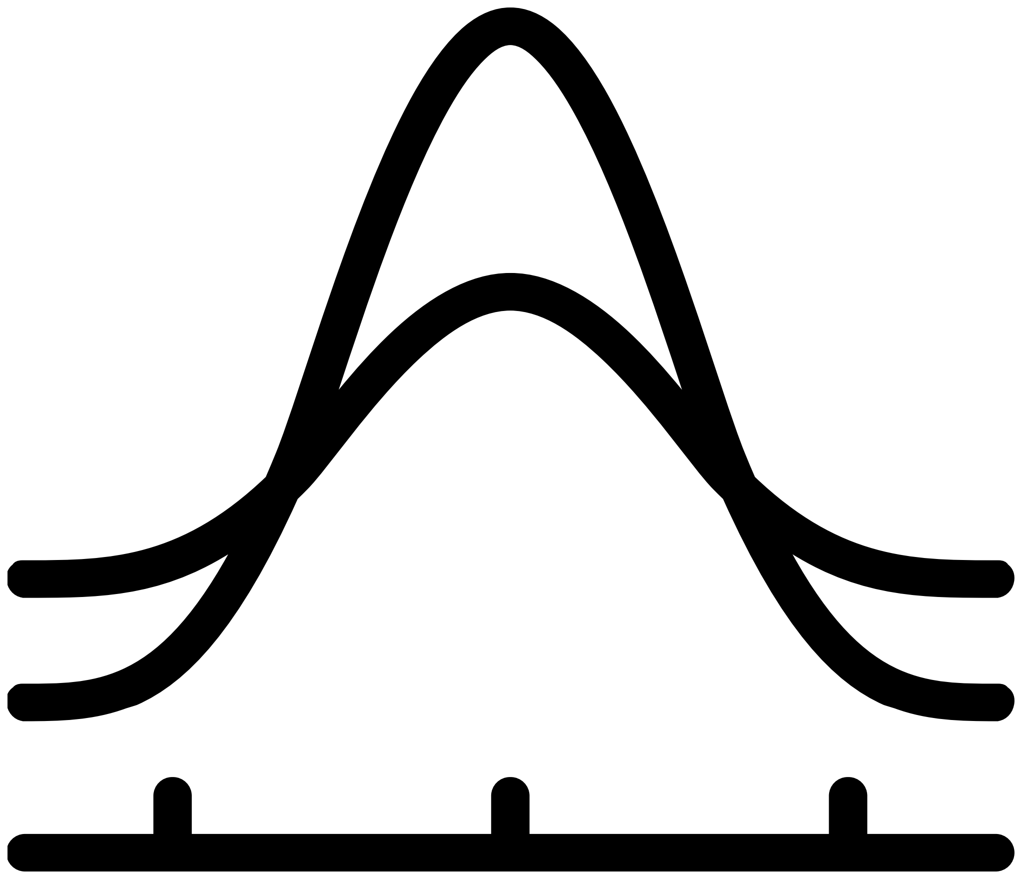
ARTIFICIAL INTELLIGENCE

BACS2003|BACS3074|BMCS2003

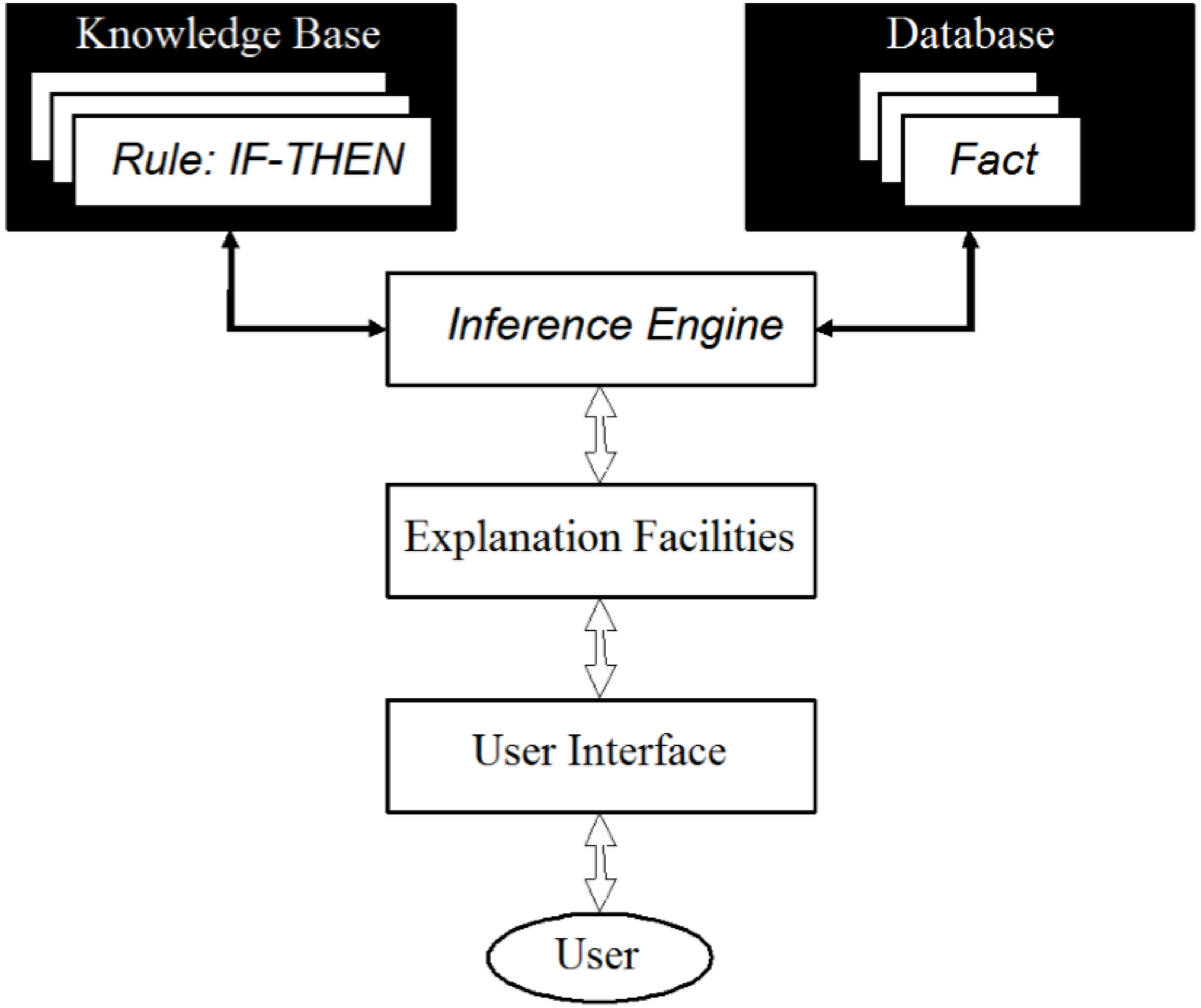
CHAPTER 12 UNCERTAINTY IN EXPERT SYSTEM

OUTCOMES

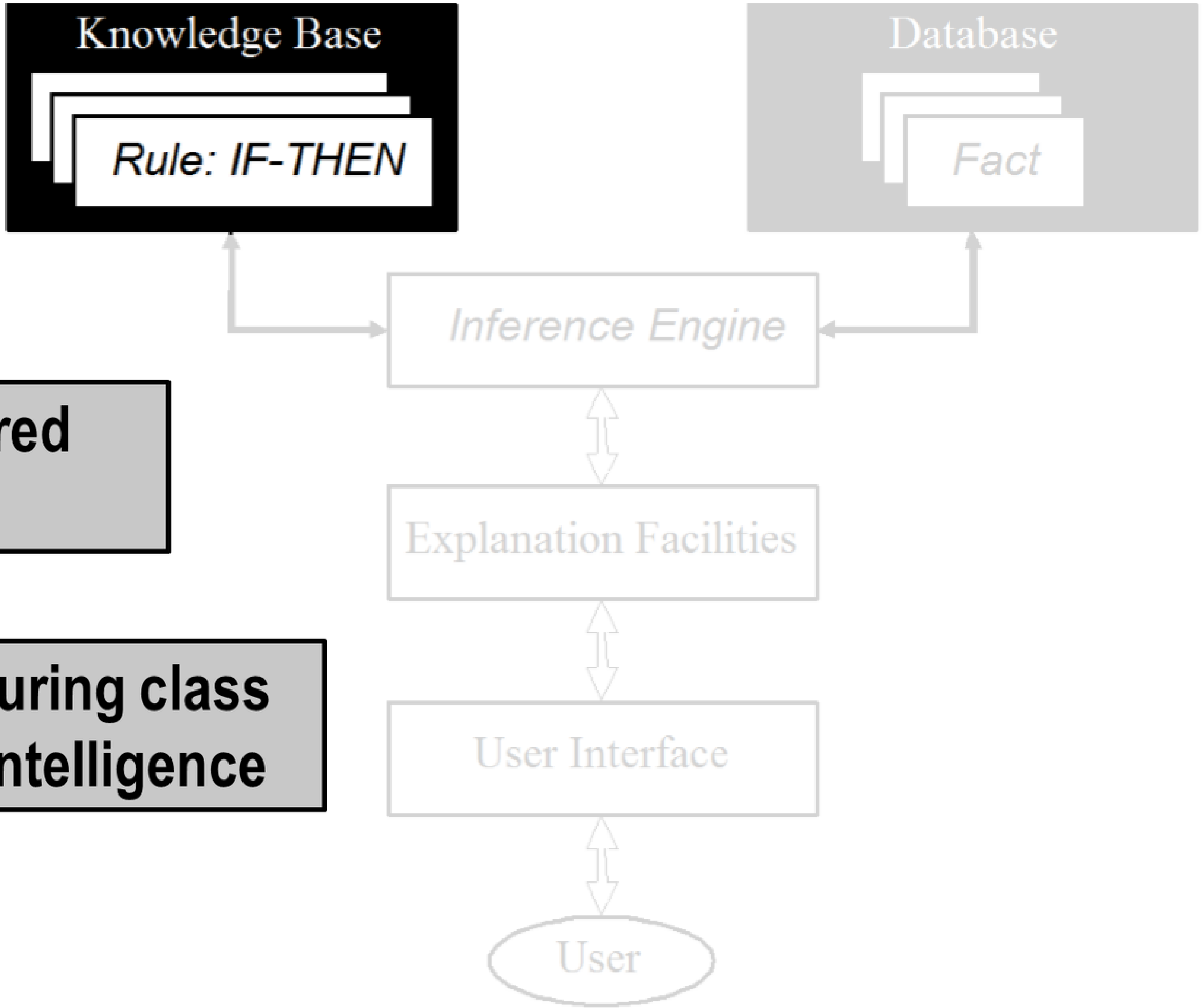
1. **Uncertainty vs. Classical Expert System**
2. **Basic Probability Theory**
3. **Bayesian Reasoning**
4. **Certainty Factors**



