Doble masa-resorte-amortiguador

Based on Señales y Sistemas Doble masa-resorte-amortiguador

Based on Señales y Sistemas Masa-resorte-amortiguador doble

Based on Señales y Sistemas Función de transferencia

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(%i2) `info:build_info()$info@version;`

(%o2)

5.38.1

(%i3) `file_search_maxima:cons(sconcat("D:/USERS/wxMaxima/pw/pw/≠≠≠.lisp,mac,mc"),file_search_maxima)$`

(%i2) `reset()$kill(all)$`

(%i1) `derivabbrev:true$`

(%i2) `ratprint:false$`

(%i3) `fpprintprec:5$`

(%i4) `if get('draw,'version)=false then load(draw)$`

(%i5) `wxplot_size:[1024,768]$`

(%i6) `load(pw)$`

(%i7) `if get('optvar,'version)=false then load(optvar)$`

(%i8) `if get('rkf45,'version)=false then load(rkf45)$`

(%i9) `declare(trigsimp,evfun)$`

(%i10) `declare(t,mainvar)$`

1 Settings

(%i11) `declare([m_1,m_2,b,K_1,K_2],constant)$`

(%i12) `assume(m_1>0,m_2>0,b>0,K_1>0,K_2>0)$`

(%i13) `params:[m_1=1,m_2=2,b=0.5,K_1=3,K_2=1]$`

(%i14) `τ:25$`

Generalized coordinates

(%i15) `ζ:[q_1,q_2]$`

(%i16) `depends(ζ,t)$`

(%i17) `dim:length(ζ)$`

2 Newtonian Formalism

(%i18) `depends(u,t)$`

(%i19) `ldisplay(N_1:m_1*diff(q_1,t,2)+b*diff(q_1,t)+K_1*q_1+K_2*(q_1-q_2)=u)$`

$$K_2\,(q_1 - q_2) + m_1\,(\ddot{q}_1) + b\,(\dot{q}_1) + K_1 q_1 = u \tag{\%t19}$$

(%i20) `ldisplay(N_2:m_2*diff(q_2,t,2)+K_2*(q_2-q_1))$`

$$N_2 = m_2\,(\ddot{q}_2) + K_2\,(q_2 - q_1) \tag{\%t20}$$

(%i21) `Newton:[N_1,N_2]$`

(%i22) `ξ:[z_1,z_2,z_3,z_4]$`

(%i23) `depends(ξ,t)$`

Three substitutions

(%i24) `ldisplay(M_1:[q_1=z_1,'diff(q_1,t)=z_2,q_2=z_3,'diff(q_2,t)=z_4])$`

$$M_1 = [q_1 = z_1, \dot{q}_1 = z_2, q_2 = z_3, \dot{q}_2 = z_4] \tag{\%t24}$$

(%i25) `ldisplay(M_2:['diff(z_1,t,2)='diff(z_2,t),'diff(z_3,t,2)='diff(z_4,t)])$`

$$M_2 = [\ddot{z}_1 = \dot{z}_2, \ddot{z}_3 = \dot{z}_4] \tag{\%t25}$$

(%i26) `ldisplay(M_3:['diff(z_1,t)=z_2,'diff(z_3,t)=z_4])$`

$$M_3 = [\dot{z}_1 = z_2, \dot{z}_3 = z_4] \tag{\%t26}$$

(%i27) `subst(M_3,subst(M_2,subst(M_1,Newton)));`

$$[K_2\,(z_1 - z_3) + m_1\,(\dot{z}_2) + b\,z_2 + K_1 z_1 = u, m_2\,(\dot{z}_4) + K_2\,(z_3 - z_1)] \tag{\%o27}$$

New equations

(%i28) `ldisplay(S_1:linsolve(%,['diff(z_2,t),'diff(z_4,t)]))$`

$$S_1 = \left[\dot{z}_2 = \frac{K_2 z_3 - b\,z_2 + (-K_2 - K_1)\,z_1 + u}{m_1}, \dot{z}_4 = -\frac{K_2 z_3 - K_2 z_1}{m_2}\right] \tag{\%t28}$$

(%i29) `ldisplay(S_2:[diff(z_1,t)=z_2,diff(z_3,t)=z_4])$`

$$S_2 = [\dot{z}_1 = z_2, \dot{z}_3 = z_4] \tag{\%t29}$$

New system

(%i30) `map(ldisp,join(S_2,S_1))$`

$$\dot{z}_1 = z_2 \tag{\%t30}$$

$$\dot{z}_2 = \frac{K_2 z_3 - b\,z_2 + (-K_2 - K_1)\,z_1 + u}{m_1} \tag{\%t31}$$

$$\dot{z}_3 = z_4 \tag{\%t32}$$

$$\dot{z}_4 = -\frac{K_2 z_3 - K_2 z_1}{m_2} \tag{\%t33}$$

(%i34) `augcoefmatrix(join(S_2,S_1),ξ);`

$$\begin{pmatrix} 0 & -1 & 0 & 0 & 0 \\ \frac{K_2+K_1}{m_1} & \frac{b}{m_1} & -\frac{K_2}{m_1} & 0 & -\frac{u}{m_1} \\ 0 & 0 & 0 & -1 & 0 \\ -\frac{K_2}{m_2} & 0 & \frac{K_2}{m_2} & 0 & 0 \end{pmatrix} \tag{\%o34}$$

Solve for second derivative of coordinates

(%i35) `linsol:linsolve(Newton,diff(ζ,t,2))$`

(%i36) `map(ldisp,linsol)$`

$$\ddot{q}_1 = \frac{u + K_2 q_2 - b\,(\dot{q}_1) + (-K_2 - K_1)\,q_1}{m_1} \tag{\%t36}$$

$$\ddot{q}_2 = -\frac{K_2 q_2 - K_2 q_1}{m_2} \tag{\%t37}$$

Transfer function

(%i41)
```
atvalue(q_1(t),[t=0],0)$
atvalue(q_2(t),[t=0],0)$
atvalue(diff(q_1(t),t),[t=0],0)$
atvalue(diff(q_2(t),t),[t=0],0)$
```

(%i42)
```
N_1:laplace(convert(N_1,append(ζ,[u]),t),t,s)$
```

(%i43)
```
N_2:laplace(convert(N_2,append(ζ,[u]),t),t,s)$
```

(%i44)
```
LQ:[laplace(q_1(t),t,s)=Q_1(s),laplace(q_2(t),t,s)=Q_2(s),laplace(u(t),t,s)=U(s)]$
```

(%i45)
```
Newton:subst(LQ,[N_1,N_2])$
```

(%i46)
```
map(ldisp,Newton)$
```

$$K_2\left(Q_1(s)-Q_2(s)\right)+m_1 s^2\,Q_1(s)+bs\,Q_1(s)+K_1 Q_1(s)=\mathrm{U}(s) \tag{\%t46}$$

$$K_2\left(Q_2(s)-Q_1(s)\right)+m_2 s^2\,Q_2(s) \tag{\%t47}$$

(%i48)
```
linsol:linsolve(Newton,[Q_1(s),Q_2(s)])$
```

(%i49)
```
map(ldisp,linsol/U(s))$
```

$$\frac{Q_1(s)}{\mathrm{U}(s)}=\frac{m_2 s^2+K_2}{m_1 m_2 s^4+b\,m_2 s^3+\left(K_2 m_2+K_1 m_2+K_2 m_1\right)s^2+K_2 bs+K_1 K_2} \tag{\%t49}$$

$$\frac{Q_2(s)}{\mathrm{U}(s)}=\frac{K_2}{m_1 m_2 s^4+b\,m_2 s^3+\left(K_2 m_2+K_1 m_2+K_2 m_1\right)s^2+K_2 bs+K_1 K_2} \tag{\%t50}$$

