Based on Math Fortress Systems of Differential Equations

Written by Daniel Volinski at danielvolinski@yahoo.es

```
(%i2) info:build_info()$info@version;
```

(%o2)

5.38.1

```
(%i3) file_search_maxima:cons(sconcat("C:/Maxima-sbcl-5.38.1/share/maxima/5.38.1/share/odepack/###.lis
(%i2) reset()$kill(all)$
(%i1) derivabbrev:true$
(%i2) ratprint:false$
(%i3) fpprintprec:5$
(%i4) if get('draw,'version)=false then load(draw)$
(%i5) wxplot_size:[1024,768]$
(%i6) load(dlsode)$
(%i7) if get('optvar,'version)=false then load(optvar)$
(%i8) if get('rkf45,'version)=false then load(rkf45)$
(%i9) declare(trigsimp,evfun)$
(%i10) declare(t,mainvar)$
```

This is a system of equations where both equations are 2nd order.

(%i11)
```
kill(labels,t,x,y)$
```
(%i1)
```
r:[x,y]$
```
(%i2)
```
depends(r,t)$
```
(%i3)
```
initial:[Vx_0=-1.0,Vy_0=2.0,x_0=0.0,y_0=-0.5]$
```
(%i4)
```
τ:2$
```
(%i5)
```
Eq1:diff(x,t,2)=4*y+exp(t);
```

$$\ddot{x} = 4y + e^t \tag{Eq1}$$

(%i6)
```
Eq2:diff(y,t,2)=4*x-exp(t);
```

$$\ddot{y} = 4x - e^t \tag{Eq2}$$

Analytical solution

(%i8)
```
atvalue(x(t),t=0,x_0)$
atvalue(y(t),t=0,y_0)$
```
(%i10)
```
atvalue(diff(x(t),t),t=0,Vx_0)$
atvalue(diff(y(t),t),t=0,Vy_0)$
```
answer p;

(%i11)
```
desol:desolve(convert([Eq1,Eq2],r,t),convert(r,r,t))$
```
Is

$$g436 - 1$$

positive, negative or zero?

$$p;$$

(%i12)
```
map(ldisp,desol)$
```

$$\begin{aligned}
\mathrm{x}(t) = &-\frac{\cos(2t)\ (5y_0 - 5x_0 + 2)}{10} + \frac{e^{2t}\ (2y_0 + 2x_0 + Vy_0 + Vx_0)}{8} \\
&+ \frac{e^{-2t}\ (2y_0 + 2x_0 - Vy_0 - Vx_0)}{8} - \frac{(5\,Vy_0 - 5\,Vx_0 + 2)\sin(2t)}{20} + \frac{e^t}{5} \\
\mathrm{y}(t) = &\frac{\cos(2t)\ (5y_0 - 5x_0 + 2)}{10} + \frac{e^{2t}\ (2y_0 + 2x_0 + Vy_0 + Vx_0)}{8} \\
&+ \frac{e^{-2t}\ (2y_0 + 2x_0 - Vy_0 - Vx_0)}{8} + \frac{(5\,Vy_0 - 5\,Vx_0 + 2)\sin(2t)}{20} - \frac{e^t}{5}
\end{aligned}$$

Verify

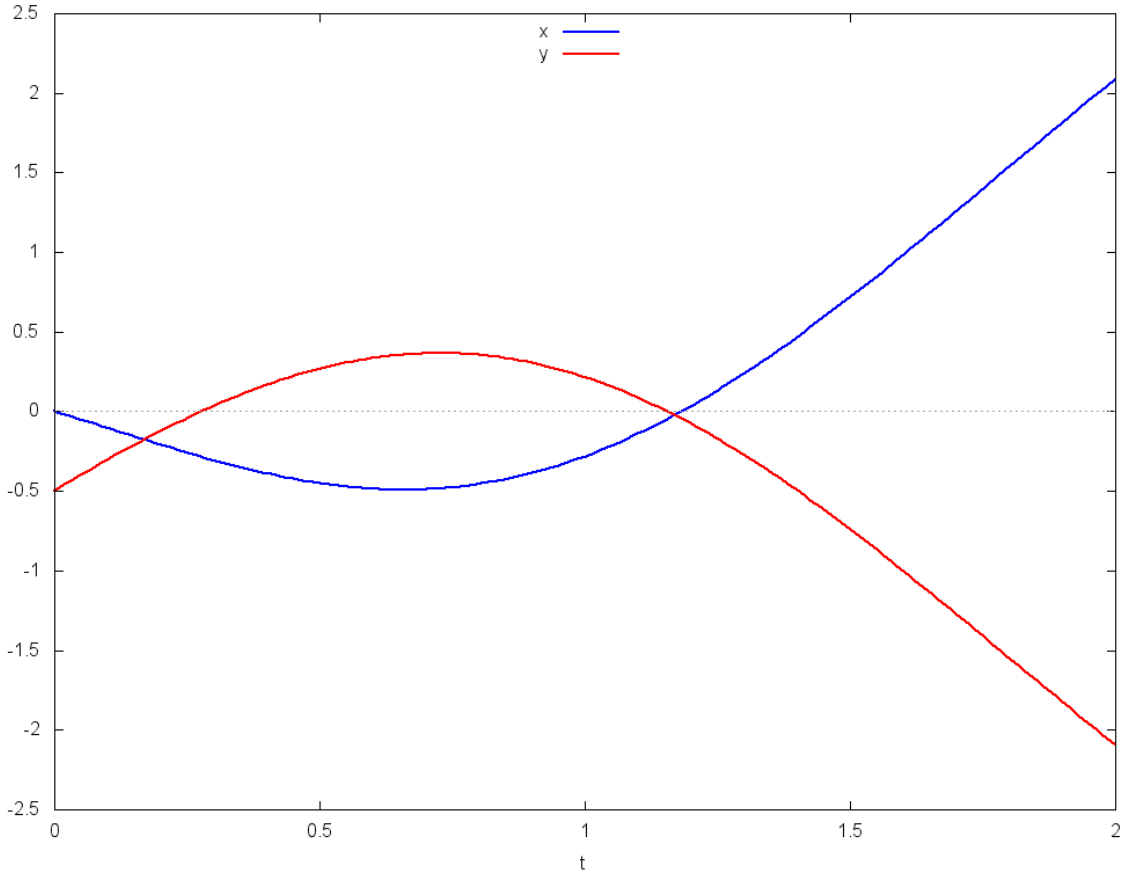
(%i14)
```
is(ev(convert(Eq1,r,t),desol,diff,eval,initial,eval,expand,eval));
```

true (%o14)

(%i15)
```
is(ev(convert(Eq2,r,t),desol,diff,eval,initial,eval,expand,eval));
```

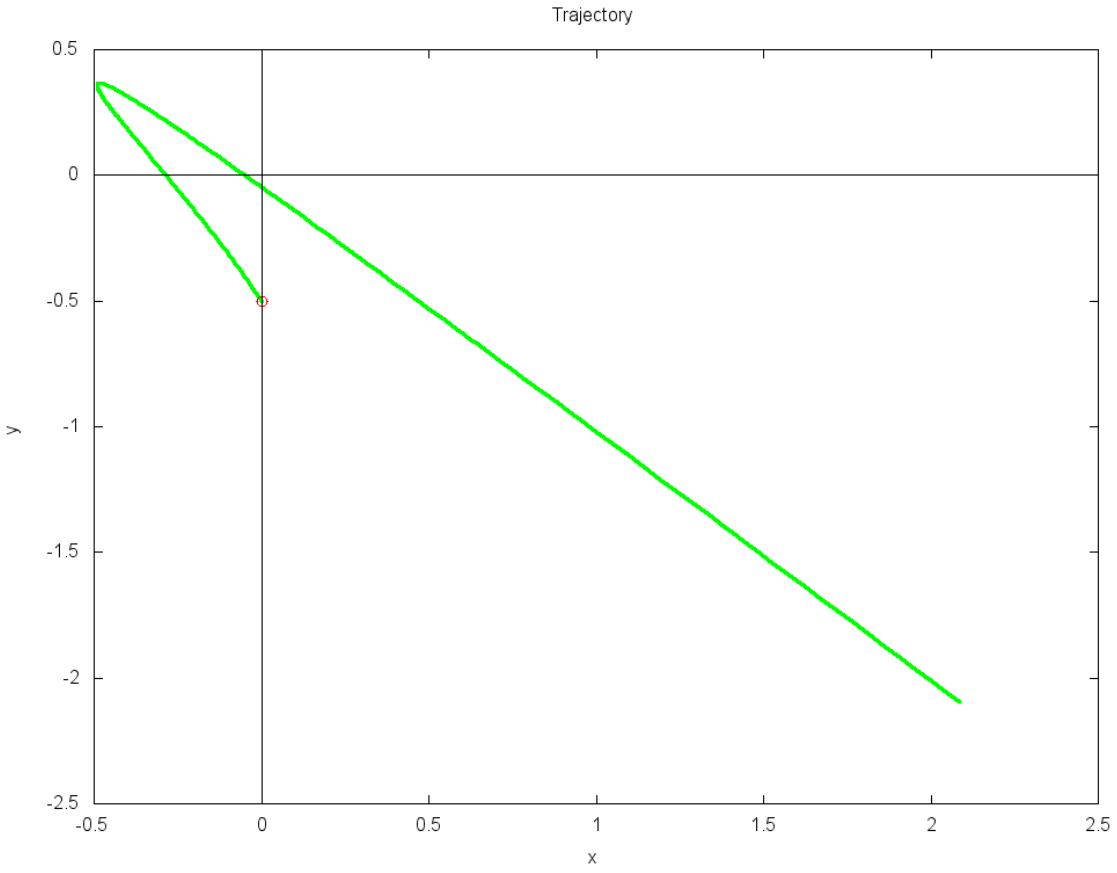
true (%o15)

(%i16) `wxplot2d([x(t),y(t)],[t,0,τ],[style,[lines,2]],[legend,"x","y"], [gnuplot_preamble,"set key top center"]),desol,initial$`



