Naive Lie Theory 2.5 Exercises

OblivionIsTheName

October 2025

Notes

This chapter is about expressing rotation and reflection of $\mathbb{R}^4$ in quaternion multiplications. Nothing much to say here additionally. I think this chapter is fairly clear and self-contained.

2.5.1

The check is easy. This map is a $\frac{\pi}{2}$ rotation in both 1-i and j-k planes. Alternatively, it is represented by the following matrix with respect to the standard basis of $\mathbb{H}$:

$$\begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

zero point is fixed.

2.5.2

Look at the the first theorem of this chapter. The map $q \mapsto -u\bar{q}u$ is the reflection in the hyperplane orthogonal to u. The wanted rotation is simply $\frac{\pi}{2}$ on $\mathbb{C}$. Recalling exercise 1.5.2, the desired rotation is a combination of reflection in lines with angles $\frac{\pi}{2}$ and $\frac{3\pi}{4}$, which are characterized by i and $\frac{i-1}{\sqrt{2}}$.

2.5.3

Completely the same process as 2.5.2, but it's a $\frac{\pi}{2}$ rotation in the j-k plane and fixing $\mathbb{C}$.

2.5.4

Just calculation.