

BAMS1633 FURTHER DISCRETE MATHEMATICS

Academic year 2022/23

Session 202205

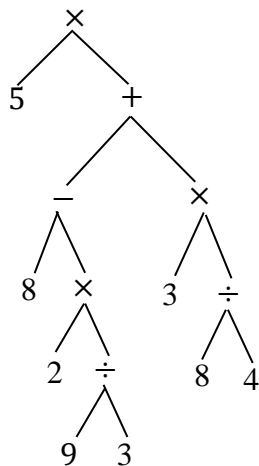
Answers

Q1. a) (i) 40

Fully parenthesized form:

$$5 \times ((8 - (2 \times (9 \div 3))) + (3 \times (8 \div 4)))$$

(ii) Binary tree



Polish form : $\times \ 5 \ + \ - \ 8 \ \times \ 2 \ \div \ 9 \ 3 \ \times \ 3 \ \div \ 8 \ 4$

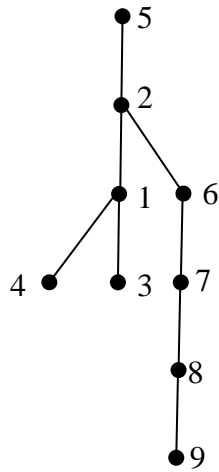
Q1. b)

| Vertex | List of adjacencies |
|--------|---------------------|
| 1 | 2 3 4 |
| 2 | 1 5 6 |
| 3 | 1 |
| 4 | 1 |
| 5 | 2 |
| 6 | 2 7 8 |
| 7 | 6 8 9 |
| 8 | 6 7 9 |
| 9 | 7 8 |

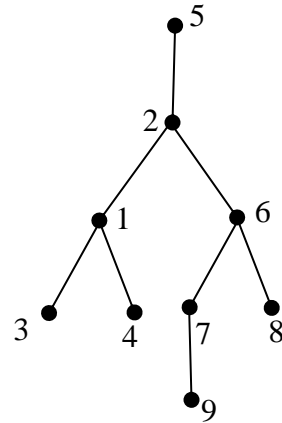
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Question 1b) (Continued)

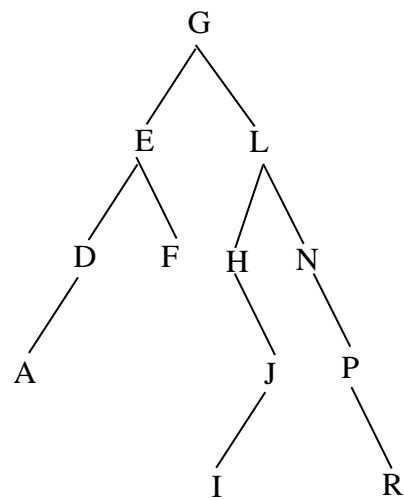
Depth-first search



Breadth-first search



c) (i)