3 Lagrangian Formalism

(%i51) `kill(labels)$`

Kinetic Energy

(%i1) `T_1:` $\frac{1}{2}$`*m_1*diff(q_1,t)`^{2}`$`

(%i2) `T_2:` $\frac{1}{2}$`*m_2*diff(q_2,t)`^{2}`$`

(%i3) `ldisplay(T:T_1+T_2)$`

$$T = \frac{m_2 (\dot{q}_2)^2}{2} + \frac{m_1 (\dot{q}_1)^2}{2} \tag{\%t3}$$

(%i4) `map(ldisp,makelist(diff(T,s),s,`ζ`))$`

$$0 \tag{\%t4}$$

$$0 \tag{\%t5}$$

(%i6) `map(ldisp,makelist(diff(T,diff(s,t)),s,`ζ`))$`

$$m_1 \left(\dot{q}_1\right) \tag{\%t6}$$

$$m_2 \left(\dot{q}_2\right) \tag{\%t7}$$

Potential Energy

(%i8) `U_1:` $\frac{1}{2}$`*K_1*q_1`^{2}`$`

(%i9) `U_2:` $\frac{1}{2}$`*K_2*(q_1-q_2)`^{2}`$`

(%i10) `ldisplay(U:U_1+U_2)$`

$$U = \frac{K_2 (q_1 - q_2)^2}{2} + \frac{K_1 {q_1}^2}{2} \tag{\%t10}$$

(%i11) `map(ldisp,makelist(diff(U,s),s,`ζ`))$`

$$K_2 \left(q_1 - q_2\right) + K_1 q_1 \tag{\%t11}$$

$$-K_2 \left(q_1 - q_2\right) \tag{\%t12}$$

(%i13) `map(ldisp,makelist(diff(U,diff(s,t)),s,`ζ`))$`

$$0 \tag{\%t13}$$

$$0 \tag{\%t14}$$

Lagrangian

(%i15) `ldisplay(L:T-U)$`

$$L = \frac{m_2 (\dot{q}_2)^2}{2} - \frac{K_2 (q_1 - q_2)^2}{2} + \frac{m_1 (\dot{q}_1)^2}{2} - \frac{K_1 {q_1}^2}{2} \tag{\%t15}$$

Momentum Conjugate

(%i16) `ldisplay(P_1:ev(diff(L,'diff(q_1,t))))$`

$$P_1 = m_1 \left(\dot{q}_1\right) \tag{\%t16}$$

(%i17) `linsolve(p_1=P_1,diff(q_1,t));`

$$\left[\dot{q}_1 = \frac{p_1}{m_1}\right] \tag{\%o17}$$

(%i18) `ldisplay(P_2:ev(diff(L,'diff(q_2,t))))$`

$$P_2 = m_2 \left(\dot{q}_2\right) \tag{\%t18}$$

(%i19) `linsolve(p_2=P_2,diff(q_2,t));`

$$\left[\dot{q}_2 = \frac{p_2}{m_2}\right] \tag{\%o19}$$

Euler-Lagrange Equations

(%i20) `aa:el(L,ζ,t)$`

(%i23) `bb:ev(aa,eval,diff)$`

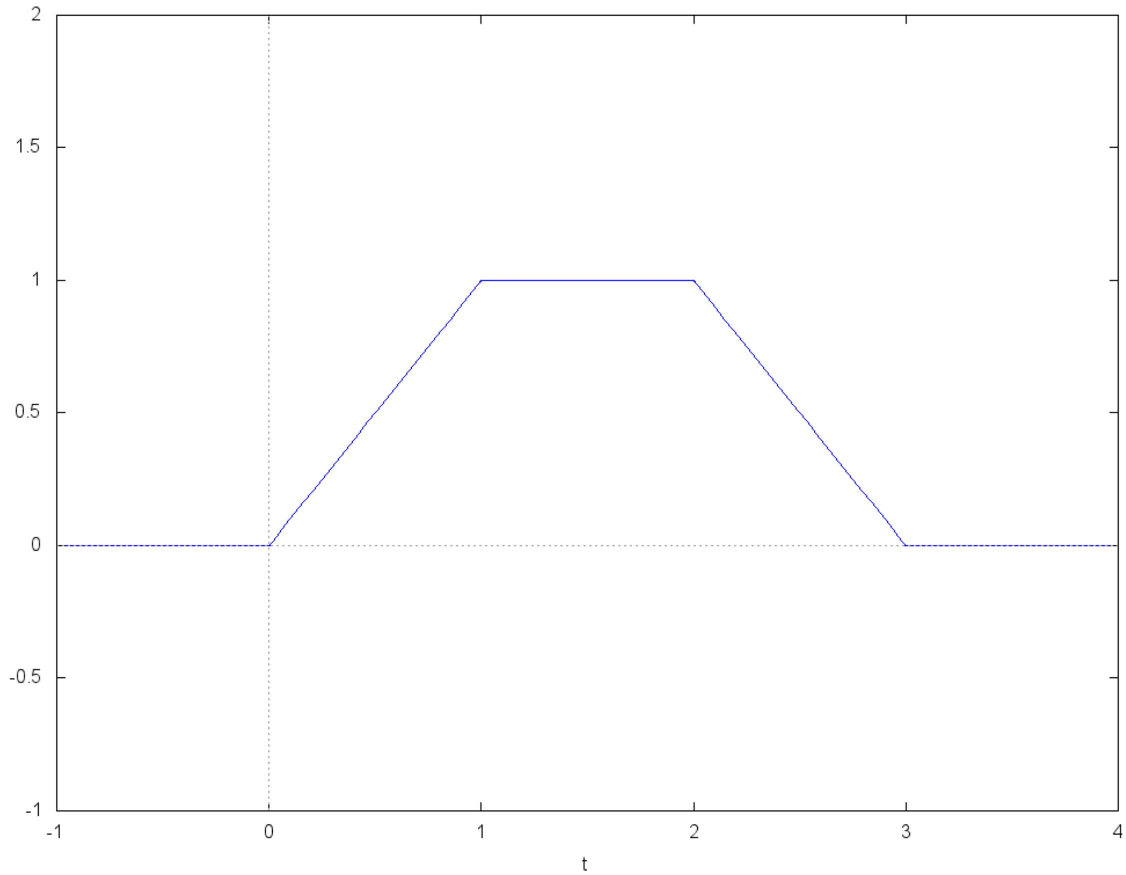
(%i24) `bb[1]:subst([k[0]=-E],-bb[1])$`

(%i26)
```
bb[2]:lhs(bb[2])-rhs(bb[2])=u-b*diff(q_1,t)$
bb[3]:lhs(bb[3])-rhs(bb[3])=0$
```

External Force

(%i27) `u:piecewise([-∞,0,0,t,1,1,2,3-t,3,0,∞],t)$`

(%i28) `wxplot2d(u,[t,-1,4],[y,-1,2])$`



(%t28)

Splice input

(%i29) `R(t):=λ_3*t^3+λ_2*t^2+λ_1*t+λ_0$`

(%i33)
```
Eq_1:at(R(t),[t=0])=0$
Eq_2:at(R(t),[t=1])=1$
Eq_3:at(diff(R(t),t),[t=0])=0$
Eq_4:at(diff(R(t),t),[t=1])=0$
```