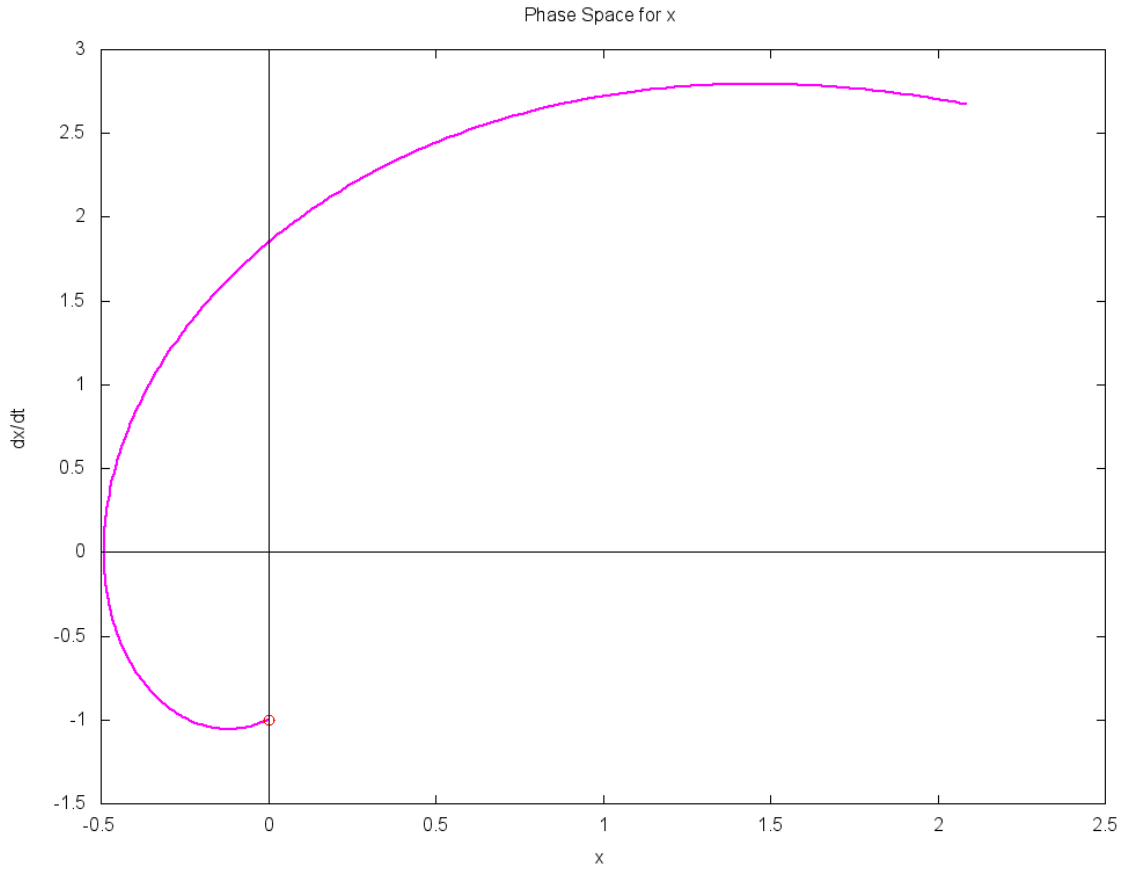
(%t16)

(%i17)
```
wxplot2d([[parametric,x(t),y(t),[t,0,τ]], [discrete,[[x_0,y_0]]],[axes,solid],
[title,"Trajectory"],[style,[lines,3],[points,3]], [color,green,red],[point_type,circle],[legend,
[xlabel,"x"],[ylabel,"y"]),desol,initial$
```



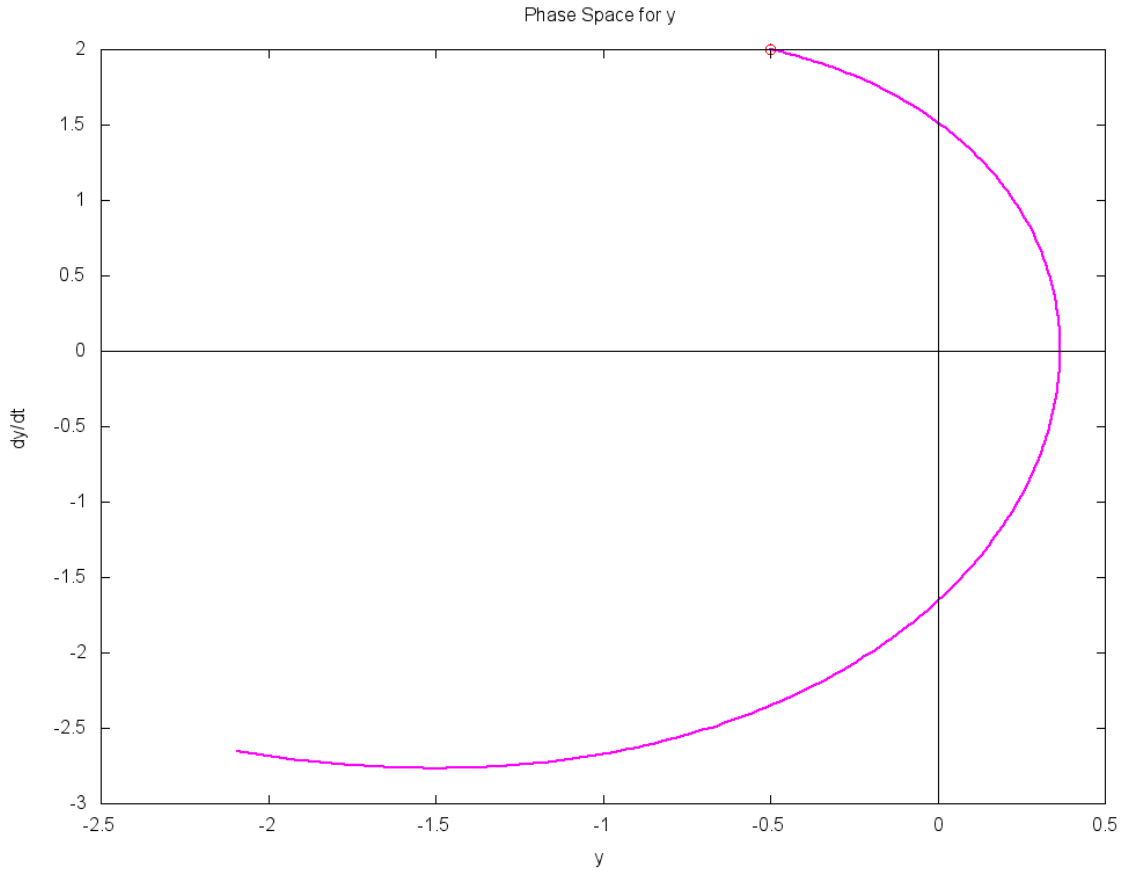
(%t17)

(%i18)
```
wxplot2d([[parametric,x(t),diff(x(t),t),[t,0,τ]], [discrete,[[x_0,Vx_0]]]],[axes,solid],
[title,"Phase Space for x"],[style,[lines,2],[points,3]], [color,magenta,red],[point_type,circle]
[xlabel,"x"],[ylabel,"dx/dt"]),desol,initial$
```



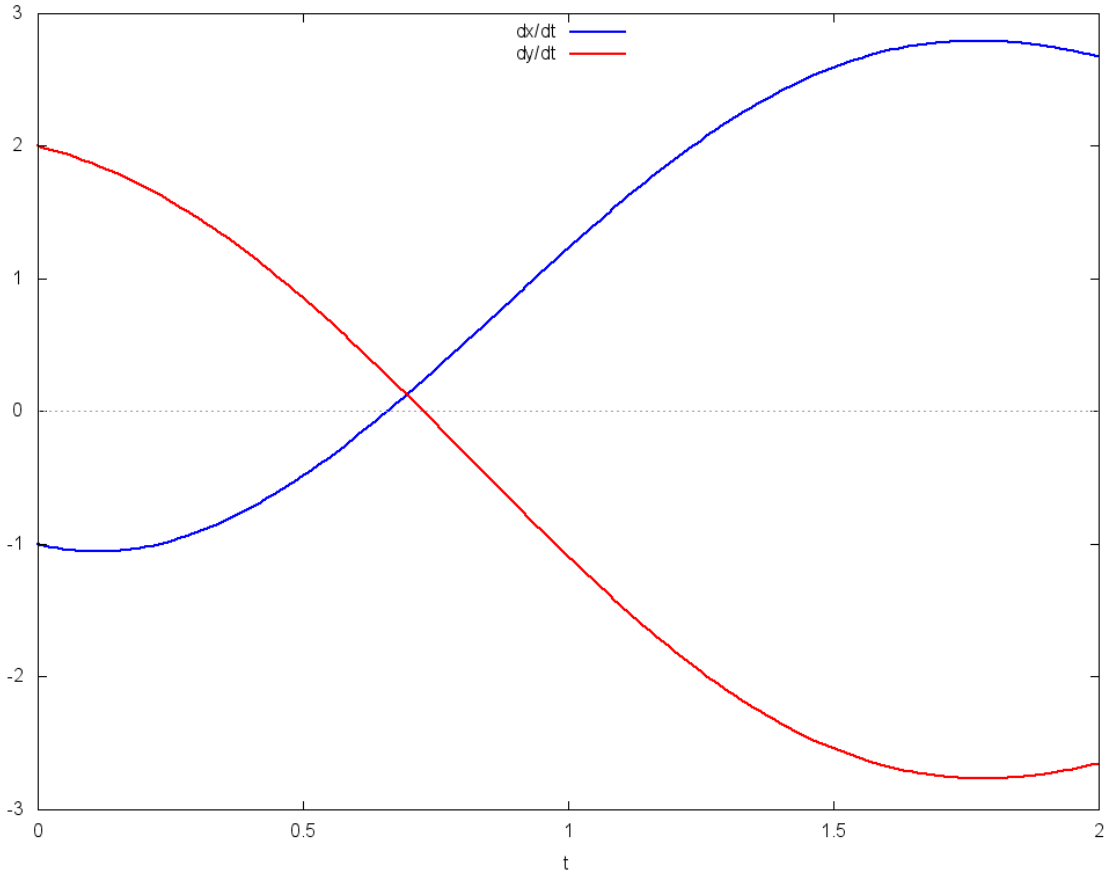
(%t18)

(%i19)
```
wxplot2d([[parametric,y(t),diff(y(t),t),[t,0,τ]], [discrete,[[y_0,Vy_0]]]],[axes,solid],
[title,"Phase Space for y"],[style,[lines,2],[points,3]], [color,magenta,red],[point_type,circle]
[xlabel,"y"],[ylabel,"dy/dt"]),desol,initial$
```



(%t19)

(%i20) `wxplot2d([diff(x(t),t),diff(y(t),t)],[t,0,τ],[style,[lines,2]],[legend,"dx/dt","dy/dt"],[gnuplot_preamble,"set key top center"]),desol,initial$`



(%t20)

Alternative initial conditions

(%i21)
```
Eq1a:ev(coeff(x(t),cos(2*t)),desol)=1;
```

$$-\frac{5y_0-5x_0+2}{10}=1 \tag{Eq1a}$$

(%i22)
```
Eq2a:ev(coeff(x(t),sin(2*t)),desol)=1;
```

$$-\frac{5\,Vy_0-5\,Vx_0+2}{20}=1 \tag{Eq2a}$$

(%i23)
```
Eq3a:ev(coeff(x(t),exp(2*t)),desol)=0;
```

$$\frac{2y_0+2x_0+Vy_0+Vx_0}{8}=0 \tag{Eq3a}$$

(%i24)
```
Eq4a:ev(coeff(x(t),exp(-2*t)),desol)=0;
```

$$\frac{2y_0+2x_0-Vy_0-Vx_0}{8}=0 \tag{Eq4a}$$

(%i25)
```
linsol:linsolve([Eq1a,Eq2a,Eq3a,Eq4a],map(lhs,initial));
```

$$\left[Vx_0=\frac{11}{5},\,Vy_0=-\frac{11}{5},\,x_0=\frac{6}{5},\,y_0=-\frac{6}{5}\right] \tag{linsol}$$