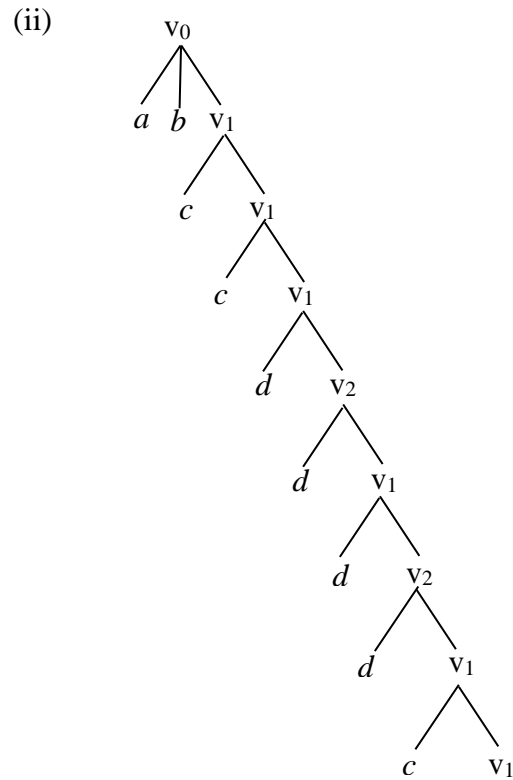


(ii) Preorder: G E D A F L H J I N P R

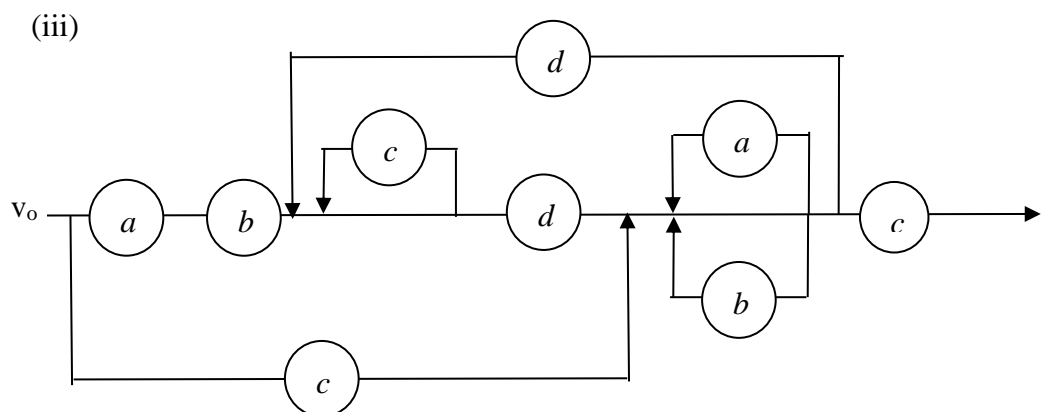
(iii) Expected number of comparisons = 3.25

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- Q2. a) (i) $\langle v_0 \rangle ::= ab\langle v_1 \rangle \mid c\langle v_2 \rangle$
 $\langle v_1 \rangle ::= c\langle v_1 \rangle \mid d\langle v_2 \rangle$
 $\langle v_2 \rangle ::= d\langle v_1 \rangle \mid a\langle v_2 \rangle \mid b\langle v_2 \rangle \mid c$



abccdddddc is not a syntactically correct sentence.



- (iv) Regular expression = $((abc^*d) \vee c)(a \vee b)^*(dc^*d(a \vee b)^*)^*c$

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Question 2 (Continued)

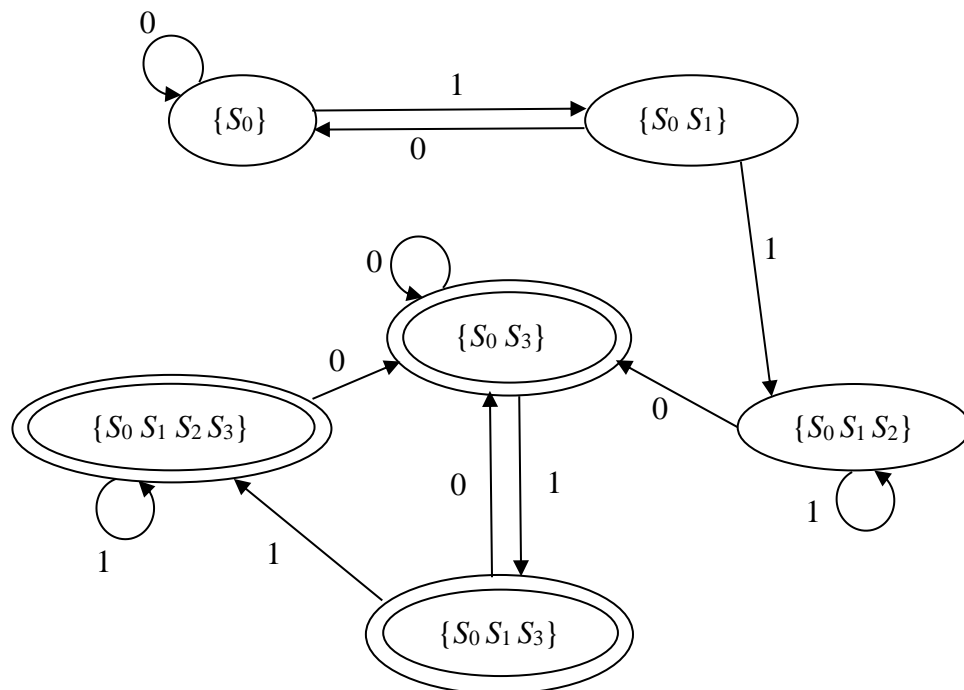
(b) (i)

| | 0 | 1 |
|-------|-------------|-------------|
| S_0 | S_0 | S_0, S_1 |
| S_1 | \emptyset | S_2 |
| S_2 | S_3 | \emptyset |
| S_3 | S_3 | S_3 |

(ii) Regular expression that can be accepted is $(0 \vee 1)^* 110(0 \vee 1)^*$.

