

## TD 1 – CALCULUS

The following exercises (1) - (3) can all be solved with mathematical induction (note that there might also be other ways to prove the statements). For the last exercise, you should make use of a clever transformation of a formula presented in today's class.

- (1) Prove that for all integers  $n \geq 1$  we have

$$8 \mid 9^n - 1 .$$

(Notation  $a \mid b$  for two integers  $a, b$  means:  $a$  divides  $b$ )

- (2) Prove that for  $n \geq 1$  we have

$$\sum_{i=1}^n 2^i = 2^{n+1} - 2 .$$

- (3) Prove by induction that for  $n \geq 4$  the inequality  $n! > 2^n$  is true.

- (4) (**Binomial Theorem**) Determine the value of the following sum (without induction)

$$\sum_{k=0}^n \binom{n}{k} = ??? .$$