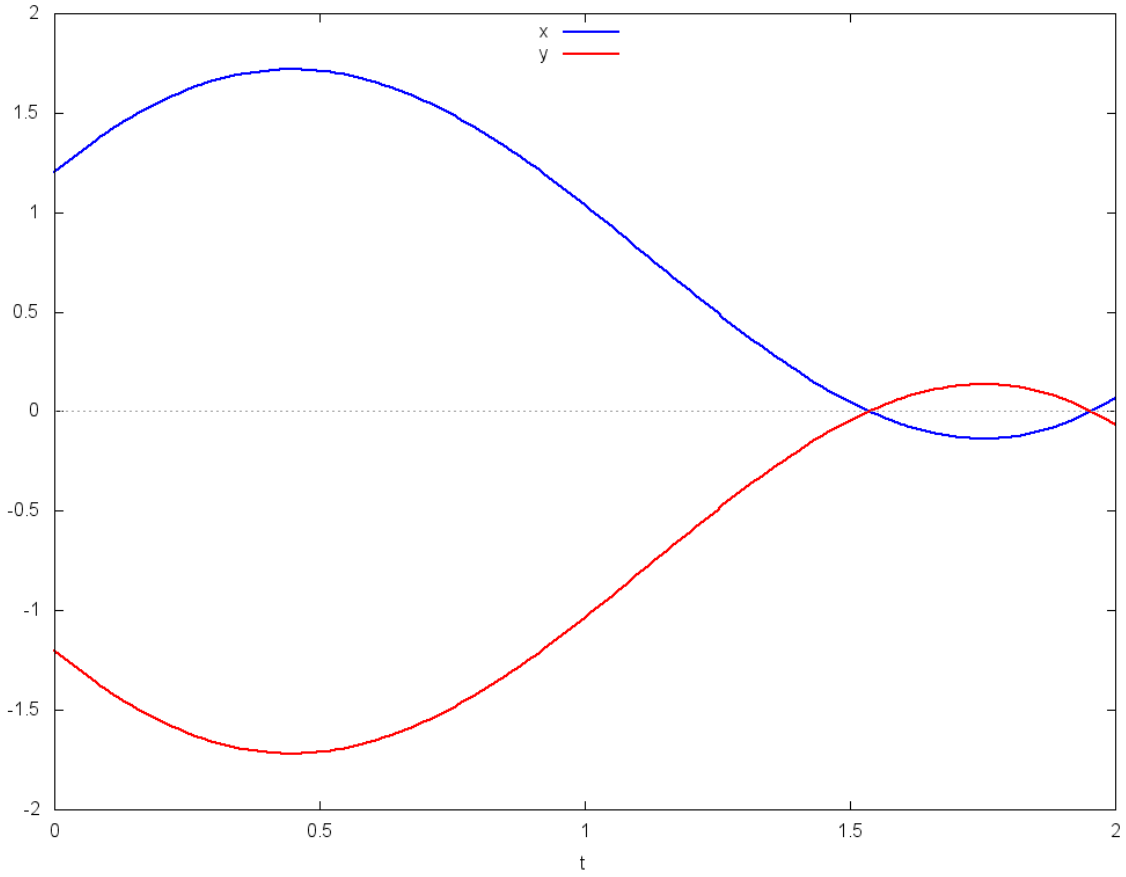
(%i26)
```
x(t),desol,linsol;
```

$$\sin\left(2t\right)+\cos\left(2t\right)+\frac{e^t}{5} \tag{%o26}$$

(%i27)
```
y(t),desol,linsol;
```

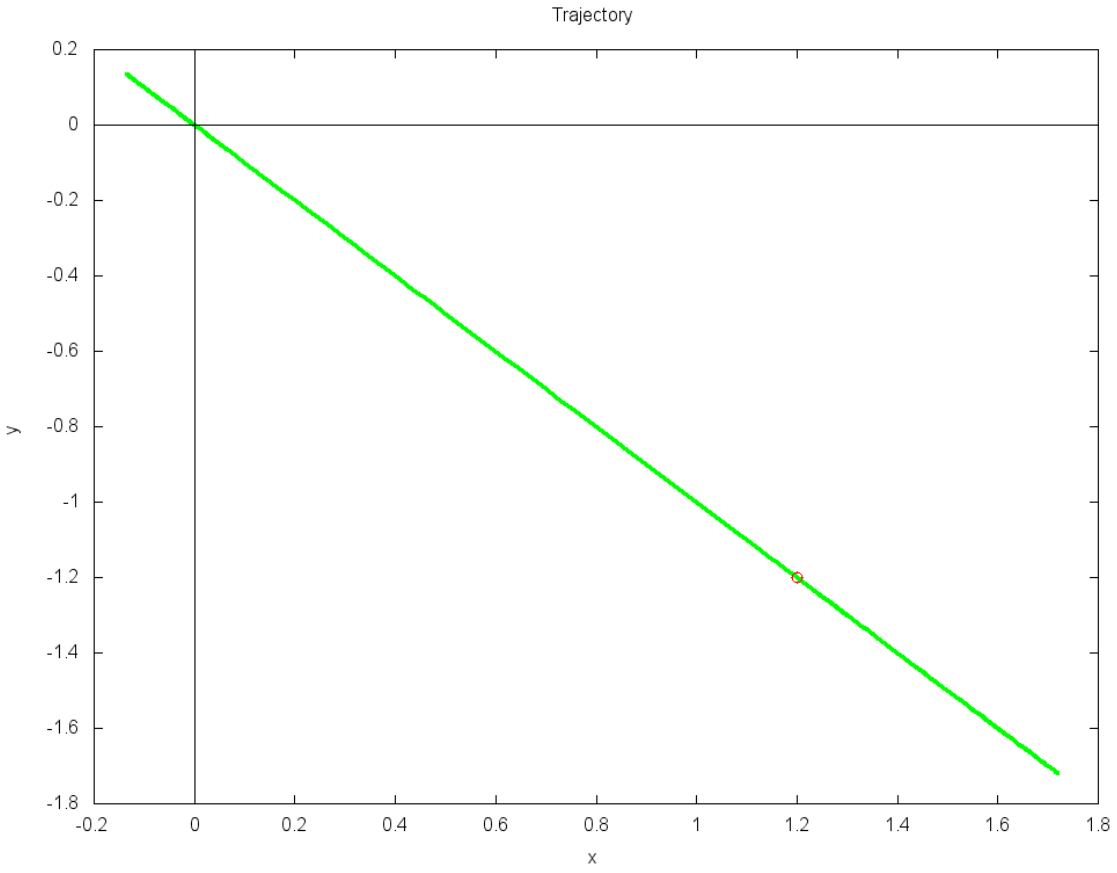
$$-\sin\left(2t\right)-\cos\left(2t\right)-\frac{e^t}{5} \tag{%o27}$$

(%i28) `wxplot2d([x(t),y(t)],[t,0,τ],[style,[lines,2]],[legend,"x","y"], [gnuplot_preamble,"set key top center"]),desol,linsol$`



(%t28)

(%i29)
```
wxplot2d([[parametric,x(t),y(t),[t,0,τ]], [discrete,[[x_0,y_0]]],[axes,solid],
[title,"Trajectory"],[style,[lines,3],[points,3]], [color,green,red],[point_type,circle],[legend,
[xlabel,"x"],[ylabel,"y"]),desol,linsol$
```



(%t29)

Reduce Order

(%i30)
```
kill(labels)$
```

(%i2)
```
R:[X,Y]$
depends(R,t)$
```

(%i4)
```
gradef(x,t,X)$
gradef(y,t,Y)$
```

(%i5)
```
Eq1:Eq1,diff,eval;
```

$$\dot{X} = 4y + e^t \tag{Eq1}$$

(%i6)
```
Eq2:Eq2,diff,eval;
```

$$\dot{Y} = 4x - e^t \tag{Eq2}$$

(%i7)
```
Eq3:'diff(x,t)=X;
```

$$\dot{x} = X \tag{Eq3}$$

(%i8)
```
Eq4:'diff(y,t)=Y;
```

$$\dot{y} = Y \tag{Eq4}$$

Analytical solution

(%i10)
```
atvalue(X(t),t=0,Vx_0)$
atvalue(Y(t),t=0,Vy_0)$
```

answer p;

(%i11)
```
desol:desolve(convert([Eq1,Eq2,Eq3,Eq4],append(R,r),t),convert(append(R,r),append(R,r),t))$
```

Is

$$g622 - 1$$

positive, negative or zero?

$$p;$$

(%i12)
```
map(ldisp,desol)$
```

$$\begin{aligned}\mathrm{X}(t) = {} & \frac{\sin(2t)\,(10y_0 - 10x_0 + 4)}{10} + \frac{e^{2t}\,(2y_0 + 2x_0 + Vy_0 + Vx_0)}{4} \\ & - \frac{e^{-2t}\,(2y_0 + 2x_0 - Vy_0 - Vx_0)}{4} - \frac{(5\,Vy_0 - 5\,Vx_0 + 2)\cos(2t)}{10} + \frac{e^t}{5} \\ \mathrm{Y}(t) = {} & -\frac{\sin(2t)\,(10y_0 - 10x_0 + 4)}{10} + \frac{e^{2t}\,(2y_0 + 2x_0 + Vy_0 + Vx_0)}{4} \\ & - \frac{e^{-2t}\,(2y_0 + 2x_0 - Vy_0 - Vx_0)}{4} + \frac{(5\,Vy_0 - 5\,Vx_0 + 2)\cos(2t)}{10} - \frac{e^t}{5}\end{aligned}$$

$$\mathrm{x}(t) = -\frac{\cos(2t)\ (5y_0 - 5x_0 + 2)}{10} + \frac{e^{2t}\ (2y_0 + 2x_0 + Vy_0 + Vx_0)}{8} + \frac{e^{-2t}\ (2y_0 + 2x_0 - Vy_0 - Vx_0)}{8} - \frac{(5\,Vy_0 - 5\,Vx_0 + 2)\sin(2t)}{20} + \frac{e^t}{5}$$

$$\mathrm{y}(t) = \frac{\cos(2t)\ (5y_0 - 5x_0 + 2)}{10} + \frac{e^{2t}\ (2y_0 + 2x_0 + Vy_0 + Vx_0)}{8} + \frac{e^{-2t}\ (2y_0 + 2x_0 - Vy_0 - Vx_0)}{8} + \frac{(5\,Vy_0 - 5\,Vx_0 + 2)\sin(2t)}{20} - \frac{e^t}{5}$$

