Based on Math Fortress Systems of Differential Equations

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```
(%i2) info:build_info()$info@version;
```

(%o2)

5.38.1

```
(%i3) file_search_maxima:cons(sconcat("C:/Maxima-sbcl-5.38.1/share/maxima/5.38.1/share/odepack/###.lis
(%i2) reset()$kill(all)$
(%i1) derivabbrev:true$
(%i2) ratprint:false$
(%i3) fpprintprec:5$
(%i4) if get('draw,'version)=false then load(draw)$
(%i5) wxplot_size:[1024,768]$
(%i6) load(dlsode)$
(%i7) if get('optvar,'version)=false then load(optvar)$
(%i8) if get('rkf45,'version)=false then load(rkf45)$
(%i9) declare(trigsimp,evfun)$
(%i10) declare(t,mainvar)$
```

This is a system of equations with a 2nd order equation for x and a 1st order equation for y.

(%i11)
```
kill(labels,t,x,y)$
```

(%i1)
```
r:[x,y]$
```

(%i2)
```
depends(r,t)$
```

(%i3)
```
initial:[v_0=2.0,y_0=1.0,x_0=1.0]$
```

(%i4)
```
τ:1$
```

(%i5)
```
Eq1:diff(x,t,2)+diff(y,t)=-5*t;
```

$$\dot{y} + \ddot{x} = -5t \tag{Eq1}$$

(%i6)
```
Eq2:diff(x,t)+diff(y,t)=-x+4*y;
```

$$\dot{y} + \dot{x} = 4y - x \tag{Eq2}$$

Analytical solution

(%i8)
```
atvalue(x(t),t=0,x_0)$
