Practical 3

Solving Cauchy Problem for first order partial differential equation
and hence plotting the integral surface with initial curve.

1 u_x - u_y =1 ; u(x,0)=x^2

```
(%i48) kill(all);
```

(%o0) done

```
(%i4) eqn1: 'diff(y,x)=-1;
sol1: ode2(eqn1,y,x);
sol1: subst([%c= c1],sol1);
solve(sol1,c1);
```

(%o1) $\frac{d}{d\,x}\,y=-1$

(%o2) $y=\%c-x$

(%o3) $y=c1-x$

(%o4) $[c1=y+x]$

```
(%i8) eqn2: 'diff(u,x)= 1;
sol2: ode2(eqn2,u,x);
sol2: subst([%c= c2],sol2);
solve(sol2,c2);
```

(%o5) $\frac{d}{d\,x}\,u=1$

(%o6) $u=x+\%c$

(%o7) $u=x+c2$

(%o8) $[c2=u-x]$

Therefore, the general solution of the PDE is given by:
u-x = f(x+y)

```
(%i9) solve(u-x=f(x+y),u);
```

(%o9) $[u=f\,(y+x)+x]$

(%i12)
```
u(x,y):= f(y+x) +x;
u(x,0)=x^2;
solve(%,f(x));
```

(%o10) $u(x,y):=f(y+x)+x$

(%o11) $f(x)+x=x^2$

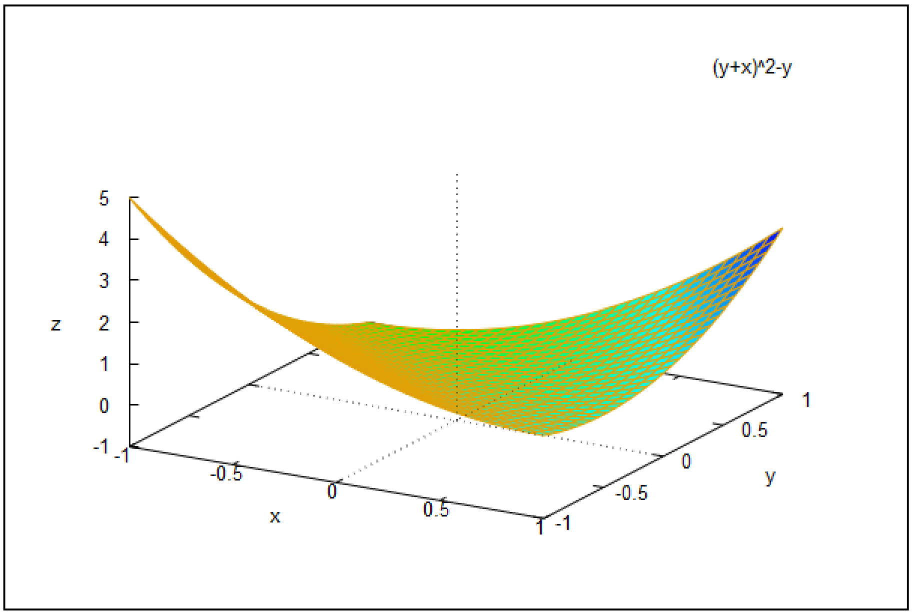
(%o12) $[f(x)=x^2-x]$

```
(%i38) f(x):= x^2 -x;
       'u(x,y)= u(x,y);

       /*plotting integral surface*/
       wxplot3d(u(x,y),[x,-1,1],[y,-1,1]);

       /*plotting initial curve*/
       wxdraw3d(xlabel="X", ylabel="Y", zlabel="U",
              title= "Initial curve",
              parametric(s,0,s^2,s, -2,2)
               );

       /*plotting integral surface with initial curve*/
       wxdraw3d(xlabel="X", ylabel="Y", zlabel="U",
              title= "Integral surface with initial curve",
              color= green, explicit((y+x)^2 -y, x,-1,1,y,-1,1),
              color= blue, parametric(s,0,s^2,s, -2,2)
               );
```

