TD 9 CALCULUS – SOLUTIONS

(1) **Determine with a suitable substitution an anti-derivative for the following functions:**

(a) $\int (x+4)^5 \, dx$:

We remark that $(x+4)^5$ is a composed function with inner function $x+4$ (and outer function x^5), so we choose $t = x+4$. Then $dt = dx$, and we get

$$\int (x+4)^5 \, dx \;=\; \int t^5 \, dt \;=\; \frac{t^6}{6} \;=\; \frac{(x+4)^6}{6} \, .$$

(b) $\int \frac{1}{1-2x} \, dx$:

The only difficulty in this exercise is the observation that this function can also be seen as composed function. We choose as inner function $1-2x$ and as outer function $\frac{1}{x}$. Therefore, we do the substitution $t = 1-2x$, from which we derive $dt = -2dx$, or equivalently $dx = \frac{-1}{2} dt$. Then we determine

$$\int \frac{1}{1-2x} \, dx \;=\; \int \frac{1}{t} \cdot \frac{-1}{2} \, dt \;=\; \frac{-1}{2} \ln(t) \;=\; \frac{-\ln(1-2x)}{2} \, .$$

(c) $\int 2x\sqrt{1+x^2} \, dx$:

In the last example, the situation is slightly more complicated since the input function consists of a product of two terms. We note that the second factor is a composed function with inner function $1+x^2$. Therefore, we try the substitution $t = 1+x^2$. This leads to $dt = 2xdx$ or equivalently $dx = \frac{dt}{2x}$. As a consequence, we have

$$\int 2x\sqrt{1+x^2} \, dx \;=\; \int 2x \sqrt{t} \, \frac{dt}{2x} \;=\; \int \sqrt{t} \, dt \;=\; \int t^{1/2} \, dt \;=\; \frac{2}{3} \cdot t^{3/2} \;=\; \frac{2}{3}\left(1+x^2\right)^{3/2} .$$