https://github.com/t-o-k/Maxima-bezier/rational_bezier_surface_3d.wxmx

```
(%i1) kill(all)$

(%i1) load("draw")$

(%i2) load("bezier")$

(%i3) tau: 2*%pi$

(%i4) angle: tau/4;
```

(angle) $\frac{\pi}{2}$

```
(%i5) w: matrix([ 1, cos(angle/2), 1 ]);
```

(w) $\begin{pmatrix} 1 & \frac{1}{\sqrt{2}} & 1 \end{pmatrix}$

```
(%i6) weights: transpose(w).w;
```

(weights) $\begin{pmatrix} 1 & \frac{1}{\sqrt{2}} & 1 \\ \frac{1}{\sqrt{2}} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ 1 & \frac{1}{\sqrt{2}} & 1 \end{pmatrix}$

The 9 points; <x, y, z> in the control grid:

```
< 0, +Rmaj+Rmin,       0>   <+Rmaj+Rmin, +Rmaj+Rmin,       0>   <+Rmaj+Rmin, 0,       0>
< 0, +Rmaj+Rmin, +Rmin>   <+Rmaj+Rmin, +Rmaj+Rmin, +Rmin>   <+Rmaj+Rmin, 0, +Rmin>
< 0, +Rmaj     , +Rmin>   <+Rmaj     ,  +Rmaj     , +Rmin>   <+Rmaj     , 0, +Rmin>
```

This will create a surface that is 1/16 of the surface of a torus, or 1/8 of the surface of a sphere if Rmaj is 0.

```
(%i7) points_x:
          matrix(
              [  0, +Rmaj+Rmin, +Rmaj+Rmin ],
              [  0, +Rmaj+Rmin, +Rmaj+Rmin ],
              [  0, +Rmaj     , +Rmaj      ]
          )
      ;
```

(points_x) $\begin{pmatrix} 0 & Rmin+Rmaj & Rmin+Rmaj \\ 0 & Rmin+Rmaj & Rmin+Rmaj \\ 0 & Rmaj & Rmaj \end{pmatrix}$

```
(%i8) points_y:
          matrix(
              [ +Rmaj+Rmin, +Rmaj+Rmin,  0 ],
              [ +Rmaj+Rmin, +Rmaj+Rmin,  0 ],
              [ +Rmaj     , +Rmaj     ,  0 ]
          )
      ;
```

(points_y) $\begin{pmatrix} Rmin+Rmaj & Rmin+Rmaj & 0 \\ Rmin+Rmaj & Rmin+Rmaj & 0 \\ Rmaj & Rmaj & 0 \end{pmatrix}$

(%i9)

```
points_z:
   matrix(
      [        0,           0,          0 ],
      [ +Rmin,      +Rmin,      +Rmin ],
      [ +Rmin,      +Rmin,      +Rmin ]
   )
;
```

(points_z)

$$\begin{pmatrix} 0 & 0 & 0 \\ Rmin & Rmin & Rmin \\ Rmin & Rmin & Rmin \end{pmatrix}$$

(%i10)

```
define(f_x(u, v), rational_bezier_function_2a(points_x, weights, u, v));
```

(%o10)

$$f_x(u,v):=\left(Rmaj\,u^2\,v^2+\sqrt{2}\,Rmaj\,(1-u)\,u\,v^2+\sqrt{2}\,(Rmin+Rmaj)\,u^2\,(1-v)\,v+2\,(Rmin+Rmaj)\,(1-u)\,u\,(1-v)\,v+(Rmin+Rmaj)\,u^2\,(1-v)^2+\sqrt{2}\,(Rmin+Rmaj)\,(1-u)\,u\,(1-v)^2\right)/\left(u^2\,v^2+\sqrt{2}\,(1-u)\,u\,v^2+(1-u)^2\,v^2+\sqrt{2}\,u^2\,(1-v)\,v+2\,(1-u)\,u\,(1-v)\,v+\sqrt{2}\,(1-u)^2\,(1-v)\,v+u^2\,(1-v)^2+\sqrt{2}\,(1-u)\,u\,(1-v)^2+(1-u)^2\,(1-v)^2\right)$$

(%i11)

```
define(f_y(u, v), rational_bezier_function_2a(points_y, weights, u, v));
```