atvalue(y(t),t=0,y_0)$
```

(%i9)
```
atvalue(diff(x(t),t),t=0,v_0)$
```

(%i10)
```
desol:desolve(convert([Eq1,Eq2],r,t),convert(r,r,t))$
```

(%i11)
```
map(ldisp,desol)$
```

$$\mathrm{x}(t) = e^{\frac{5t}{2}} \left(\frac{\sinh\left(\frac{\sqrt{29}t}{2}\right) \left(2\left(20y_0 - 5x_0 + 21v_0 - 2830\right) + 5\left(-4y_0 + x_0 - 4v_0 + 545\right)\right)}{\sqrt{29}} + \cosh\left(\frac{\sqrt{29}t}{2}\right) \left(-4y_0 + x_0 - 4v_0 + 545\right) \right) + 4y_0 + 4v_0 - 10t^2 + 105t - 545$$

$$\mathrm{y}(t) = e^{\frac{5t}{2}} \left(\frac{\sinh\left(\frac{\sqrt{29}t}{2}\right) \left(2\left(4y_0 - x_0 + 4v_0 - 545\right) + 5\left(105 - v_0\right)\right)}{\sqrt{29}} + \cosh\left(\frac{\sqrt{29}t}{2}\right) \left(105 - v_0\right) \right) + y_0 + v_0 - \frac{5t^2}{2} + 20t - 105$$

Verify

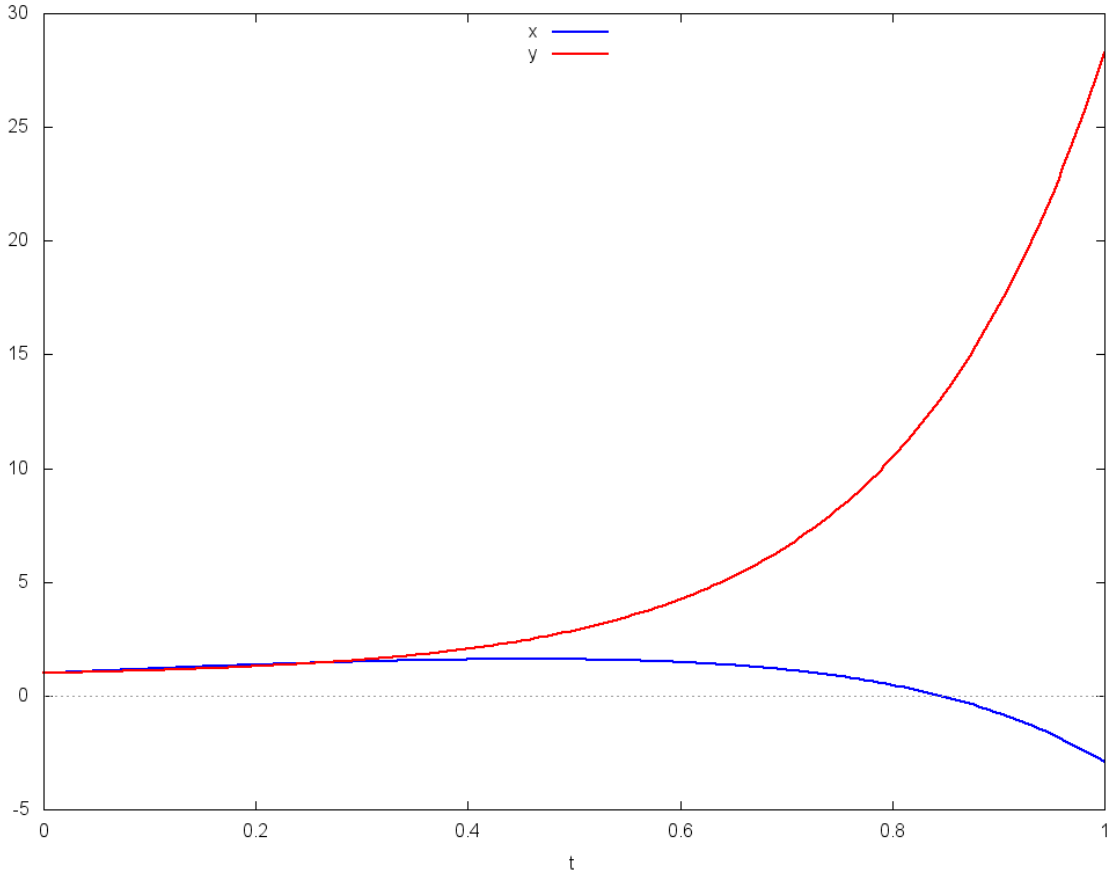
(%i13)
```
is(ev(convert(Eq1,r,t),desol,diff,eval,initial,eval,ratsimp,eval));
```

true (%o13)

(%i14)
```
is(ev(convert(Eq2,r,t),desol,diff,eval,initial,eval,ratsimp,eval));
```

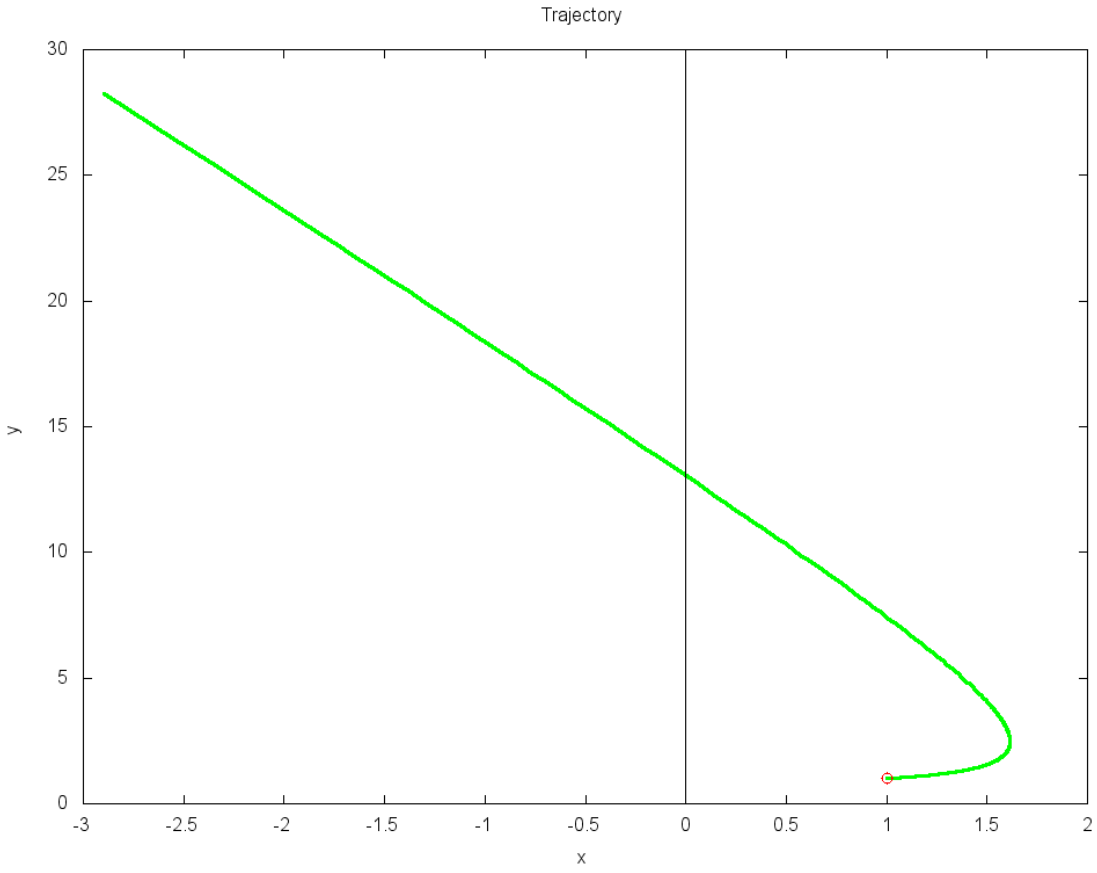
true (%o14)