(%i34) `f1:linsolve([Eq_1,Eq_2,Eq_3,Eq_4],[λ_3,λ_2,λ_1,λ_0]);`

$$[\lambda_3 = -2, \lambda_2 = 3, \lambda_1 = 0, \lambda_0 = 0] \qquad \text{(f1)}$$

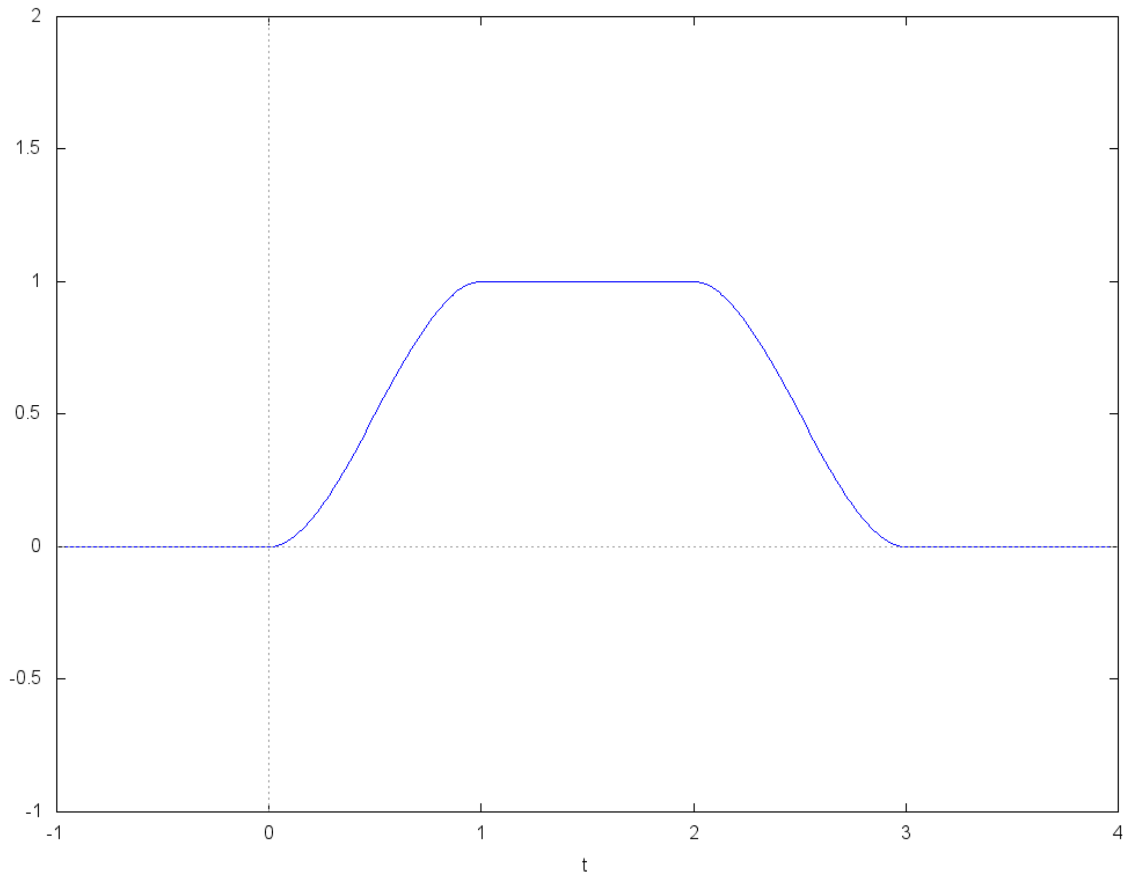
(%i38)
```
Eq_1:at(R(t),[t=2])=1$
Eq_2:at(R(t),[t=3])=0$
Eq_3:at(diff(R(t),t),[t=2])=0$
Eq_4:at(diff(R(t),t),[t=3])=0$
```

(%i39) `f2:linsolve([Eq_1,Eq_2,Eq_3,Eq_4],[λ_3,λ_2,λ_1,λ_0]);`

$$[\lambda_3 = 2, \lambda_2 = -15, \lambda_1 = 36, \lambda_0 = -27] \tag{f2}$$

(%i40) `u:piecewise([-∞,0,0,ev(R(t),f1),1,1,2,ev(R(t),f2),3,0,∞],t)$`

(%i41) `wxplot2d(u,[t,-1,4],[y,-1,2])$`



(%t41)

Conservation Laws

(%i42) `bb[1];`

$$\frac{m_2(\dot{q}_2)^2}{2} + \frac{K_2(q_1 - q_2)^2}{2} + \frac{m_1(\dot{q}_1)^2}{2} + \frac{K_1 {q_1}^2}{2} = E \tag{\%o42}$$

Equations of Motion

(%i43) `map(ldisp,part(bb,[2,3]))$`

$$K_2\,(q_1 - q_2) + m_1\,(\ddot{q}_1) + K_1 q_1 = u - b\,(\dot{q}_1) \tag{\%t43}$$

$$m_2\,(\ddot{q}_2) - K_2\,(q_1 - q_2) = 0 \tag{\%t44}$$

Solve for second derivative of coordinates

(%i45) `linsol:linsolve(part(bb,[2,3]),diff(ζ,t,2))$`

(%i46) `map(ldisp,linsol)$`

$$\ddot{q}_1 = \frac{u + K_2 q_2 - b\,(\dot{q}_1) + (-K_2 - K_1)\, q_1}{m_1} \tag{\%t46}$$

$$\ddot{q}_2 = -\frac{K_2 q_2 - K_2 q_1}{m_2} \tag{\%t47}$$

Check Conservation of Energy

(%i48) `subst(linsol,diff(lhs(bb[1]),t)),fullratsimp;`

$$-((2\,(\dot{q}_1)\,t^3 - 3\,(\dot{q}_1)\,t^2)\,\mathrm{signum}(t) + (-2\,(\dot{q}_1)\,\mathrm{signum}\,(t-1) - 2\,(\dot{q}_1)\,\mathrm{signum}\,(t-2) + 2\,(\dot{q}_1)\,\mathrm{signum}\,(t-3))\,t^3 + (3\,(\dot{q}_1)\,\mathrm{signum}\,(t - \tag{\%o48}$$

4 Hamiltonian Formalism

Legendre Transformation

(%i49)
```
Legendre:linsolve([p_1=P_1,p_2=P_2],diff(ζ,t));
```

$$\left[\dot{q}_1 = \frac{p_1}{m_1}, \dot{q}_2 = \frac{p_2}{m_2}\right] \tag{Legendre}$$

Hamiltonian

(%i50)
```
H:subst(Legendre,p_1*'diff(q_1,t)+p_2*'diff(q_2,t)-L);
```

$$\frac{K_2 (q_1 - q_2)^2}{2} + \frac{K_1 {q_1}^2}{2} + \frac{{p_2}^2}{2m_2} + \frac{{p_1}^2}{2m_1} \tag{H}$$

Equations of Motion

(%i51)
```
Hq:makelist(Hq[i],i,1,2*dim)$
```

(%i55)
```
Hq[1]:'diff(q_1,t)=trigsimp(diff(H,p_1))$
Hq[2]:'diff(q_2,t)=trigsimp(diff(H,p_2))$
Hq[3]:'diff(p_1,t)=-trigsimp(diff(H,q_1))$
Hq[4]:'diff(p_2,t)=-trigsimp(diff(H,q_2))$
```

