Chapter 6 Hypothesis Testing

In hypothesis testing, we test an assumed (hypothesised) value of a population parameter. Using some sample information, we may want to know whether a given claim (or statement) about a population parameter is true or not. Then, we can either accept or reject the hypothesised value.

Statistical Hypothesis

⇨ A statement, assumption or belief about a population parameter(s)

⇨ Experimental or sample evidence is required to verify the statement

Null Hypothesis (H_0)

⇨ A claim (or statement) about a population parameter that is assumed to be true until it is declared false

⇨ Specifies the value of the population parameter to be tested

Alternative Hypothesis (H_1)

⇨ A claim (or statement) about a population parameter that will be true if the null hypothesis is false

⇨ The rejection of H_0 means to accept the alternative hypothesis H_1

Results of hypothesis testing

There are only four possible results when we test a given hypothesis.

1. We accept a true hypothesis
   ⇨ a correct decision
2. We reject a false hypothesis
   ⇨ a correct decision
3. We reject a true hypothesis
   ⇨ an incorrect decision
   ⇨ known as Type I error (denoted by α --- “alpha”)
4. We accept a false hypothesis
   ⇨ an incorrect decision
   ⇨ known as Type II error (denoted by β --- “beta”)

Significance Levels

⇨ The maximum probability of making Type I error in hypothesis testing

- ⇨ Denoted by α
- ⇨ Usually specified before a hypothesis test is made
- ⇨ The value of 5% ($\alpha = 0.05$) or 1% ($\alpha = 0.01$) is frequently used

Example:
If we select 5% significance level, we will expect that the probability of making an error of rejecting the hypothesis when it is true is 5%. In other words, we are about 95% confidence that we will make a correct decision although we could be wrong with a probability of 5%.

Type I error (denoted by α)
- ⇨ Occurs when a null hypothesis is rejected as false when in fact it is true
- ⇨ α = P(H_0 is rejected | H_0 is true)
- ⇨ The value of α represents the significance level of the test

Type II error (denoted by β)
- ⇨ Occurs when a null hypothesis is accepted as true when in fact it is false
- ⇨ β = P(H_0 is not rejected | H_0 is false)
- ⇨ The value of β represents the probability of committing a Type II error
- ⇨ The value of 1-β is called the power of the test; it represents the probability of not making a Type II error

Critical Region / Rejection Region
- ⇨ The region which corresponds to a predetermined levels of significance
- ⇨ When the sample statistic falls in the critical region, we reject the hypothesis as it will be considered to be false

The critical region may be represented by a portion of the area under the normal curve in two ways:
1. Two tails or sides under the curve
2. One tail or side under the curve which is either the right tail or left tail

Two-tailed test
- ⇨ The test of hypothesis which are based on a critical region represented by both tails under the normal curve

☐ Rejection Region ☐ Acceptance Region * Critical Value

One-tailed test

⇨ The test of hypothesis which are based on a critical region represented by only one tails under the normal curve

★ Right-tailed test

★ Left-tailed test

☐ Rejection Region ☐ Acceptance Region * Critical Value

Critical value

⇨ The value that separates the rejection region from the acceptance region

Sign of H_1	Type of test
>	Right-tailed test
$\neq$	Two-tailed test
<	Left-tailed test

There are different notations for mean, standard deviation, proportion and size of a population and a sample.

Statistic	Population	Sample
Mean	μ	$\overline{X}$
Standard deviation	σ	S
Proportion	π	P_s
Size	N	n

Steps to perform a hypothesis testing

1. State the null and alternative hypothesis
2. Determine the significance level α and the critical value
3. Select the distribution (test statistic) to use
4. Determine the rejection and non-rejection regions
5. Calculate the value of the test statistic
6. Make a decision (reject H_0 or fail to reject H_0)

Test Statistic

⇨ can be defined as a rule/criterion to determine acceptance/rejection of H_0

Hypothesis testing about a population mean: Large sample

★ The null and alternative hypothesis

	Null Hypothesis H_0	Alternative Hypothesis H_1	Type of test
(i)	$\mu = \mu_0$	$\mu \neq \mu_0$	Two-tailed test
(ii)	$\mu = \mu_0$ or $\mu \geq \mu_0$	$\mu < \mu_0$	Left-tailed test
(iii)	$\mu = \mu_0$ or $\mu \leq \mu_0$	$\mu > \mu_0$	Right-tailed test

★ Test statistic

i) $Z = \dfrac{\overline{X} - \mu_0}{\sigma / \sqrt{n}}$ if σ is known

ii) $Z = \dfrac{\overline{X} - \mu_0}{S / \sqrt{n}}$ if σ is unknown and $n \geq 30$

