

Naive Lie Theory 3.1 Exercises

OblivionIsTheName

September 2025

Notes

Nothing much to say here.

3.1.1

Anything with determinant -1 . For example $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

3.1.2

Same. $-I$. Notice that $-I$ has determinant -1 as the dimension is odd. It represents a reflection through the origin.

3.1.3

Let $n = (a, b, c)$. We want a reflection through a plane orthogonal to n . Say an arbitrary vector x , it goes in the direction of (a, b, c) until it hits the plane, then then continues the same distance on the other side. Therefore

$$R(x) = x - 2(x \cdot n)n$$

$x \cdot n$ can be written as $n^T x$. As $n^T x$ is a scalar, we can move n to the left, and $(x \cdot n)n$ can be written as $nn^T x$. So now we know that $\mathcal{M}(R) = I - 2nn^T$. One should check that

$$\mathcal{M}(R) = \begin{pmatrix} 1 - 2a^2 & -2ab & -2ac \\ -2ab & 1 - 2b^2 & -2bc \\ -2ac & -2bc & 1 - 2c^2 \end{pmatrix}$$