BMMS2633 Advanced Discrete Mathematics

Tutorial 5

(1) Draw the digraph of the machine whose state transition table is shown. Remember to label the edges with the appropriate inputs.

(a)

	a	b	c
s_o	s_1	s_o	s_2
s_1	s_o	s_o	s_1
s_2	s_2	s_o	s_2

(b)

	0	1
s_o	s_o	s_1
s_1	s_1	s_2
s_2	s_2	s_3
s_3	s_3	s_o

(2) Consider the machine whose state transition table is given as below.

	a	b	c
s_o	s_o	s_1	s_3
s_1	s_o	s_1	s_2
s_2	s_2	s_3	s_o
s_3	s_2	s_2	s_o

Draw the digraph for the machine above. Let $R = \{(s_o, s_1), (s_o, s_o), (s_1, s_1), (s_1, s_o), (s_3, s_2), (s_2, s_2), (s_3, s_3), (s_2, s_3)\}$, and construct the digraph for the corresponding quotient machine.

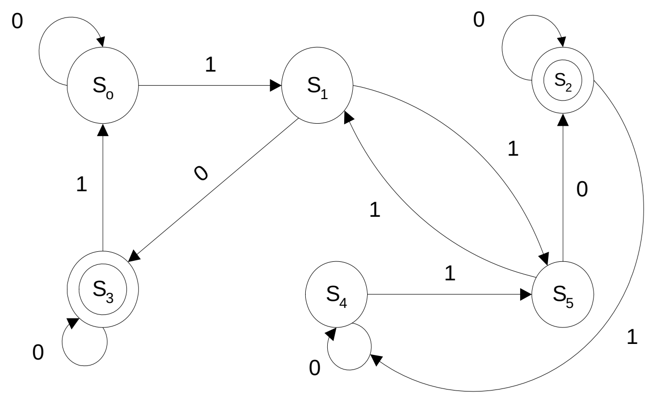
(3) Consider the machine whose state transition table is shown below.

	0	1
1	1	4
2	3	2
3	2	3
4	4	1

Here S = {1, 2, 3, 4}.
Show that R = {(1,1), (1,4), (4,1), (4,4), (2,2), (2,3), (3,2), (3,3)} is a machine congruence, and construct the state transition table for the corresponding quotient machine. Draw the digraph of the given machine and the digraph of the quotient machine.

(4) Consider the Moore machine whose digraph is shown below. Show that the relation R on S whose matrix is

$$M_R = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$



is a machine congruence.
Draw the digraph of the corresponding quotient Moore machine.

(5) Refer to the finite-state machine whose state transition table is given in Question 1(b).

(a) Find $f_{00101}(s_o)$

(b) Tabulate the transition function f_w corresponding to the input string w, where

(i) w = 01001

(ii) w = 11100

(c) Describe the set of binary words w (sequences of 0's and 1's) having the property that $f_w(s_o) = s_o$.

(6) Construct the digraph of a Moore machine that accepts the input strings described, and no others.

(a) Inputs a, b. Accepts strings where the number of b's is divisible by 3 (including strings without b).

(b) Inputs 0, 1. Accepts strings that contain 0011.

(c) Inputs 0, 1. Accepts strings that end with 0011.

(7) (a) The state transition table of a Moore machine, M, is given below. Find the partition corresponding to a machine congruence relation R, and construct the state transition table of the corresponding quotient machine which is equivalent to the given Moore machine. Draw the state transition diagrams of M and of M/R.

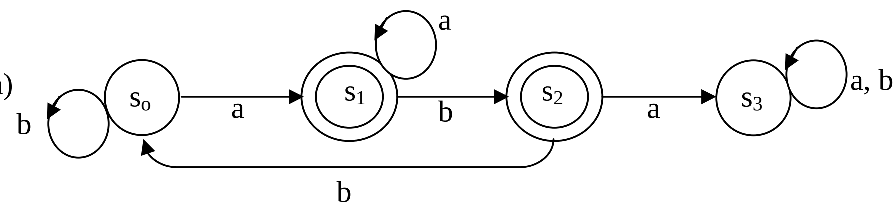
	0	1
a	a	c
b	g	d
c	f	e
d	a	d
e	a	d
f	g	f
g	g	c

$s_o = a$, $T = \{d, e\}$

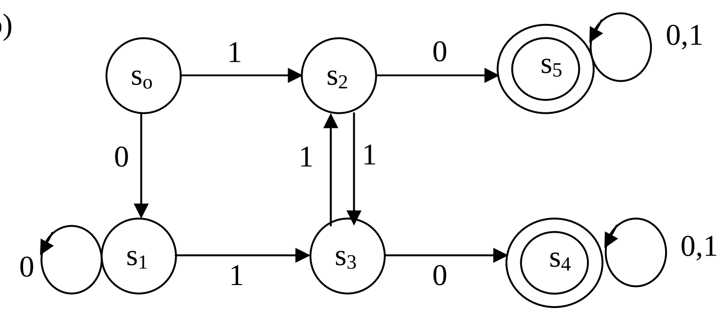
(b) Consider the Moore machine of Question 4. Suppose the matrix of the relation R is not given, apply the partitioning procedure as in part (a) above to simplify the machine.