★ Critical value and rejection region

	H_0	H_1	Critical Value	Critical Region
(i)	$\mu = \mu_0$	$\mu \neq \mu_0$	$\pm Z_{\alpha/2}$	$Z < -Z_{\alpha/2}$ or $Z > Z_{\alpha/2}$
(ii)	$\mu \geq \mu_0$	$\mu < \mu_0$	$-Z_{\alpha}$	$Z < -Z_{\alpha}$
(iii)	$\mu \leq \mu_0$	$\mu > \mu_0$	Z_{α}	$Z > Z_{\alpha}$

(i) Two-tailed test

(ii) Left-tailed test

(iii) Right-tailed test

Rejection Region
- reject H_0 / Accept H_1

Non-rejection / Acceptance Region
- do not reject H_0

Example 1:
A company markets car tyres. Their lives are normally distributed with a mean of 40,000 km and standard deviation of 3,000 km. A change in the production process is believed to result in a better product. A test sample of 64 new tyres has a mean life of 41,200 km. Can you conclude that the new product is significantly better than the current one? ($\alpha = 0.05$)

Solution:

Example 2:

A large retailer wants to determine whether the mean income of families living within two miles of a proposed building site exceeds RM14400. What can he conclude at the 0.05 level of significance, if the mean income of a random sample of 60 families living within two miles of the proposed site is RM14524 and the standard deviation is RM763?

Solution:

Example 3:
It is thought that a certain Normal population has a mean of 1.6. A sample of 50 gives a mean of 1.51 and a standard deviation of 0.3. Does this provide evidence, at the 5% level, that the population mean is less than 1.6?

Solution:

Example 4:
The expected lifetime of electric light bulbs produced by a given process was 1500 hours. To test a new batch, a sample of 40 was taken which showed a mean lifetime of 1410 hours. The standard deviation is 90 hours. Test the hypothesis that the mean lifetime of the electric light bulbs has not changed, using a level of significance of 0.05.

Solution:

Example 5:

The management of a club claims that its members lose an average of 2 kilograms or more within the first month after joining the club. A consumer agency wanted to check this claim took a random sample of 36 members of this health club and found that they lost an average of 1.8 kilograms within the first month of membership with a standard deviation of 0.5 kilograms. Test the management's claim at the 0.05 level of significance.

Solution:

<u>Hypothesis testing about a population proportion: Large sample</u>

★ The null and alternative hypothesis

	Null Hypothesis H_0	Alternative Hypothesis H_1	Type of test
(i)	$\pi = \pi_0$	$\pi \neq \pi_0$	Two-tailed test
(ii)	$\pi = \pi_0$ or $\pi \geq \pi_0$	$\pi < \pi_0$	Left-tailed test
(iii)	$\pi = \pi_0$ or $\pi \leq \pi_0$	$\pi > \pi_0$	Right-tailed test

★ Test statistic

$$Z = \frac{P_s - \pi_0}{\sqrt{\frac{\pi_0(1-\pi_0)}{n}}} \quad \text{or} \quad Z = \frac{nP_s - n\pi_0}{\sqrt{n\pi_0(1-\pi_0)}} \quad \text{or} \quad Z = \frac{X - n\pi_0}{\sqrt{n\pi_0(1-\pi_0)}}$$

★ Critical value and rejection region

	H_0	H_1	Critical Value	Critical Region
(i)	$\pi = \pi_0$	$\pi \neq \pi_0$	$\pm Z_{\alpha/2}$	$Z < -Z_{\alpha/2}$ or $Z > Z_{\alpha/2}$
(ii)	$\pi \geq \pi_0$	$\pi < \pi_0$	$-Z_{\alpha}$	$Z < -Z_{\alpha}$
(iii)	$\pi \leq \pi_0$	$\pi > \pi_0$	Z_{α}	$Z > Z_{\alpha}$

(i) Two-tailed test

(ii) Left-tailed test

(iii) Right-tailed test

Rejection Region
- reject H_0 / Accept H_1

Non-rejection Region
- do not reject H_0

Note:
1. $\pi_0 \equiv$ the population proportion (predetermined constant)
2. $P_s \equiv$ the sample proportion
3. Population proportion π, is used instead of sample proportion because the population proportion is known

Example 6:
In an investigation into ownership of calculators, 200 randomly chosen school students were interviewed, 163 of them owned a calculator. Using the evidence of this sample, test at the 5% level of significance, hypothesis that the proportion of school students owning a calculator is more than 80%.

Solution:

Example 7:
An election candidate claims that 60 percent of the voters support him. A random sample of 2500 voters show that 1400 support him. Test his claim at 0.10 level of significance.

Solution:

Example 8:
A coin is tossed 100 times and 38 heads are obtained. Is there evidence, at the 2% level that the coin is biased in favour of tails?

Solution:

Example 9:
ABC Mailing Company sells computers and computer parts by mail. The company claims that at least 90% of all orders are mailed within 72 hours after they are received. The quality control department at the company often takes samples to check if this claim is valid. A recently taken sample of 150 orders showed that 129 of them were mailed within 72 hours. Do you think the company's claim is true? Use a 2. 5% significance level.