RECALL EXPERT SYSTEM



KNOWLEDGE BASE



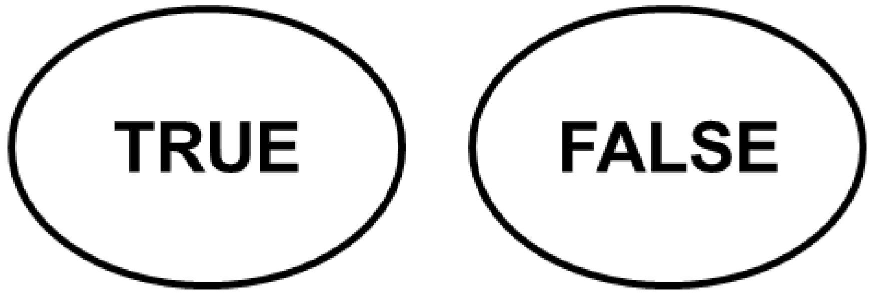
IF	**traffic light red**
THEN	**STOP!**

IF	**pay attention during class**
THEN	**pass Artificial Intelligence**

CLASSICAL ES VS UNCERTAINTY

Classic logic

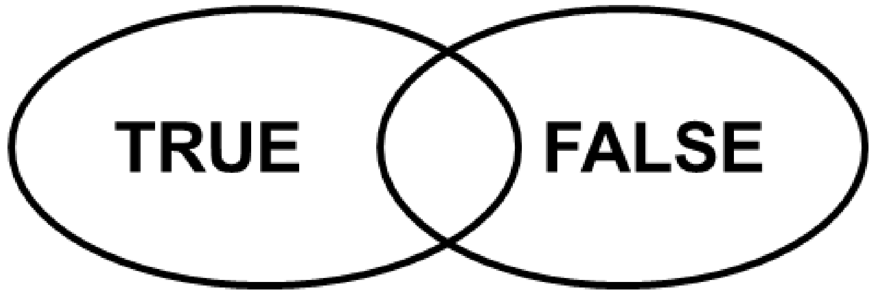
- permits only exact reasoning
- *law of the excluded middle*
- A → B