(8) For each of the following Moore machine M whose digraph is given, draw the digraph of the corresponding quotient Moore machine.

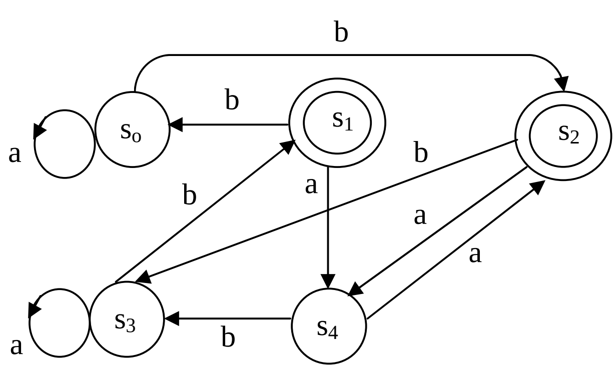
(a)



(b)



(c)



(9) In a Moore machine, M, the input set is {x, y}, the initial state is s_o = A and the acceptance states are D and E. The state transition functions are tabulated below:

	x	y
A	C	A
B	F	D
C	A	F
D	F	D
E	E	F
F	F	F

(a) Tabulate the word transition function f_{xx}. Determine whether $f_{xx}(C)$ can be accepted by the machine.

(b) Obtain a partition of the state set so that the corresponding equivalence relation is a machine congruence. Draw the state transition diagram of the resulting quotient machine.

(10) Consider the Moore Machine with the input set {a, b, c}, initial state S_o and acceptance states are S_1 and S_3. The state transition table is shown below.

	a	b	c
S_o	S_3	S_2	S_2
S_1	S_1	S_1	S_1
S_2	S_3	S_1	S_2
S_3	S_3	S_3	S_3

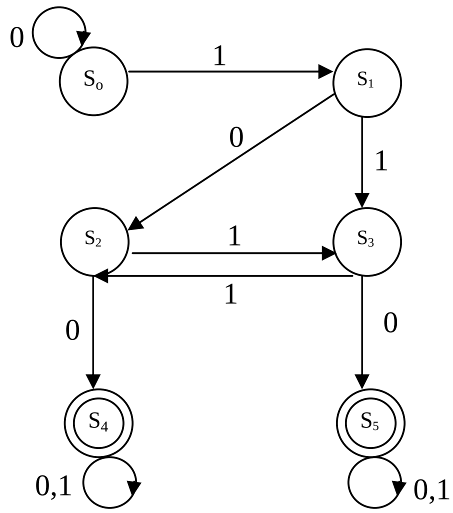
(a) Draw the state transition diagram for the Moore machine.

(b) Find f_{bcacb} and f_{bbacb} and determine which can be accepted by the machine.

(c) Find the partition corresponding to a machine congruence relation R.

(d) Draw the state transition diagram of the quotient machine M/R.

(11) Let M be the Moore machine given by the following transition diagram with S_o be the initial state.



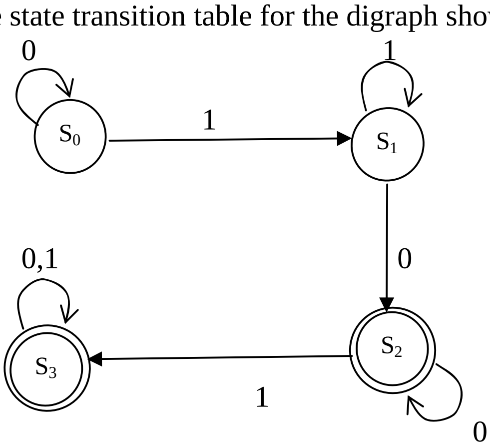
(a) Construct the state transition table.

(b) Find the partition corresponding to a machine congruence relation R with as few classes as possible.

(c) Draw the digraph of the corresponding quotient machine.

(12) In a Moore Machine, M, the input set is {0,1} and the initial state is S_o.