(%i15) `wxplot2d([x(t),y(t)],[t,0,τ],[style,[lines,2]],[legend,"x","y"], [gnuplot_preamble,"set key top center"]),desol,initial$`



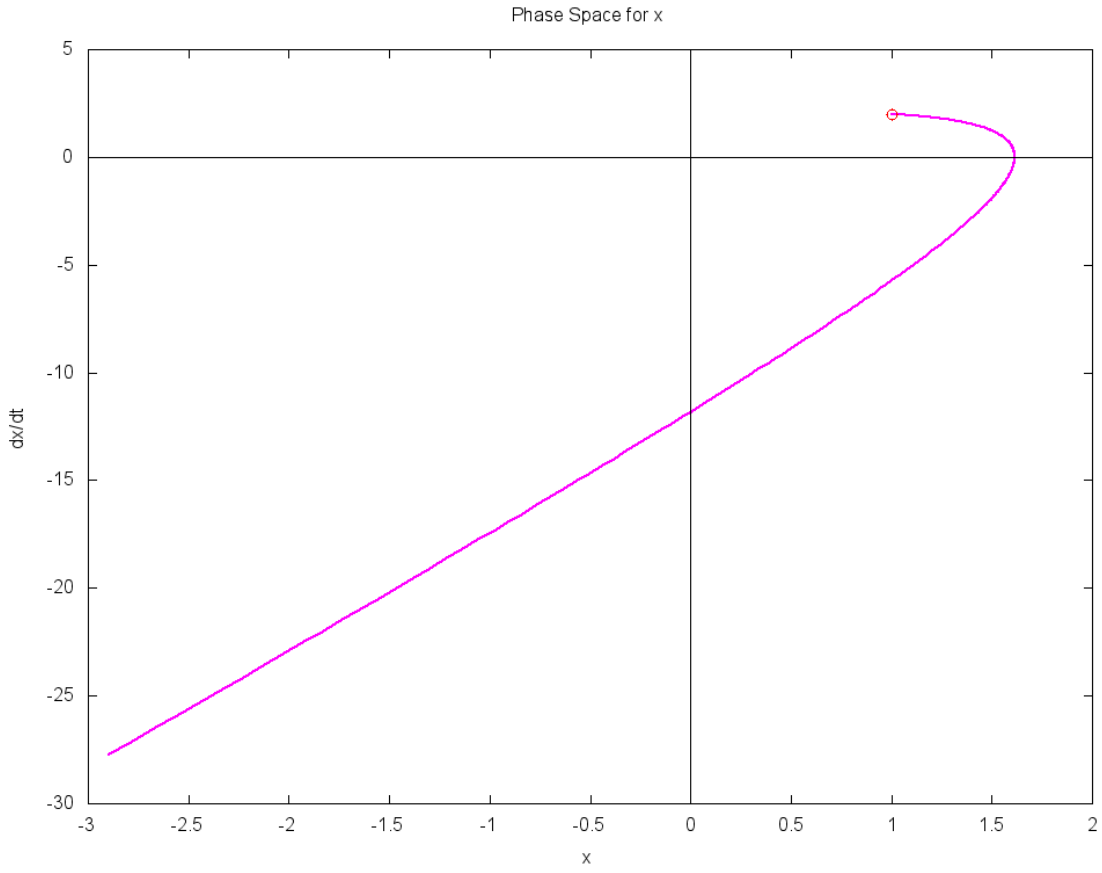
(%t15)

(%i16)
```
wxplot2d([[parametric,x(t),y(t),[t,0,τ]], [discrete,[[x_0,y_0]]],[axes,solid],
[title,"Trajectory"],[style,[lines,3],[points,3]], [color,green,red],[point_type,circle],[legend,
[xlabel,"x"],[ylabel,"y"]),desol,initial$
```



(%t16)

(%i17)
```
wxplot2d([[parametric,x(t),diff(x(t),t),[t,0,τ]], [discrete,[[x_0,v_0]]],[axes,solid],
[title,"Phase Space for x"],[style,[lines,2],[points,3]], [color,magenta,red],[point_type,circle]
[xlabel,"x"],[ylabel,"dx/dt"]),desol,initial$
```

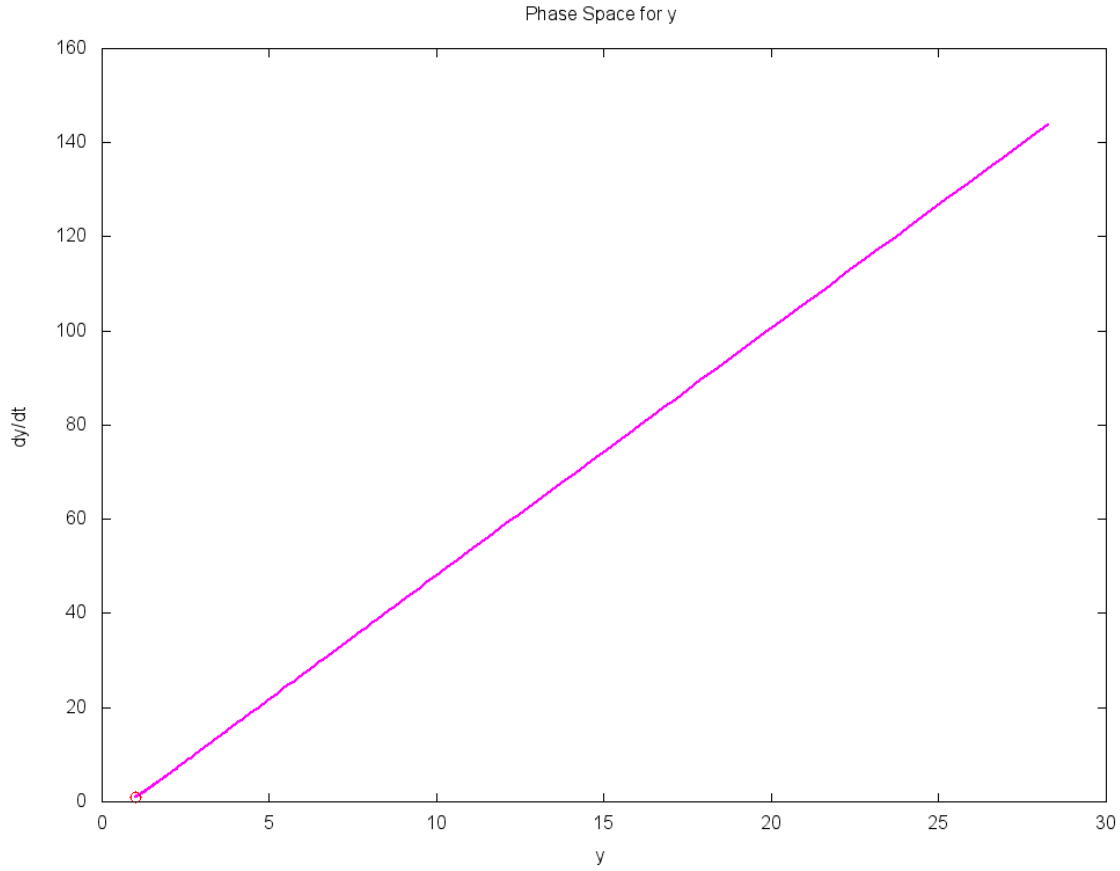


(%t17)

(%i18)
```
Dy0:at(ev(diff(y(t),t),desol),[t=0]),ratsimp;
```

$$4y_0 - x_0 - v_0 \tag{Dy0}$$

(%i19)
```
wxplot2d([[parametric,y(t),diff(y(t),t),[t,0,τ]], [discrete,[[y_0,Dy0]]],[axes,solid],
[title,"Phase Space for y"],[style,[lines,2],[points,3]], [color,magenta,red],[point_type,circle]
[xlabel,"y"],[ylabel,"dy/dt"]),desol,initial$
```



(%t19)

Reduce Order

```
(%i20) kill(labels)$
(%i1)  depends(X,t)$
(%i2)  gradef(x,t,X)$
