

https://github.com/t-o-k/Maxima-bezier/bezier_differentiated.wmxm

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```
(%i1) kill(all)$
```

```
(%i1) load("bezier")$
```

```
(%i2) define(
      g(u, v),
      bezier_function_2a(
        matrix(
          [ 4, 2, 3, 5 ],
          [ 5, 3, 2, 4 ],
          [ 2, 5, 4, 3 ],
          [ 3, 4, 5, 2 ]
        ),
        u, v
      )
)$
```

```
(%i3) define(der_u_g(u, v), diff(g(u, v), u))$
```

```
(%i4) define(der_v_g(u, v), diff(g(u, v), v))$
```

```
(%i5) define(der2_u_g(u, v), diff(g(u, v), u, 2))$
```

```
(%i6) define(der2_v_g(u, v), diff(g(u, v), v, 2))$
```

```
(%i7) define(der_u_der_v_g(u, v), diff(diff(g(u, v), u), v))$
```

```
(%i8) define(der_v_der_u_g(u, v), diff(diff(g(u, v), v), u))$
```

```
(%i9) expand(g(u, v));
```

```
(%o9) -8 u3 v3 + 36 u2 v3 - 36 u v3 + 8 v3 - 6 u3 v2 - 27 u2 v2 + 45
      u v2 - 12 v2 + 12 u3 v - 18 u2 v + 3 v - 2 u3 + 9 u2 - 6 u + 4
```

```
(%i10) expand(der_u_g(u, v));
```

```
(%o10) -24 u2 v3 + 72 u v3 - 36 v3 - 18 u2 v2 - 54 u v2 + 45 v2 + 36
      u2 v - 36 u v - 6 u2 + 18 u - 6
```

```
(%i11) expand(der_v_g(u, v));
```

```
(%o11) -24 u3 v2 + 108 u2 v2 - 108 u v2 + 24 v2 - 12 u3 v - 54 u2 v
+ 90 u v - 24 v + 12 u3 - 18 u2 + 3
```

```
(%i12) expand(der2_u_g(u, v));
```

```
(%o12) -48 u v3 + 72 v3 - 36 u v2 - 54 v2 + 72 u v - 36 v - 12 u + 18
```

```
(%i13) expand(der2_v_g(u, v));
```

```
(%o13) -48 u3 v + 216 u2 v - 216 u v + 48 v - 12 u3 - 54 u2 + 90 u -
24
```

```
(%i14) expand(der_u_der_v_g(u, v));
```

```
(%o14) -72 u2 v2 + 216 u v2 - 108 v2 - 36 u2 v - 108 u v + 90 v + 36
u2 - 36 u
```

```
(%i15) expand(der_v_der_u_g(u, v));
```

```
(%o15) -72 u2 v2 + 216 u v2 - 108 v2 - 36 u2 v - 108 u v + 90 v + 36
u2 - 36 u
```

```
(%i16) define(
```

```
    f(s, t),
```

```
    bezier_function_2a(
```

```
        matrix(
```

```
            [ a_11, a_12, a_13, a_14 ],
```

```
            [ a_21, a_22, a_23, a_24 ],
```

```
            [ a_31, a_32, a_33, a_34 ]
```

```
        ),
```

```
        s, t
```

```
    )
```

```
)$
```

(%i17) expand(f(s, t));

(%o17)
$$\begin{aligned} & a_{34} s^3 t^2 - 3 a_{33} s^3 t^2 + 3 a_{32} s^3 t^2 - a_{31} s^3 t^2 - 2 a_{24} s^3 t^2 + 6 \\ & a_{23} s^3 t^2 - 6 a_{22} s^3 t^2 + 2 a_{21} s^3 t^2 + a_{14} s^3 t^2 - 3 a_{13} s^3 t^2 + 3 a_{12} s^3 \\ & t^2 - a_{11} s^3 t^2 + 3 a_{33} s^2 t^2 - 6 a_{32} s^2 t^2 + 3 a_{31} s^2 t^2 - 6 a_{23} s^2 t^2 + 12 \\ & a_{22} s^2 t^2 - 6 a_{21} s^2 t^2 + 3 a_{13} s^2 t^2 - 6 a_{12} s^2 t^2 + 3 a_{11} s^2 t^2 + 3 a_{32} s \\ & t^2 - 3 a_{31} s t^2 - 6 a_{22} s t^2 + 6 a_{21} s t^2 + 3 a_{12} s t^2 - 3 a_{11} s t^2 + a_{31} t^2 \\ & - 2 a_{21} t^2 + a_{11} t^2 + 2 a_{24} s^3 t - 6 a_{23} s^3 t + 6 a_{22} s^3 t - 2 a_{21} s^3 t - 2 \\ & a_{14} s^3 t + 6 a_{13} s^3 t - 6 a_{12} s^3 t + 2 a_{11} s^3 t + 6 a_{23} s^2 t - 12 a_{22} s^2 t + \\ & 6 a_{21} s^2 t - 6 a_{13} s^2 t + 12 a_{12} s^2 t - 6 a_{11} s^2 t + 6 a_{22} s t - 6 a_{21} s t - \\ & 6 a_{12} s t + 6 a_{11} s t + 2 a_{21} t - 2 a_{11} t + a_{14} s^3 - 3 a_{13} s^3 + 3 a_{12} s^3 - \\ & a_{11} s^3 + 3 a_{13} s^2 - 6 a_{12} s^2 + 3 a_{11} s^2 + 3 a_{12} s - 3 a_{11} s + a_{11} \end{aligned}$$

(%i18) expand(f(0, 0));

(%o18) a_{11}

(%i19) expand(f(0, 1));

(%o19) a_{31}

(%i20) expand(f(1, 0));

(%o20) a_{14}

(%i21) expand(f(1, 1));

(%o21) a_{34}

(%i22) expand(f(u, 0));

(%o22)
$$\begin{aligned} & a_{14} u^3 - 3 a_{13} u^3 + 3 a_{12} u^3 - a_{11} u^3 + 3 a_{13} u^2 - 6 a_{12} u^2 + 3 \\ & a_{11} u^2 + 3 a_{12} u - 3 a_{11} u + a_{11} \end{aligned}$$

(%i23) expand(f(u, 1));

(%o23)
$$\begin{aligned} & a_{34} u^3 - 3 a_{33} u^3 + 3 a_{32} u^3 - a_{31} u^3 + 3 a_{33} u^2 - 6 a_{32} u^2 + 3 \\ & a_{31} u^2 + 3 a_{32} u - 3 a_{31} u + a_{31} \end{aligned}$$

```
(%i24) expand(f(0, v));
```

```
(%o24)  $a_{31} v^2 - 2 a_{21} v^2 + a_{11} v^2 + 2 a_{21} v - 2 a_{11} v + a_{11}$ 
```

```
(%i25) expand(f(1, v));
```

```
(%o25)  $a_{34} v^2 - 2 a_{24} v^2 + a_{14} v^2 + 2 a_{24} v - 2 a_{14} v + a_{14}$ 
```