}\right)$$

$$= 10\left(\frac{1-2048}{-1}\right)$$

FINAL REsult $= 10(2047) = 20470$

(b)

$$12 + 4 + \frac{4}{3} + \frac{4}{9} + \frac{4}{27} + \cdots$$

$a_1 = a$ $\quad \frac{1}{3} \quad \frac{1}{3} \quad \frac{1}{3}$

$r = \frac{1}{3}$ Sum of serie:

$$\sum_{i=1}^{\infty} a_i = \frac{a}{1-r} = \frac{12}{1-\frac{1}{3}} = \frac{12}{\frac{2}{3}} = 12 \cdot \frac{3}{2} =$$

$$= 18$$

(tried to use $\frac{1}{1-x}$, got nonsense results; same for (c))

(c)

$$2 - 1 + \frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} + \dots$$

$$\cdot\left(-\frac{1}{2}\right) \quad \cdot\left(-\frac{1}{2}\right) \quad \left(-\frac{1}{2}\right)$$

$a_1 = a = 2$

$r = -\frac{1}{2}$

$$\sum_{i=1}^{n} a_i = \frac{a}{1-r} = \frac{2}{1-\left(-\frac{1}{2}\right)} =$$

$$= \frac{2}{1+\frac{1}{2}} = \frac{2}{\frac{2}{2}+\frac{1}{2}} = \frac{2}{\frac{3}{2}}$$

$$= 2 \cdot \frac{2}{3} = \boxed{\frac{4}{3}}$$

(d)

1st
$$\sum_{n=1}^{\infty} \frac{(-1)^n 2^n}{3^{n-1}} = \sum_{n=0}^{\infty} \frac{(-1)^{n+1} \cdot 2^{n+1}}{3^{n-1+1}} = \sum_{n=0}^{\infty} \frac{-1 \cdot 2^1 \cdot 2^n}{3^n} =$$

$$= \sum_{n=0}^{\infty} \frac{-2 \cdot 2^n}{3^n} = -2 \sum_{n=0}^{\infty} \frac{2^n}{3^n} = -2 \sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n$$

2nd
$$\frac{1}{1-x} = \frac{1}{1-\frac{2}{3}} = \frac{1}{\frac{3}{3}-\frac{2}{3}} = \frac{1}{\frac{1}{3}} =$$

$= 1 \cdot \frac{3}{1} = \boxed{3}$ 3rd $-2 \sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n = -6$ FINAL Result

(4)

$$\sum_{i=0}^{n} 2-\frac{1}{3^{2i}} = \lim_{n\to\infty} \sum_{i=0}^{n} 2-\frac{1}{3^{2i}} =$$

$$= \lim_{n\to\infty} \sum_{i=0}^{n} \frac{6^{2i}-1}{3^{2i}} = \lim_{n\to\infty} \sum_{i=0}^{n} 2^{2i} - \frac{1}{3^{2i}} =$$

$$= 2^2 . 2^i - 0 = 4 . 2^i = 8^i$$

$$\frac{1}{1-x} = \frac{1}{1-8} = \frac{1}{-7}$$ The serie converges