(%i3)  Eq1:Eq1,diff,eval;
```

$$\dot{y} + \dot{X} = -5t \tag{Eq1}$$

```
(%i4)  Eq2:Eq2,diff,eval;
```

$$\dot{y} + X = 4y - x \tag{Eq2}$$

```
(%i5)  linsol:linsolve([Eq1,Eq2],[diff(x,t,2),diff(y,t)])$
(%i6)  map(ldisp,linsol)$
```

$$\dot{X} = -4y + x - 5t + X \tag{\%t6}$$

$$\dot{y} = 4y - x - X \tag{\%t7}$$

```
(%i8)  Eq3:'diff(x,t)=X;
```

$$\dot{x} = X \tag{Eq3}$$

Analytical solution

```
(%i9)  atvalue(X(t),t=0,v_0)$
(%i10) desol:desolve(convert(append(linsol,[Eq3]),[X,y,x],t),convert([X,y,x],[X,y,x],t))$
(%i11) map(ldisp,desol)$
```

$$\mathrm{X}(t) = e^{\frac{5t}{2}} \left(\frac{\sinh\left(\frac{\sqrt{29}t}{2}\right) \left(2\left(-4y_0 + x_0 - 4v_0 + 545\right) + 5\left(v_0 - 105\right)\right)}{\sqrt{29}} + \cosh\left(\frac{\sqrt{29}t}{2}\right) \left(v_0 - 105\right) \right) - 20t + 105$$

$$\mathrm{y}(t) = e^{\frac{5t}{2}} \left(\frac{\sinh\left(\frac{\sqrt{29}t}{2}\right) \left(2\left(4y_0 - x_0 + 4v_0 - 545\right) + 5\left(105 - v_0\right)\right)}{\sqrt{29}} + \cosh\left(\frac{\sqrt{29}t}{2}\right) \left(105 - v_0\right) \right) + y_0 + v_0 - \frac{5t^2}{2} + 20t - 105$$

