Practical 7

Solving a system of ordinary differential equations.

1 $x' = 2x + 3y$ $y' = 4x + 3y$

(%i8)
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eqn1: diff(x(t),t)= 2* x(t) + 3* y(t);
eqn2: diff(y(t),t)= 4* x(t) + 3* y(t);
desolve([eqn1,eqn2],[x(t),y(t)]);
A: matrix([2,3], [4,3]);
eigenvalues(A);
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(%o4) $$\frac{d}{d\,t}\,x(t)=3\,y(t)+2\,x(t)$$

(%o5) $$\frac{d}{d\,t}\,y(t)=3\,y(t)+4\,x(t)$$

(%o6) $$\left[x(t)=\frac{(3\,y(0)+3\,x(0))\,\%e^{6\,t}}{7}-\frac{(3\,y(0)-4\,x(0))\,\%e^{-t}}{7},\;y(t)=\frac{(4\,y(0)+4\,x(0))\,\%e^{6\,t}}{7}+\frac{(3\,y(0)-4\,x(0))\,\%e^{-t}}{7}\right]$$

(%o7) $$\begin{pmatrix}2 & 3\\ 4 & 3\end{pmatrix}$$

(%o8) $[[6,-1],[1,1]]$

2 $x' = 4x + 8y + 2\cos(t) - 16\sin(t)$ $y' = 6x + 2y + \cos(t) - 14\sin(t)$

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eqn1: diff(x(t),t)= 4* x(t) + 8* y(t)+ 2*cos(t) - 16* sin(t);
eqn2: diff(y(t),t)= 6* x(t) + 2* y(t)+ cos(t) - 14* sin(t);

atvalue(x(t),t=0,15)$
atvalue(y(t),t=0,13)$

desolve([eqn1,eqn2],[x(t),y(t)]);
A: matrix([4,8], [6,2]);
eigenvalues(A);
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(%o10) $\frac{d}{d\,t}\,x(t)=-16\sin(t)+2\cos(t)+8\,y(t)+4\,x(t)$

(%o11) $\frac{d}{d\,t}\,y(t)=-14\sin(t)+\cos(t)+2\,y(t)+6\,x(t)$

(%o12) 15

(%o13) 13

(%o14) $[x(t)=2\sin(t)+16\,\%e^{10\,t}-\%e^{-4\,t},y(t)=\sin(t)+12\,\%e^{10\,t}+\%e^{-4\,t}]$

(%o15) $\begin{pmatrix}4 & 8\\ 6 & 2\end{pmatrix}$

(%o16) $[[10,-4],[1,1]]$

3 y'''' - 5 y'' + 4y =0

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(%i50) eqn1: diff(y1(t),t)= y2(t);
eqn2: diff(y2(t),t)= y3(t);
eqn3: diff(y3(t),t)= y4(t);
eqn4: diff(y4(t),t)= 5*y3(t)- 4* y1(t);

atvalue(y1(t),t=0,8)$
atvalue(y2(t),t=0,7)$
atvalue(y3(t),t=0,-8)$
atvalue(y4(t),t=0,-7)$

desolve([eqn1,eqn2,eqn3,eqn4],[y1(t),y2(t),y3(t),y4(t)]);
A: matrix([0,1,0,0], [0,0,1,0],[0,0,0,1],[-4,0,5,0]);
eigenvalues(A);
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(%o40) $$\frac{d}{d\,t}\,y1(t)=y2(t)$$

(%o41) $$\frac{d}{d\,t}\,y2(t)=y3(t)$$

(%o42) $$\frac{d}{d\,t}\,y3(t)=y4(t)$$

(%o43) $$\frac{d}{d\,t}\,y4(t)=5\,y3(t)-4\,y1(t)$$

(%o48) $$\left[y1(t)=-\frac{23\,\%e^{2\,t}}{6}+\frac{25\,\%e^{t}}{2}+\frac{5\,\%e^{-t}}{6}-\frac{3\,\%e^{-2\,t}}{2},\ y2(t)=-\frac{23\,\%e^{2\,t}}{3}+\frac{25\,\%e^{t}}{2}-\frac{5\,\%e^{-t}}{6}+3\,\%e^{-2\,t},\ y3(t)=-\frac{46\,\%e^{2\,t}}{3}+\frac{25\,\%e^{t}}{2}+\frac{5\,\%e^{-t}}{6}-6\,\%e^{-2\,t},\ y4(t)=-\frac{92\,\%e^{2\,t}}{3}+\frac{25\,\%e^{t}}{2}-\frac{5\,\%e^{-t}}{6}+12\,\%e^{-2\,t}\right]$$

(%o49) $$\begin{pmatrix}0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ -4 & 0 & 5 & 0\end{pmatrix}$$

(%o50) $[[-2,2,-1,1],[1,1,1,1]]$

4 x' = x-y+8z
y' = 10y -2z
z' = 9z

5 x' = 8x-y
y' = x + 10y

6 $x' = -x+y+0.4z$

$y' = x-0.1y + 1.4z$

$z' = 0.4x + 1.4y + 0.2z$

7 $x' = -x+y + \exp(-2t)$

$y' = -x -y - 324t$