(%i56)
```
map(ldisp,Hq)$
```

$$\dot{q}_1 = \frac{p_1}{m_1} \tag{\%t56}$$

$$\dot{q}_2 = \frac{p_2}{m_2} \tag{\%t57}$$

$$\dot{p}_1 = K_2 q_2 - (K_2 + K_1)\, q_1 \tag{\%t58}$$

$$\dot{p}_2 = K_2 q_1 - K_2 q_2 \tag{\%t59}$$

5 Reduce Order

(%i60) `kill(Q_1,Q_2)$`

(%i61) `ξ:[Q_1,Q_2]$`

(%i62) `depends(ξ,t)$`

(%i64)
```
gradef(q_1,t,Q_1)$
gradef(q_2,t,Q_2)$
```

Euler-Lagrange Equations

(%i65) `bb:ev(aa,eval,diff)$`

(%i66) `bb[1]:subst([k[0]=-E],-bb[1])$`

(%i68)
```
bb[2]:lhs(bb[2])-rhs(bb[2])=u-b*diff(q_1,t)$
bb[3]:lhs(bb[3])-rhs(bb[3])=0$
```

Conservation Laws

(%i69) `bb[1];`

$$\frac{K_2(q_1-q_2)^2}{2}+\frac{K_1{q_1}^2}{2}+\frac{m_2{Q_2}^2}{2}+\frac{m_1{Q_1}^2}{2}=E \qquad (\%o69)$$

Equations of Motion

(%i70) `map(ldisp,part(bb,[2,3]))$`

$$K_2\left(q_1-q_2\right)+K_1q_1+m_1\left(\dot{Q}_1\right)=\frac{\left(3t^2-2t^3\right)\left(\mathrm{signum}(t)-\mathrm{signum}\left(t-1\right)\right)}{2}+\frac{\left(\mathrm{signum}\left(t-2\right)-\mathrm{signum}\left(t-3\right)\right)\left(2t^3-15t^2+36\right.}{2} \qquad (\%t70)$$

$$m_2\left(\dot{Q}_2\right)-K_2\left(q_1-q_2\right)=0 \qquad (\%t71)$$

Solve for second derivative of coordinates

(%i72) `linsol:linsolve(part(bb,[2,3]),diff(ζ,t,2))$`

(%i73) `map(ldisp,linsol)$`

$$\dot{Q}_1=-(\left(2t^3-3t^2\right)\mathrm{signum}(t)+\left(-2\,\mathrm{signum}\left(t-1\right)-2\,\mathrm{signum}\left(t-2\right)+2\,\mathrm{signum}\left(t-3\right)\right)t^3+(3\,\mathrm{signum}\left(t-1\right)+15\,\mathrm{signum}\left(t-\right. \qquad (\%t73)$$

$$\dot{Q}_2=-\frac{K_2q_2-K_2q_1}{m_2} \qquad (\%t74)$$

Numerical solution (Lagrangian)

(%i75)
```
kill(labels)$
```

(%i7)
```
funcs:[q_1,q_2,Q_1,Q_2]$ldisplay(funcs)$
initial:[0,0,0,0]$ldisplay(initial)$
odes:append(ξ,map(rhs,linsol))$
interval:[t,0,τ]$ldisplay(interval)$
```

$$funcs = [q_1, q_2, Q_1, Q_2] \tag{\%t2}$$

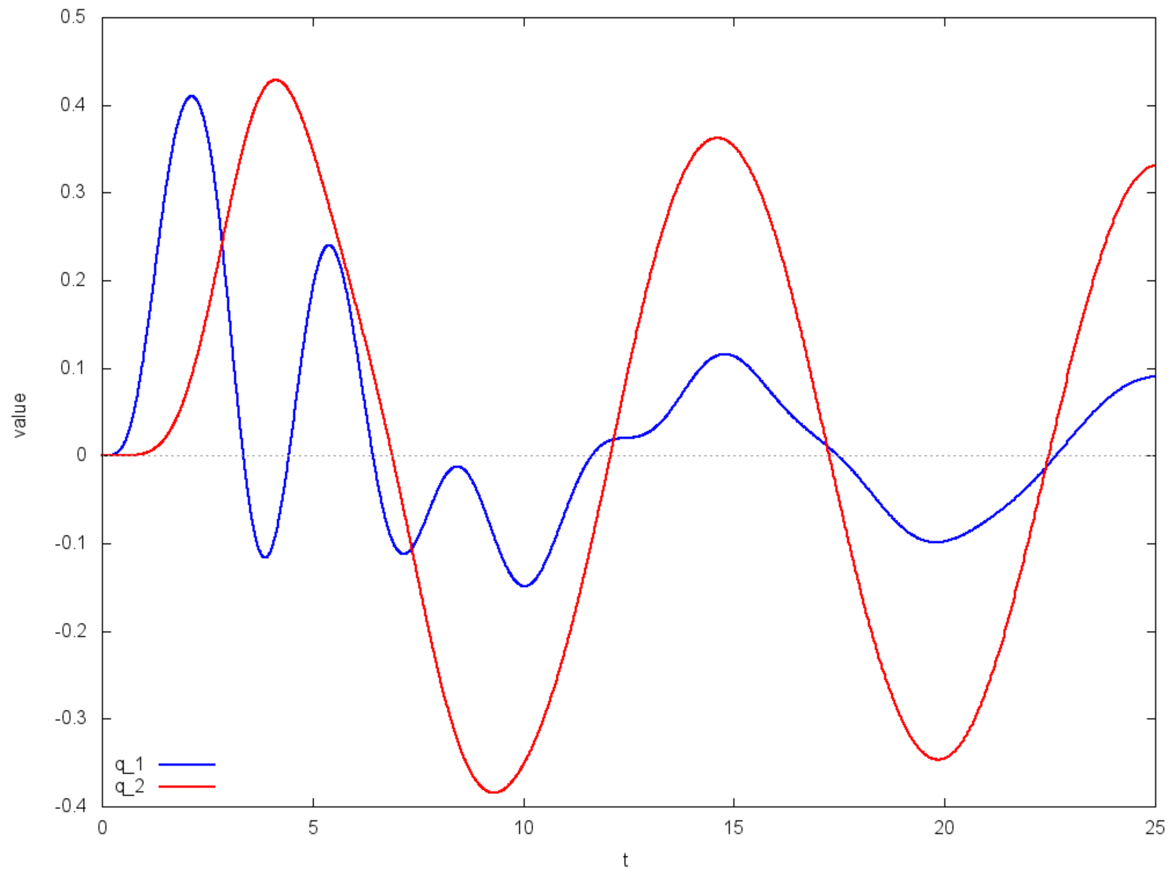
$$initial = [0, 0, 0, 0] \tag{\%t4}$$

$$interval = [t, 0, 25] \tag{\%t7}$$

(%i8)
```
rksol:rkf45(odes,funcs,initial,interval, absolute_tolerance=1E-10,report=true),params$
```