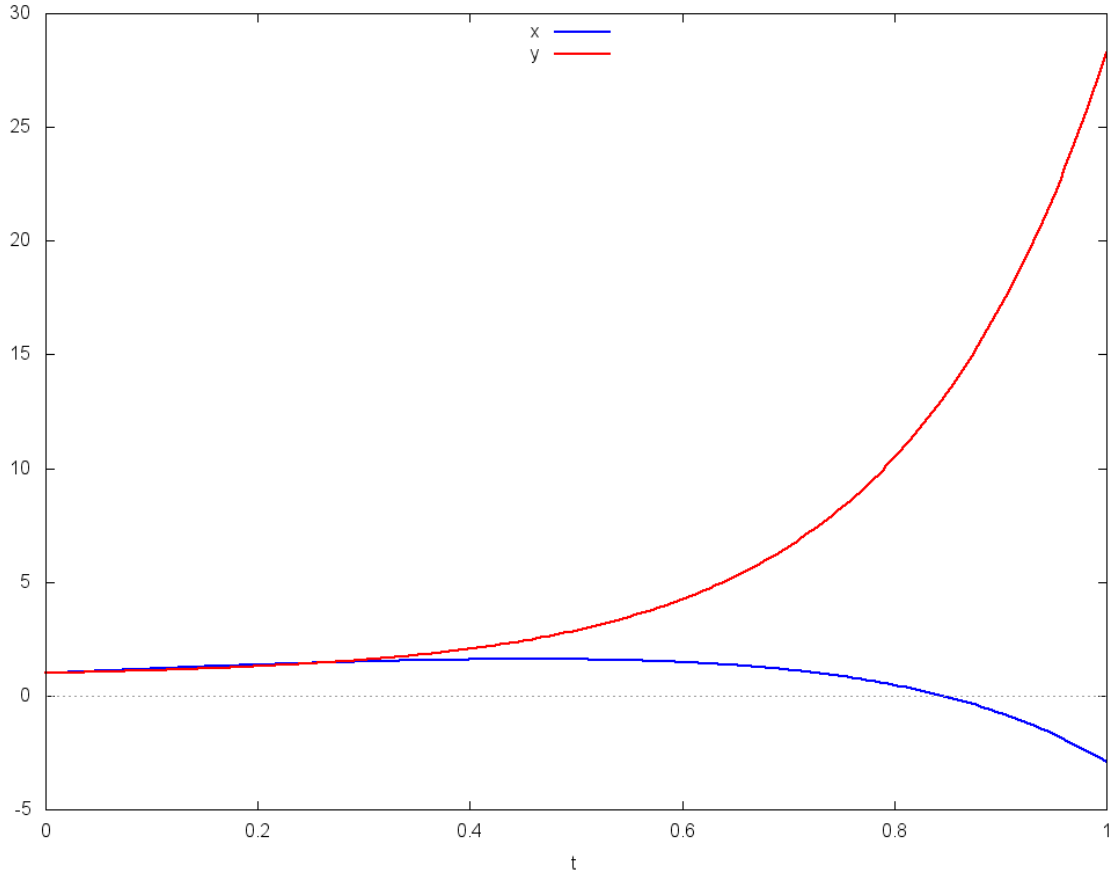
$$\mathrm{x}(t) = e^{\frac{5t}{2}} \left(\frac{\sinh\left(\frac{\sqrt{29}t}{2}\right) \left(2\left(20y_0 - 5x_0 + 21v_0 - 2830\right) + 5\left(-4y_0 + x_0 - 4v_0 + 545\right)\right)}{\sqrt{29}} + \cosh\left(\frac{\sqrt{29}t}{2}\right) \left(-4y_0 + x_0 - 4v_0 + 545\right) \right) + 4y_0 + 4v_0 - 10t^2 + 105t - 545$$

Verify

```
(%i14) is(ev(convert(Eq3,[X,y,x],t),desol,diff,eval,initial,eval,expand,eval));
```

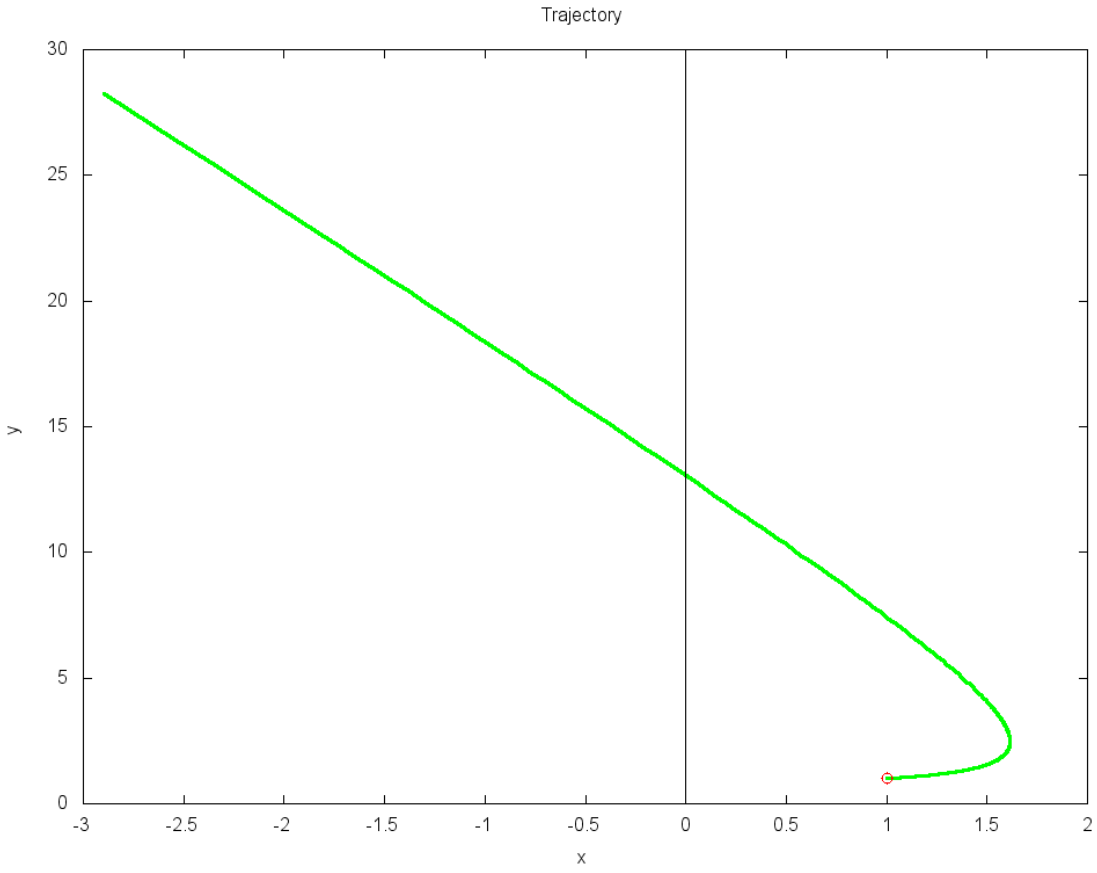
true (%o14)

(%i15)
```
wxplot2d([x(t),y(t)],[t,0,τ],[style,[lines,2]],[legend,"x","y"], [gnuplot_preamble,"set key top center"]),desol,initial$
```



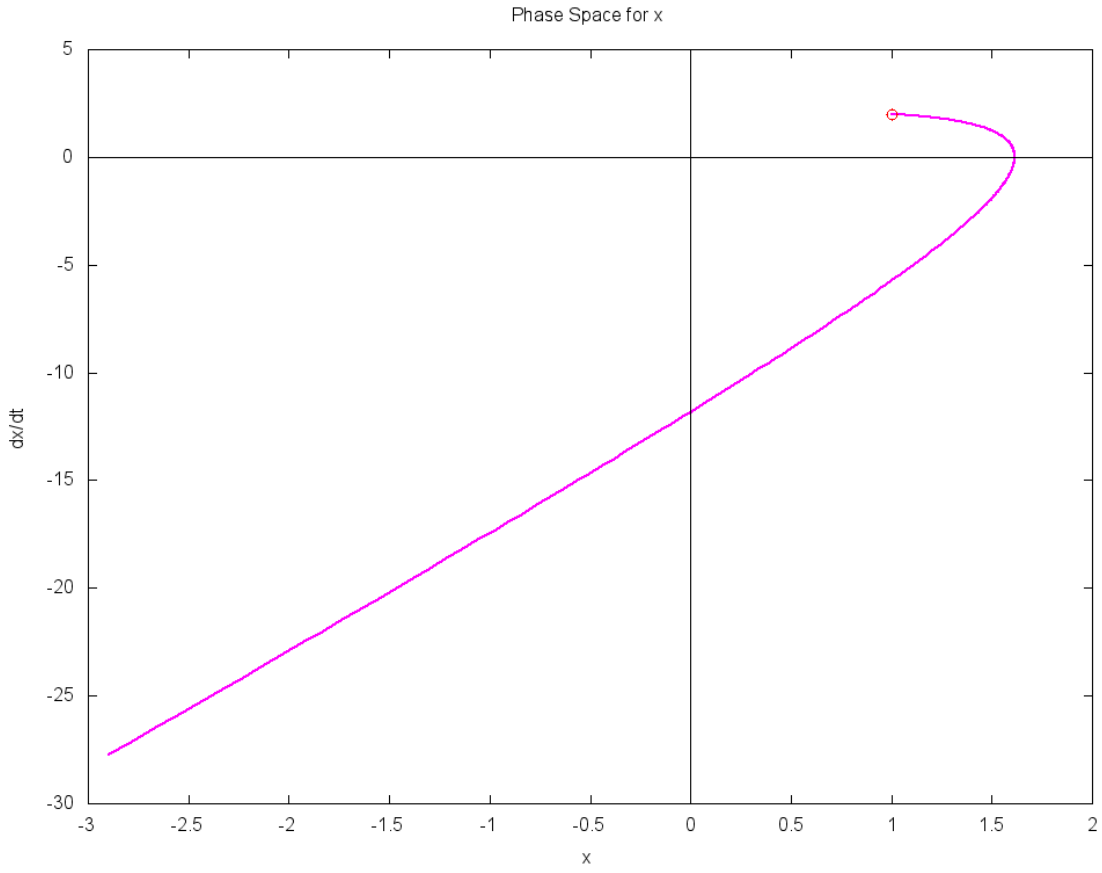
(%t15)

```
(%i16) wxplot2d([[parametric,x(t),y(t),[t,0,τ]], [discrete,[[x_0,y_0]]],[axes,solid],
       [title,"Trajectory"],[style,[lines,3],[points,3]], [color,green,red],[point_type,circle],[legend,
       [xlabel,"x"],[ylabel,"y"]),desol,initial$
```



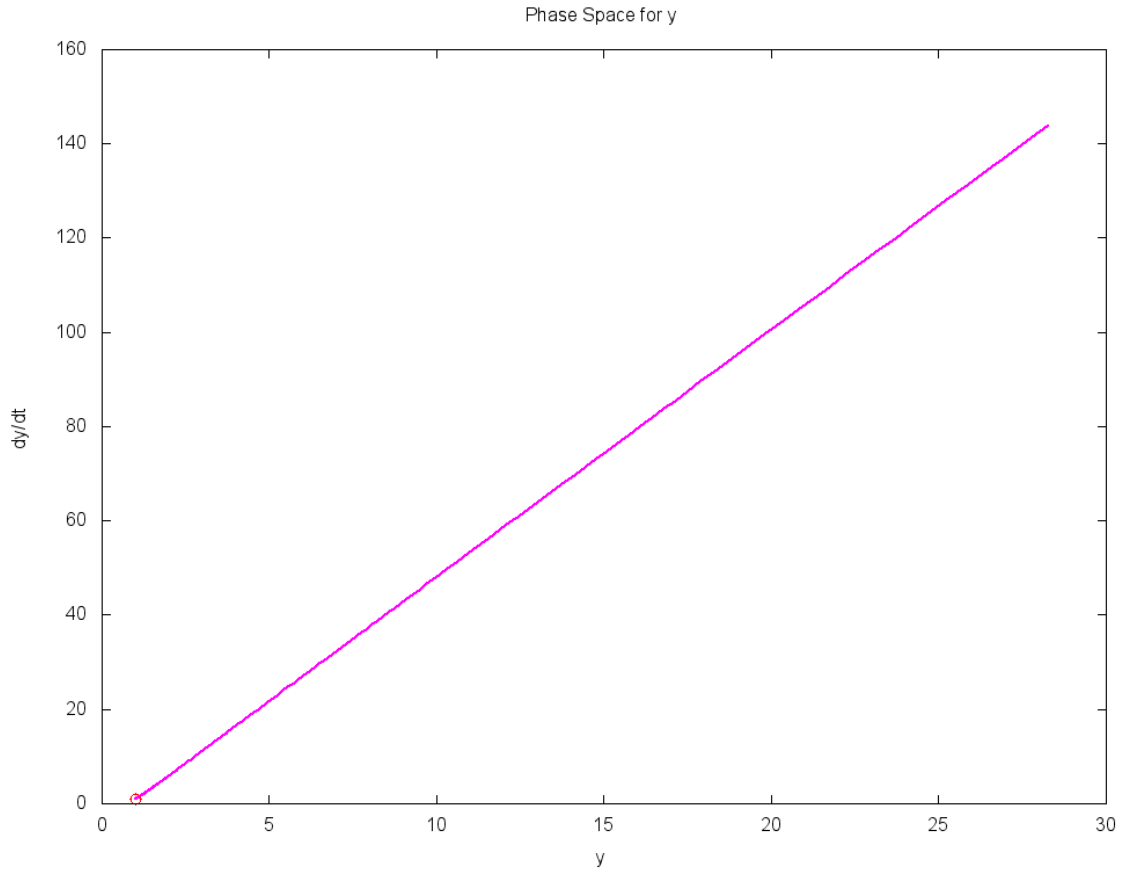
(%t16)

(%i17)
```
wxplot2d([[parametric,x(t),X(t),[t,0,τ]], [discrete,[[x_0,v_0]]],[axes,solid],
[title,"Phase Space for x"],[style,[lines,2],[points,3]], [color,magenta,red],[point_type,circle]
