Naive Lie Theory 3.2 Exercises

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Notes

I wanted to give a more formal definition of path-connectedness here. We say a metric (or topological) space is path-connected if for any $x, y \in V$ we have a continuous function $f : I \to V$, where $I = [0, 1]$, $f(0) = x$ and $f(1) = y$. A common norm for matrices is the Frobenius norm:

$$||A|| = \sqrt{\Sigma_i^n \Sigma_j^m a_{ij}^2}$$

3.2.1

Say we have a path $P(x)$, where $x \in [0, 1]$. $P(0) = 1$ and $P(1) = B$, and P is continuous. We simply define $Q(x) = AP(x)$. As matrix multiplication is continuous, Q is continuous. $Q(0) = A$ and $Q(1) = AB$, so we have a continuous path from A to AB thus from 1 to AB, as we have one from 1 to A.

3.2.2

Similarly, we have $P(x)$ where $P(0) = 1$ and $P(1) = A$. We define $Q(x) = (P(x))^{-1}$, which is a continuous path from 1 to A^{-1}.

3.2.3

3.2.1 and 3.2.2 showed that the set of identity components are closed under group operation and inverse, thus it's a subgroup. Note that simply being closed under group operation is not sufficient, as that only works for finite groups.