IF	**traffic light red**
THEN	**STOP!**

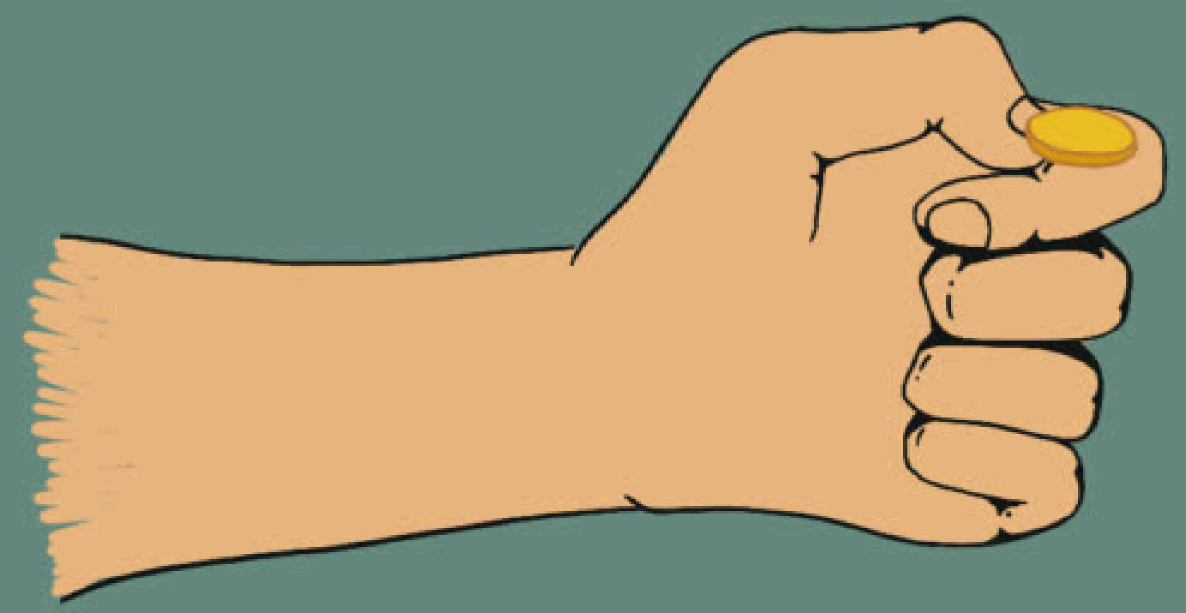
Uncertainty

- lack of the exact knowledge that still enable us to reach a perfectly reliable conclusion



IF	**pay attention during class**
THEN	**pass Artificial Intelligence**

CLASSICAL LOGIC



Toss a Coin:

IF it is HEAD
THEN it is NOT TAIL

IF it is TAIL
THEN it is NOT HEAD

Any chance it becomes HEAD and TAIL at the same time?

SOURCES OF UNCERTAIN KNOWLEDGE

Imprecise language

- natural language is ambiguous and imprecise.
- e.g. "*often*", "*sometimes*", "*frequently*", "*hardly ever*"

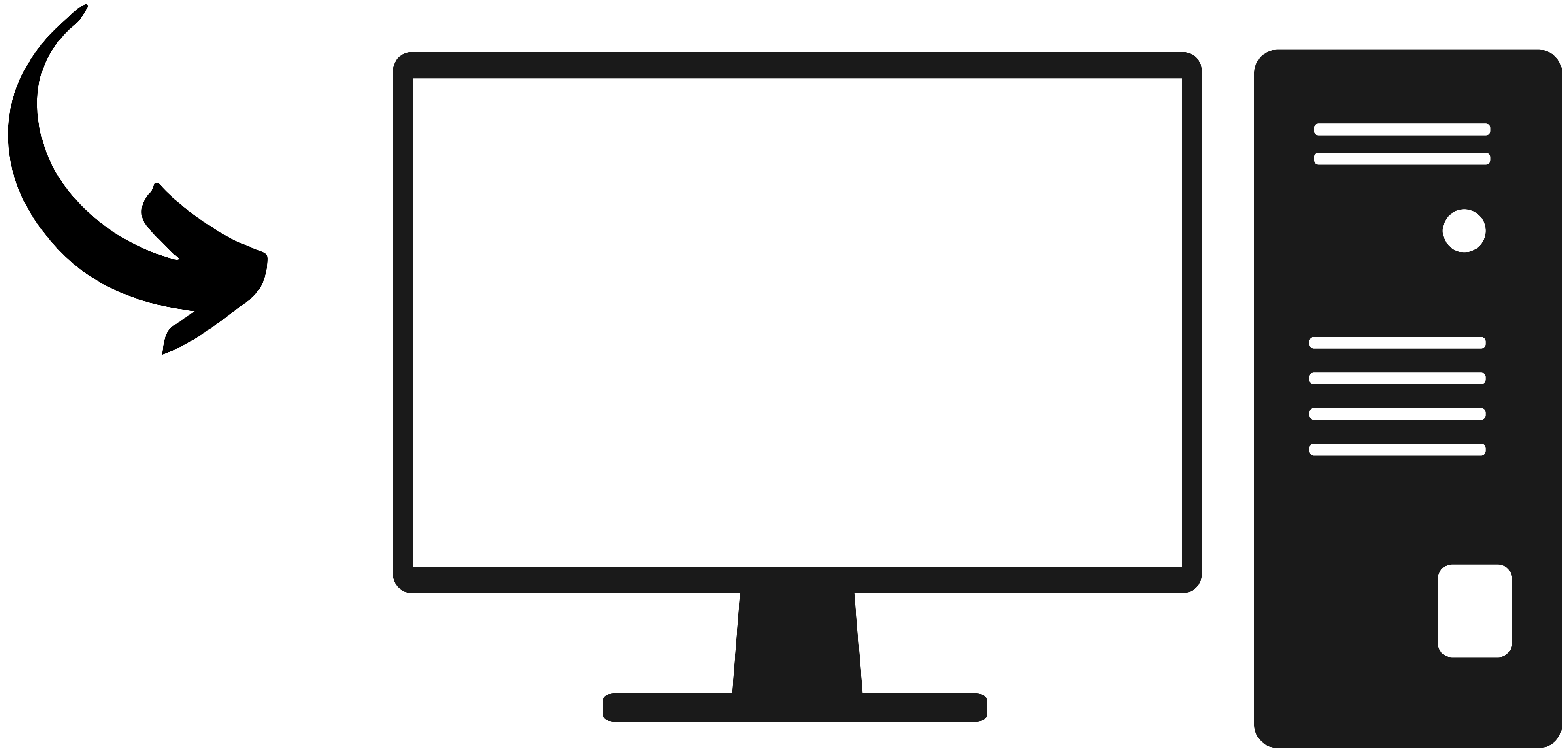
Unknown data

- When the data is incomplete or missing, the only solution is to accept the value "unknown" and proceed to an approximate reasoning with this value.
- e.g. "*I don't know*", "*I am not sure*", "*I don't understand*"

