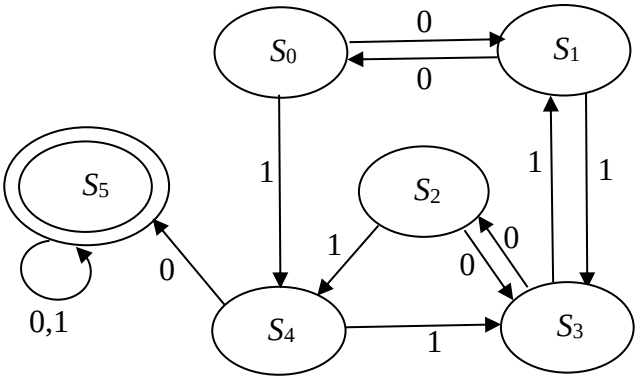
**BMMS2633 Advanced Discrete Mathematics**
**Academic year 2024/25**
**Session 202405**
**Assignment**

## Question 1

In designing a traffic light control system for an intersection among different roads crossing each other, the traffic light system should handle multiple states to manage the green, yellow, and red lights for each direction. We will model the traffic light control system with six states and then partition it to simplify the design. Consider the finite state machine, $M$ where the initial state is $S_0$, the input set is $\{0, 1\}$, and the state transition diagram is shown below.




a) Organise the state transition diagram in a comprehensive state transition table. (3 marks)

b) Find the word transition function $f_{101110}$ for $S_0$ to $S_5$ based on the given state transition diagram. (3 marks)

c) Identify a partition of the state set corresponding to a machine congruence relation $R$ with as few classes as possible. (3 marks)

d) Develop the state transition table of the corresponding quotient machine. (2 marks)

[Total: 11 marks]

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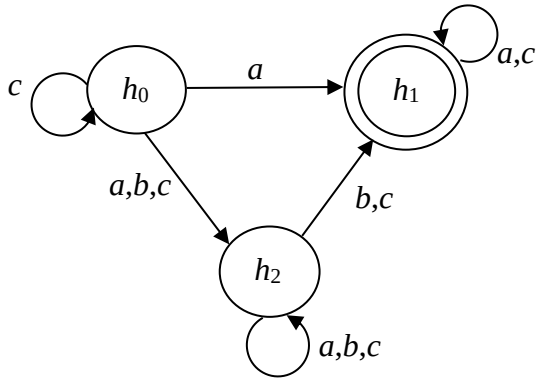
## Question 2

Suppose you are designing a simple spell-checker that recognizes basic patterns in text input. The nondeterministic finite state machine represents different pathways to reach correct spelling patterns. The deterministic finite state machine is then the optimized version used by the spell-checker to efficiently verify if a word is correctly spelled. The state transition diagram for a nondeterministic finite state machine is shown below.




a) Based on the nondeterministic finite state machine transition diagram, find the state transition table. (2 marks)

b) Develop a state transition table of the deterministic finite state machine that recognises the same language as the nondeterministic finite state machine in the diagram shown above. (3 marks)

c) Identify the regular expression that can be accepted for the nondeterministic finite state machine above. (4 marks)

[Total: 9 marks]

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## Question 3

A secure access control system for a restricted area can only grant access to authorised personnel whose ID badges end with the sequence "01101". Develop a digraph of a Moore machine that can validate ID badge numbers as personnel approaches the access point.

[Total: 5 marks]

## Question 4

Examine whether the following description of * is valid define a binary operation on the set given. Provide a reason or show a counterexample if it is not a valid definition of binary operation on the set.

a) On $\{1, 2, 3, 4\}$, where $a * b = \gcd(a, b) + 1$. (3 marks)

b) On $\{0, 1, 2, 4\}$, where $a * b = (a^2 + b^2) \bmod 5$. (3 marks)

[Total: 6 marks]

## Question 5

Let $*$ be a binary operation on $\mathbb{R} - \{-1\}$ defined by $p * q = \frac{pq}{q+1}$ for all $p, q \in \mathbb{R} - \{-1\}$.

a) Examine whether the binary operation is commutative or associative. Provide the reason for your answer or show a counterexample if it is not. (8 marks)

b) Identify the identity element, if it exists, or else explain why the identity does not exist. (5 marks)

c) Solve the inverse of *p*, if it exists, else explain why the inverse does not exist. (1 mark)

d) Examine whether the binary operation, $*$ on $\mathbb{R} - \{-1\}$ defined by $p * q = \frac{pq}{q+1}$ for all $p, q \in \mathbb{R} - \{-1\}$ gives a group structure on the set. Justify your answer. (2 marks)

[Total: 16 marks]

## Question 6

Let $G = (\mathbb{R}^*, \times)$, where $\mathbb{R}^*$ is the set of all the non-zero real numbers under multiplication. Consider the subset $K = \{x \in \mathbb{R}^* | x = 2^n,\ n \in \mathbb{Z}\}$. Identify if *K* is a subgroup of *G*.

[Total: 3 marks]