Verify

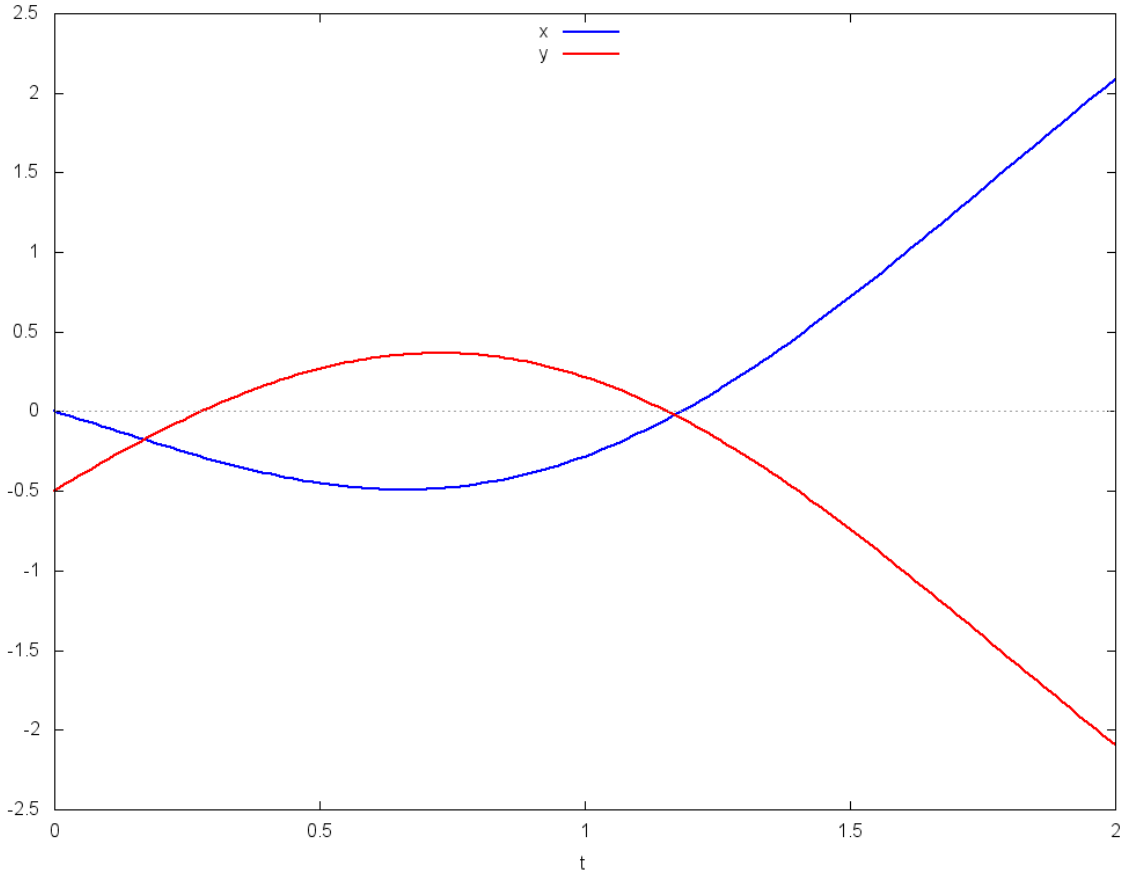
(%i16)
```
is(ev(convert(Eq3,append(R,r),t),desol,diff,eval,initial,eval,expand,eval));
```

true (%o16)

(%i17)
```
is(ev(convert(Eq4,append(R,r),t),desol,diff,eval,initial,eval,expand,eval));
```

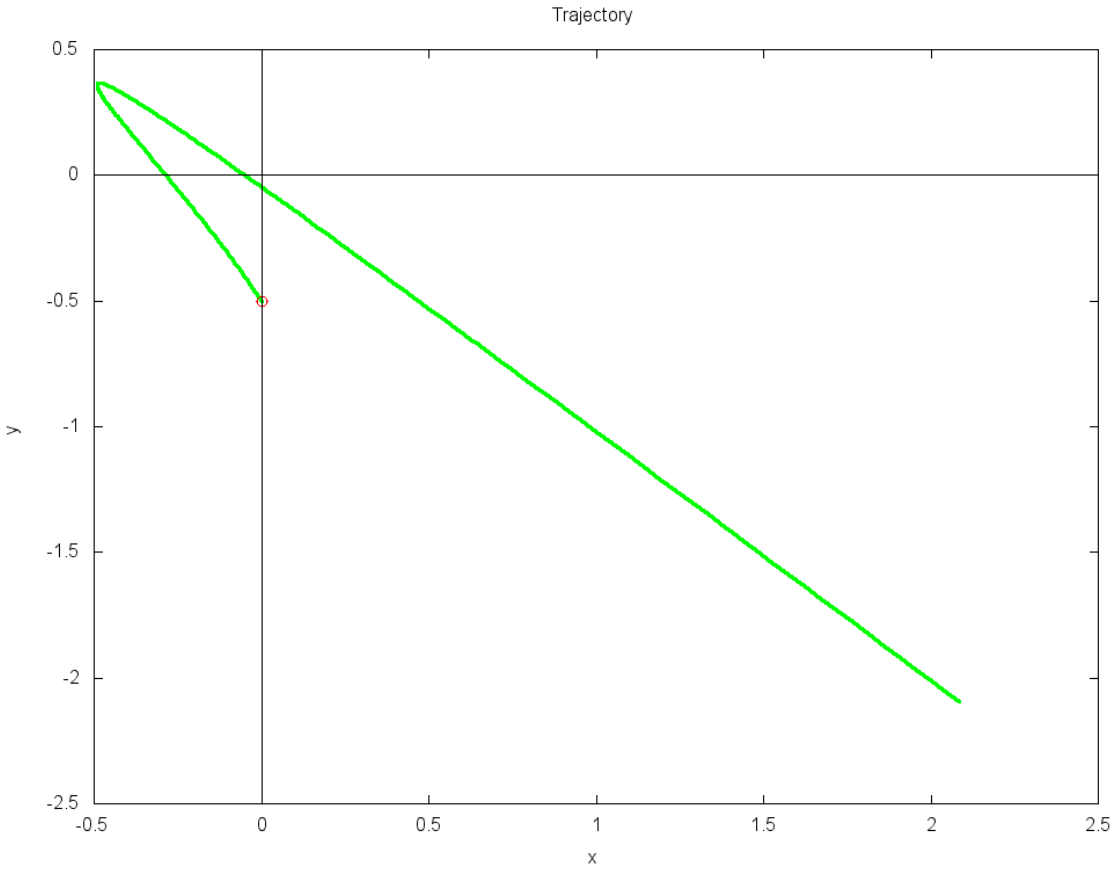
true (%o17)

(%i18) `wxplot2d([x(t),y(t)],[t,0,τ],[style,[lines,2]],[legend,"x","y"], [gnuplot_preamble,"set key top center"]),desol,initial$`



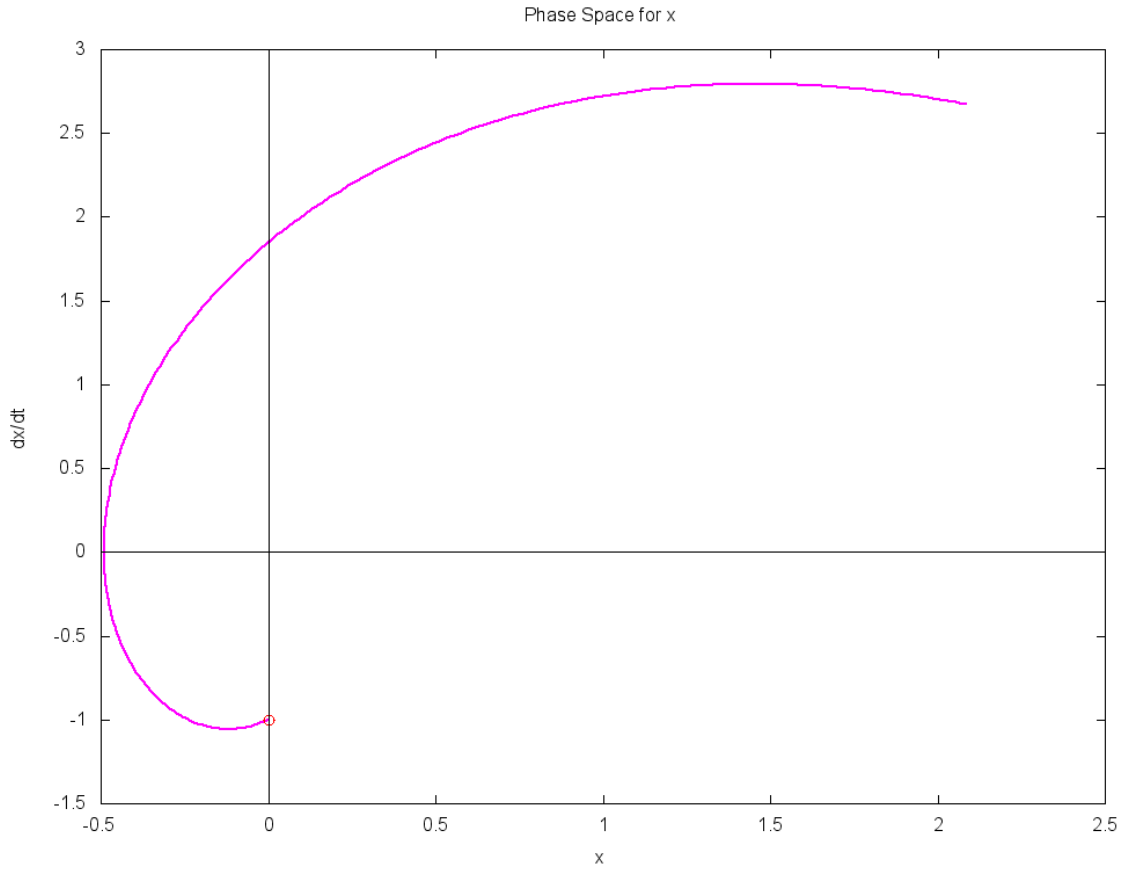
(%t18)

(%i19)
```
wxplot2d([[parametric,x(t),y(t),[t,0,τ]], [discrete,[[x_0,y_0]]],[axes,solid],
[title,"Trajectory"],[style,[lines,3],[points,3]], [color,green,red],[point_type,circle],[legend,
[xlabel,"x"],[ylabel,"y"]),desol,initial$
```