EXAMPLE OF UNCERTAIN DATA COLLECTION

An expert system may gather information through the following questions:

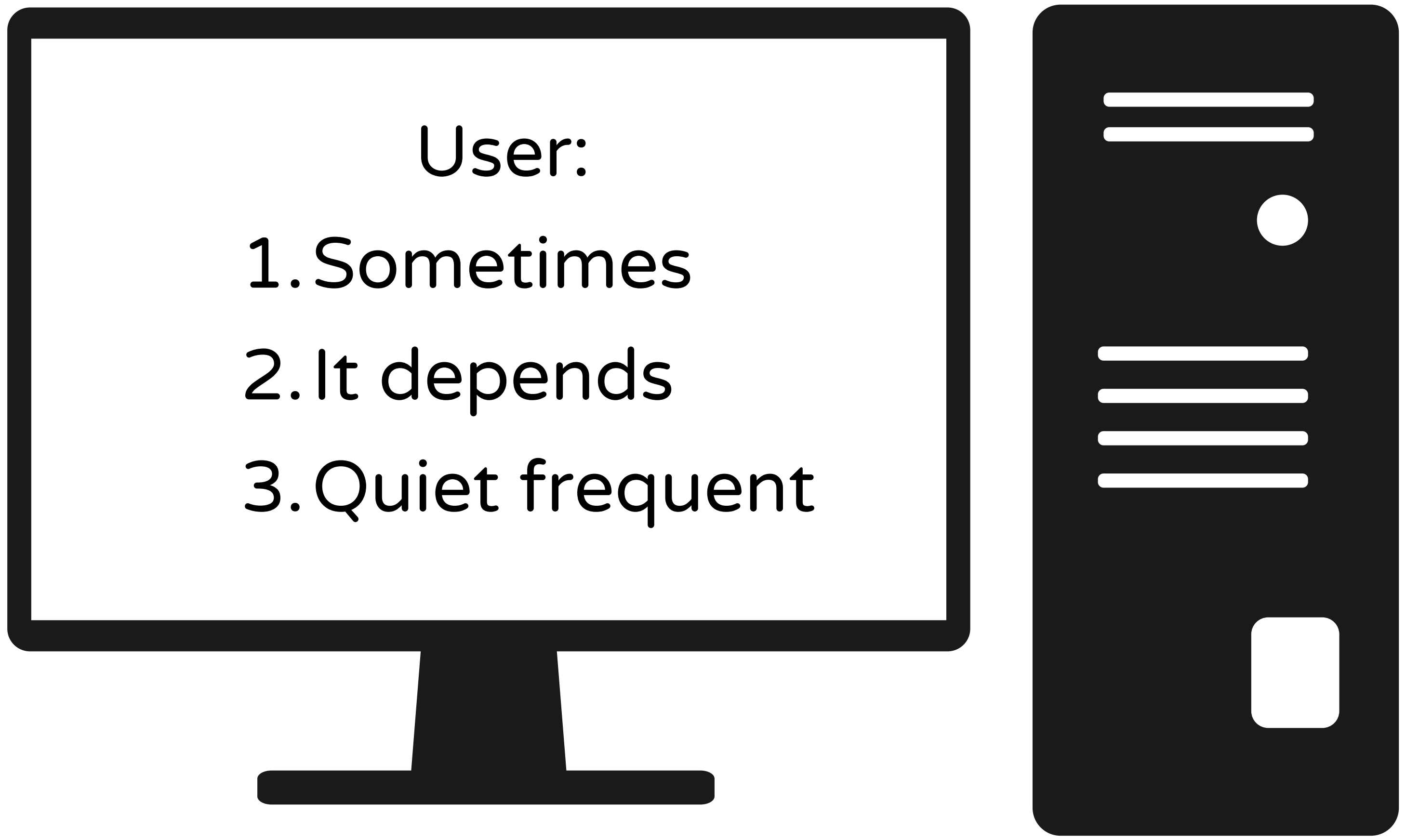
1. Do you feel dizzy?
2. Is your heart beat unordinary fast?
3. Do you have insomnia?



EXAMPLE OF UNCERTAIN DATA COLLECTION

An expert system may gather information through the following questions:

