BMMS2633 Advanced Discrete Mathematics

Tutorial 1

(1) In each case, determine whether the relation R defined on the set A is a tree, and if it is, find the root.

(a) A = {a,b,c,d,e,f}
R = {(a,d), (b,c), (c,a), (d,e)}

(b) A = {a,b,c,d,e}
R = {(a,b), (b,e), (c,d), (d,b), (c,a)}

(c) A = {a,b,c,d,e,f}
R = {(a,b), (c,e), (f,a), (f,c), (f,d)}

(d) A = {1,2,3,4,5,6}
R = {(2,1), (3,4), (5,2), (6,5), (6,3)}

(e) A = {1,2,3,4,5,6}
R = {(1,1), (2,1), (2,3), (3,4), (4,5), (4,6)}

(2) Construct the tree of each of the following algebraic expressions, and obtain the prefix (Polish) form by applying a preorder search.

(a) $(7 + (6 - 2)) - (x - (y - 4))$

(b) $(x + (y - (x + y))) \times ((3 \div (2 \times 7)) \times 4)$

(c) $3 - (x + (6 \times (4 \div (2 - 3))))$

(3) Draw a binary tree to represent the following expression.

$$(s \div t) - (u \div (v + w))$$

Write the expression in infix and postfix notations.

(4) For the following expression in Polish form, draw its binary tree, and write the corresponding fully parenthesized expression:

$+ + \times 2\ 5 - 3 \times 4\ 6 - 9 \times 7\ 1$

Evaluate the expression.

(5) The following expression is given in Polish (prefix) form.

$+ \times + u\ v\ w \div - x\ y\ z$

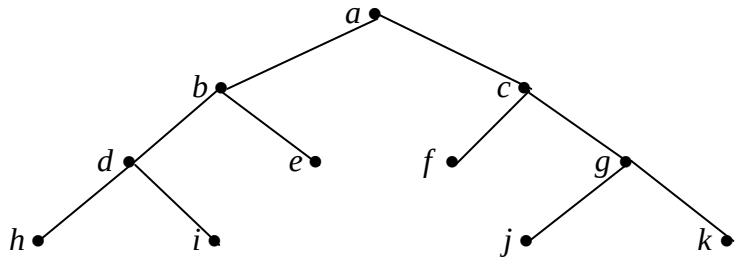
Draw its binary tree, write the corresponding fully parenthesized algebraic expression and rewrite the expression in its postorder (postfix) form.

(6) Consider the following parenthesised expression:

$$(x + (y - (x + y))) \text{ x } ((3 \div (2 \text{ x } 7)) \text{ x } 4)$$

Draw a binary tree and apply inorder and postorder search.

(7) The digraph of a labelled, binary positional tree is shown below. Suppose visiting a node results in printing out the label of that node. Write the sequence printed as a result of performing a preorder search of the tree.



(8) (a) Draw a binary search tree for the following list, in the given order:
STOP, LET, THERE, TAPE, NONE, YOU, ANT, NINE, OAT, NUT.
(b) Write the preorder and postorder traversals of the tree.
(c) Explain step by step how you would search for the word TEST in your tree.

(9) Build a binary search tree for the words banana, peach, apple, pear, coconut, mango, and papaya using alphabetical order.
(a) Write the sorted list by applying inorder search.
(b) Supposing all the items are equally likely to be searched for, calculate the expected number of comparisons to locate an item.

(10) Build a binary search tree for the numbers 50, 17, 76, 9, 23, 54, 14, 19, 72, 12, 67.
(a) Write the sorted list by applying inorder search.
(b) Supposing all the items are equally likely to be searched for, calculate the expected number of comparisons to locate an item.

Answers

(1) (a) not a tree
(b) not a tree
(c) tree
(d) tree
(e) not a tree

(2) (a) $- + 7 - 6\ 2 - x - y\ 4$
(b) $\times + x - y + x\ y \times \div 3 \times 2\ 7\ 4$
(c) $- 3 + x \times 6 \div 4 - 2\ 3$

(4) -9

(9)(b) $\frac{20}{7}$

(10)(b) $\frac{36}{11}$