**BMMS2633 Advanced Discrete Mathematics**

**Tutorial 3**

(1) Let $G = (V, S, v_o, \mapsto)$, where $V = \{v_o, v_1, v_2, a, b, c\}$, $S = \{a, b, c\}$

$\mapsto$ : $v_o \mapsto aav_o$
$v_o \mapsto bv_1$
$v_1 \mapsto cv_2b$
$v_1 \mapsto cb$
$v_2 \mapsto bbv_2$
$v_2 \mapsto bb$

(a) State which of the following are in L(G).
(i) aabcb
(ii) abbcb
(iii) aaaabcbb
(iv) aaaabcbbb
(v) abcbbbbb

(b) Draw a master syntax diagram to illustrate the productions given above.

(c) If G is the grammar of productions given above, describe L(G).

(2) Give two distinct derivations (sequences of substitutions that start at $v_o$) for the string $xyz \in L(G)$, where G is the grammar specified below.

$G = (V, S, v_o, \mapsto)$
$V = \{v_o, v_1, v_2, x, y, z\}$, $S = \{x, y, z\}$

$v_o \mapsto v_ov_1$
$v_ov_1 \mapsto v_2v_o$
$v_2v_o \mapsto xy$
$v_2 \mapsto x$
$v_1 \mapsto z$

(3) For each grammar in part (a) to (d) below, state whether the grammar is type 1, 2, or 3. and write the productions in BNF notation.

(a) $G = (V, S, v_o, \mapsto)$
$V = \{v_o, x\}$, $S = \{x\}$
$v_o \mapsto xxv_o$
$v_o \mapsto xx$

(b) $G = (V, S, v_o, \mapsto)$
$V = \{v_o, x, y, z\}$, $S = \{x, y, z\}$
$v_o \mapsto xv_o$
$v_o \mapsto yv_o$
$v_o \mapsto z$

(c) $G = (V, S, v_o, \mapsto)$
$V = \{v_o, v_1, v_2, a, +, (, )\}$, $S = \{a, +, (, )\}$
$v_o \mapsto (v_o)$ (where left and right parentheses are symbols from S)
$v_o \mapsto a + v_1$
$v_1 \mapsto a + v_2$
$v_2 \mapsto a + v_2$
$v_2 \mapsto a$

(d) $G = (V, S, v_o, \mapsto)$
$V = \{v_o, v_1, v_2, x, y, z\}$, $S = \{x, y, z\}$
$v_o \mapsto xv_o$
$v_o \mapsto yv_1$
$v_1 \mapsto xyv_1$
$v_1 \mapsto xv_2$
$v_2 \mapsto z$

(4) A type 3 grammar G is given as follows.
$G = (V, S, v_o, \mapsto)$
$V = \{v_o, v_1, v_2, 0, 1, 2\}$, $S = \{0, 1, 2\}$
$v_o \mapsto 1v_o$
$v_o \mapsto 1v_1$
$v_o \mapsto 2v_2$
$v_1 \mapsto 0v_2$
$v_2 \mapsto 0v_2$
$v_2 \mapsto 1$

(a) Write the productions of G in BNF notation.
(b) Draw a master syntax diagram to illustrate the productions of G.
(c) Write a regular expression that corresponds to the language defined by G, L(G).

(5) Let $G = (V, S, v_o, \mapsto)$ be a phrase structure grammar defined as follows:
$V = \{v_o, v_1, v_2, 0, 1\}$, $S = \{0, 1\}$
$v_o \mapsto 1v_o$
$v_o \mapsto 0v_1$
$v_1 \mapsto 11v_2$
$v_2 \mapsto 0$

(a) Write the BNF notation for the above productions.
(b) Draw a master syntax diagram to illustrate the productions given above.
(c) Determine L(G), the language defined by G.

(6) For each grammar G specified, give the BNF (Backus-Naur Form), draw the corresponding syntax diagrams for the productions of the grammar, and draw the master diagram. Describe precisely the language, L(G), produced by the grammar; that is, describe all syntactically correct sentences. Then, state whether the grammar is Type 1, 2, or 3.