1. Do you feel dizzy?
2. Is your heart beat unordinary fast?
3. Do you have insomnia?

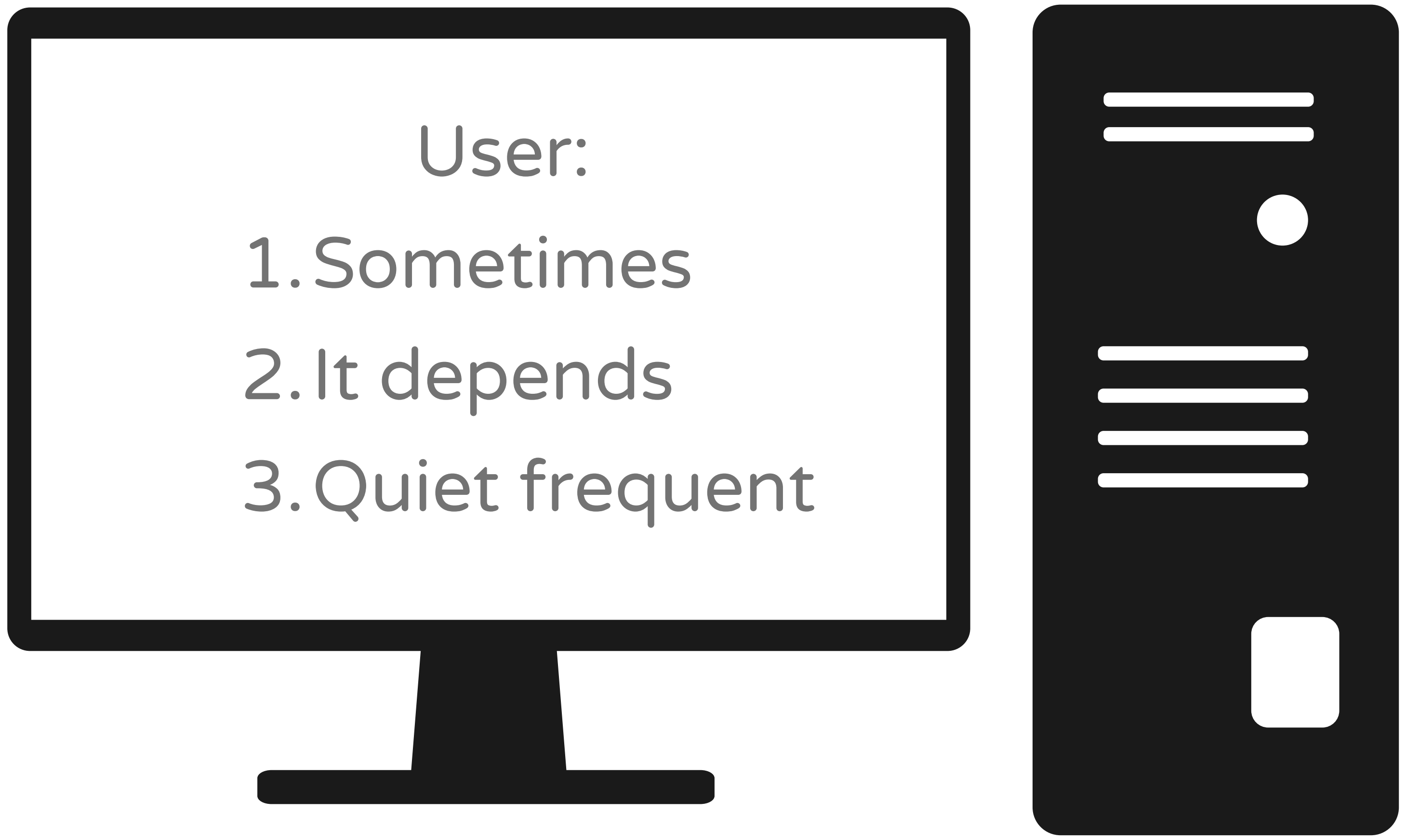


EXAMPLE OF UNCERTAIN DATA COLLECTION

An expert system may gather information through the following questions:

1. Do you feel dizzy?
2. Is your heart beat unordinary fast?
3. Do you have insomnia?

Conclusion: Probably you are falling in love



QUANTIFICATION OF AMBIGUOUS AND IMPRECISE TERMS

Ray Simpson (1944)		*Milton Hakel* (1968)	
Term	*Mean value*	*Term*	*Mean value*
Always	99	Always	100
Very often	88	Very often	87
Usually	85	Usually	79
Often	78	Often	74
Generally	78	Rather often	74
Frequently	73	Frequently	72
Rather often	65	Generally	72
About as often as not	50	About as often as not	50
Now and then	20	Now and then	34
Sometimes	20	Sometimes	29
Occasionally	20	Occasionally	28
Once in a while	15	Once in a while	22
Not often	13	Not often	16
Usually not	10	Usually not	16
Seldom	10	Seldom	9
Hardly ever	7	Hardly ever	8
Very seldom	6	Very seldom	7
Rarely	5	Rarely	5
Almost never	3	Almost never	2
Never	0	Never	0

THREE APPROACHES TO DEAL WITH UNCERTAINTY

1. Probability Theory
2. Certainty Factors
3. Fuzzy sets

Expressed as a numerical index

Range

0 ------------------------------------ 1

absolute impossibility absolute certainty

Example: Is it raining?

0	0.5	1
Not rain	I don't know	Rain

Conditional probability refers to the probability of an event A occurring given that another event B has already occurred.

Consider 2 events: A and B:

A = Road is wet, B = raining

- Assumption: A and B are not mutually exclusive
- A occur conditionally on the occurrence of B (Road is wet because it was raining)

The probability that event A will occur if event B occurs is called the conditional probability.

$p(A|B)$ = "Conditional probability of event A occurring given that event B has occurred".

PROBABILITY THEORY

Conditional probability

A = Road is wet, B = raining
p(A|B) = "Conditional probability of event A occurring given that event B has occurred".
where,
the probability that A and B can occur = p(A∩B)
the number times B can occur = p(B)

Therefore

$$p(A|B) = \frac{p(A \cap B)}{p(B)}$$

Similarly, the conditional probability of event B occurring given that event A has occurred equals

$$p(B|A) = \frac{p(B \cap A)}{p(A)}$$

Hence,

$$p(B \cap A) = p(B|A) \times p(A)$$

$$p(A \cap B) = p(B|A) \times p(A)$$

Substituting the last equation into the first equation

$$p(A|B) = \frac{p(A \cap B)}{p(B)}$$

where,
the probability that A and B can occur = p(A∩B)
the number times B can occur = p(B)

PROBABILITY THEORY

BAYESIAN RULE / BAYE'S THEOREM

$$p(A|B) = \frac{p(B|A) \times p(A)}{p(B)}$$

where:

- p(A|B) is the conditional probability that event A occurs given that event B has occurred;
- p(B|A) is the conditional probability of event B occurring given that event A has occurred;
- p(A) is the probability of event A occurring;
- p(B) is the probability of event B occurring.

PROBABILITY THEORY

Bayesian rule / Baye's Theorem:

$$p(A|B) = \frac{p(B|A) \times p(A)}{p(B)}$$

$$\boxed{P(H \mid E) = \frac{P(E \mid H) * P(H)}{P(E)}}$$

where:

- p(H|E) is the posterior probability that hypothesis H occurs given that evidence E has occurred;
- p(E|H) is the conditional probability of evidence E occurring given that hypothesis H has occurred;
- p(H) is the prior probability of hypothesis H occurring;
- p(E) is the probability of evidence E occurring.

EXERCISE

(Single evidence with Single hypothesis)

$$P(H \mid E) = \frac{P(E \mid H) * P(H)}{P(E)}$$

IF it was raining yesterday night

THEN the backyard is wet in the next morning (P=0.7)

It rains 200 days in a year.

There is 60% chance that the backyard will be wet

Question: If the backyard is wet, what is the chance that yesterday night rained? P(rain|wet) = ?

