Practical 6

Solving a differential equation by
variation of parameter method

1 y'' + y = tan(x)

The general solution is given by:
y = yc +yp
where yc is a complementary
function and yp is a particular
integral

For yc: we solve the corresponding
homogeneous equation using ode2

```
(%i1) ode2('diff(y,x,2) + y = 0, y, x);
```

$$(\%o1)\quad y = \%k1\,\sin(x) + \%k2\,\cos(x)$$

```
(%i13) y1(x):= sin(x);
       y2(x):= cos(x);

       A: matrix ( [y1(x), y2(x)],
                   [diff(y1(x),x), diff(y2(x),x)]
                 );

       W: determinant(A);      /*Wronskian*/
```

$$(\%o10)\quad y1(x) := \sin(x)$$

$$(\%o11)\quad y2(x) := \cos(x)$$

$$(\%o12)\quad \begin{pmatrix} \sin(x) & \cos(x) \\ \cos(x) & -\sin(x) \end{pmatrix}$$

$$(\%o13)\quad -\sin(x)^2 - \cos(x)^2$$

Let yp = v1(x)y1(x) + v2(x)y2(x)
where v1 and v2 are given by

```
➔      v1 : integrate(-tan(x)* y2(x)/ W , x);
       v2 :  integrate(tan(x)* y1(x)/ W , x);
```

$$(\%o16)\quad -\cos(x)$$

$$(\%o17)\quad -\frac{\log(\sin(x)+1)}{2}+\frac{\log(\sin(x)-1)}{2}+\sin(x)$$

(%i18)
```
yp: v1* y1(x) + v2 * y2(x);
```

(%o18)
$$\cos(x)\left(-\frac{\log(\sin(x)+1)}{2}+\frac{\log(\sin(x)-1)}{2}+\sin(x)\right)-\cos(x)\sin(x)$$

```
ratsimp(%);                    /* simplify previous expression*/
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(%o19)
$$-\frac{\cos(x)\log(\sin(x)+1)-\cos(x)\log(\sin(x)-1)}{2}$$

Therefore, general solution is
y= %k1 sin(x) + %k2 cos(x) -
(cos(x)log(sin(x)+1)-
cos(x)log(sin(x)-1))/2

2 x^2 y'' + x y' -y = x^2 exp(x)

(%i20)
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ode2(x^2 * 'diff(y,x,2) + x* 'diff(y,x) -y = 0, y, x);
```

(%o20)
$$y=\%k2\,x-\frac{\%k1}{2\,x}$$

(%i24)
```
y1(x):= x;
y2(x):= 1/x;

A: matrix ( [y1(x), y2(x)],
            [diff(y1(x),x), diff(y2(x),x)]
          );

W: determinant(A);
```

(%o21)
$$y1(x):=x$$

(%o22)
$$y2(x):=\frac{1}{x}$$

(%o23)
$$\begin{pmatrix} x & \frac{1}{x} \\ 1 & -\frac{1}{x^2} \end{pmatrix}$$

(%o24)
$$-\frac{2}{x}$$

```
(%i26) v1 : integrate(-x^2 * exp(x)* y2(x)/ (x^2 * W) , x);
       v2 :  integrate(x^2 * exp(x)* y1(x)/ (x^2 * W) , x);
```

$$(\%o25)\quad \frac{\%e^{x}}{2}$$

$$(\%o26)\quad -\frac{(x^{2}-2\,x+2)\,\%e^{x}}{2}$$

```
(%i29) yp: v1* y1(x) + v2 * y2(x);
       ratsimp(%);
```

$$(\%o28)\quad \frac{x\,\%e^{x}}{2}-\frac{(x^{2}-2\,x+2)\,\%e^{x}}{2\,x}$$

$$(\%o29)\quad \frac{(x-1)\,\%e^{x}}{x}$$

3 y'' + a^2 y = sec(ax) a is non-zero

4 y'' - 2 y' + y = x exp(x) log(x) ; x >0

5 y'' - y = 2 exp(x)/ (1+ exp(x))

6 y'' -2 y' + y = exp(x) arcsin(x) (Hint: Use asin(x))

➔