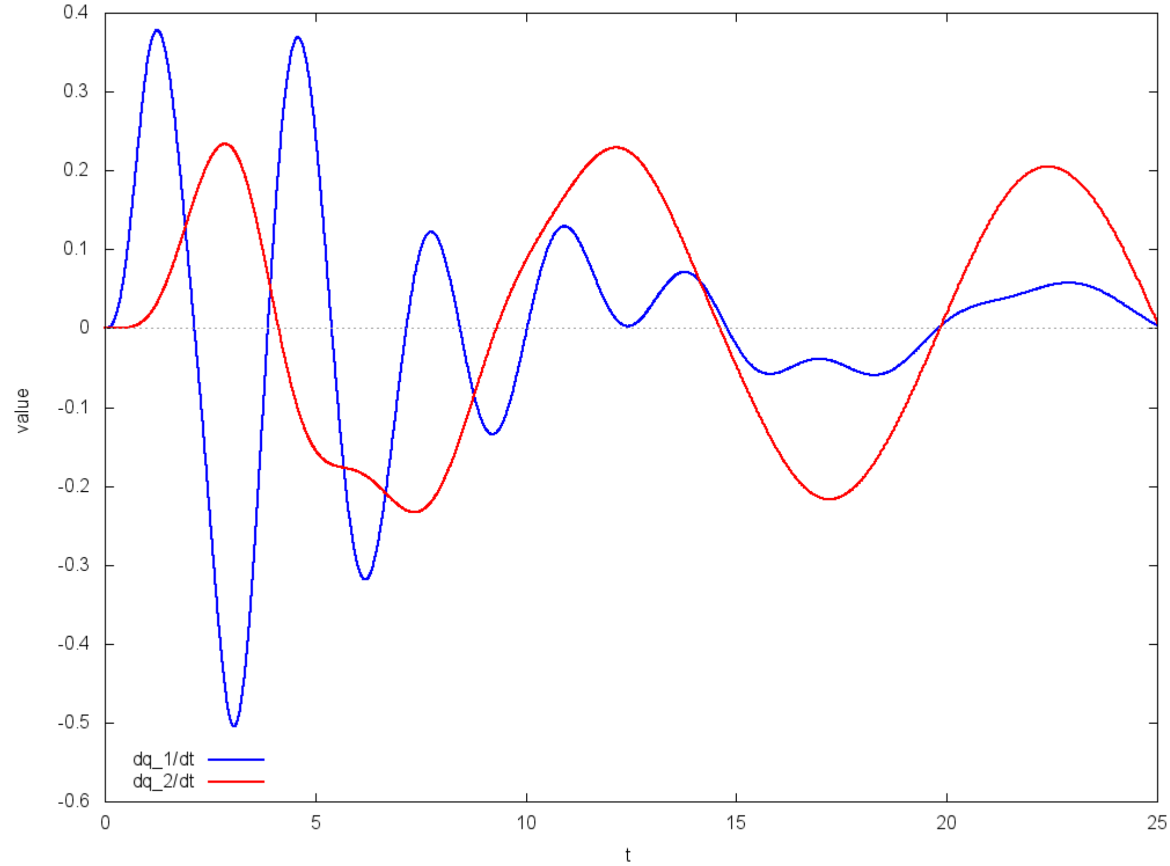
Info: rkf45:
Integration points selected:2037
Total number of iterations:2054
Bad steps corrected:18
Minimum estimated error:$1.4992 10^{-16}$
Maximum estimated error:$6.1553 10^{-11}$
Minimum integration step taken:$3.4465 10^{-4}$
Maximum integration step taken:0.030479

(%i9) wxplot2d([[discrete,map(lambda([u],part(u,[1,2])),rksol)], [discrete,map(lambda([u],part(u,[1,3]
[style,[lines,2]],[xlabel,"t"],[ylabel,"value"], [legend,"q_1","q_2"],[gnuplot_preamble,"set
key bottom left"])$



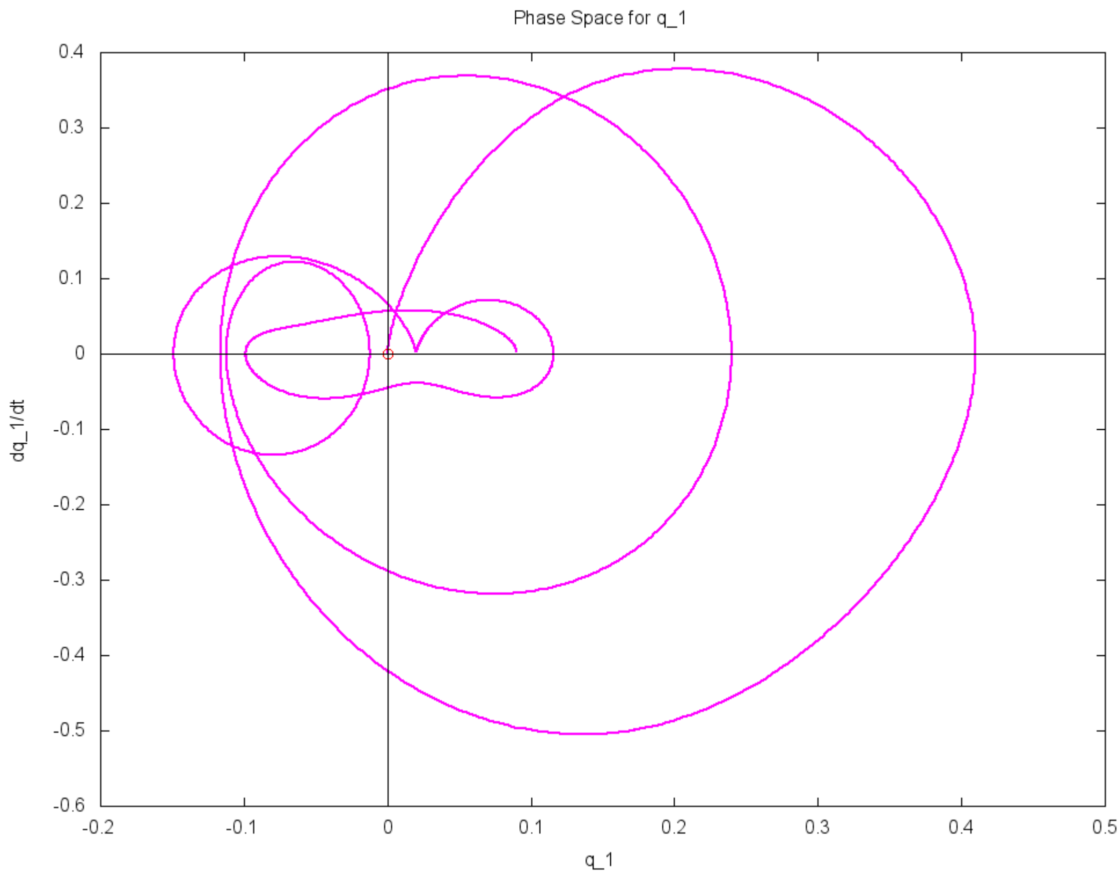
(%t9)

(%i10) wxplot2d([[discrete,map(lambda([u],part(u,[1,4])),rksol)], [discrete,map(lambda([u],part(u,[1,5]
[style,[lines,2]],[xlabel,"t"],[ylabel,"value"], [legend,"dq_1/dt","dq_2/dt"],[gnuplot_preamble,"
key bottom left"])$