EXERCISE

(Single evidence with Single hypothesis)

$$P(H \mid E) = \frac{P(E \mid H) * P(H)}{P(E)}$$

IF it was raining yesterday night

THEN the backyard is wet in the next morning (P=0.7)

It rains 200 days in a year.

There is 60% chance that the backyard will be wet

Question: If the backyard is wet, what is the chance that yesterday night rained? P(rain|wet) = ?

P(rain|wet) = P(wet|rain)*P(rain) / P(wet)

= 0.7*(200/365) / 0.6

= 0.64

Conclusion: 64% it was raining yesterday night

EXERCISE (E IS UNKNOWN)

(Single evidence with Multiple hypothesis)

$$P(H|E) = \frac{P(E|H) * P(H)}{P(E)}$$

R1: If there was drizzle last night, then there is 60% chance that the grass in the backyard is wet the next morning.

R2: If there was rain last night, then there is 80% chance that the grass in the backyard is wet the next morning.

Drizzle occurs in 160 days in a year.
Rain occurs in 120 days in a year.

Question: If you see the grass in the backyard is wet in the morning, by using Bayes Theorem, what is the event that most probably occurred last night?

P(drizzle,H1 | wet) = ?

P(rain,H2 | wet) = ?

P(wet E|drizzle H1) = 0.6

P(wet E|rain H2) = 0.8

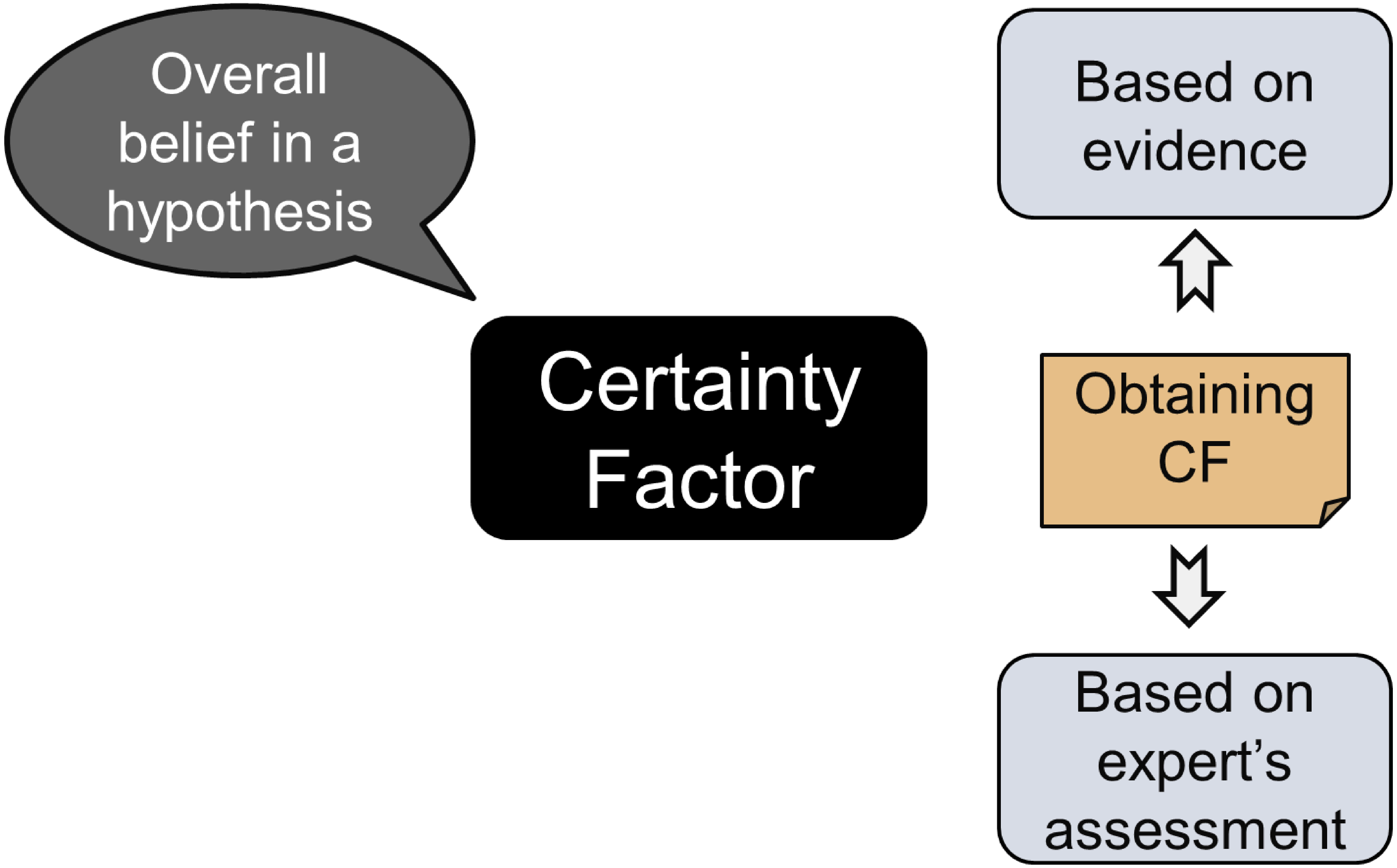
P(drizzle H1) =160/365 = 0.438

P(rain H2) = 120/365 = 0.329

$$P(drizzle\ H|wet\ E) = \frac{P(wet|drizzle) * P(drizzle)}{P(wet|drizzle) * P(drizzle) + P(wet|rain) * P(rain)}$$

$$P(rain|wet) = \frac{P(wet|rain) * P(rain)}{P(wet|drizzle) * P(drizzle) + P(wet|rain) * P(rain)}$$

CERTAINTY FACTOR THEORY



REASONING WITH UNCERTAINTY

$$\text{IF E1 THEN H } (CF = CF_i)$$

where,

E1 is available evidence,

H is conclusion (hypothesis),

CF_i is the level (value) of belief in H given the evidence, E1.

- Note: CFi bounded to range between -1 and +1

REASONING WITH UNCERTAINTY

IF E1 AND E2 THEN H ($CF = CF_i$)

where,

E1 and E2 is available evidence

H is conclusion (hypothesis)

CF_i is the level of belief in H given the evidence, E1 and E2

MEASURE OF BELIEF (MB) AND DISBELIEF (MD)

Measure of belief (MB) and measure of disbelief (MD)

- each bounded to lie between “0” and “1”

CF = MB - MD

- CF bounded to lie between -1 and +1

Negative indicates disbelief.

