Answer ALL **SEVEN (7)** questions. Time allocated: 1 hour.
Write your answers **in the space provided.**

$x+3$
$x-4$

$x=-3$
$x=4$

$\frac{2}{10}$

408
-352
56

56
× 3
168

Question 1

$U = \{4, 5, 6, 7, 8, 9, 10\}$

Let $U = \{x : x \in \mathbb{Z}$ and $4 \le x \le 10\}$ be the universal set, $A = \{x : x \in U$ and $x^2 - x - 12 = 0\}$, $B = \{x : x \in U$ and x is a factor of $20\}$ and $C = \{x : x \in U$ and x is an even number$\}$.

a) List all the elements of sets A, B and C. (3 marks)

$A = \{4\}$

$B = \{4, 5, 10\}$

$C = \{4, 6, 8, 10\}$

b) Find $\overline{(B \oplus C)} - A$. (4 marks)

$B \cap \overline{C} = \{5\}$

$\overline{B} \cap C = \{6, 8\}$

$\overline{A} = \{5, 6, 7, 8, 9, 10\}$

$\overline{(B \oplus C)} - A = \overline{(B \cap \overline{C}) \cup (\overline{B} \cap C)} \cap \overline{A}$

$= \overline{\{5\} \cup \{6, 8\}} \cap \{5, 6, 7, 8, 9, 10\}$

$= \overline{\{5, 6, 8\}} \cap \{5, 6, 7, 8, 9, 10\}$

$= \{4, 7, 9, 10\} \cap \{5, 6, 7, 8, 9, 10\}$

$= \{7, 9, 10\}$

Question 2

Compute the greatest common divisor of $a = 1808$ and $b = 408$ by using Euclidean algorithm and express it in the form of $sa + tb$, where $s, t \in \boldsymbol{Z}$. Hence, obtain the least common multiple of a and b. (7 marks)

Euclidean Algorithm:

$1808 = 4(408) + 176$

$408 = 2(176) + 56$

$176 = 3(56) + 8$

$56 = 7(8) + 0$

$\Rightarrow$

$176 = 1808 - 4(408)$

$56 = 408 - 2(176)$

$8 = 176 - 3(56)$

2 | 1808
2 | 904
2 | 452
2 | 226
2 | 113

2 | 408
2 | 204
2 | 102
3 | 51
17

Start from $8 = 176 - 3(56)$

$8 = 176 - 3[408 - 2(176)]$

$= 176 - 3(408) + 6(176)$

$= \cancel{7(176)}\ 7(176) - 3(408)$

$= 7[1808 - 4(408)] - 3(408)$

$= 7(1808) - 28(408) - 3(408)$

$= 7(1808) - 31(408)$ #

$1808 = 2^5 \cdot 113$

$408 = 2^3, 3 \cdot 17$

$\because$ LCM of a and b $= 2^5 \cdot 3 \cdot 17 \cdot 113$

$\therefore s = 7, b = -31,$

LCM of a and b is $2^5 \cdot 3 \cdot 17 \cdot 113$, LCM$(1808, 408) = 184416$.

Question 3

(5 marks)

Prove by induction that $8^n - 5^n$ is divisible by 3 for all positive integer $n \geq 1$.

① When $n=1$, $8^1 - 5^1 = 3$

$\therefore$ This statement is true, where result of $n=1$ is divisible by 3

② When $n=k$, $8^k - 5^k = 3a$

$\therefore$ assume this statement is true, when $n=k$.

③ When $n=k+1$, $8^{k+1} - 5^{k+1} = 8^k(8) - 5^k(5)$

$= [8^k(7+1)] - [5^k(4+1)]$

$= (8^k(7) + 8^k) - [5^k(4) + 5^k]$

$= 8^k(7) + 8^k - 5^k(4) - 5^k$

$= 8^k(7) - 5^k(4) + 8^k - 5^k$

$= 8^k(7) - 5^k(4) + 3a$

$= 8^k(6+1) - 5^k(3+1) + 3a$

$= 8^k(6) - 5^k(3) + 6a$

$= 3(8^k(2) - 5^k + 2a)$

$(8^k(7+1)) - (5^k(4+1))$

$= (8^k(7) + 8^k) - (5^k(4) + 5^k)$

$= 8^k(7) + 8^k - 5^k(4) - 5^k$

$= 8^k(7) - 5^k(4) + 8^k - 5^k$

$= 8^k(7) - 5^k(4) + 3a$

$= 8^k(6+1) - 5^k(3+1) + 3a$

$= 8^k(6) - 5^k(3) + 6a$

$= 3(8^k(2) - 5^k + 2a)$

$\therefore$ The statement $8^n - 5^n$ is divisible by 3 for all positive integer $n \geq 1$ is true.

Question 4

Given that $A = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$, compute

a) $A \vee B$ (2 marks)

$$\begin{bmatrix} 0101 \\ 1010 \\ 0011 \\ 1101 \end{bmatrix} \vee \begin{bmatrix} 0110 \\ 1101 \\ 1001 \\ 0101 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

b) $A \wedge B$ (2 marks)

$$\begin{bmatrix} 0101 \\ 1010 \\ 0011 \\ 1101 \end{bmatrix} \wedge \begin{bmatrix} 0110 \\ 1101 \\ 1001 \\ 0101 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

c) $A \odot B$ (2 marks)

$$\begin{bmatrix} 0101 \\ 1010 \\ 0011 \\ 1101 \end{bmatrix} \odot \begin{bmatrix} 0110 \\ 1101 \\ 1001 \\ 0101 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Question 5

A logical statement is given by $A \equiv q \rightarrow (\sim p \wedge r)$.