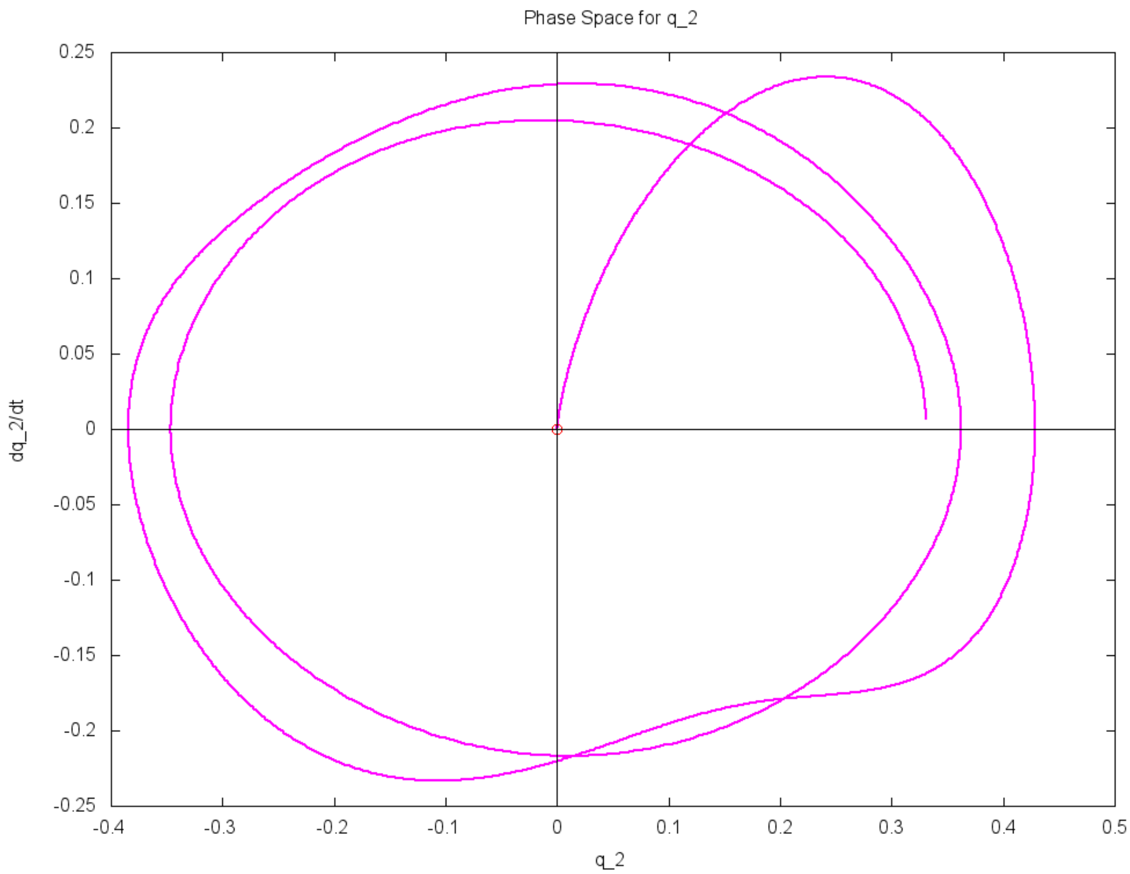
(%t10)

(%i11)
```
wxplot2d([[discrete,map(lambda([u],part(u,[2,4])),rksol)], [discrete,[part(initial,[1,3])]]],[axe
[title,"Phase Space for q_1"],[point_type,circle], [style,[lines,2],[points,3]],[color,magenta,re
[xlabel,"q_1"],[ylabel,"dq_1/dt"],[legend,false])$
```



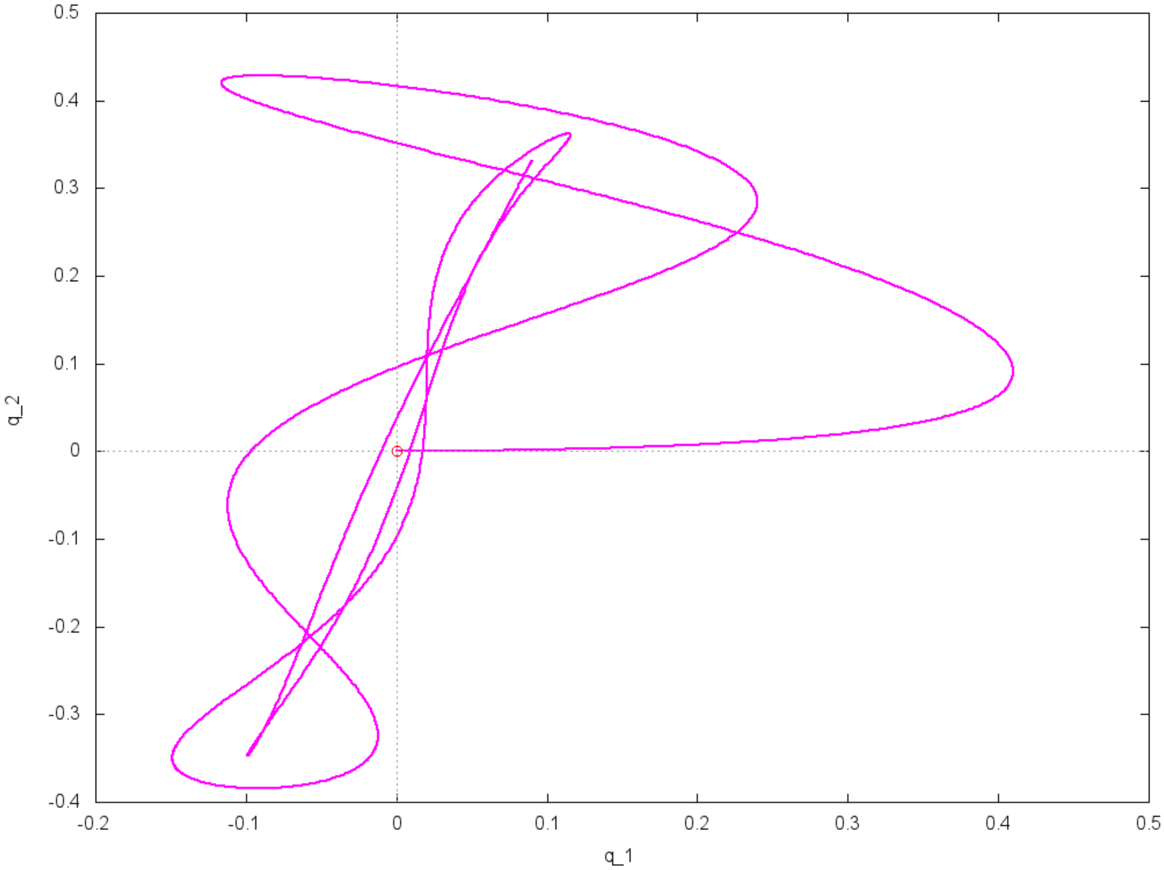
(%t11)

(%i12)
```
wxplot2d([[discrete,map(lambda([u],part(u,[3,5])),rksol)], [discrete,[part(initial,[2,4])]]],[axe
[title,"Phase Space for q_2"],[point_type,circle], [style,[lines,2],[points,3]],[color,magenta,re
[xlabel,"q_2"],[ylabel,"dq_2/dt"],[legend,false])$
```