- -1 indicates maximum disbelief

Positive indicates belief

- 1 indicates maximum belief

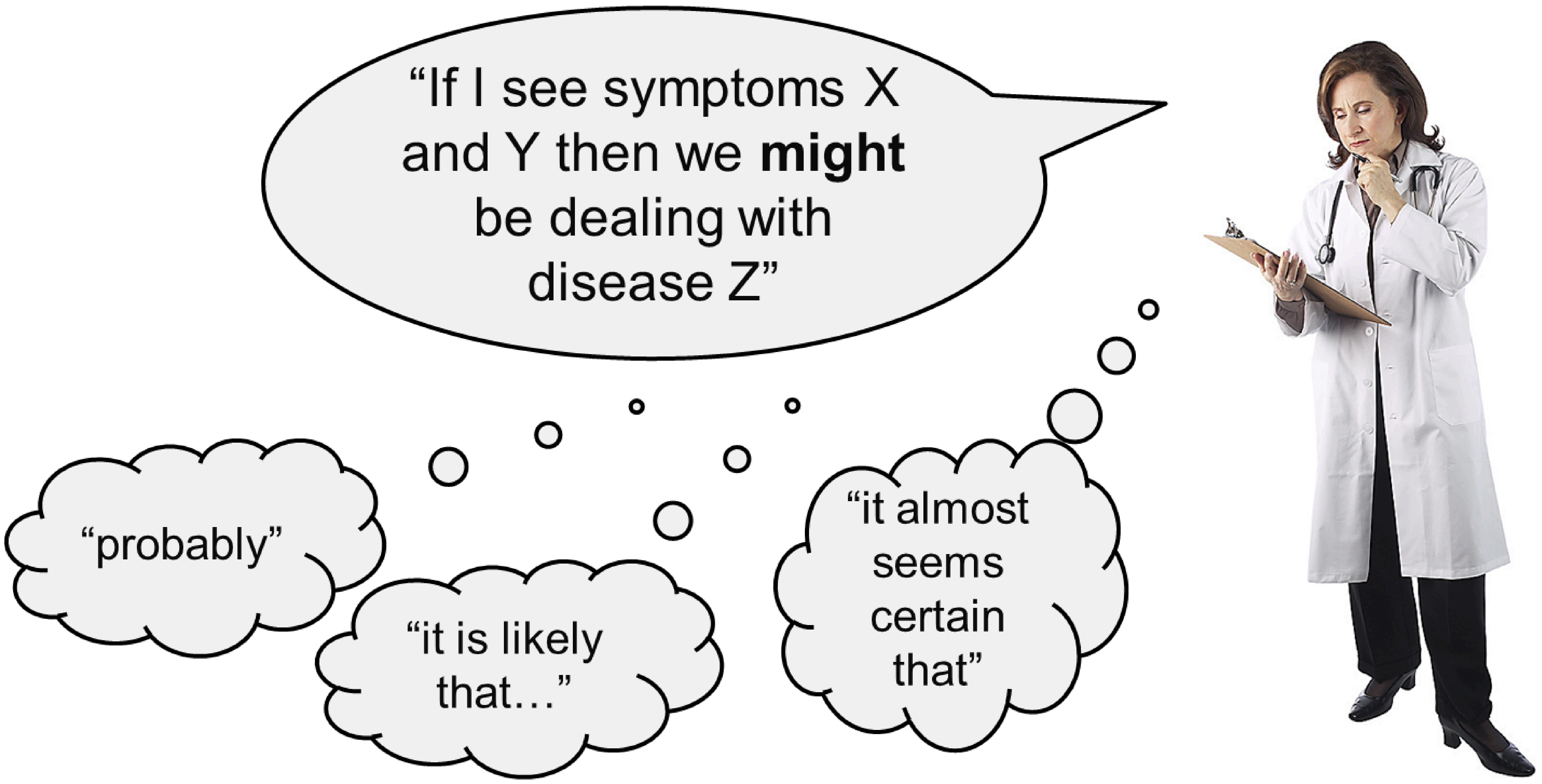
0 indicates ‘don't know’

UNCERTAIN TERMS AND THEIR INTERPRETATION

Term	Certainty Factor
Definitely not	-1.0
Almost certainly not	-0.8
Probably not	-0.6
Maybe not	-0.4
Unknown	-0.2 to +0.2
Maybe	+0.4
Probably	+0.6
Almost certainly	+0.8
Definitely	+1.0

EXAMPLE

Medical diagnosis and inexact reasoning



EXAMPLE

Term	Certainty Factor
Definitely not	-1.0
Almost certainly not	-0.8
Probably not	-0.6
Maybe not	-0.4
Unknown	-0.2 to +0.2
Maybe	+0.4
Probably	+0.6
Almost certainly	+0.8
Definitely	+1.0

Medical diagnosis and inexact reasoning

IF patient has fever and red spots
THEN dengue CF = ?

How certain are you that the patient has dengue? 50%, 80%, or…? Based on expert's assessment

Note: A patient that has fever and spots may not necessary has dengue (may be infected by measles or lassa fever or chicken pox).

CERTAINTY FACTOR

An expert system can propagate CFs via rules to give CFs to conclusions. For this, rules too may have CFs associated with them. E.g.

if (B or C) then A (CF 0.8)

This says that if we know **B (cf = 1)** or **C (cf=1)** to be true then this knowledge will give us a degree of belief that **A** is true.

But what if B and C themselves are subject to uncertainty?

CERTAINTY FACTOR

Term	Certainty Factor
Definitely not	-1.0
Almost certainly not	-0.8
Probably not	-0.6
Maybe not	-0.4
Unknown	-0.2 to +0.2
Maybe	+0.4
Probably	+0.6
Almost certainly	+0.8
Definitely	+1.0

Rule 1:
IF E1 THEN H *(CF=0.7)*

Observation:

E1 (CF=0.6)

Conclusion: $CF_{E1} \times CF_{R1} = 0.6 \times 0.7$
$= 0.42$

FORMULA

We must provide rules which allow CFs of logical combinations to be calculated.

<u>Rules:</u>

1. CF(A and B) = min(CF(A),CF(B))
2. CF(A or B) = max(CF(A),CF(B))

EXAMPLE

Rules:

1. CF(A and B) = min(CF(A),CF(B))
2. CF(A or B) = max(CF(A),CF(B))

Term	Certainty Factor
Definitely not	-1.0
Almost certainly not	-0.8
Probably not	-0.6
Maybe not	-0.4
Unknown	-0.2 to +0.2
Maybe	+0.4
Probably	+0.6
Almost certainly	+0.8
Definitely	+1.0

IF you did exercise and did assignment,

THEN you will pass exam.