(%o11)

$$f_y(u,v):=\left(\sqrt{2}\,Rmaj\,(1-u)\,u\,v^2+Rmaj\,(1-u)^2\,v^2+2\,(Rmin+Rmaj)\,(1-u)\,u\,(1-v)\,v+\sqrt{2}\,(Rmin+Rmaj)\,(1-u)^2\,(1-v)\,v+\sqrt{2}\,(Rmin+Rmaj)\,(1-u)\,u\,(1-v)^2+(Rmin+Rmaj)\,(1-u)^2\,(1-v)^2\right)/\left(u^2\,v^2+\sqrt{2}\,(1-u)\,u\,v^2+(1-u)^2\,v^2+\sqrt{2}\,u^2\,(1-v)\,v+2\,(1-u)\,u\,(1-v)\,v+\sqrt{2}\,(1-u)^2\,(1-v)\,v+u^2\,(1-v)^2+\sqrt{2}\,(1-u)\,u\,(1-v)^2+(1-u)^2\,(1-v)^2\right)$$

(%i12)

```
define(f_z(u, v), rational_bezier_function_2a(points_z, weights, u, v));
```

(%o12)

$$f_z(u,v):=\left(Rmin\,u^2\,v^2+\sqrt{2}\,Rmin\,(1-u)\,u\,v^2+Rmin\,(1-u)^2\,v^2+\sqrt{2}\,Rmin\,u^2\,(1-v)\,v+2\,Rmin\,(1-u)\,u\,(1-v)\,v+\sqrt{2}\,Rmin\,(1-u)^2\,(1-v)\,v\right)/\left(u^2\,v^2+\sqrt{2}\,(1-u)\,u\,v^2+(1-u)^2\,v^2+\sqrt{2}\,u^2\,(1-v)\,v+2\,(1-u)\,u\,(1-v)\,v+\sqrt{2}\,(1-u)^2\,(1-v)\,v+u^2\,(1-v)^2+\sqrt{2}\,(1-u)\,u\,(1-v)^2+(1-u)^2\,(1-v)^2\right)$$

(%i14)

```
Rmaj: 0$
Rmin: 1$
```

(%i15)

```
f_x(u, v);
```

(%o15)

$$\left(\sqrt{2}\,u^2\,(1-v)\,v+2\,(1-u)\,u\,(1-v)\,v+u^2\,(1-v)^2+\sqrt{2}\,(1-u)\,u\,(1-v)^2\right)/\left(u^2\,v^2+\sqrt{2}\,(1-u)\,u\,v^2+(1-u)^2\,v^2+\sqrt{2}\,u^2\,(1-v)\,v+2\,(1-u)\,u\,(1-v)\,v+\sqrt{2}\,(1-u)^2\,(1-v)\,v+u^2\,(1-v)^2+\sqrt{2}\,(1-u)\,u\,(1-v)^2+(1-u)^2\,(1-v)^2\right)$$

(%i16)

```
f_y(u, v);
```

(%o16)

$$\left(2\,(1-u)\,u\,(1-v)\,v+\sqrt{2}\,(1-u)^2\,(1-v)\,v+\sqrt{2}\,(1-u)\,u\,(1-v)^2+(1-u)^2\,(1-v)^2\right)/\left(u^2\,v^2+\sqrt{2}\,(1-u)\,u\,v^2+(1-u)^2\,v^2+\sqrt{2}\,u^2\,(1-v)\,v+2\,(1-u)\,u\,(1-v)\,v+\sqrt{2}\,(1-u)^2\,(1-v)\,v+u^2\,(1-v)^2+\sqrt{2}\,(1-u)\,u\,(1-v)^2+(1-u)^2\,(1-v)^2\right)$$

(%i17)