(a) Construct the state transition table for the digraph shown below:

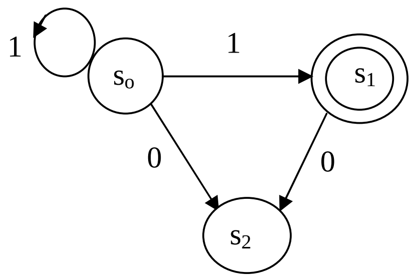


(b) Determine which of the following input strings are accepted by M:
  (1) 1110011
  (2) 001100

(c) Determine the state transition table of the quotient machine M/R.

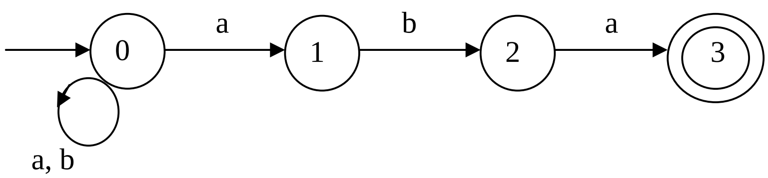
(d) Draw the state transition diagram for M/R.

(13) Let M be the nondeterministic finite state machine whose state diagram is shown below.



(a) Construct a state transition table for M.
(b) Find L(M).
(c) Find the corresponding deterministic finite state machine of M.

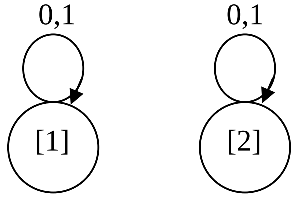
(14) Consider a nondeterministic finite state machine M shown as below.



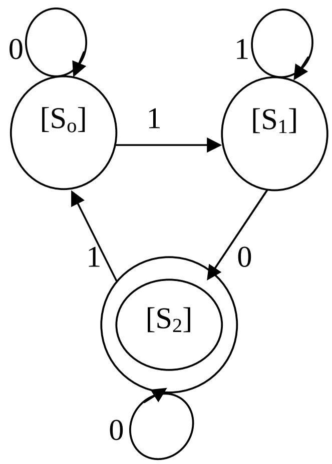
(a) Find L(M).
(b) Construct the state transition table.
(c) Find a deterministic finite state machine that recognizes the same language as the nondeterministic finite state machine given above.

Answers

(3) Quotient machine



(4)



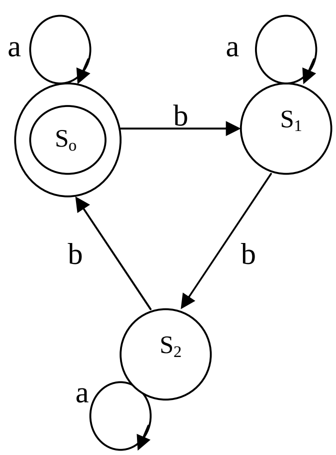
(5) (a) S_2

(b)

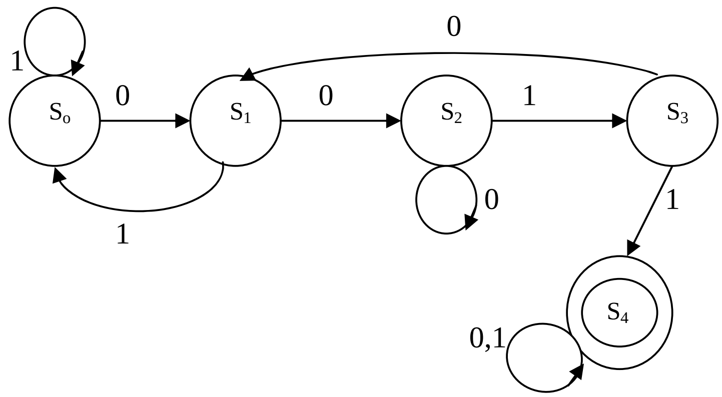
	(i) f_{01001}	(ii) f_{11100}
S_o	S_2	S_3
S_1	S_3	S_o
S_2	S_o	S_1
S_3	S_1	S_2

(c) $f_w(s_o) = s_o$ if the number of 1's in w is divisible by 4.