[xlabel,"x"],[ylabel,"dx/dt"]),desol,initial$
```



(%t17)

(%i18)
```
wxplot2d([[parametric,y(t),diff(y(t),t),[t,0,τ]], [discrete,[[y_0,Dy0]]]],[axes,solid],
[title,"Phase Space for y"],[style,[lines,2],[points,3]], [color,magenta,red],[point_type,circle]
[xlabel,"y"],[ylabel,"dy/dt"]),desol,initial$
```



(%t18)

Numerical solution with rfk45

(%i19)
```
kill(labels)$
```

(%i6)
```
funcs:[X,y,x]$ldisplay(funcs)$
odes:map('rhs,append(linsol,[Eq3]))$ldisplay(odes)$
interval:[t,0,7]$ldisplay(interval)$
```

$$funcs = [X, y, x] \tag{\%t2}$$

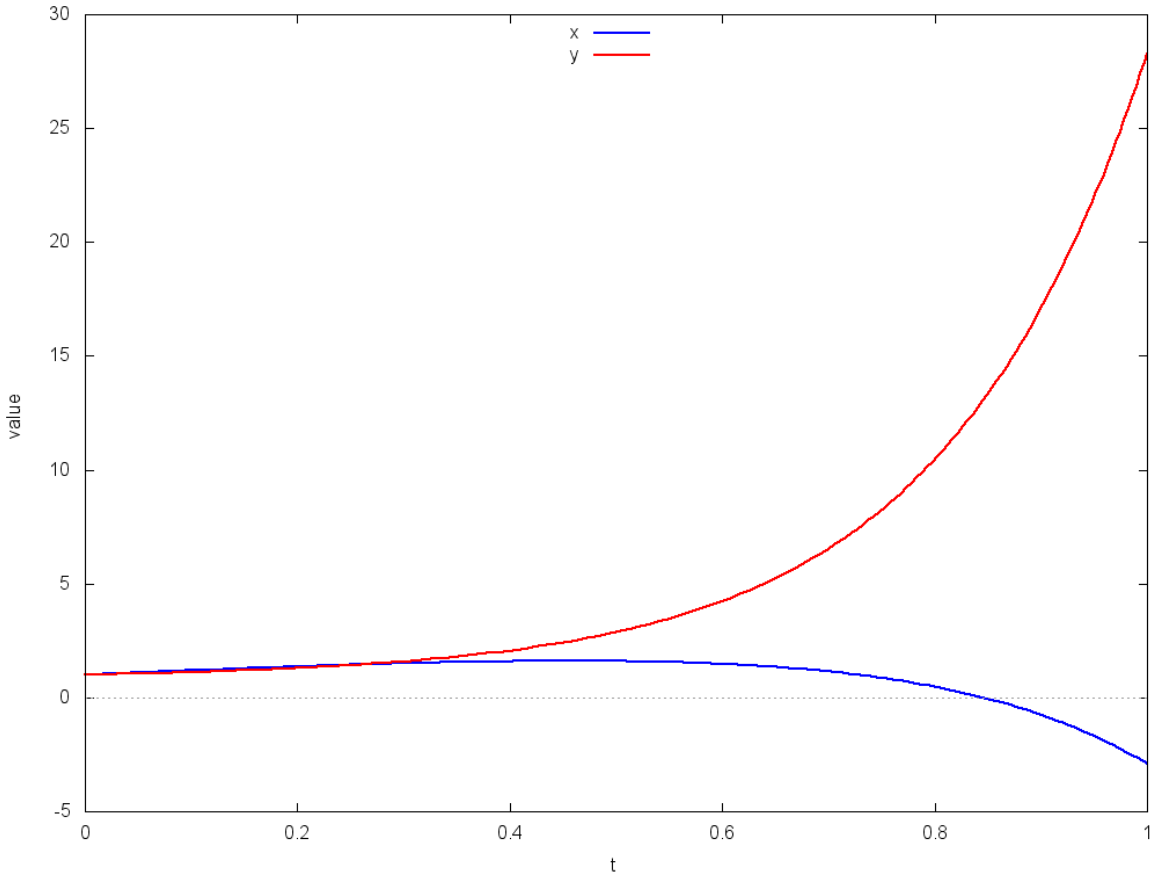
$$odes = [-4y + x - 5t + X, 4y - x - X, X] \tag{\%t4}$$

$$interval = [t, 0, 1] \tag{\%t6}$$

(%i7)
```
rksol:rkf45(odes,funcs,map('rhs,initial),interval, absolute_tolerance=5d-8,report=true)$
```

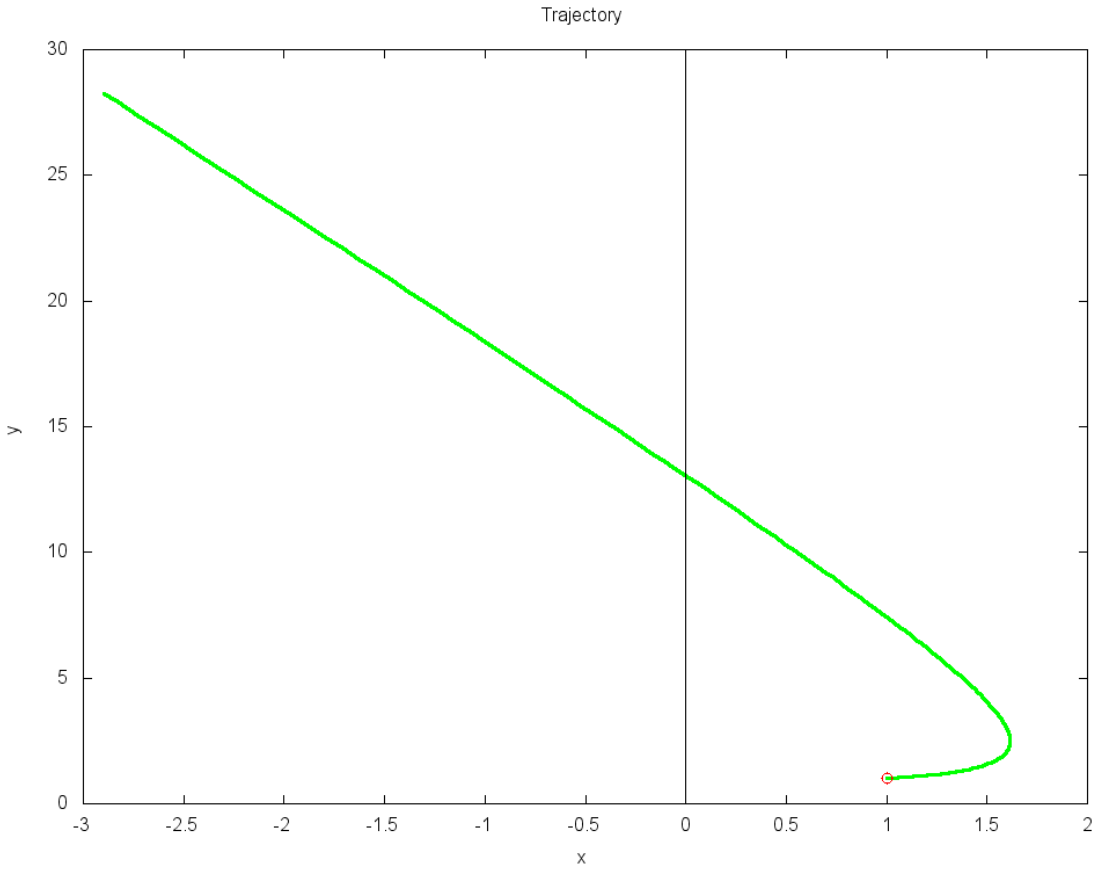
Info: rkf45:
Integration points selected:151
Total number of iterations:150
Bad steps corrected:0
Minimum estimated error:4.990810^{-9}
Maximum estimated error:2.683410^{-8}
Minimum integration step taken:0.0024759
Maximum integration step taken:0.013573

