Naive Lie Theory 1.5 Exercises

OblivionIsTheName

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1.5.1

Congruent triangles.

1.5.2

Angle chasing.

1.5.3

Let the rotations be $R_p(\theta)$, $R_Q(\varphi)$, and $R_R(\chi)$, and reflections be $r_{\mathcal{L}}$, $r_{\mathcal{M}}$, and $r_{\mathcal{N}}$. According to 1.5.2, we have

$$\begin{cases} R_P(\theta) = r_{\mathcal{L}} r_{\mathcal{M}} \\ R_Q(\varphi) = r_{\mathcal{M}} r_{\mathcal{N}} \\ R_R(\chi) = r_{\mathcal{L}} r_{\mathcal{M}} \end{cases}$$

(One should notice that the reflection in the line with smaller angle in the direction of rotation should be done first, as rotations are not commutative.) Then we have:

$$R_P(\theta) R_Q(\varphi) = (r_{\mathcal{L}} r_{\mathcal{M}})(r_{\mathcal{M}} r_{\mathcal{N}}) = r_{\mathcal{L}}(r_{\mathcal{M}} r_{\mathcal{M}}) r_{\mathcal{N}} = r_{\mathcal{L}} r_{\mathcal{M}} = R_R(\chi)$$

1.5.4

According to 1.5.3, $R_P(\theta)R_Q(\varphi)$ still holds, and $\mathcal{L} \parallel \mathcal{N}$. It becomes a translation perpendicular to both lines.

1.5.5

Nothing changes except what 1.5.6 suggests.

1.5.6

Each ”line segment” in previous problems is an arc on a greatest circle on the spherical surface, which uniquely corresponds to a plane that contains center O. You can create a bijection between the set of all lines containing O and the set of all pairs of planes containing O. That said, the two points that the line intersect with spherical surface is the intersections of the two greatest circles, so any two greatest circles must intersect at two points on the spherical surfaces.