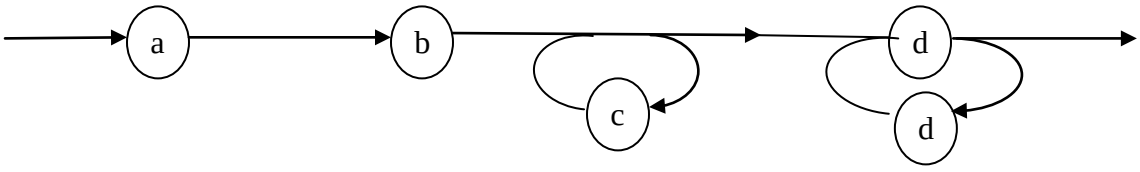
(a) $G = (V, S, v_o, \mapsto)$
$V = \{v_o, v_1, x, y, z\}$, $S = \{x, y, z\}$
$v_o \mapsto xv_o$
$v_o \mapsto yv_1$
$v_1 \mapsto yv_1$
$v_1 \mapsto z$

(b) $G = (V, S, v_o, \mapsto)$
$V = \{v_o, a, b\}$, $S = \{a, b\}$
$v_o \mapsto aav_o$
$v_o \mapsto a$
$v_o \mapsto b$

(c) $G = (V, S, v_o, \mapsto)$
$V = \{v_o, v_1, a, b\}$, $S = \{a, b\}$
$v_o \mapsto av_1$
$v_1 \mapsto bv_o$
$v_1 \mapsto a$

(7) Draw a derivation tree for
(a) the string $x^2y^2z$ in the grammar of Question 6(a);
(b) the string $aba^2$ in the grammar of Question 6(c).

(8) Give a BNF representation for the syntax diagram shown below. The symbols a, b, c, and d are terminal symbols of some grammar. You may provide nonterminal symbols as needed (in addition to $v_o$), to use in the BNF productions. You may use several BNF statements if needed.



(9) Let $G = (V, S, v_o, \mapsto)$ be a phrase structure grammar with $V = \{v_o, v_1, v_2, a, b, c\}$, $S = \{a, b, c\}$ and

$\mapsto$ : $v_o \mapsto aav_o$
$v_o \mapsto bv_1$
$v_1 \mapsto bv_2$
$v_2 \mapsto ccv_2$
$v_2 \mapsto c$

(a) Give the BNF (Backus – Naur Form) for the productions of G.
(b) Draw the master syntax diagram for G.
(c) Draw a derivation tree for the sentence $a^2b^2c^5$.

(10) Let $G = (V, S, v_o, \mapsto)$ be a grammar where $V = \{v_o, v_1, v_2, a, b, c\}$, $S = \{a, b, c\}$ and the production relation $\mapsto$ is described as:

$v_o \mapsto av_1$
$v_o \mapsto bv_1$
$v_o \mapsto cv_1$
$v_1 \mapsto bbv_1$
$v_1 \mapsto av_o$
$v_1 \mapsto cv_2$
$v_2 \mapsto a$
$v_2 \mapsto b$

(a) Rewrite the production relation in Backus – Naur Form.
(b) Draw the master syntax diagram for the productions of G.

(11) Let $G = (V, S, v_o, \mapsto)$ where $V = \{ v_o, v_1, v_2, v_3, a, b\}$, $S = \{a, b\}$ and the production relation $\mapsto$ is described as:

$v_o \mapsto av_1$
$v_1 \mapsto av_2$
$v_1 \mapsto b$
$v_2 \mapsto bv_3$
$v_2 \mapsto a$
$v_3 \mapsto b$

(a) Give the Backus-Naur form (BNF) representation for the productions of G.
(b) Draw the master syntax diagram for G.

(12) Let $G = (V, S, v_o, \mapsto)$ where $V = \{ v_o, v_1, 0, 1\}$, $S = \{0, 1\}$ and the production relation $\mapsto$ is described as:

$v_o \mapsto 1v_1$
$v_1 \mapsto 1\ 0v_1$
$v_1 \mapsto 1\ 0\ 1v_1$
$v_1 \mapsto 1$

(a) Give the BNF (Backus-Naur Form) representation for the production of G.
(b) Draw the master syntax diagram for G.
(c) Write an expression to describe the sentences that can be formed according to the master syntax diagram.

**Answers**

(1) (c) $L(G) = (aa)^*bc(bb)^*b$

(5) (c) $L(G) = 1^*0110$

(6) (a) $L(G) = \{x^n y^m z \mid n \geq 0, m \geq 1\} = \{x^* yy^* z\}$

(b) $L(G) = \{a^{2n} b^m a^{1-m} \mid n \geq 0, m = 0 \text{ or } 1\} = \{(a^2)^* (a \vee b)\}$

(c) $L(G) = \{(ab)^n a^2 \mid n \geq 0\}$ or $\{a(ba)^n a \mid n \geq 0\} = \{(ab)^* a^2\}$

(12) (c) $L(G) = 1(101 \vee 10)^* 1$