(iii)

| | 0 | 1 |
|-----------------------|---------------|-----------------------|
| $\{S_0\}$ | $\{S_0\}$ | $\{S_0 S_1\}$ |
| $\{S_0 S_1\}$ | $\{S_0\}$ | $\{S_0 S_1 S_2\}$ |
| $\{S_0 S_1 S_2\}$ | $\{S_0 S_3\}$ | $\{S_0 S_1 S_2\}$ |
| $\{S_0 S_3\}$ | $\{S_0 S_3\}$ | $\{S_0 S_1 S_3\}$ |
| $\{S_0 S_1 S_3\}$ | $\{S_0 S_3\}$ | $\{S_0 S_1 S_2 S_3\}$ |
| $\{S_0 S_1 S_2 S_3\}$ | $\{S_0 S_3\}$ | $\{S_0 S_1 S_2 S_3\}$ |



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Q3. a) Associative.

Identity element, $e = -6 \in \mathbb{Z}$.

Inverse, $I = -12 - p \in \mathbb{Z}$.

The binary operation, $*$ on \mathbb{Z} defined by $p * q = p + q + 6$ gives a group structure on the set.

Q3. b) (i) $r * s = 2$
 $s * r = -5$

Since $r * s \neq s * r$ the binary operation, $*$ on the set A defined by $r * s = rs + s - 2r^2$ is not commutative.

(ii) Since $1 * 1 = 0 \notin A$
Hence the binary operation, $*$ on the set A defined by $r * s = rs + s - 2r^2$ is not closed.

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Q4 a) (i)
$$\begin{pmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 1 \\ 1 & 1 \end{pmatrix}$$

| w | e(w) |
|----|------|
| 00 | 0000 |
| 01 | 0100 |
| 10 | 1011 |
| 11 | 1111 |

(ii) The minimum distance of this (2, 4) encoding function $e_H = 1$

(iii)

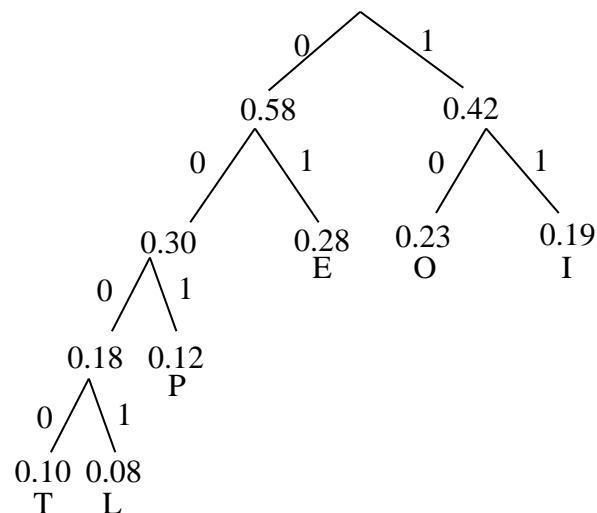
| \oplus | 0000 | 0100 | 1011 | 1111 |
|----------|------|------|------|------|
| 0001 | 0001 | 0101 | 1010 | 1110 |
| 0010 | 0010 | 0110 | 1001 | 1101 |
| 1000 | 1000 | 1100 | 0011 | 0111 |

(iv) (1) $d(1100) = 01$

(2) $d(1010) = 10$

Q4. b) (i) Process of merging two lowest probabilities.

0.28 0.23 0.19 0.12 0.10 0.08
 0.28 0.23 0.19 0.18 0.12
 0.30 0.28 0.23 0.19
 0.42 0.30 0.28
 0.58 0.42



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Question 4 b) (i) (Continued)

| | | | | | | |
|-----------------------|------|------|------|------|------|------|
| Letter, (x_i) | L | E | T | O | I | P |
| Probability, $P(x_i)$ | 0.08 | 0.28 | 0.10 | 0.23 | 0.19 | 0.12 |
| Codeword, C_i | 0001 | 01 | 0000 | 10 | 11 | 001 |

001 10 0001 11 0000 01
P O L I T E

Q4. b) (ii) Average code length, $L(C) = 2.48$ bits

Entropy, $H(x) = 2.4479$

Efficiency = 0.9871

The efficiency of this code is 98.71%.