(%i8) `wxplot2d([[discrete,map(lambda([u],part(u,[1,4])),rksol)], [discrete,map(lambda([u],part(u,[1,3])),rksol)], [style,[lines,2]],[xlabel,"t"],[ylabel,"value"], [legend,"x","y"],[gnuplot_preamble,"set key top center"])$`



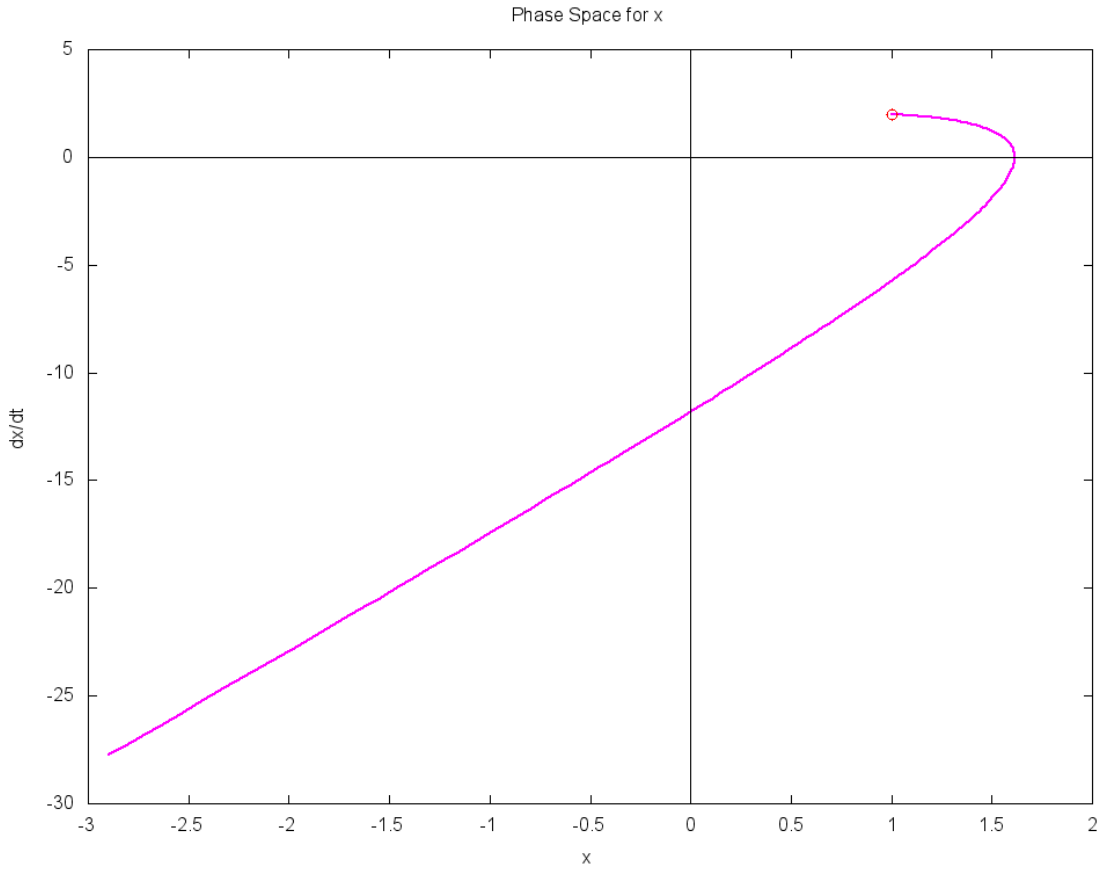
(%t8)

(%i9) `wxplot2d([[discrete,map(lambda([u],part(u,[4,3])),rksol)], [discrete,[[x_0,y_0]]],[axes,solid], [title,"Trajectory"],[point_type,circle], [style,[lines,3],[points,3]],[color,green,red], [xlabel,"x"],[ylabel,"y"],[legend,false]),initial$`



(%t9)

```
(%i10) wxplot2d([[discrete,map(lambda([u],part(u,[4,2])),rksol)], [discrete,[[x_0,v_0]]]],[axes,solid],
       [title,"Phase Space for x"],[point_type,circle], [style,[lines,2],[points,3]],[color,magenta,red]
       [xlabel,"x"],[ylabel,"dx/dt"],[legend,false]),initial$
```



(%t10)

Numerical solution with dlsode

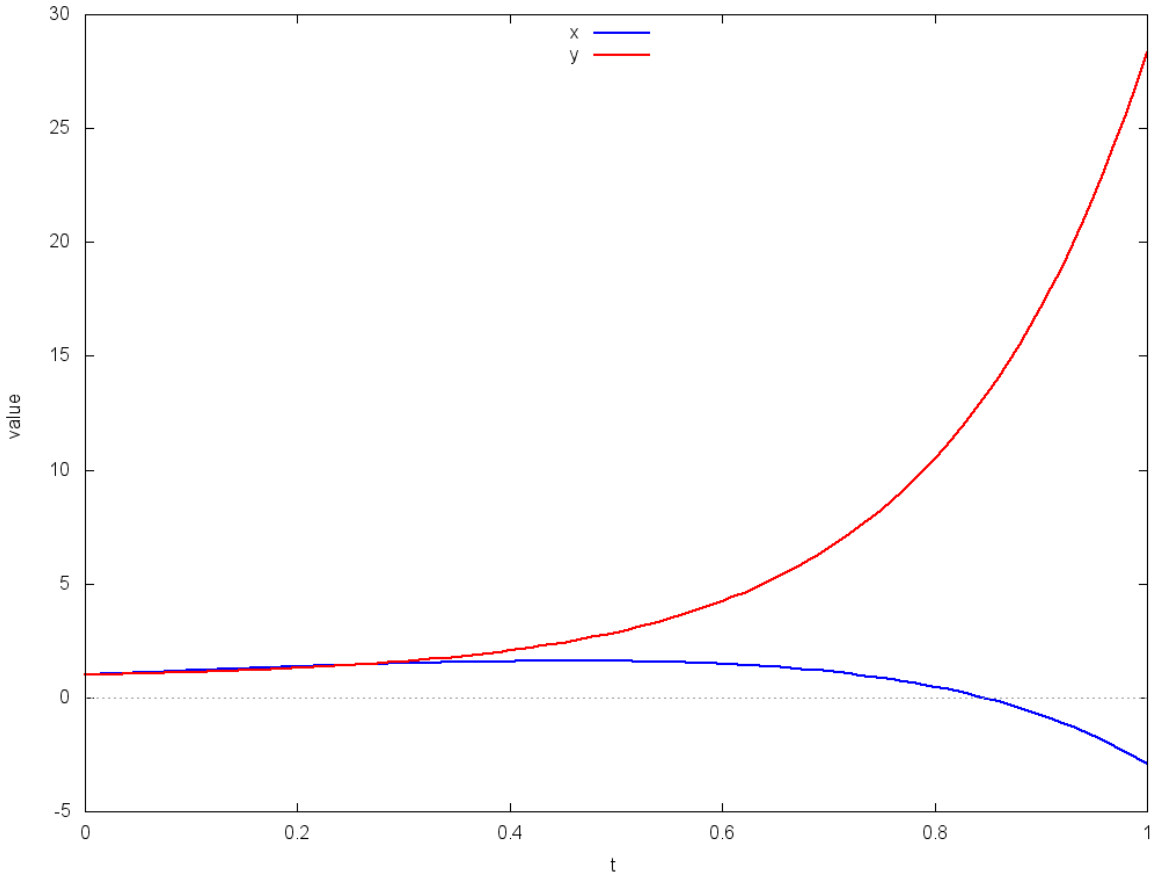
(%i11)
```
kill(labels,t,X,y,x)$
```

(%i1)
```
state:dlsode_init(map('rhs,append(linsol,[Eq3])),['t,'X,'y,'x],21)$
```

(%i9)
```
t:0d0$
init:map(rhs,initial)$
rtol:1d-4$
atol:[5d-8,5d-8,5d-8]$
result:[]$
