Practical 2

Plotting a family of
characteristic curves and the
respective characteristic
base curves of first order PDE

1 $(y+u)\,u_x + y\,u_y = x-y$

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eqn1: diff(x(t),t)= y(t) + u(t);  /*Characteristic equations*/
eqn2: diff(y(t),t)= y(t);
eqn3: diff(u(t),t)= x(t)-y(t);
sol: desolve([eqn1,eqn2,eqn3],[x(t),y(t),u(t)]);
psol1: ev(sol, x(0)=1, y(0)=20, u(0)=30);
psol2: ev(sol, x(0)=5, y(0)=9, u(0)=8);
psol3: ev(sol, x(0)=0, y(0)=19, u(0)=80);
wxdraw3d(grid=true, nticks= 500, xlabel="X",ylabel="Y", zlabel="U",
         line_width=3, title="Characteristic curves",
         color=blue, key="Curve 1",
         parametric(rhs(psol1[1]),rhs(psol1[2]),rhs(psol1[3]),t,-1,1),
         color=red, key="Curve 2",
         parametric(rhs(psol2[1]),rhs(psol2[2]),rhs(psol2[3]),t,-1,1),
         color=magenta, key="Curve 3",
         parametric(rhs(psol3[1]),rhs(psol3[2]),rhs(psol3[3]),t,-1,1)
        );
/*nticks sets the number of points used in plotting, so it controls
the smmothness of the curve*/

wxdraw2d(grid=true, nticks= 500, xlabel="X",ylabel="Y",
         line_width=3, title="Characteristic base curves",
         color=blue, key="Curve 1",
         parametric(rhs(psol1[1]),rhs(psol1[2]),t,-1,1),
         color=red, key="Curve 2",
         parametric(rhs(psol2[1]),rhs(psol2[2]),t,-1,1),
         color=magenta, key="Curve 3",
         parametric(rhs(psol3[1]),rhs(psol3[2]),t,-1,1)
        );
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(%o49) $$\frac{d}{dt}x(t)=y(t)+u(t)$$

(%o50) $$\frac{d}{dt}y(t)=y(t)$$

(%o51) $$\frac{d}{dt}u(t)=x(t)-y(t)$$

(%o52) $$\left[x(t)=\frac{(y(0)+x(0)+u(0))\,\%e^{t}}{2}-\frac{(y(0)-x(0)+u(0))\,\%e^{-t}}{2},y(t)=y(0)\,\%e^{t},u(t)=\frac{(y(0)-x(0)+u(0))\,\%e^{-t}}{2}-\frac{(y(0)-x(0)-u(0))\,\%e^{t}}{2}\right]$$

(%o53) $$\left[x(t)=\frac{51\,\%e^{t}}{2}-\frac{49\,\%e^{-t}}{2},y(t)=20\,\%e^{t},u(t)=\frac{11\,\%e^{t}}{2}+\frac{49\,\%e^{-t}}{2}\right]$$

(%o54) $$\left[x(t)=11\,\%e^{t}-6\,\%e^{-t},y(t)=9\,\%e^{t},u(t)=2\,\%e^{t}+6\,\%e^{-t}\right]$$

(%o55) $$\left[x(t)=\frac{99\,\%e^{t}}{2}-\frac{99\,\%e^{-t}}{2},y(t)=19\,\%e^{t},u(t)=\frac{61\,\%e^{t}}{2}\right.$$

(%i2) $3\,u_x + 4\,u_y = u$

(%i3) $-x\,u_x + y\,u_y = 1$