TD 4 – CALCULUS

(1) Consider the recurrent sequence defined for $n > 0$ by $u_n = u_{n-1} + \frac{1}{u_{n-1}}$ and $u_0 = 1$. Examine whether this sequence converges to a finite limit or not.

(2) Do the following series converge?

(a) $\sum_{n\geq 0} \frac{(n+2)^2}{2^{n+2}}$,

(b) $\sum_{n\geq 0} 0.9999^n$,

(c) $\sum_{n\geq 2} (-1)^{n-1} \frac{n^2-n}{n^2+n}$ (Note: kind of tricky!).

(3) **Nature of functions.** Are the functions given below injective from their domain to $\mathbb{R}$? Surjective? Continuous on $\mathbb{R}$?

(a) $f(x) = x^3$,

(b) $g(x) = x^3 - x$,

(c) $h(x) = \lfloor x \rfloor$ (floor of x is defined as the largest integer at most equal to x),

(d) $j(x) = |x|$,

(e) $k(x) = \frac{1}{x}$.