$(\wedge) \vee (\wedge) \vee (\wedge)$

a) Simplify statement A to the principle disjunctive normal form (PDNF) by using the laws of algebra of propositions. Hence, determine whether A is a tautology, contradiction, or contingency. (7 marks)

p	q	r	$\sim p$	$\sim p \wedge r$	$\sim q$	$\sim q \vee \sim p \wedge r$
T	T	T	F	F	F	F
T	T	F	F	F	F	F
T	F	T	F	F	T	T
T	F	F	F	F	T	T
F	T	T	T	T	F	T
F	T	F	T	F	F	F
F	F	T	T	T	T	T
F	F	F	T	F	T	T

$A \equiv q \rightarrow (\sim p \wedge r)$

$= \sim q \vee \sim p \wedge r$

$= ((p \vee \sim p) \wedge \sim q \wedge (r \vee \sim r)) \vee (\sim p \wedge (q \vee \sim q) \wedge r)$

$= (p \wedge \sim q \wedge r) \vee (\sim p \wedge \sim q \wedge r) \vee (p \wedge \sim q \wedge \sim r) \vee (\sim p \wedge \sim q \wedge \sim r) \vee (\sim p \wedge q \wedge r) \vee (\sim p \wedge \sim q \wedge r)$

PDNF of A $= (p \wedge \sim q \wedge r) \vee (\sim p \wedge \sim q \wedge r) \vee (p \wedge \sim q \wedge \sim r) \vee (\sim p \wedge \sim q \wedge \sim r) \vee (\sim p \wedge q \wedge r)$

$\therefore$ PDNF of $A = (p \wedge \sim q \wedge r) \vee (\sim p \wedge \sim q \wedge r) \vee (p \wedge \sim q \wedge \sim r) \vee (\sim p \wedge \sim q \wedge \sim r) \vee (\sim p \wedge q \wedge r)$. and

It is contigency.

b) Obtain the principal disjunctive normal form (PDNF) and principal conjunctive normal form (PCNF) of the statement $\sim A$. (3 marks)

PDNF of $\sim A = (p \wedge q \wedge r) \vee (p \wedge q \wedge \sim r) \vee (\sim p \wedge q \wedge \sim r)$

~~PCNF of A = ~ [PDNF of ~A]~~

PCNF of $\sim A$ $= \sim$ [PDNF of A]

$= (\sim p \vee q \vee \sim r) \wedge (p \vee q \vee \sim r) \wedge (\sim p \vee q \vee r) \wedge (p \vee q \vee r) \wedge (p \vee \sim q \vee \sim r)$

c) Let p, q and r be the following propositions:

$\sim(\sim q \vee (\sim p \wedge r))$

$= q \wedge p \vee \sim r$

$= p \wedge q \rightarrow \sim r$

$\sim p \vee \sim q \rightarrow \sim r$

p : n is even.
q : n is prime.
r : $n - 1$ is even.

Write the converse, inverse, contrapositive, and negation forms for statement A, in sentence form. (4 marks)

converse, $(\sim p \wedge r) \rightarrow q$ = If n is not even and n-1 is even, then n is prime

Inverse, $\sim q \rightarrow \sim(\sim p \wedge r) = \sim q \rightarrow (p \vee \sim r)$ If n is not prime, then n is even or n-1 is not even

Contrapositive $(p \vee \sim r) \rightarrow \sim q$ = If n is even or n-1 is not even, then n is not prime

negation $\sim[q \rightarrow (\sim p \wedge r)] = (\sim p \vee \sim q \rightarrow \sim r)$

= If n is not even or n is not prime, then n-1 is not even.

Question 6

Let the universe of discourse be {2, 3, 4}. The predicate $P(x, y)$ is defined below:

6 divided by 2
1 divide 6
5 divided by 5
6 divided by 3
3 divides 6

$$P(x, y) : x \text{ divides } y.$$

Rewrite the expression $\forall x[\exists y P(x, y)]$ by eliminating the quantifiers. Hence, determine the truth value of the statement. **(6 marks)**

$\forall x[\exists y P(x,y)] = [P(2,2) \vee P(2,3) \vee P(2,4)] \wedge$
$[P(3,2) \vee P(3,3) \vee P(3,4)] \wedge$
$[P(4,2) \vee P(4,3) \vee P(4,4)]$
$= [T \vee F \vee T] \wedge [F \vee T \vee F] \wedge [F \vee \cancel{F} \vee T]$
$= T \wedge T \wedge T$
$= T$

Question 7

Show that the following argument form is valid:

$p \vee \sim r$
$\sim q \rightarrow p$
$\therefore \sim p \vee q$

$\sim q \rightarrow p = q \vee p$

(5 marks)

p	q	r	$\sim q$	$\sim r$	$p \vee \sim r$	$q \vee p$	$\sim p \vee q$
T	T	T	F	F	T	T	T
T	T	F	F	T	T	T	T
T	F	T	T	F	T	T	F
T	F	F	T	T	T	T	F
F	T	T	F	F	F	T	T
F	T	F	F	T	T	T	T
F	F	T	T	F	F	F	T
F	F	F	T	T	T	F	T

$\therefore$ The form is invalid.

{End of Question Paper}