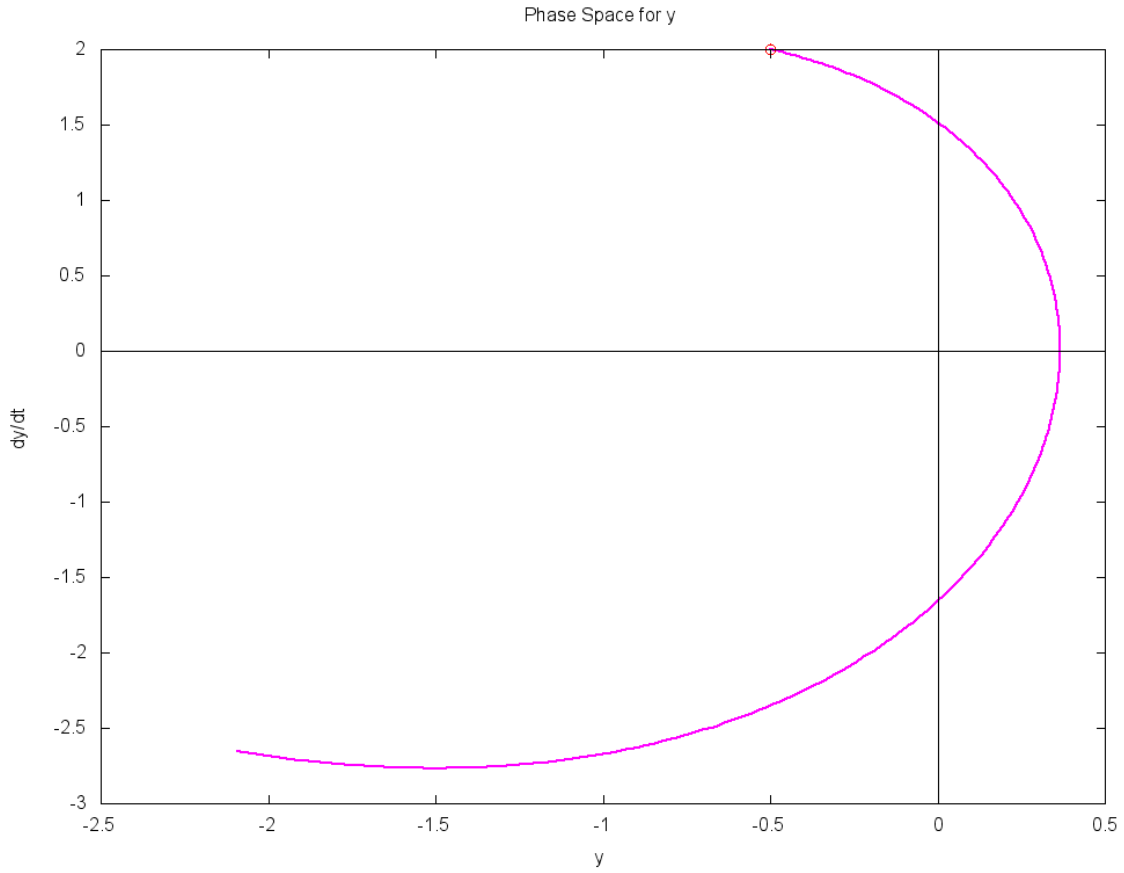
(%t19)

(%i20)
```
wxplot2d([[parametric,x(t),X(t),[t,0,τ]], [discrete,[[x_0,Vx_0]]],[axes,solid],
[title,"Phase Space for x"],[style,[lines,2],[points,3]], [color,magenta,red],[point_type,circle]
[xlabel,"x"],[ylabel,"dx/dt"]),desol,initial$
```



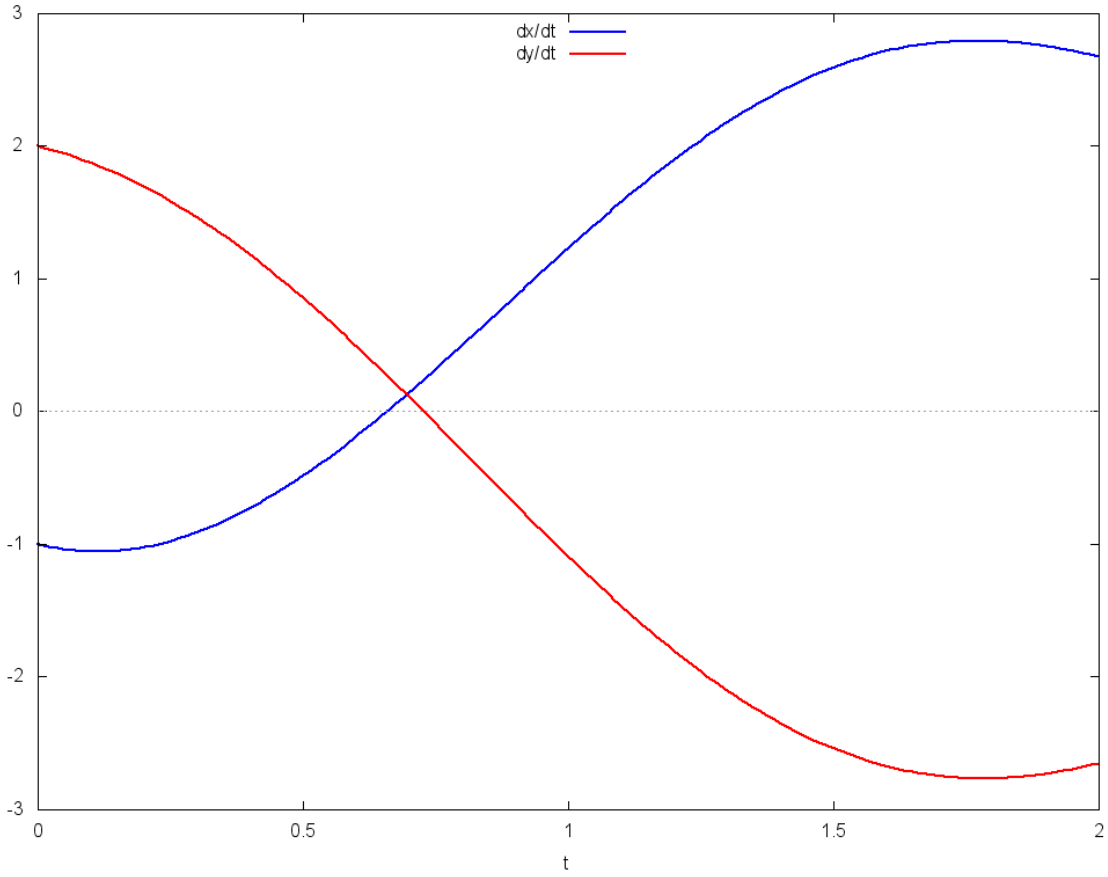
(%t20)

(%i21)
```
wxplot2d([[parametric,y(t),Y(t),[t,0,τ]], [discrete,[part(map('lhs,initial),[4,2])]]],[axes,soli
[title,"Phase Space for y"],[style,[lines,2],[points,3]], [color,magenta,red],[point_type,circle]
[xlabel,"y"],[ylabel,"dy/dt"]),desol,initial$
```



(%t21)

(%i22) `wxplot2d([X(t),Y(t)],[t,0,τ],[style,[lines,2]],[legend,"dx/dt","dy/dt"],`
`[gnuplot_preamble,"set key top center"]),desol,initial$`



(%t22)

Numerical solution with rfk45

(%i23)
```
kill(labels)$
```

(%i6)
```
funcs:append(R,r)$ldisplay(funcs)$
odes:map('rhs,[Eq1,Eq2,Eq3,Eq4])$ldisplay(odes)$
interval:[t,0,7]$ldisplay(interval)$
```

$$funcs = [X, Y, x, y] \tag{\%t2}$$

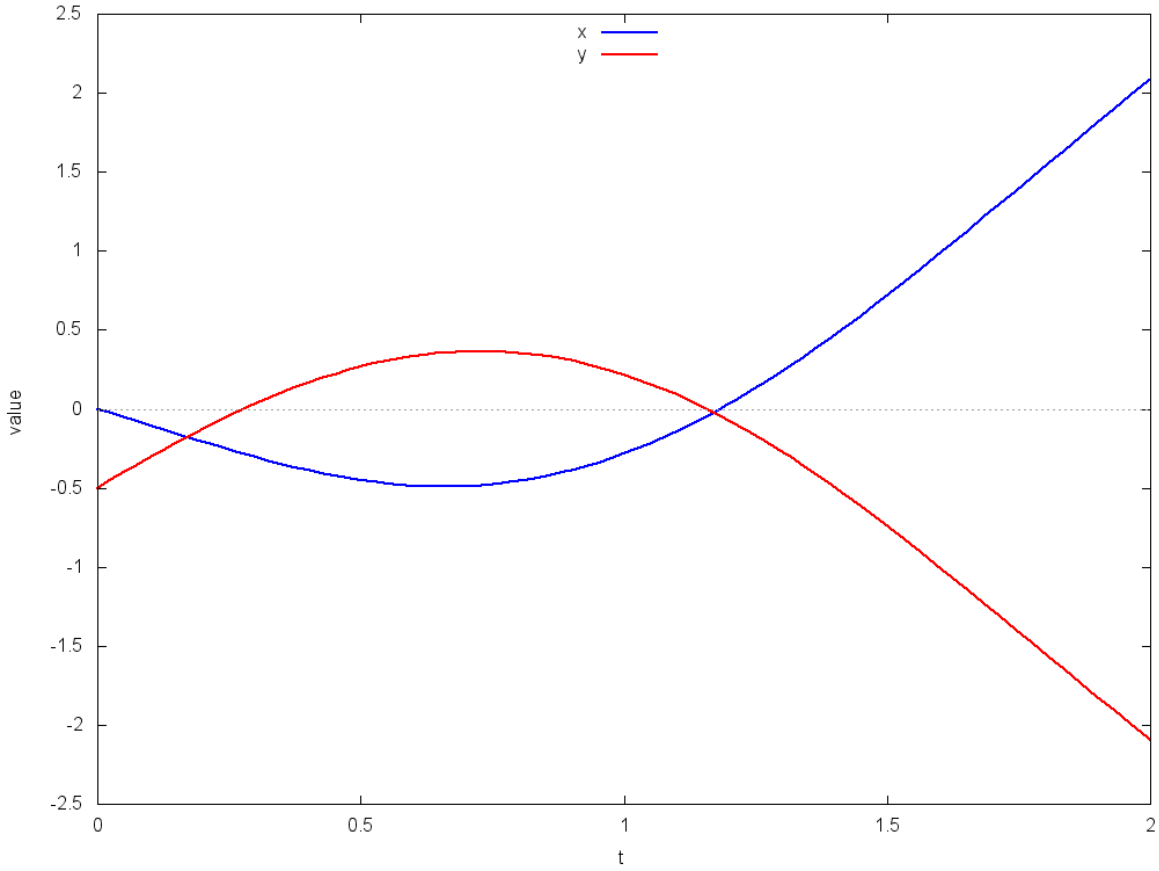
$$odes = [4y + e^t, 4x - e^t, X, Y] \tag{\%t4}$$

$$interval = [t, 0, 2] \tag{\%t6}$$

(%i7)
```
rksol:rkf45(odes,funcs,map('rhs,initial),interval, absolute_tolerance=5d-8,report=true)$
```

