BMMS2633 Advanced Discrete Mathematics

<u>Tutorial 6</u>

(1) Find the weights of the given words:
(a) 1011 (b) 011101 (c) 010101 (d) 100000101

(2) Consider the (3, 4) parity check code. For each of the received words, determine whether an error will be detected:
(a) 0100 (b) 1100 (c) 0010 (d) 1001

(3) Consider the (6, 7) parity check code. For each of the received words, determine whether an error will be detected.
(a) 1101010 (b) 1010011 (c) 0011111 (d) 1001101

(4) Find the distance between x and y.
(a) $x = 1100010$, $y = 1010001$
(b) $x = 0100110$, $y = 0110010$

(5) Find the minimum distance of the (2,4) encoding function e:
e(00) = 0000 e(10) = 0110 e(01) = 1011 e(11) = 1100

(6) Consider the (2, 6) encoding function e.
e(00) = 000000 e(10) = 101010 e(01) = 011110 e(11) = 111000
(a) Find the minimum distance of e.
(b) How many errors will e detect?

(7) Show that the (2,5) encoding function e: $B^2 \rightarrow B^5$ defined by
e(00) = 00000 e(01) = 01110
e(10) = 10101 e(11) = 11011
is a group code.
Find the minimum distance of this group code.

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(8) Show that the (3, 7) encoding function e: $B^3 \rightarrow B^7$ defined by
e(000) = 0000000 e(100) = 1000101
e(001) = 0010110 e(101) = 1010011
e(010) = 0101000 e(110) = 1101101
e(011) = 0111110 e(111) = 1111011
is a group code.
Find the minimum distance of this group code.

(9) Compute $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \oplus \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$.

(10) Compute $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \oplus \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$.

(11) Compute $\begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$.

(12) Compute $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$.

(13) Let $H = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ be a parity check matrix. Determine the (2, 5) group code function $e_H : B^2 \to B^5$. Determine the minimum distance of this group code by finding the distance between every pair of code words.

(14) Consider the (2, 4) group encoding function e: $B^2 \to B^4$ defined by
e(00) = 0000 e(10) = 1001
e(01) = 0111 e(11) = 1111.
Decode the following words relative to a maximum likelihood decoding function.
(a) 0011 (b) 1011 (c) 1111

(15) Consider the (3, 5) group encoding function e: $B^3 \to B^5$ defined by
e(000) = 00000 e(100) = 10011
e(001) = 00110 e(101) = 10101
e(010) = 01001 e(110) = 11010
e(011) = 01111 e(111) = 11100
Decode the following words relative to a maximum likelihood decoding function.
(a) 11001 (b) 01010 (c) 00111

(16) Determine the coset leaders for $N = e_H(B^m)$ for the given parity check matrix H. Then determine the syndrome for each coset leader.

(a) $H = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$ (b) $H = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(17) Consider the (3, 5) encoding function $e : B^3 \to B^5$ defined by

e(000) = 00000 e(001) = 00110
e(010) = 01001 e(011) = 01111
e(100) = 10011 e(101) = 10101
e(110) = 11010 e(111) = 11100

(a) Show that this encoding function is a group code.
(b) Determine the minimum distance of this group code.
(c) Obtain a decoding table.
(d) Decode the following words relative to a maximum likelihood decoding function:
(i) 11001 (ii) 01010 (iii) 00111

(18) Let $H = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ be a parity check matrix. Decode the following words relative to a maximum likelihood decoding function associated with e_H :

(a) 011001 (b) 101011 (c) 111010

(19) Let $H = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$ be a parity check matrix. Decode the following words relative to a maximum likelihood decoding function.

(a) 0101 (b) 1010 (c) 1101

(20) Let $H = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ be a parity check matrix. Decode the following words relative to a maximum likelihood decoding function associated with e_H :

(a) 10100 (b) 01101 (c) 11011

Answers

(4) (a) 4 (b) 2

(6) (a) 2

(7) 3

(8) 2

(13) 00000, 01011, 10011, 11000

(14) (a) 01 (b) 10 (c) 11

(15) (a) 010 (b) 110 (c) 001 or 011

(16) (a) syndrome = 00, 01, 10, 11
(b) syndrome = 000, 001, 010, 100, 101, 011, 110, 111

(17) (b) 2

(c)

C	00000	00110	01001	01111	10011	10101	11010	11100
00001⊕C	00001	00111	01000	01110	10010	10100	11011	11101
00010⊕C	00010	00100	01011	01101	10001	10111	11000	11110
10000⊕C	10000	10110	11001	11111	00011	00101	01010	01100

(d) (i) 010 (ii) 110 (iii) 001

(18) (a) 011 or 111 (b) 001 or 101 (c) 110

(19) (a) 11 (b) 10 (c) 11

(20) (a) 00 or 11 (b) 01 (c) 10