Given:

CF(did exercise) = 0.8

CF(did assignment)=0.9

MIN [0.8, 0.9] X 1 = 0.8

EXAMPLE

Rules:

1. CF(A and B) = min(CF(A),CF(B))
2. CF(A or B) = max(CF(A),CF(B))

Term	Certainty Factor
Definitely not	-1.0
Almost certainly not	-0.8
Probably not	-0.6
Maybe not	-0.4
Unknown	-0.2 to +0.2
Maybe	+0.4
Probably	+0.6
Almost certainly	+0.8
Definitely	+1.0

IF you did exercise OR did assignment,

THEN you will pass exam. (CF 0.9)

Given:

CF(did exercise) = 0.8

CF(did assignment)=0.7

MAX [0.8, 0.7] X 0.9 = 0.72

TRY THIS!

Rules:

1. CF(A and B) = min(CF(A),CF(B))
2. CF(A or B) = max(CF(A),CF(B))

Term	Certainty Factor
Definitely not	-1.0
Almost certainly not	-0.8
Probably not	-0.6
Maybe not	-0.4
Unknown	-0.2 to +0.2
Maybe	+0.4
Probably	+0.6
Almost certainly	+0.8
Definitely	+1.0

Here is a set of rules:

1. if (B or C) then A (CF 0.6)
2. if (D and E and F) then B (CF 1.0)
3. if (G or H) then C (CF 0.75)

Suppose that we know CFs for D, E, F, G and H as follows:

CF(D) = 0.8 CF(E) = 0.5 CF(F) = 0.9 CF(G) = 0.2 CF(H) = 0.8

Find the CF for A.

CERTAINTY FACTOR(DIFFERENT EVIDENCE/RULE WITH THE SAME CONCLUSION)

1. It may happen that a conclusion is supported by more than one rule. For example, in addition to the rules (if (B or C) then A (CF 0.6)), we might have another rule for A:
   - if (P or Q) and R then A (CF 0.8)
2. This could lead to a quite independent calculation of a CF for A. How should this be combined with the other one to obtain an overall CF?
3. Suppose that we can find certainty factors CF1 and CF2 for a conclusion by application of two independent rules.
4. We can then calculate an overall CF by applying one of three formulas:

CERTAINTY FACTOR(DIFFERENT EVIDENCE/RULE WITH THE SAME CONCLUSION)

$$
\begin{cases}
CF1 + CF2 - CF1*CF2 & \text{if both are positive,} \\
CF1 + CF2 + CF1*CF2 & \text{if both are negative,} \\
\dfrac{CF1 + CF2}{1 - min(|\,CF1\,|,|\,CF2\,|)} & \text{otherwise}
\end{cases}
$$

EXAMPLE

$$
\begin{cases}
CF1 + CF2 - CF1*CF2 & \text{if both are positive,} \\
CF1 + CF2 + CF1*CF2 & \text{if both are negative,} \\
\dfrac{CF1 + CF2}{1 - min(|CF1|, |CF2|)} & \text{otherwise}
\end{cases}
$$

R1: if you studied hard, then you pass exam.(CF=0.36)

R2: if you did tutorials, then you pass exam.(CF=0.75)

Observation

'you studied hard' = {1.0}

'you did tutorials' = {1.0}

What is CF(pass exam)??

$$= 0.36 + 0.75 – (0.36 \text{ X } 0.75)$$

$$= 0.36 + 0.75 – 0.27$$

$$= 0.84$$

TRY THIS!

Rules:

1. CF(A and B) = min(CF(A),CF(B))
2. CF(A or B) = max(CF(A),CF(B))

$$
\begin{cases}
CF1 + CF2 - CF1*CF2 & \text{if both are positive,} \\
CF1 + CF2 + CF1*CF2 & \text{if both are negative,} \\
\dfrac{CF1 + CF2}{1 - \min(|CF1|,|CF2|)} & \text{otherwise}
\end{cases}
$$

R1: IF she smiles at you AND she blinks at you

THEN she likes you CF = 0.4

R2: IF she invites you for dinner AND she gives you present

THEN she likes you CF = 0. 8

Observation

CF(she smiles at you) = 0.9

CF(she blinks at you) = 0.8

CF(she invites you for dinner) = 0.6

CF(she gives you present) = 0.6

What is CF(She likes you)?

TRY THIS!

<u>Rules:</u>

1. CF(A and B) = min(CF(A),CF(B))
2. CF(A or B) = max(CF(A),CF(B))

$$CF = \begin{cases} CF1 + CF2 - CF1*CF2 & \text{if both are positive,} \\ CF1 + CF2 + CF1*CF2 & \text{if both are negative,} \\ \dfrac{CF1 + CF2}{1 - \min(|CF1|,|CF2|)} & \text{otherwise} \end{cases}$$

R1: IF nose is stuffy [P1.1] **AND** stuffy nose resolves within week [P1.2] THEN symptom of cold [H1]	**R3:** IF sneezing present [P3.1] **AND** headaches not present [P3.2] **AND** sore throat presents [P3.3] THEN symptom of cold (cf(H3) = 0.9)
R2: IF cough present [P2.1] **AND** cough is productive [P2.2] THEN symptom of cold [H2]	**R4:** IF fever not present [P4.1] **OR** fever is mild [P4.2] THEN symptom of cold (cf(H4) = 0.8)

Question: identify the certainty factor that the user is getting a cold.

- His nose is stuffy CF = 0
- Stuffy nose resolves within week CF = 0
- Cough present CF = 0.8
- Cough is productive CF = -0.4
- Sneezing present CF= 1.0

- **Headaches present CF = 1.0**
- Sore throat present CF = 0.8
- **Fever present CF = 1.0**
- Fever is mild CF = 0.4

ADVANTAGES

Easy to compute

- Can be used to easily propagate uncertainty through the system.

It has been found to be remarkably resilient

- Systems which use them seem to work reasonably well in practice, in comparison with human experts.

DRAWBACKS

Assumption of independent probabilities

- The certainty factor of two rules in an inference chain is calculated as independent probabilities.

CF is not being rigorously founded

- The use of CFs has been criticized (with some justification) for not being rigorously founded.

THE END

NEXT LECTURE

Fuzzy Logic