Answer ALL **SEVEN (7)** questions. Time allocated: 1 hour.
Write your answers **in the space provided**.

$(x+1)(x-2)=0$

Question 1

$x=-1$ $x=2$

$0,1,2,3,4,5,6$

Let $U=\{x : x \in \mathbb{Z}$ and $0 \leq x \leq 6\}$ be the universal set, $A=\{x : x \in U$ and $x^2-x-2=0\}$, $B=\{x : x \in U$ and x is a factor of $8\}$ and $C=\{x : x \in U$ and x is an even number$\}$. the following sets.

a) List all the elements of sets A, B and C. (3 marks)

$A=\{2\}$ $C=\{0,2,4,6\}$

$B=\{1,2,4\}$

b) Find $\overline{(B \oplus C)}-A$. (4 marks)

$B \oplus C=(B-C)\cup(C-B)$

$=\{1\}\cup\{0,6\}$

$=\{0,1,6\}$

$\overline{(B\oplus C)}=\{2,3,4,5\}$

$\overline{(B\oplus C)}-A=\{2,3,4,5\}-\{2\}$

$=\{3,4,5\}$

Question 2

Compute the greatest common divisor of $a=1981$ and $b=315$ by using Euclidean algorithm and express it in the form of $sa+tb$, where $s, t \in \mathbb{Z}$. Hence, obtain the least common multiple of a and b. (7 marks)

$1981=6(315)+91$

$315=3(91)+42$

$91=2(42)+7 \leftarrow$ GCD

$42=6(7)+0$

$\text{GCD}(1981,315)=7$

Back substitution

$91=1981-6(315)$

$42=315-3(91)$

$7=91-2(42)$

$\text{GCD}(1981,315)\times\text{LCM}(1981,315)=1981\times315$

$7\times\text{LCM}(1981,315)=1981\times315$

$\text{LCM}(1981,315)=89145$

$7=91-2(42)$

$=1981-6(315)-2(315-3(91))$

$=1981-8(315)+6(91)$

$=1981-8(315)+6(1981-6(315))$

$=7(1981)-44(315)$

$s=7$ $t=-44$

(5 marks)

Question 3

Prove by induction that $5^n - 3^n$ is divisible by 2 for all positive integer $n \geq 1$.

when $n=1$, $5^1 - 3^1 = 2$ is divisible by 2.

The statement is true for $n=1$.

when $n=k$, assume $5^k - 3^k = 2a$, $a \in \mathbb{Z}$ is true.

$5^k = 2a + 3^k$

When $n = k+1$,

$5^{k+1} - 3^{k+1} = 2p \rightarrow 5^k \times 5^1 - 3^k \times 3$

$5(2a + 3^k) - 3^k \times 3 = 2p$

$10a + 2(3^k) = 2p$

$2(5a + 3^k) = 2p$, $p \in \mathbb{Z}$

divisible by 2

the statement is true for $n = k+1$.

Therefore, $5^n - 3^n$ is divisible by 2 for all positive integer $n \geq 1$

Question 4

Given that $A = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$, compute

(2 marks)

or

a) $A \vee B = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$

(2 marks)

b) $A \wedge B = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$

(2 marks)

c) $A \odot B = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

Question 5

A logical statement is given by $A \equiv r \to (\sim p \wedge q)$.

a) Simplify statement A to the principle disjunctive normal form (PDNF) by using the laws of algebra of propositions. Hence, determine whether A is a tautology, contradiction, or contingency. (7 marks)