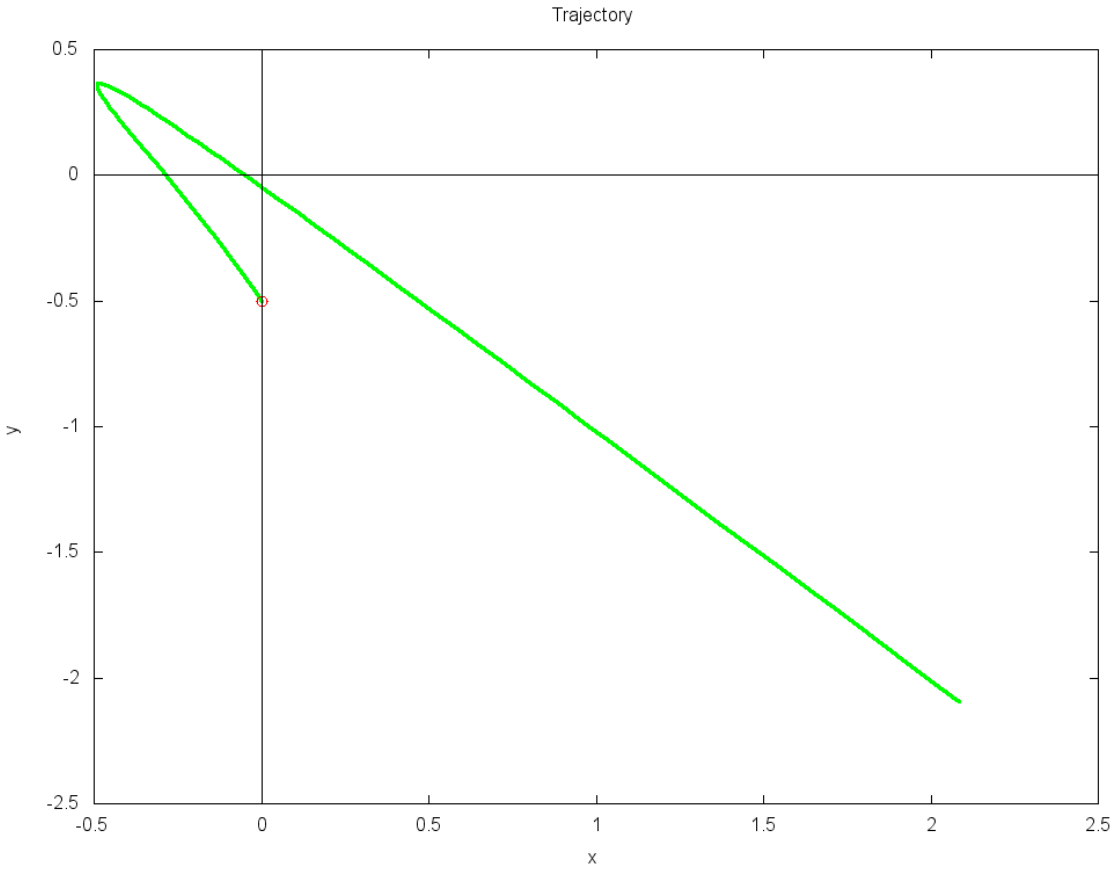
Info: rkf45:
Integration points selected:78
Total number of iterations:77
Bad steps corrected:0
Minimum estimated error:3.842510^{-12}
Maximum estimated error:2.679610^{-8}
Minimum integration step taken:0.003622
Maximum integration step taken:0.032861

(%i8)
```
wxplot2d([[discrete,map(lambda([u],part(u,[1,4])),rksol)], [discrete,map(lambda([u],part(u,[1,5])
[style,[lines,2]],[xlabel,"t"],[ylabel,"value"], [legend,"x","y"],[gnuplot_preamble,"set
key top center"])$
```



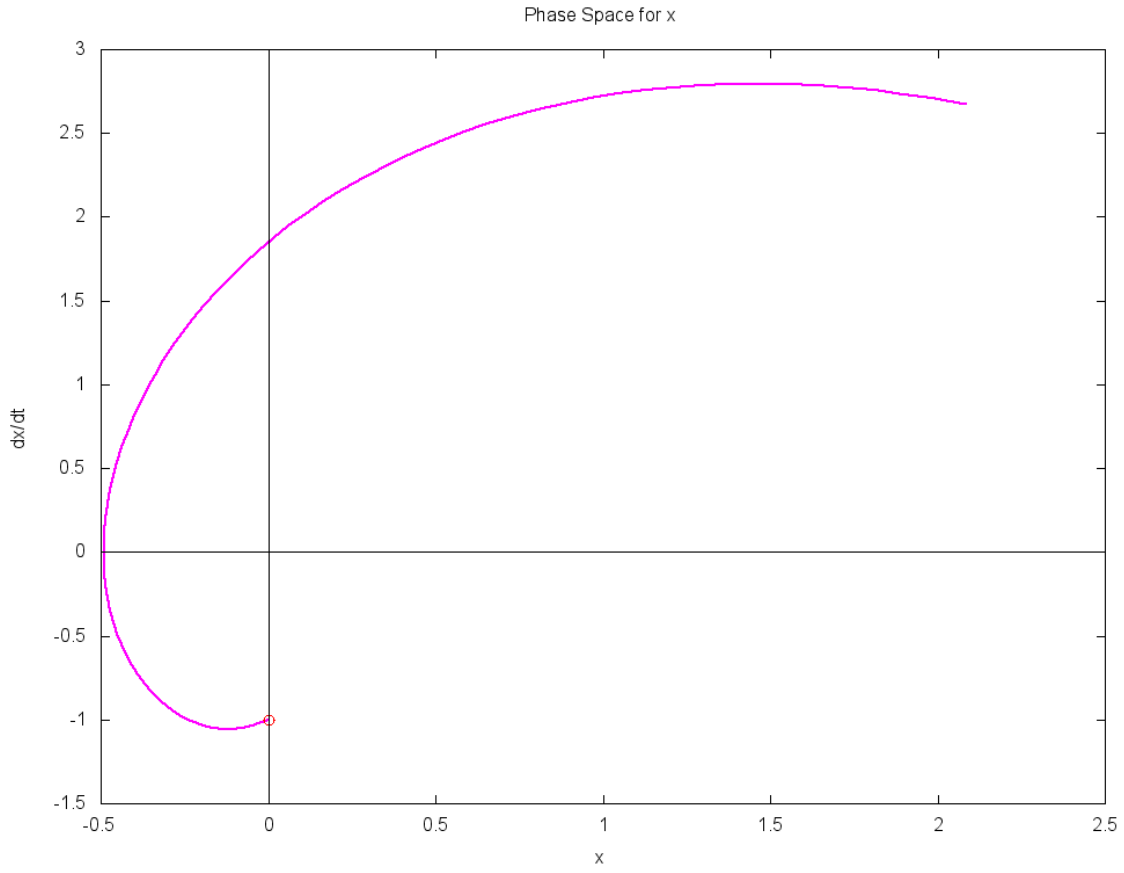
(%t8)

(%i9) `wxplot2d([[discrete,map(lambda([u],part(u,[4,5])),rksol)], [discrete,[[x_0,y_0]]],[axes,solid], [title,"Trajectory"],[point_type,circle], [style,[lines,3],[points,3]],[color,green,red], [xlabel,"x"],[ylabel,"y"],[legend,false]),initial$`



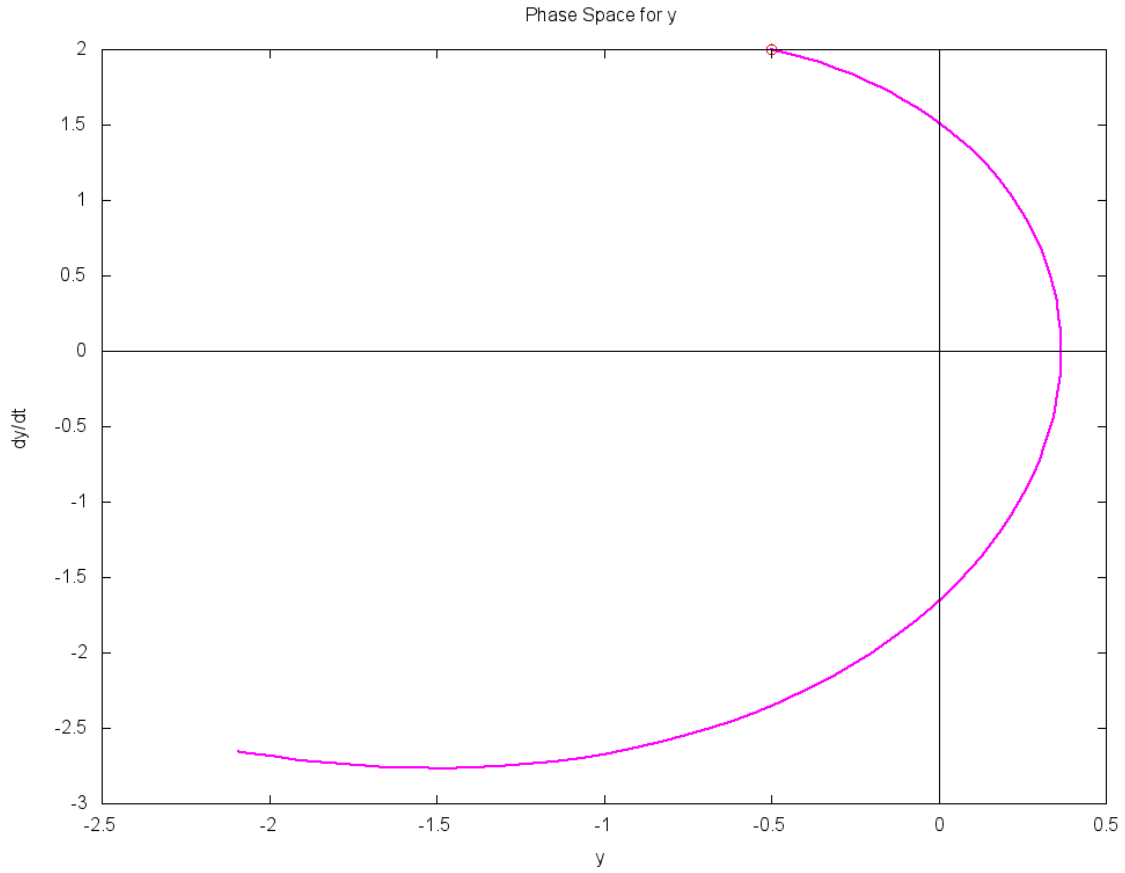
(%t9)

```
(%i10) wxplot2d([[discrete,map(lambda([u],part(u,[4,2])),rksol)], [discrete,[[x_0,Vx_0]]],[axes,solid],
       [title,"Phase Space for x"],[point_type,circle], [style,[lines,2],[points,3]],[color,magenta,red]
       [xlabel,"x"],[ylabel,"dx/dt"],[legend,false]),initial$
```



