BMMS2633 Advanced Discrete Mathematics

Tutorial 4

(1) Determine whether the description of * is a valid definition of binary operation on the set.

(a) On $\mathbb{R}$, where $a*b = ab$

(b) On $\mathbb{Z}^+$, where $a*b = \frac{a}{b}$

(c) On $\mathbb{Z}$, where $a*b = a^b$

(d) On $\mathbb{Z}^+$, where $a*b = a^b$

(e) On $\mathbb{Z}^+$, where $a*b = a - b$

(f) On $\mathbb{R}$, where $a*b = a\sqrt{b}$

(g) On $\mathbb{Z}$, where $a*b = 2a + b$

(2) Determine whether the binary operation * on the set S is commutative and whether it is associative:

(a) $S = \mathbb{R}$, $a * b = \min\{a, b\}$

(b) $S = \mathbb{R}$, $a * b = \frac{ab}{3}$

(3) Fill in the following table so that the binary operation * is commutative.

*	a	b	c
a	b		
b	c	b	a
c	a		c

(4) Fill in the following table so that the binary operation * is commutative and has the idempotent property.

*	a	b	c
a		c	
b			
c	c	a	

(5) Consider the binary operation * defined on the set A = {a, b, c} by the following table.

*	a	b	c
a	b	c	b
b	a	b	c
c	c	a	b

(a) Is * a commutative operation?

(b) Compute a*(b*c) and (a*b)*c.

(c) Is * an associative operation?

(6) Complete the following tables so that the binary operation * is associative.

(a)

*	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	a	b
d				

(b)

*	a	b	c	d
a	a	b	c	d
b	b	a	c	d
c				
d	d	c	c	d

(7) Let A = {a, b}. Which of the following tables define a semigroup on A? Which define a monoid on A?

(a)

*	a	b
a	a	b
b	a	a

(b)

*	a	b
a	a	b
b	b	b

(8) Does the following table define a semigroup?

*	a	b	c
a	c	b	a
b	b	c	b
c	a	b	c

(9) Complete the following table to obtain a semigroup.

*	a	b	c
a	c	a	b
b	a	b	c
c			a

(10) Let S = {a,b}. Write the operation table for the semigroup (P(S), $\cup$).

(11) Determine whether the set together with the binary operation is a group.
   (a) $\mathbb{Z}$, where * is ordinary subtraction.
   (b) $\mathbb{Z}^+$, under the operation of addition.
   (c) $\mathbb{Q}$, the set of all rational numbers under the operation of addition.

(12) Let G be the group of integers under the operation of addition. Determine whether the following subsets of G are subgroups of G.
   (a) the set of all even integers;
   (b) the set of all odd integers;
   (c) the set of all multiples of 3.

Answer

(2) (a) Commutative, associative.
(b) Commutative, associative.

(5) (a) Not commutative.
(b) b
(c) Not associative.

(8) Not associative and not semigroup.

(11) (a) Not a group.
(b) Not a group.
(c) A group.

(12) (a) Subgroup of G.
(b) Not a subgroup of G.
(c) Subgroup of G.