$$A \equiv r \to (\sim p \wedge q)$$
$$\equiv \sim r \vee (\sim p \wedge q)$$
$$\equiv (\sim r \wedge (p \vee \sim p)) \vee [(\sim p \wedge q) \wedge (r \vee \sim r)]$$
$$\equiv [(\sim r \wedge p) \vee (\sim r \wedge \sim p)] \vee [(\sim p \wedge q \wedge r) \vee (\sim p \wedge q \wedge \sim r)]$$
$$\equiv [(\sim r \wedge p) \wedge (q \vee \sim q)] \vee [(\sim r \wedge \sim p) \wedge (q \vee \sim q)] \vee (\sim p \wedge q \wedge r) \vee (\sim p \wedge q \wedge \sim r)$$
$$\equiv (p \wedge q \wedge \sim r) \vee (p \wedge \sim q \wedge \sim r) \vee (\sim p \wedge q \wedge \sim r) \vee (\sim p \wedge \sim q \wedge \sim r) \vee (\sim p \wedge q \wedge r) \vee (\sim p \wedge q \wedge \sim r)$$
$$\equiv (p \wedge q \wedge \sim r) \vee (p \wedge \sim q \wedge \sim r) \vee (\sim p \wedge q \wedge \sim r) \vee (\sim p \wedge \sim q \wedge \sim r) \vee (\sim p \wedge q \wedge r) \leftarrow \text{PDNF of } A$$

A is contingency.

b) Obtain the principal disjunctive normal form (PDNF) and principal conjunctive normal form (PCNF) of the statement $\sim A$. (3 marks)

PCNF of $A \equiv (\sim p \vee \sim q \vee \sim r) \wedge (\sim p \vee q \vee \sim r) \wedge (p \vee q \vee r)$

PDNF of $\sim A \equiv (p \wedge q \wedge r) \vee (p \wedge \sim q \wedge r) \vee (\sim p \wedge \sim q \wedge r)$

PCNF of $\sim A \equiv (\sim p \vee \sim q \vee r) \wedge (\sim p \vee q \vee r) \wedge (p \vee \sim q \vee r) \wedge (p \vee q \vee r) \wedge (p \vee \sim q \vee \sim r)$

c) Let p, q and r be the following propositions:

p : n is even.
q : n is prime.
r : $n-1$ is even. $q \to p$ $\sim p \to \sim q$ $\sim q \to \sim p$

Write the converse, inverse, contrapositive, and negation forms for statement A, in sentence form. (4 marks)

Converse : If n is odd and n is prime, then n−1 is even.

Inverse : If n−1 is odd, then n is even or n is not prime.

Contrapositive : If n is even or n is not prime, then n−1 is odd.

Negation : n−1 is even and, n is even or n is not prime.

Question 6

Let the universe of discourse be {2, 3, 4}. The predicate $P(x, y)$ is defined below:

$$P(x, y) : x \text{ divides } y. \quad y \text{ is divisible by } x$$

Rewrite the expression $\exists y[\forall x P(x, y)]$ by eliminating the quantifiers. Hence, determine the truth value of the statement. (6 marks)

$\exists y\,[\forall x\, P(x,y)]$

$= [P(2,2) \wedge P(3,2) \wedge P(4,2)] \vee [P(2,3) \wedge P(3,3) \wedge P(4,3)] \vee [P(2,4) \wedge P(3,4) \wedge P(4,4)]$ ← eliminate quantifiers

$= [T \wedge F \wedge T] \vee [F \wedge T \wedge F] \vee [T \wedge F \wedge T]$

$= F \vee F \vee F$

$= F$ ← truth value

Question 7

Show that the following argument form is valid:

$\sim q \vee \sim r$

$\sim p \leftrightarrow r$

$\therefore r \rightarrow \sim q$ (5 marks)

p	q	r	$\sim q \vee \sim r$	$\sim p \rightarrow r$	$r \rightarrow \sim p$	$\sim p \leftrightarrow r$	$r \rightarrow \sim q$
0	0	0	1	0	1	$\boxed{0}$	1
0	0	1	1	1	1	1	1
0	1	0	1	0	1	$\boxed{0}$	1
0	1	1	0	1	1	1	0
1	0	0	1	1	1	1	1
1	0	1	1	1	0	$\boxed{0}$	1
1	1	0	1	1	1	1	1
1	1	1	0	1	0	0	0

~~It is not valid because not the premises are not both 1 when conclusion~~

<End of Question Paper>