(6) (a)



(b)



(7) (a) $P_o = \{\{a,b,c,f,g\}, \{d,e\}\}$

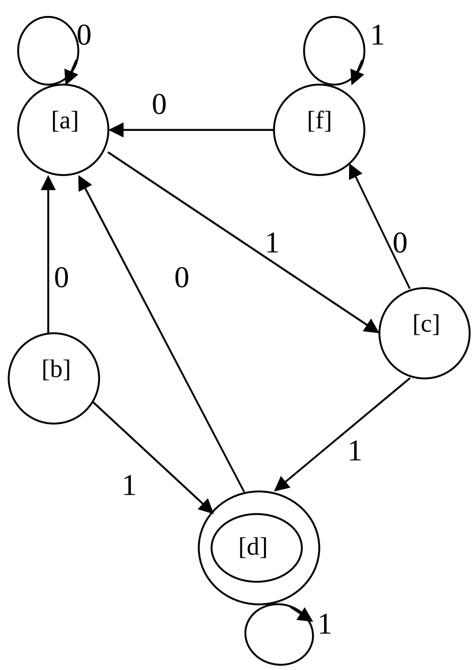
$P_1 = \{\{a,f,g\}, \{b,c\}, \{d,e\}\}$

$P_2 = \{\{a,g\}, \{f\}, \{b,c\}, \{d,e\}\}$

$P_3 = \{\{a,g\}, \{f\}, \{b\}, \{c\}, \{d,e\}\}$

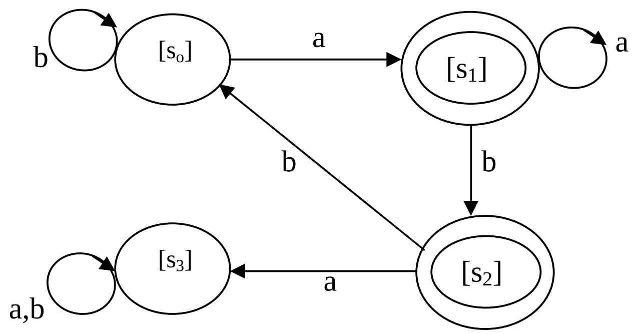
$P_4 = P_3$ is the required partition

	0	1
[a]	[a]	[c]
[b]	[a]	[d]
[c]	[f]	[d]
[d]	[a]	[d]
[f]	[a]	[f]



Note: As [b] is inaccessible from s_o , it may be dropped.

(8) (a)



(9) (a) $f_{xx}(C)$ not accepted by the machine.

(b)

	A	C	F	B	D	E
x	C	A	F	F	F	E
y	A	F	F	D	D	F

(10) (b) f_{bcacb} and f_{bbacb} are accepted by the machine.

(c)

	S_o	S_2	S_1	S_3
a	S_3	S_3	S_1	S_3
b	S_2	S_1	S_1	S_3
c	S_2	S_2	S_1	S_3

(11) (b)

	S_o	S_1	S_2	S_3	S_4	S_5
0	S_o	S_2	S_4	S_5	S_4	S_5
1	S_1	S_3	S_3	S_2	S_4	S_5

(12) (b) (1) Accepted by M.

(2) Accepted by M.

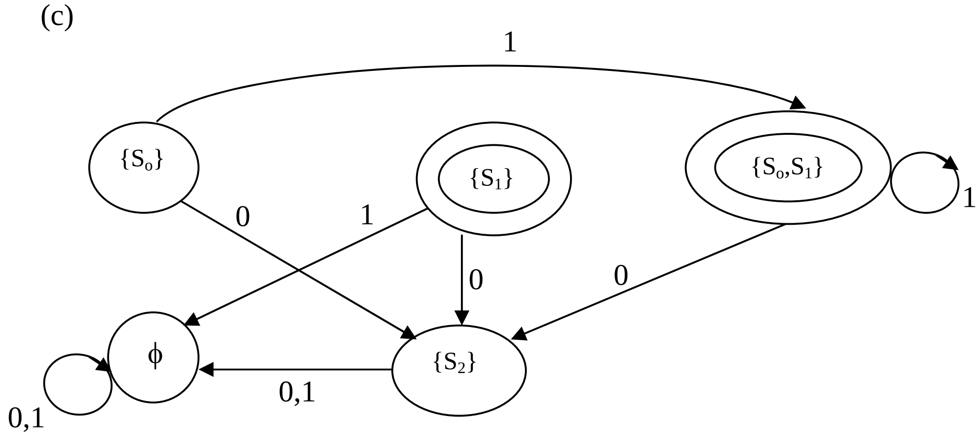
(c)

	0	1
$[S_o]$	$[S_o]$	$[S_1]$
$[S_1]$	$[S_2]$	$[S_1]$
$[S_2]$	$[S_2]$	$[S_2]$

(13) (a)

	0	1
S_o	S_2	S_o,S_1
S_1	S_2	ϕ
S_2	ϕ	ϕ

(c)



(14) (b)

F	a	b
0	0,1	0
1	ϕ	2
2	3	ϕ
3	ϕ	ϕ