April 2024

BAMS1623 Discrete Mathematics: Test (50%)

Name: ______________________

Marks: ______ /50

Programme/Group: ______________________

Answer ALL **SEVEN (7)** questions. Time allocated: 1 hour.
Write your answers **in the space provided.**

Question 1

2 3 4 5 6 7 8

Let $U = \{x : x \in \mathbb{Z} \text{ and } 2 \le x \le 8\}$ be the universal set, $A = \{x : x \in U \text{ and } x^2 + x - 2 = 0\}$, $B = \{x : x \in U \text{ and } x \text{ is a factor of } 12\}$ and $C = \{x : x \in U \text{ and } x \text{ is an odd number}\}$.

1, 2, 3, 4, 6, 12

(3 marks)

a) List all the elements of sets A, B and C.

$A = \emptyset$

$B = \{2, 3, 4, 6\}$

$C = \{3, 5, 7\}$

(4 marks)

b) Find $\overline{(B \oplus C)} - A$.

$\overline{(B \oplus C)} - A = \overline{\{2, 4, 5, 6, 7\}} - \{\}$

$= \{3, 8\}$ #

Question 2

Compute the greatest common divisor of $a = 1391$ and $b = 312$ by using Euclidean algorithm and express it in the form of $sa + tb$, where $s, t \in \mathbf{Z}$. Hence, obtain the least common multiple of a and b. (7 marks)

$1391 = 4(312) + 143$

$312 = 2(143) + 26$

$143 = 5(26) + 13$

$26 = 2(13) + 0$

$GCD(1391, 312) = 13$ #

$LCM(1391, 312) = \dfrac{1391 \times 312}{13}$

$= 33384$ #

$143 = 1391 - 4(312)$

$26 = 312 - 2(143)$

$13 = 143 - 5(26)$

$GCD(1391, 312) = 13 = 143 - 5(26)$

$= 143 - 5(312 - 2(143))$

$= 11(143) - 5(312)$

$= 11(1391 - 4(312)) - 5(312)$

$= 11(1391) - 49(312)$ #

$sa + tb$

50

Question 3

Prove by induction that $6^n - 2^n$ is divisible by 4 for all positive integer $n \geq 1$. (5 marks)

when $n = 1$, $P(1) = 6^1 - 2^1 = 4$, $P(1)$ is divisible by 4

when $n = k$, assume $6^k - 2^k = 4a$, divisible by 4

when $n = k+1$, $P(k+1) = 6^{k+1} - 2^{k+1}$

$= 6(6^k) - 2(2^k)$

$= 2[3(6^k) - (2^k)]$

$= 2[2(6^k) + (6^k) - (2^k)]$

$= 2[2(6^k) + 4a]$

$= 4(6^k) + 8a$

$= 4(6^k + 2a)$ divisible by 4

+4

$\therefore$ As $4(6^k + 2a)$ is divisible by 4, it is proven that $6^n - 2^n$ is divisible by 4 for all positive intege $n \geq 1$.

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Question 4

Given that $A = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$, compute

a) $A \vee B$ (2 marks)

$= \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{pmatrix}$

b) $A \wedge B$ (2 marks)

$= \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$

c) $A \odot B$ (2 marks)

$= \begin{pmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$

$p \wedge t \equiv p$ $\quad p \vee t \equiv t$ $\quad p \vee \sim p \equiv t$

$p \wedge c \equiv c$ $\quad p \wedge \sim p \equiv c$

Question 5

A logical statement is given by $A \equiv p \rightarrow (q \wedge \sim r)$.

a) Simplify statement A to the principle disjunctive normal form (PDNF) by using the laws of algebra of propositions. Hence, determine whether A is a tautology, contradiction, or contingency. (7 marks)

$A \equiv p \rightarrow (q \wedge \sim r)$

$\therefore$ contigency

$\equiv \sim p \vee (q \wedge \sim r)$

$\equiv [\sim p \wedge (q \vee \sim q) \wedge (r \vee \sim r)] \vee [(p \vee \sim p) \wedge q \wedge \sim r]$

$\equiv [((\sim p \wedge q) \vee (\sim p \wedge \sim q)) \wedge (r \vee \sim r)] \vee [((p \wedge q) \vee (\sim p \wedge q)) \wedge \sim r]$

$\equiv [(\sim p \wedge q \wedge r) \vee (\sim p \wedge q \wedge \sim r) \vee (\sim p \wedge \sim q \wedge r) \vee (\sim p \wedge \sim q \wedge \sim r)] \vee$
$[(p \wedge q \wedge \sim r) \vee (\sim p \wedge q \wedge \sim r)]$

$\equiv (\sim p \wedge q \wedge r) \vee (\sim p \wedge q \wedge \sim r) \vee (\sim p \wedge \sim q \wedge r) \vee (\sim p \wedge \sim q \wedge \sim r) \vee (p \wedge q \wedge \sim r)$

b) Obtain the principal disjunctive normal form (PDNF) and principal conjunctive normal form (PCNF) of the statement $\sim A$. (3 marks)

PDNF of $\sim A \equiv (p \wedge q \wedge r) \vee (p \wedge \sim q \wedge r) \vee (p \wedge \sim q \wedge \sim r)$

PCNF of $\sim A \equiv \sim [\text{PDNF of } A]$

$\equiv (p \vee \sim q \vee \sim r) \wedge (p \vee \sim q \vee r) \wedge (p \vee q \vee \sim r) \wedge (p \vee q \vee r) \wedge (\sim p \vee \sim q \vee r)$

c) Let p, q and r be the following propositions:
p : n is even.
q : n is prime.
r : $n-1$ is even.
Write the converse, inverse, contrapositive, and negation forms for statement A, in sentence form. (4 marks)

Converse : If n is prime and n-1 is not even, then n is even.

Inverse : If n is not even, then n is not prime or n-1 is even.

Contrapositive : If n is not prime or n-1 is even, then n is not even.

Negation : ~~n is even and n is not prime or n-1 is even.~~

If n is not even then q is not prime and n-1 is even.

Question 6

Let the universe of discourse be $\{2, 3, 4\}$. The predicate $P(x, y)$ is defined below:

$$P(x,y) : x \text{ divides } y. \qquad y \text{ mul of } x$$

Rewrite the expression $\forall y[\exists x P(x,y)]$ by eliminating the quantifiers. Hence, determine the truth value of the statement.

(6 marks)

$\forall y\,[\exists x\, P(x,y)]$

$= \forall y\,[P(2,y) \vee P(3,y) \vee P(4,y)]$

$= [P(2,2) \vee P(3,2) \vee P(4,2)] \wedge [P(2,3) \vee P(3,3) \vee P(4,3)] \wedge [P(2,4) \vee P(3,4) \vee P(4,4)]$

$= (T \vee F \vee F) \wedge (F \vee T \vee F) \wedge (T \vee F \vee T)$

$= T \wedge T \wedge T$

$= T$

$\therefore$ True

Question 7

Show that the following argument form is valid:

$q \leftrightarrow \sim r$
$\sim r \to p$
$\therefore q \wedge r$

(5 marks)

p	q	r	$\sim r$	$q \leftrightarrow \sim r$	$\sim r \to p$	$q \wedge r$
0	0	0	1	0	0	0
0	0	1	0	0	1	0
0	1	0	1	1	0	0
0	1	1	0	0	1	1
1	0	0	1	0	1	0
1	0	1	0	1 ✓	1 ✓	0
1	1	0	1	1 ✓	1 ✓	0
1	1	1	0	0	1	1

$\therefore$ Invalid argument