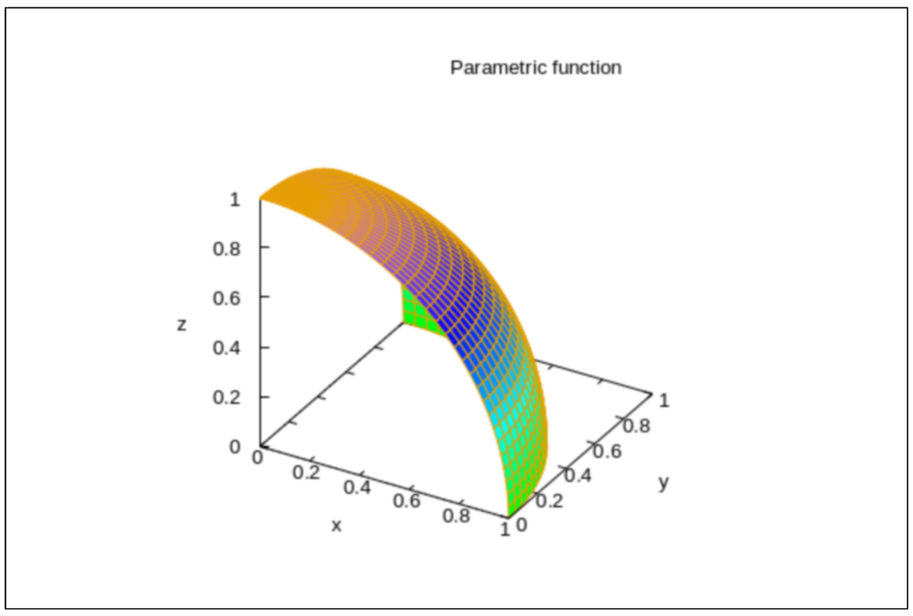
```
f_z(u, v);
```

(%o17)

$$\left(u^2\,v^2+\sqrt{2}\,(1-u)\,u\,v^2+(1-u)^2\,v^2+\sqrt{2}\,u^2\,(1-v)\,v+2\,(1-u)\,u\,(1-v)\,v+\sqrt{2}\,(1-u)^2\,(1-v)\,v\right)/\left(u^2\,v^2+\sqrt{2}\,(1-u)\,u\,v^2+(1-u)^2\,v^2+\sqrt{2}\,u^2\,(1-v)\,v+2\,(1-u)\,u\,(1-v)\,v+\sqrt{2}\,(1-u)^2\,(1-v)\,v+u^2\,(1-v)^2+\sqrt{2}\,(1-u)\,u\,(1-v)^2+(1-u)^2\,(1-v)^2\right)$$

```
(%i18) wxplot3d(
           [ f_x(u, v), f_y(u, v), f_z(u, v) ],
           [ u, 0, 1 ],
           [ v, 0, 1 ],
           same_xyz
       );
```

(%t18)

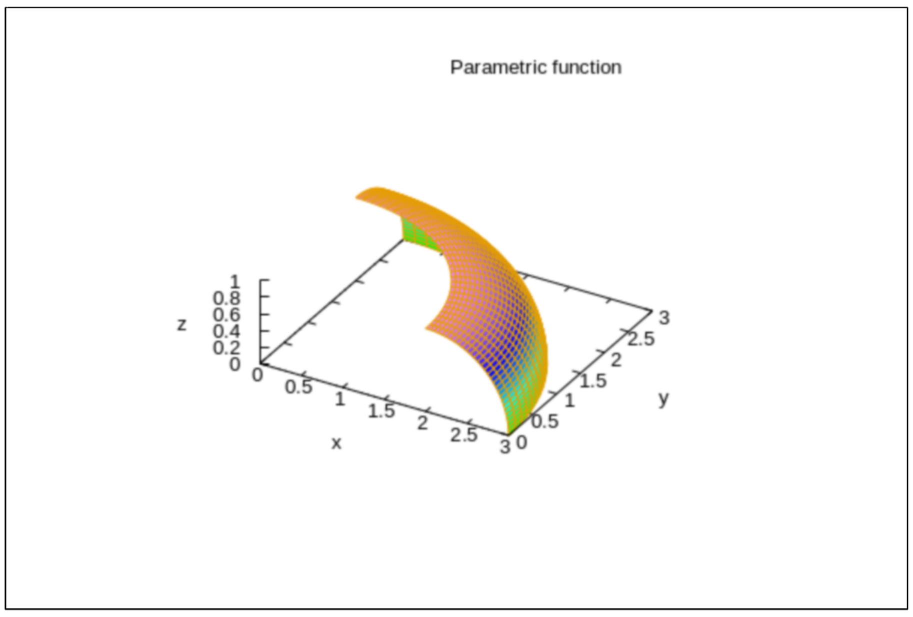


(%o18)

```
(%i20) Rmaj: 2$
       Rmin: 1$
```

```
(%i21) wxplot3d(
           [ f_x(u, v), f_y(u, v), f_z(u, v) ],
           [ u, 0, 1 ],
           [ v, 0, 1 ],
           same_xyz
       );
```

(%t21)



(%o21)