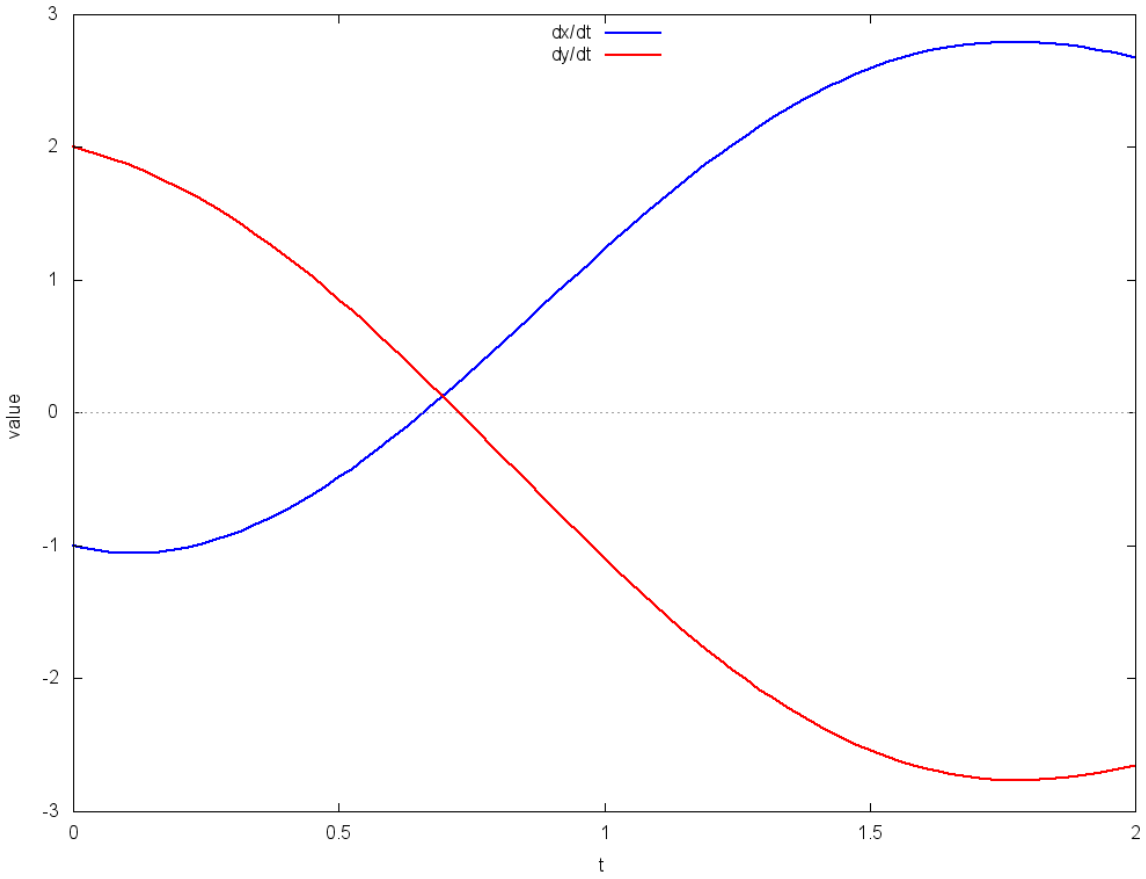
(%t10)

```
(%i11) wxplot2d([[discrete,map(lambda([u],part(u,[5,3])),rksol)], [discrete,[[y_0,Vy_0]]],[axes,solid],
       [title,"Phase Space for y"],[point_type,circle], [style,[lines,2],[points,3]],[color,magenta,red]
       [xlabel,"y"],[ylabel,"dy/dt"],[legend,false]),initial$
```



(%t11)

```
(%i12) wxplot2d([[discrete,map(lambda([u],part(u,[1,2])),rksol)], [discrete,map(lambda([u],part(u,[1,3])
       [style,[lines,2]],[xlabel,"t"],[ylabel,"value"], [legend,"dx/dt","dy/dt"],[gnuplot_preamble,"set
       key top center"])$
```



(%t12)

Numerical solution with dlsode

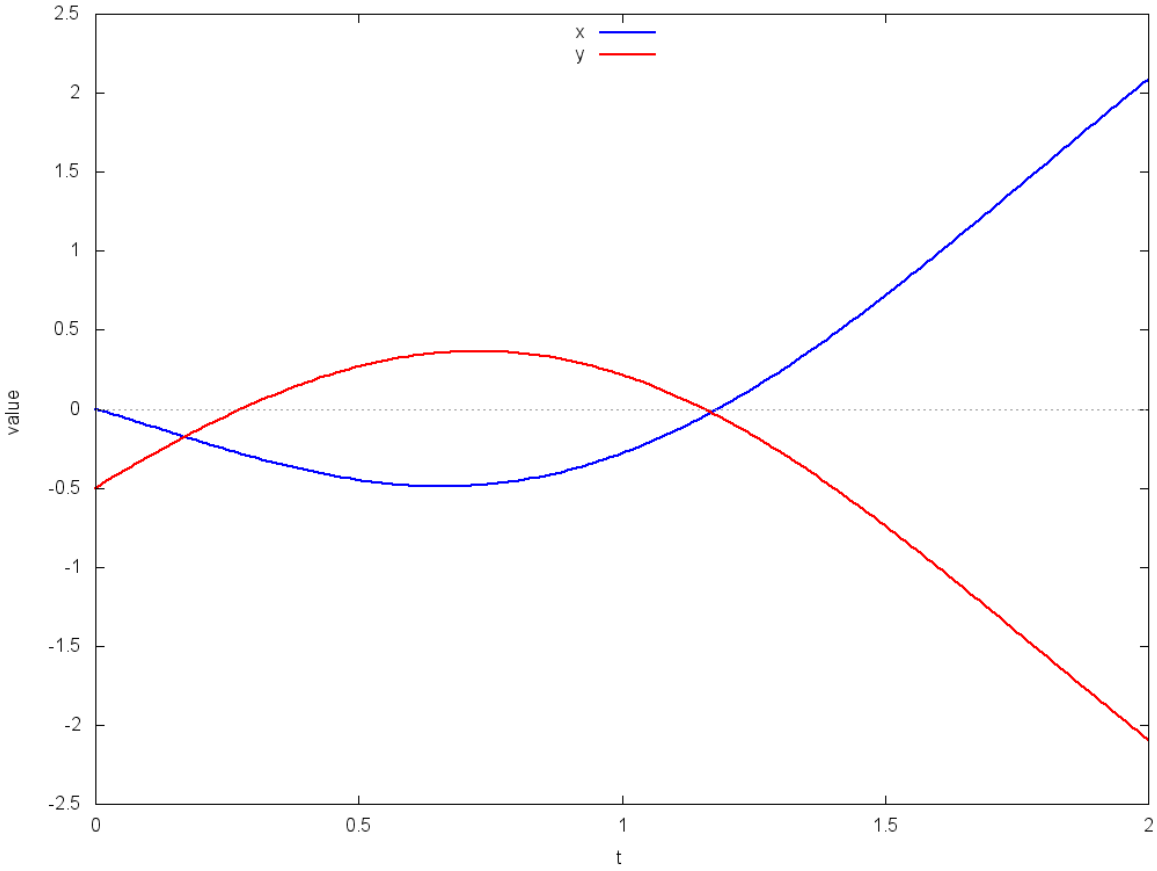
(%i13)
```
kill(labels,t,X,Y,x,y)$
```

(%i1)
```
state:dlsode_init(odes,['t,'X,'Y,'x,'y],21)$
```

(%i9)
```
t:0d0$
init:map(rhs,initial)$
rtol:1d-4$
atol:[5d-8,5d-8,5d-8,5d-8]$
result:[]$
dlsol:[cons(t,init)]$
tout:δ:0.01d0$
istate:1$
```

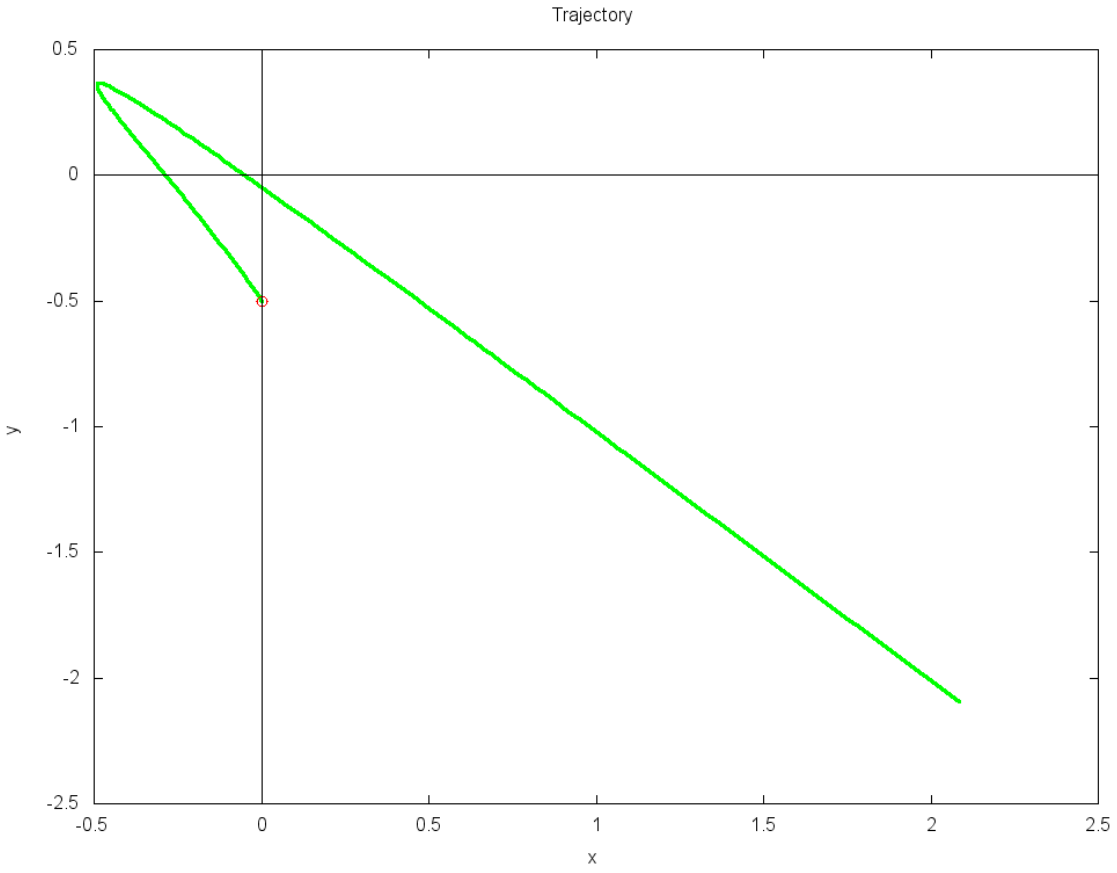
(%i10)
```
for k thru τ/δ do
block([],
result:dlsode_step(init,t,tout,rtol,atol,istate,state),
dlsol:append(dlsol,[cons(first(result),second(result))]),
istate:result[3],
tout:tout+δ)$
```

(%i11)
```
wxplot2d([[discrete,map(lambda([u],part(u,[1,4])),dlsol)], [discrete,map(lambda([u],part(u,[1,5])
[style,[lines,2]],[xlabel,"t"],[ylabel,"value"],[x,0,7], [legend,"x","y"],[gnuplot_preamble,"set
key top center"])$
```



