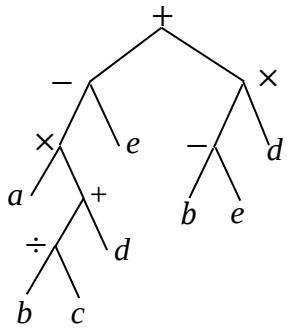
BMMS2633 ADVANCED DISCRETE MATHEMATICS

Academic year 2023/24
Session 202401

Q1. a) (i) Fully parenthesised form:
$((a \times ((b \div c) + d)) - e) + ((b - e) \times d)$

(ii) Binary tree

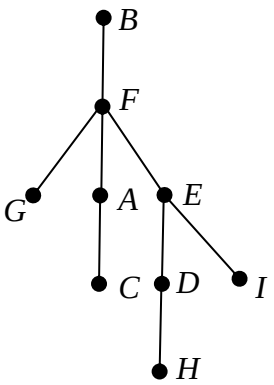


(iii) Preorder search : $+ \quad - \quad \times \quad a \quad + \quad \div \quad b \quad c \quad d \quad e \quad \times \quad - \quad b \quad e \quad d$

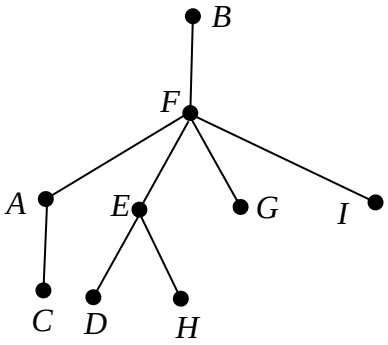
b) (i)

Vertex	List of adjacencies
A	*C F*
B	*F*
C	*A*
D	*E H*
E	*D F H I*
F	*A B E G I*
G	*F*
H	*D E*
I	*E F*

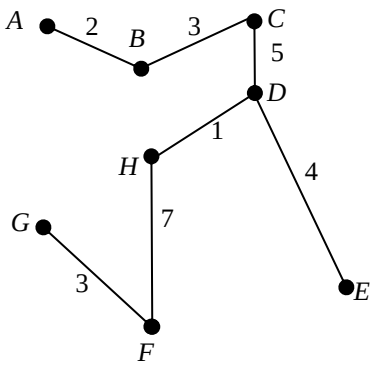
(ii) Depth-first search



Breadth-first search



c)



Minimum total weight = 25

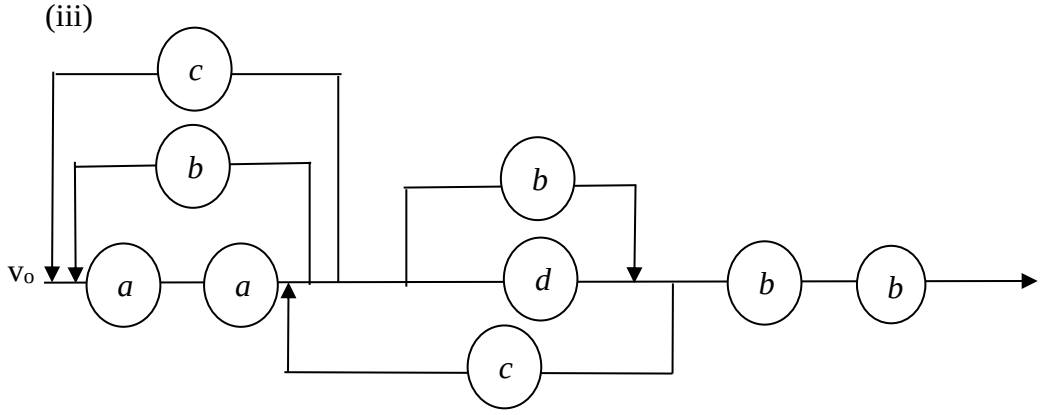
Q2. a) (i) $\langle v_o \rangle ::= aa\langle v_1 \rangle$

$\langle v_1 \rangle ::= b\langle v_0 \rangle \mid c\langle v_0 \rangle \mid b\langle v_2 \rangle \mid d\langle v_2 \rangle$

$\langle v_2 \rangle ::= c\langle v_1 \rangle \mid bb$

(ii) *aacaabcbb* is not a syntactically correct sentence.

(iii)



(iv) Regular expression= $a^2\left((b\vee c)a^2\right)^*\left((b\vee d)c\left((b\vee c)a^2\right)^*\right)^*(b\vee d)b^2$

(b) (i) State transition table of f_{00101}

	f_{00101}
S_a	S_g
S_b	S_b
S_c	S_b
S_d	S_d
S_e	S_b
S_f	S_b
S_g	S_b

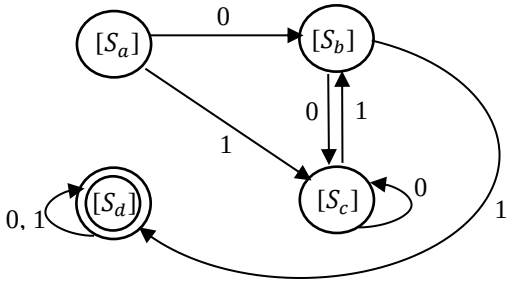
(ii)

	S_a	S_c	S_e	S_f	S_b	S_g	S_d
0	S_g	S_c	S_f	S_f	S_c	S_e	S_d
1	S_e	S_b	S_g	S_b	S_d	S_d	S_d

(iii)

	0	1
$[S_a]$	$[S_b]$	$[S_c]$
$[S_b]$	$[S_c]$	$[S_d]$
$[S_c]$	$[S_c]$	$[S_b]$
$[S_d]$	$[S_d]$	$[S_d]$

(iv)



(v) $f_{1001}(S_a) = S_b$

$f_{1001}([S_a]) = [S_b]$

Input string 1001 is not accepted by *M* and *M/R*.

Q3. a) (i) The description of * on $\mathbb{Z}$ is a valid definition of a binary operation.

(ii) Description of $*$ on $\mathbb{Z}$ is commutative.

(iii) Description of $*$ on $\mathbb{Z}$ is associative.

(iv) Since $e = 0 \in \mathbb{Z}$, description of $*$ on $\mathbb{Z}$ has an identity.

b) (i) Subset *A* is not a subgroup of G.

(ii) Subset *Y* is not a subgroup of G.

Q4 a) (i)

w	e(w)
00	0000
01	0110
10	1011
11	1101

(ii) The minimum distance of this (2, 4) encoding function $e_H = 2$

(iii)

$\oplus$	0000	0110	1011	1101
0001	0001	0111	1010	1100
0010	0010	0100	1001	1111
1000	1000	1110	0011	0101

(iv) (1) d(0011) = 10

(2) d(1110) = 01

b) (i)

Letter, (x_i)	Y	O	D	N	B	E
Number of occurrences	160	480	180	360	280	540
Probability, P(x_i)	0.08	0.24	0.09	0.18	0.14	0.27

(ii)

Letter, (x_i)	Y	O	D	N	B	E
Codeword, C_i	0001	10	0000	11	001	01

(iii) Average code length, $L(C)$ = 2.48 bits

Entropy, $H(x)$ = 2.4507

The efficiency of this code is 98.82%.