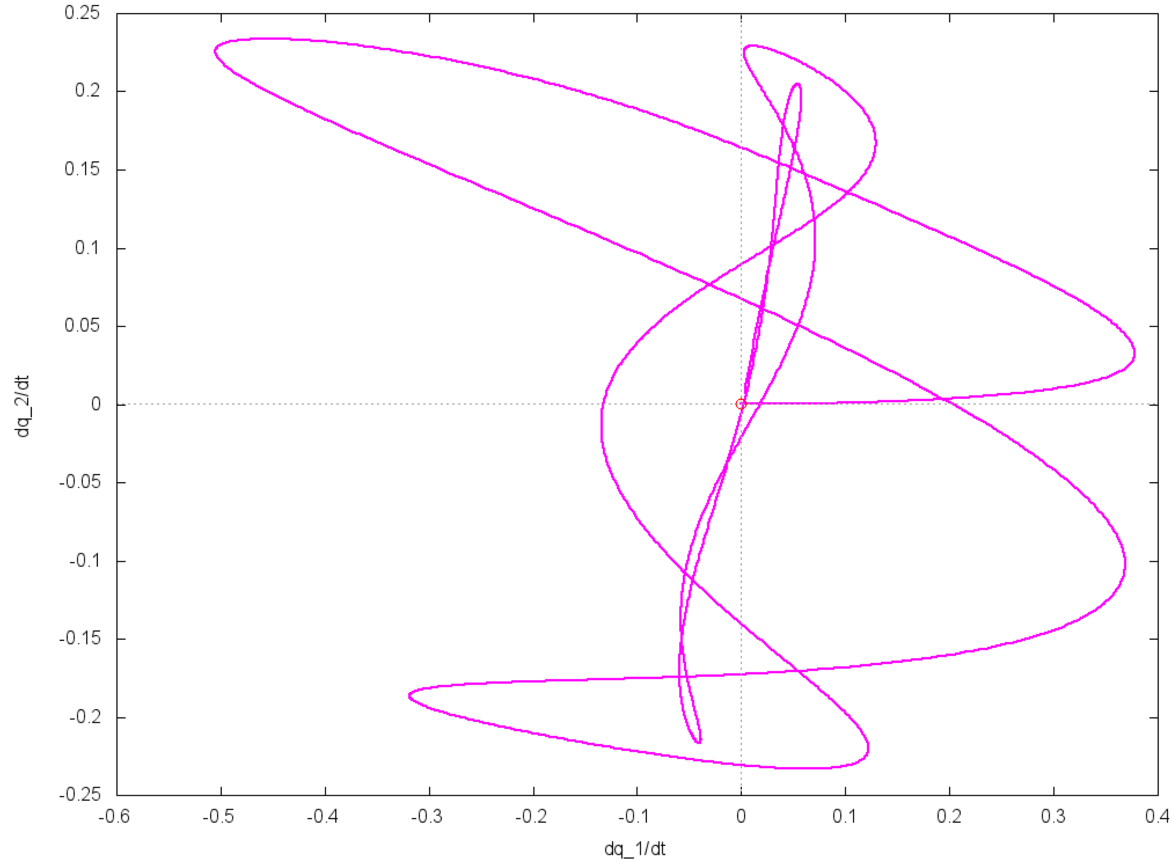
(%t12)

(%i13) `wxplot2d([[discrete,map(lambda([u],part(u,[2,3])),rksol)], [discrete,[part(initial,[1,2])]]],[poi`
`[style,[lines,2],[points,3]],[color,magenta,red], [xlabel,"q_1"],[ylabel,"q_2"],[legend,false])$`



(%t13)

(%i14) `wxplot2d([[discrete,map(lambda([u],part(u,[4,5])),rksol)], [discrete,[part(initial,[3,4])]]],[poi`
`[style,[lines,2],[points,3]],[color,magenta,red], [xlabel,"dq_1/dt"],[ylabel,"dq_2/dt"],[legend,f`



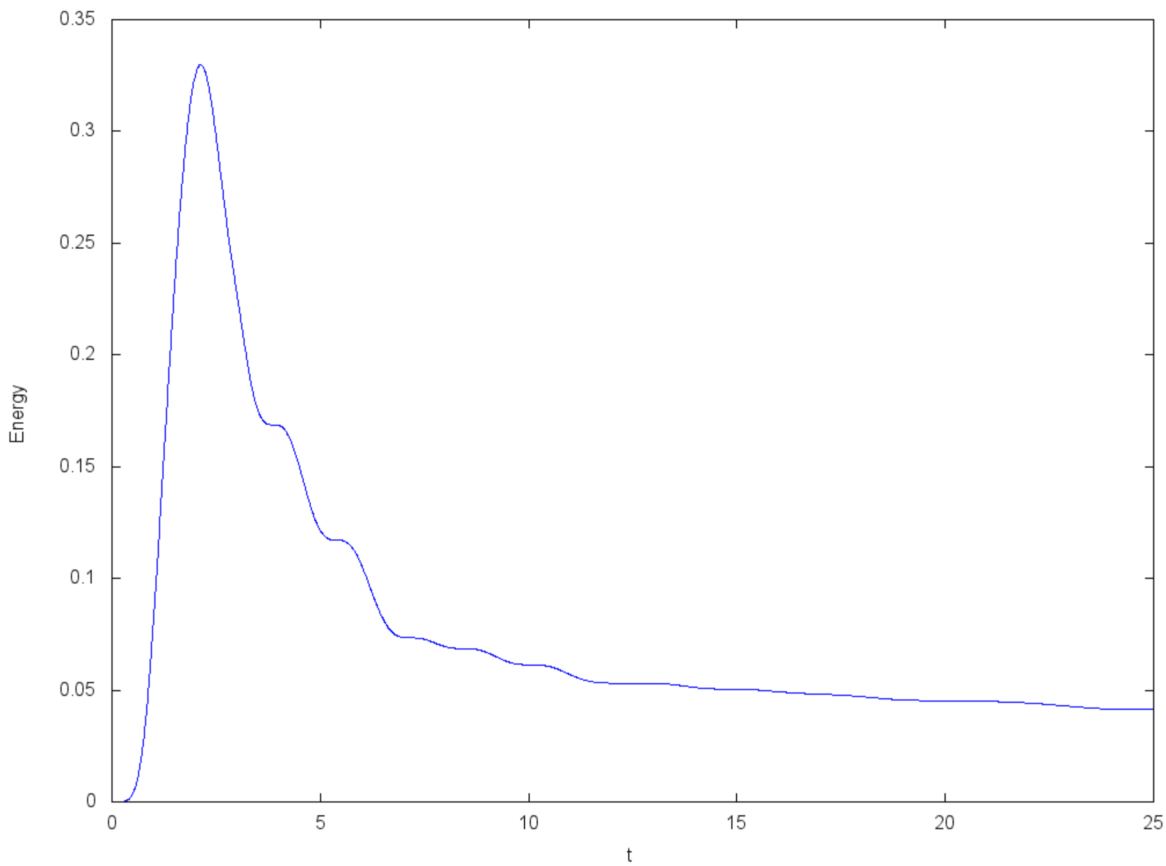
(%t14)

Check Conservation of Energy using the Numerical Data

(%i15)
```
Y:bb[1],map("=",funcs,initial),params,numer,eval;
```

$$0 = E \tag{Y}$$

(%i16)
```
wxplot2d([discrete,makelist([first(rkline), ev(lhs(bb[1]),map("=",funcs,rest(rkline)),params)],rl
[xlabel,"t"],[ylabel,"Energy"])$
```



(%t16)

6 Graphics

(%i17) `wxanimate_framerate:60$`

(%i18) `wxanimate_autoplay:true$`

(%i19) `rksol:rkf45(odes,funcs,initial,interval, absolute_tolerance=1E-6,report=true),params$`

Info: rkf45:
Integration points selected:207
Total number of iterations:212
Bad steps corrected:6
Minimum estimated error:2.603410^{-8}
Maximum estimated error:9.552810^{-7}
Minimum integration step taken:0.030096
Maximum integration step taken:0.26477