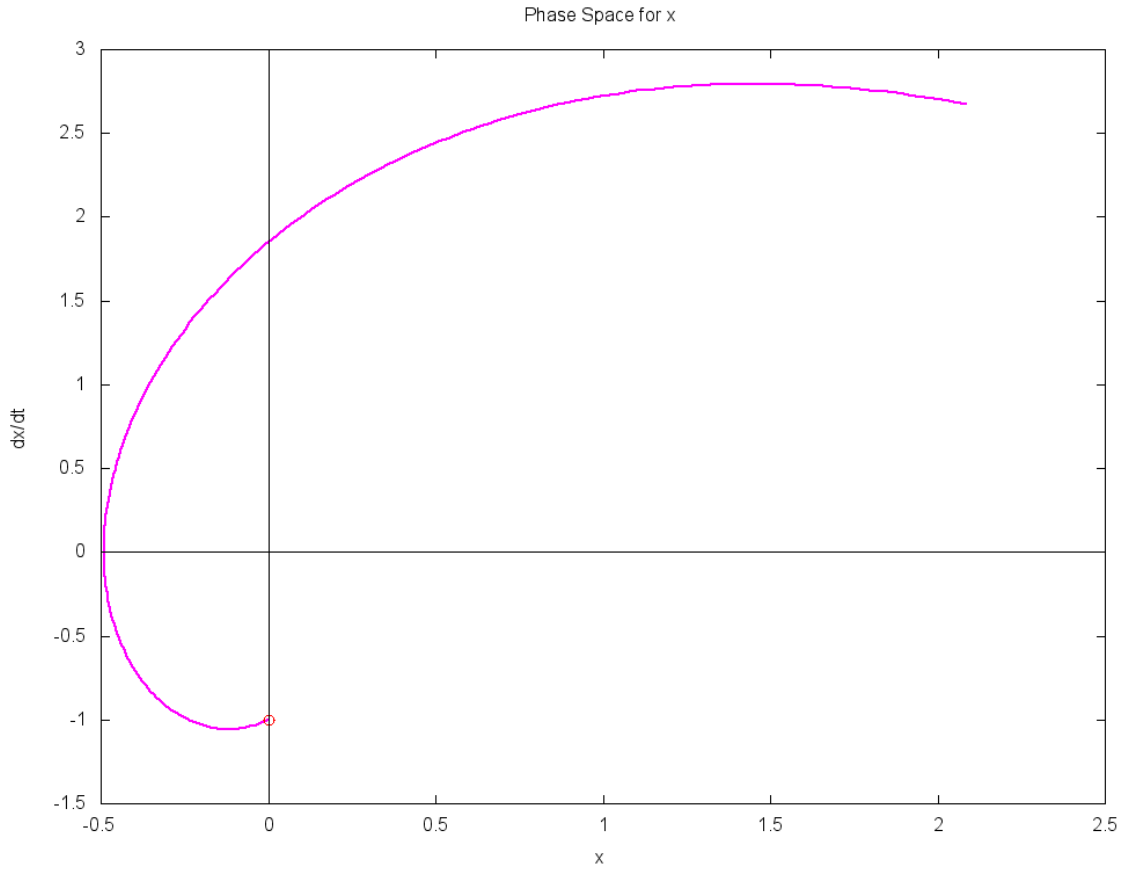
(%t11)

```
(%i12) wxplot2d([[discrete,map(lambda([u],part(u,[4,5])),dlsol)], [discrete,[[x_0,y_0]]]],[axes,solid],
[title,"Trajectory"],[point_type,circle], [style,[lines,3],[points,3]],[color,green,red],
[xlabel,"x"],[ylabel,"y"],[legend,false]),initial$
```



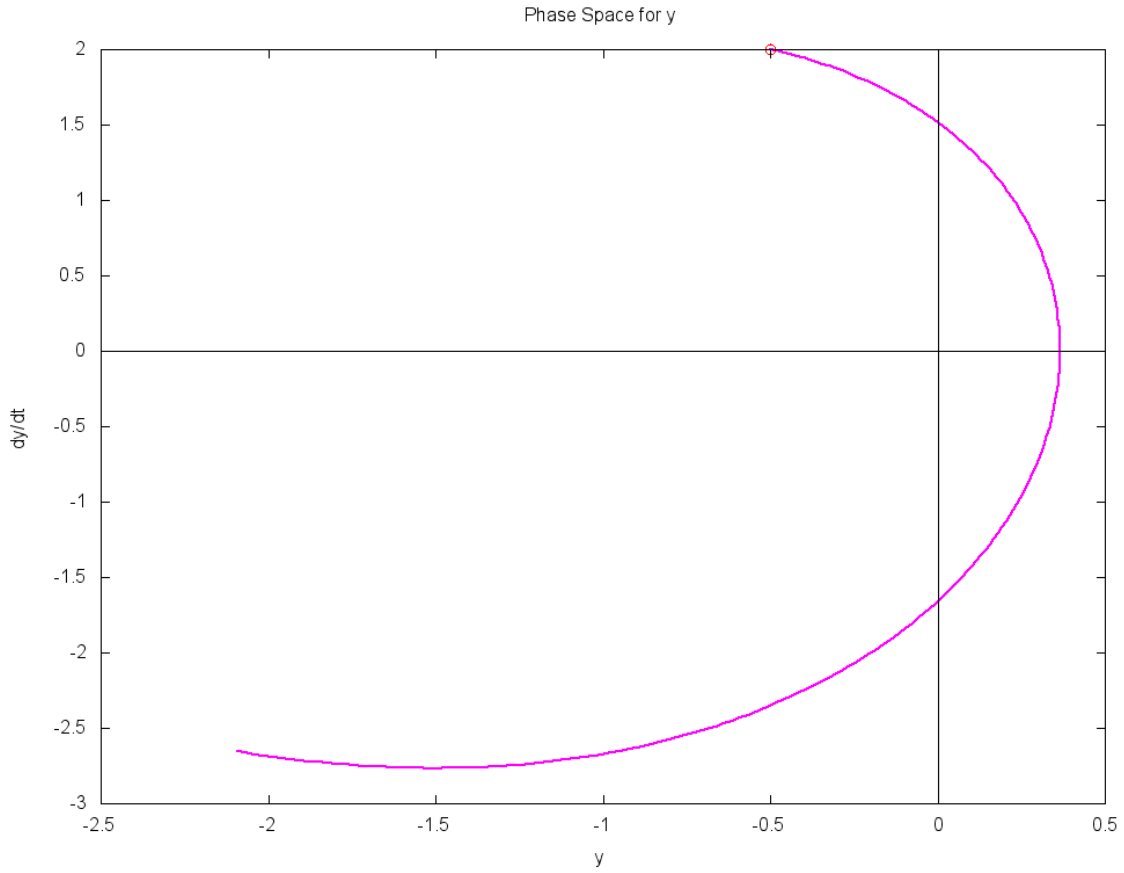
(%t12)

(%i13)
```
wxplot2d([[discrete,map(lambda([u],part(u,[4,2])),dlsol)], [discrete,[[x_0,Vx_0]]],[axes,solid],
[title,"Phase Space for x"],[point_type,circle], [style,[lines,2],[points,3]],[color,magenta,red]
[xlabel,"x"],[ylabel,"dx/dt"],[legend,false]),initial$
```



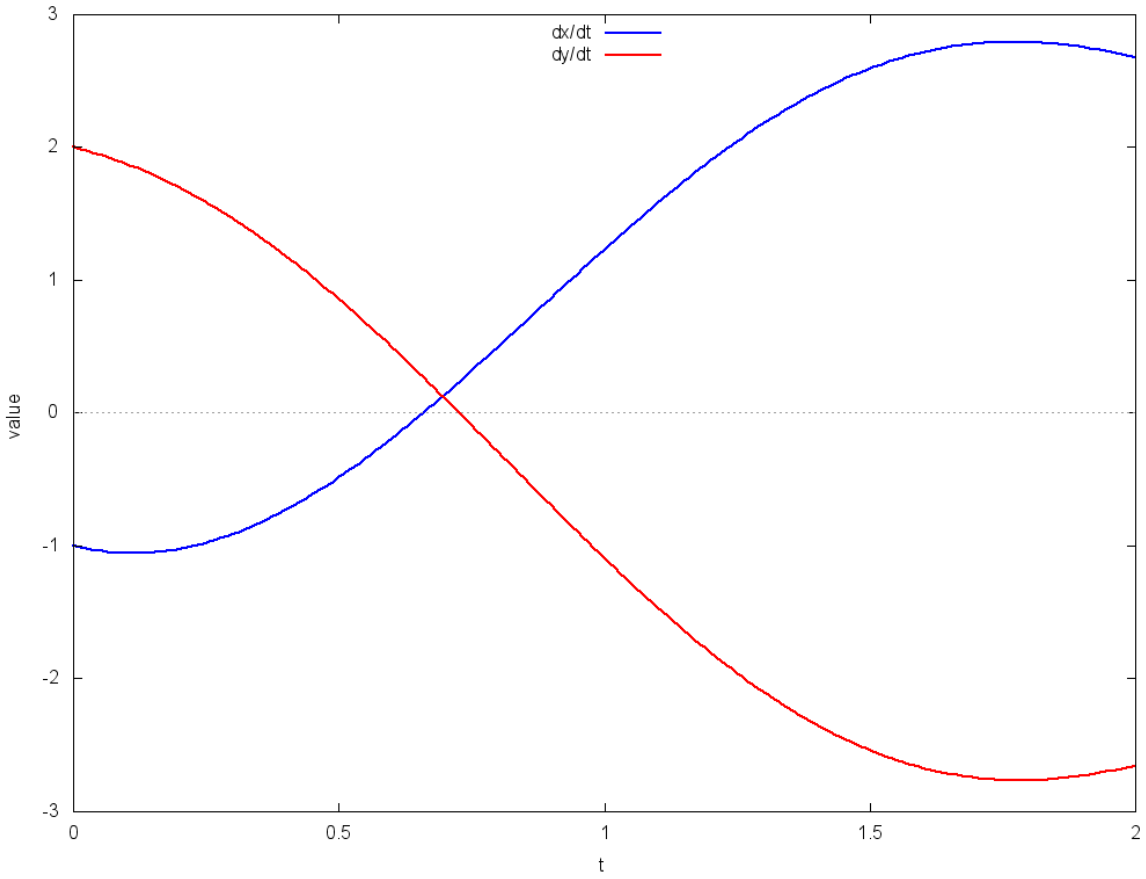
(%t13)

```
(%i14) wxplot2d([[discrete,map(lambda([u],part(u,[5,3])),dlsol)], [discrete,[[y_0,Vy_0]]],[axes,solid],
       [title,"Phase Space for y"],[point_type,circle], [style,[lines,2],[points,3]],[color,magenta,red]
       [xlabel,"y"],[ylabel,"dy/dt"],[legend,false]),initial$
```



(%t14)

(%i15)
```
wxplot2d([[discrete,map(lambda([u],part(u,[1,2])),dlsol)], [discrete,map(lambda([u],part(u,[1,3])
[style,[lines,2]],[xlabel,"t"],[ylabel,"value"],[x,0,7], [legend,"dx/dt","dy/dt"],[gnuplot_preamb
key top center"])$
```



(%t15)