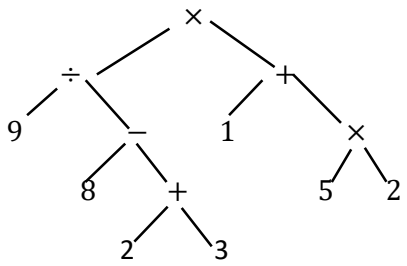
BMMS2633 ADVANCED DISCRETE MATHEMATICS
Academic Year 2023/24
Session 202305

Question 1

a) (i) 33

(ii) Binary tree

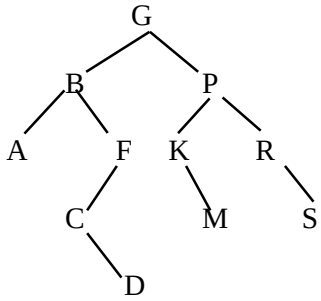


(iii) Fully parenthesized

$$\left[9 \div \left(8 - (2+3)\right)\right] \times [1 + (5 \times 2)]$$

(iv) Postorder search: 9 8 2 3 + − ÷ 1 5 2 × + ×

b) (i) G P B K R F M C A S D



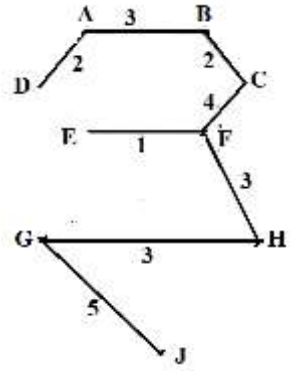
(ii) Expected number of comparisons

$$= \frac{34}{11}$$

(iii) Preorder search: G B A F C D P K M R S

Postorder search: A D C F B M K S R P G

c)



Minimum total weight
= 23

Question 2

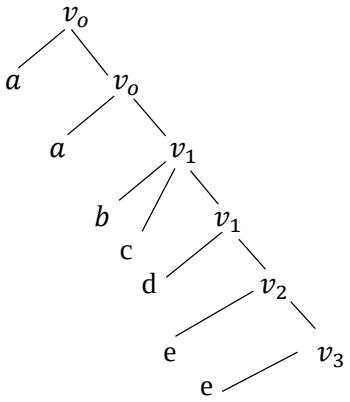
a) (i) $\langle v_o \rangle ::= a\langle v_o \rangle \mid a\langle v_1 \rangle$

$\langle v_1 \rangle ::= bc\langle v_1 \rangle \mid de\langle v_2 \rangle \mid d\langle v_2 \rangle \mid ff\langle v_1 \rangle$

$\langle v_2 \rangle ::= e\langle v_3 \rangle \mid b\langle v_1 \rangle$

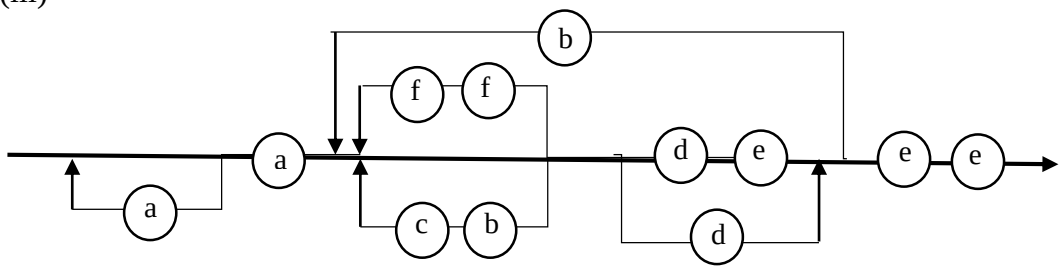
$\langle v_3 \rangle ::= e$

(ii)



aabcdee is a syntactically correct sentence.

(iii)



(iv) Regular expression = a*a (bc ∨ ff)* (de ∨ d) [b(bc ∨ ff)* (de ∨ d)]* ee

Question 2

(b) (i) State transition table of f_{0110}

	f_{0110}
S_0	S_2
S_1	S_3
S_2	S_3
S_3	S_0
S_4	S_2
S_5	S_3

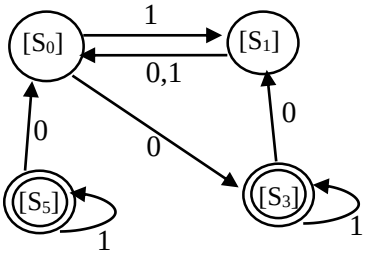
(ii)

	S_0	S_4	S_1	S_2	S_3	S_5
0	S_3	S_3	S_0	S_4	S_2	S_4
1	S_1	S_2	S_4	S_0	S_3	S_5

(iii)

	0	1
$[S_0]$	$[S_3]$	$[S_1]$
$[S_1]$	$[S_0]$	$[S_0]$
$[S_3]$	$[S_1]$	$[S_3]$
$[S_5]$	$[S_0]$	$[S_5]$

(iv)



(v) Input string 11011 is accepted by *M* and *M/R* since S_3 and $[S_3]$ are acceptance states.

Question 3

a) (i) $*$ is not a valid binary operation defined on $\mathbb{R}$.

(ii) $*$ is not a valid binary operation defined on $\mathbb{Z}$.

b) $*$ is commutative on $\mathbb{R}$
Since $(p * q) * r \neq p * (q * r)$, the binary operation on $\mathbb{R}$ defined by $(3pq - 2)$ is not associative.

c) (i) $a = u$
$b = w$

(ii) Identity element is ***t***.

(iii) Inverse of *s* is *s*
Inverse of *t* is *t*
Inverse of *u* is *u*
Inverse of *w* is *w*

Question 4

a) (i)

w	e(w)
00	00000
01	01011
10	10101
11	11110

(ii) The minimum distance of this (2, 5) encoding function $e_H = 3$.

(iii)

Coset leader	Syndrome
10000	101
01000	011
00100	100
00001	001

(iv) (1) d(11101) = 10

(2) d(11010) = 11

Question 4

b) (i)

Letter, (x_i)	W	M	C	E	O	L	P
Probability, P(x_i)	0.18	0.01	0.03	0.26	0.11	0.21	0.20
Codeword, C_i	000	00111	00110	01	0010	10	11

000 01 10 00110 0010 00111 01
W E L C O M E

(ii) Average code length, $L(C)$ = 2.52 bits

Entropy $H(x)$ = 2.4563

Efficiency = 0.9747

The efficiency of this code is 97.47%