

TD1 - LINEAR ALGEBRA; Pedro Gomes

(1)

$$(a) V_1 = \{(x, 2x + z - 1, z); x, z \in \mathbb{Q}\}$$

$$a \cdot v_1 + b \cdot v_2 \in V_1$$

Assume:

$$v_1 = (2, 6, 3)$$

$$v_2 = (3, 9, 4)$$

$$a \cdot v_1 + b \cdot v_2 \Leftrightarrow a(2, 6, 3) + b(3, 9, 4) =$$

$$= (2a, 6a, 3a) + (3b, 9b, 4b) =$$

$$= (2a + 3b, 6a + 9b, 3a + 4b)$$

$$2(2a + 3b) + 3a + 4b - 1$$

$$4a + 6b + 3a + b - 1 = 7a + 7b - 1 \neq 6a + 9b$$

Response: V_1 is not a vector space as $7a + 7b - 1 \neq 6a + 9b$

$$(b) V_2 = \{(x, y, z); x, y, z \in \mathbb{Q}, y \cdot z = 0\}$$

Assume: $v_1 = (1, 0, 2)$

$$v_2 = (2, 3, 0)$$

$$a \cdot v_1 + b \cdot v_2 \Leftrightarrow a(1, 0, 2) + b(2, 3, 0) =$$

$$= (a, 0, 2a) + (2b, 3b, 0) = (2b + a, 3b, 2a)$$

Response: V_2 is not a vector space, because we know that $y \parallel z$ must be $= 0$, and neither of them does.

$$(c) V_3 = \{(x, x+z, x-2z) \mid x, z \in \mathbb{Q}\}$$

$$v_1(2, 5, -4) \text{ ; where } z=3$$

$$v_2(3, 5, -1) \text{ ; where } z=2$$

$$\begin{aligned} a \cdot v_1 + b \cdot v_2 &\Leftrightarrow a(2, 5, -4) + b(3, 5, -1) = \\ &= (2a, 5a, -4a) + (3b, 5b, -b) = \\ &= (2a+3b, 5a+5b, -4a+(-b)) \end{aligned}$$

$$\begin{aligned} x+z &= 5a+5b \Rightarrow 2a+3b+z = 5a+5b \\ z &= 5a - 2a + 5b - 3b \\ z &= 3a + 2b \end{aligned}$$

$$\boxed{\text{if } z = 3a + 2b, \text{ then } x - 2z = -4a + (-b)} \quad ?$$

$$\begin{aligned} x - 2z &\Leftrightarrow 2a + 3b - 2(3a + 2b) = \\ &= 2a + 3b - 6a - 4b = -4a - b, \text{ which is equal} \\ &\text{to } -4a - b \end{aligned}$$

Response: The V_3 is a vector space.

(2) Generating system for $V_4 = \{(x, x+y, x-y); x, y \in \mathbb{Q}\}$

1st Separate vars:

$$V = \left\{ \begin{pmatrix} x \\ x \\ x \end{pmatrix} + \begin{pmatrix} 0 \\ y \\ -y \end{pmatrix} \right\}$$

2nd Mettre en évidence:

$$x \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + y \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

GS found as

$$\{(1, 1, 1), (0, 1, -1)\}$$