(%i20)
```
set_draw_defaults(proportional_axes = xy, delay = 1, xtics = 1, ytics = 1, xrange = [0,3],
yrange = [-1,1])$
```

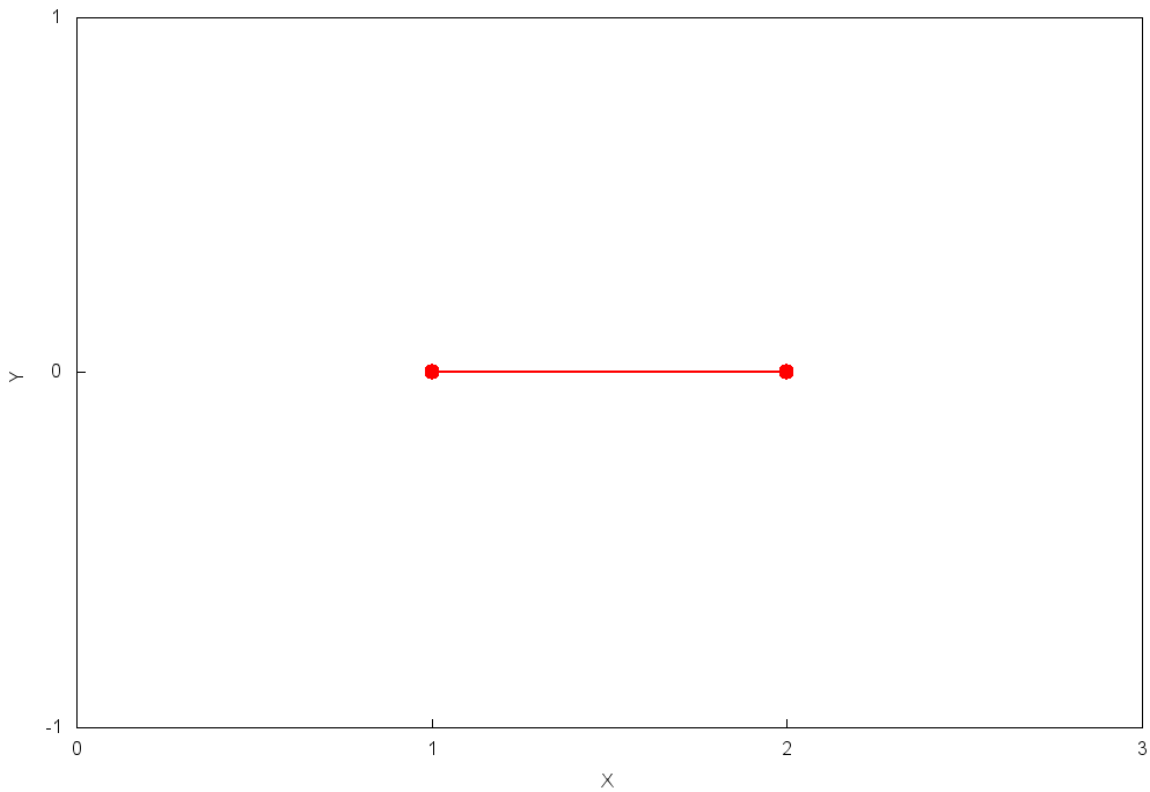
(%i21)
```
draw(terminal = 'animated_gif, file_name = "Masa-resorte-amortiguador doble",
makelist(gr2d( color = red, point_type = filled_circle, point_size = 2, points_joined
= true, line_width = 2, points([[1+rksol[t][2],0.0], [2+rksol[t][3],0.0]])),
t,1,length(rksol))),params$
```

(%i22) `time(%);`

$$[0.031] \tag{\%o22}$$

(%i24) `print("Click the figure to start animation")$ with_slider_draw( t,makelist(i,i,1,length(rksol)), color = red, point_type = filled_circle, point_size = 2, points_joined = true, line_width = 2, points([[1+rksol[t][2],0.0], [2+rksol[t][3],0.0]])),params$`

Click the figure to start animation



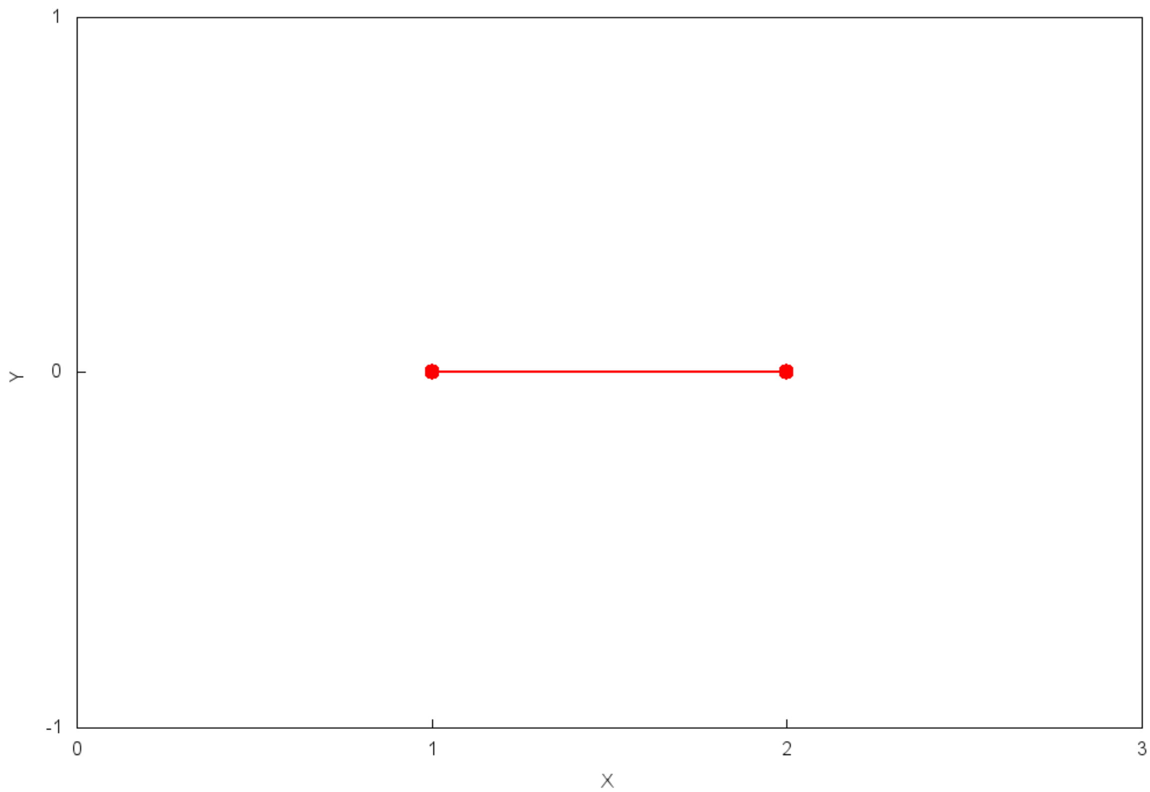
(%t24)

(%i25) `time(%);`

$$[0.516] \quad (\%o25)$$

(%i27) `print("Click the figure to start animation")$ wxanimate_draw( t,length(rksol), color = red, point_type = filled_circle, point_size = 2, points_joined = true, line_width = 2, points([[1+rksol[t][2],0.0], [2+rksol[t][3],0.0]])),params$`

Click the figure to start animation



(%t27)

(%i28) `time(%);`

$$[0.469] \tag{\%o28}$$