dlsol:[cons(t,init)]$
tout:δ:0.01d0$
istate:1$
```

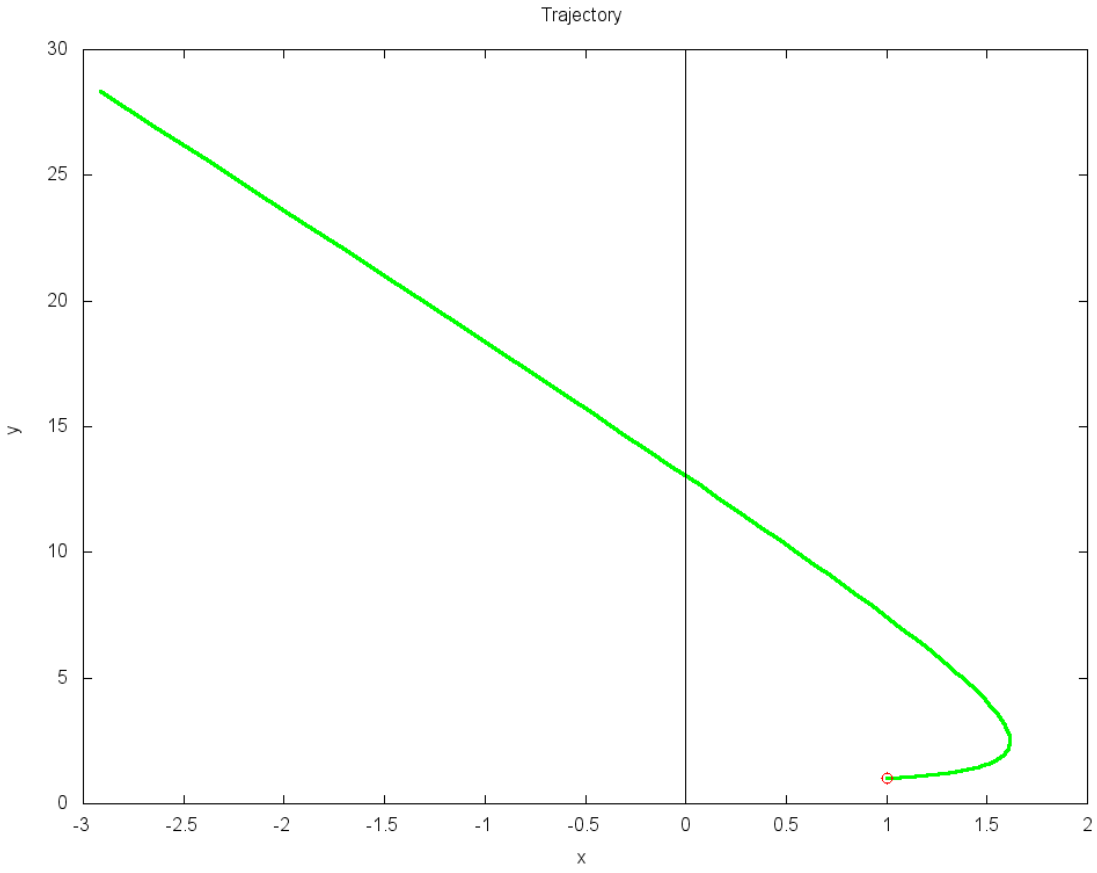
(%i10)
```
for k thru τ/δ do
block([],
result:dlsode_step(init,t,tout,rtol,atol,istate,state),
dlsol:append(dlsol,[cons(first(result),second(result))]),
istate:result[3],
tout:tout+δ)$
```

```
(%i11) wxplot2d([[discrete,map(lambda([u],part(u,[1,4])),dlsol)], [discrete,map(lambda([u],part(u,[1,3])
       [style,[lines,2]],[xlabel,"t"],[ylabel,"value"],[x,0,τ], [legend,"x","y"],[gnuplot_preamble,"set
       key top center"])$
```



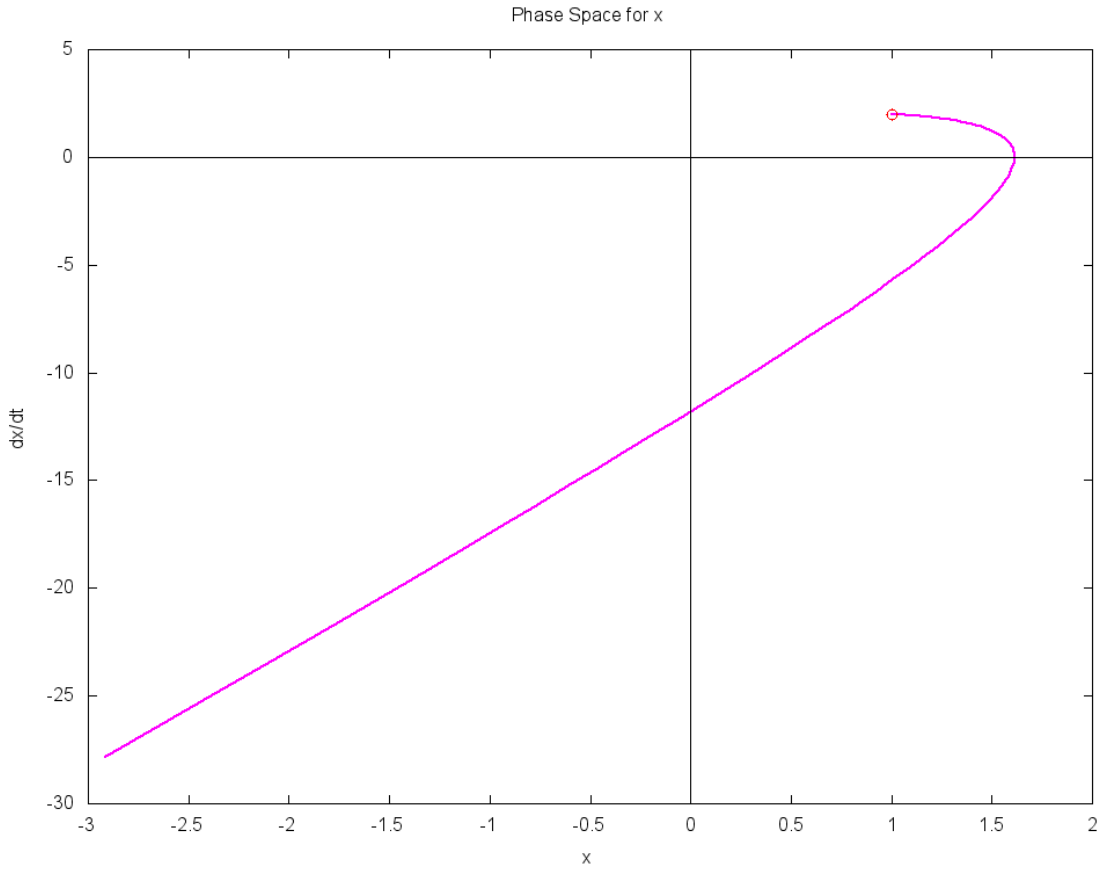
(%t11)

(%i12)
```
wxplot2d([[discrete,map(lambda([u],part(u,[4,3])),dlsol)], [discrete,[[x_0,y_0]]]],[axes,solid],
[title,"Trajectory"],[point_type,circle], [style,[lines,3],[points,3]],[color,green,red],
[xlabel,"x"],[ylabel,"y"],[legend,false]),initial$
```



(%t12)

```
(%i13) wxplot2d([[discrete,map(lambda([u],part(u,[4,2])),dlsol)], [discrete,[[x_0,v_0]]]],[axes,solid],
       [title,"Phase Space for x"],[point_type,circle], [style,[lines,2],[points,3]],[color,magenta,red]
       [xlabel,"x"],[ylabel,"dx/dt"],[legend,false]),initial$
```



(%t13)