(%o34) $f(x) := x^2 - x$

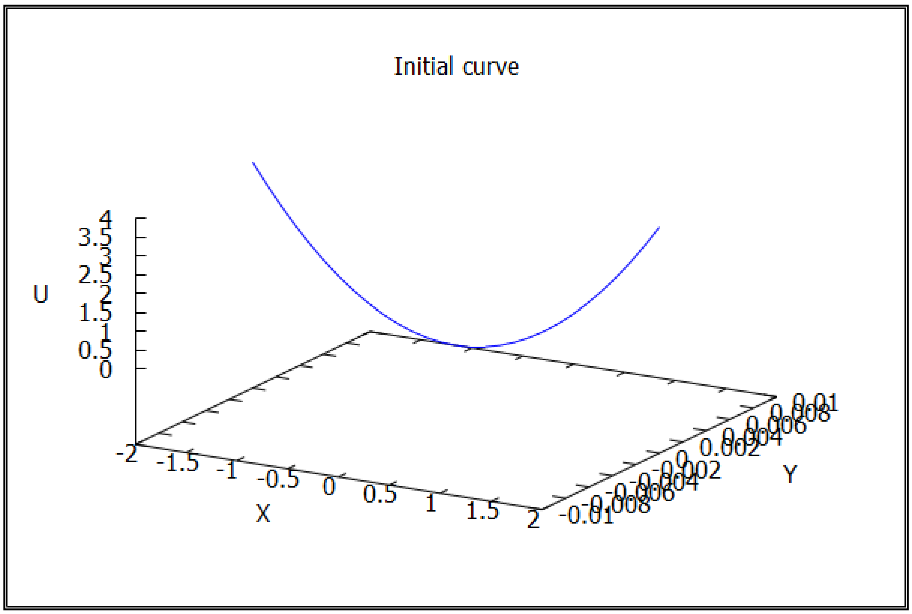
(%o35) $u(x,y) = (y+x)^2 - y$

(%t36)



(%o36)

(%t37)



2 x u_x + u_t =t ; u(x,0)= x^2 (Exercise)

3 x u_x + y u_y = x e^(-y); u=0 on y=x^2

```
(%i13) kill(all);
```
(%o0) done

```
(%i4) eqn1: 'diff(y,x)=y/x;
sol1: ode2(eqn1,y,x);
sol1: subst([%c =c1],sol1);
solve(sol1,c1);
```
(%o1) $\frac{d}{d\,x}\,y=\frac{y}{x}$

(%o2) $y=\%c\,x$

(%o3) $y=c1\,x$

(%o4) $[c1=\frac{y}{x}]$

```
(%i8) eqn2: 'diff(u,y)= c1 * %e^(-y);
sol2: ode2(eqn2,u,y);
sol2: subst([%c =c2],sol2);
solve(sol2,c2);
```
(%o5) $\frac{d}{d\,y}\,u=c1\,\%e^{-y}$

(%o6) $u=\%c-c1\,\%e^{-y}$

(%o7) $u=c2-c1\,\%e^{-y}$

(%o8) $[c2=\%e^{-y}\,(u\,\%e^{y}+c1)]$

Therefore, the general solution of the PDE is given by:
e^(-y)(u e^y + y/x) = f(y/x)

```
(%i9) solve(%e^(-y)*(u * %e^y + y/x) = f(y/x),u);
```
(%o9) $[u=\frac{\%e^{-y}\left(x\,\%e^{y}\,f\left(\frac{y}{x}\right)-y\right)}{x}]$

(%i12)
```
u(x,y):= (exp(-y)*(x* exp(y)*f(y/x)-y))/x;
u(x,x^2)=0;
solve(%,f(x));
```

(%o10) $$u(x,y):=\frac{\exp(-y)\left(x\exp(y)f\left(\frac{y}{x}\right)-y\right)}{x}$$

(%o11) $$\frac{\%e^{-x^2}\left(x\,\%e^{x^2}f(x)-x^2\right)}{x}=0$$

(%o12) $$[f(x)=x\,\%e^{-x^2}]$$

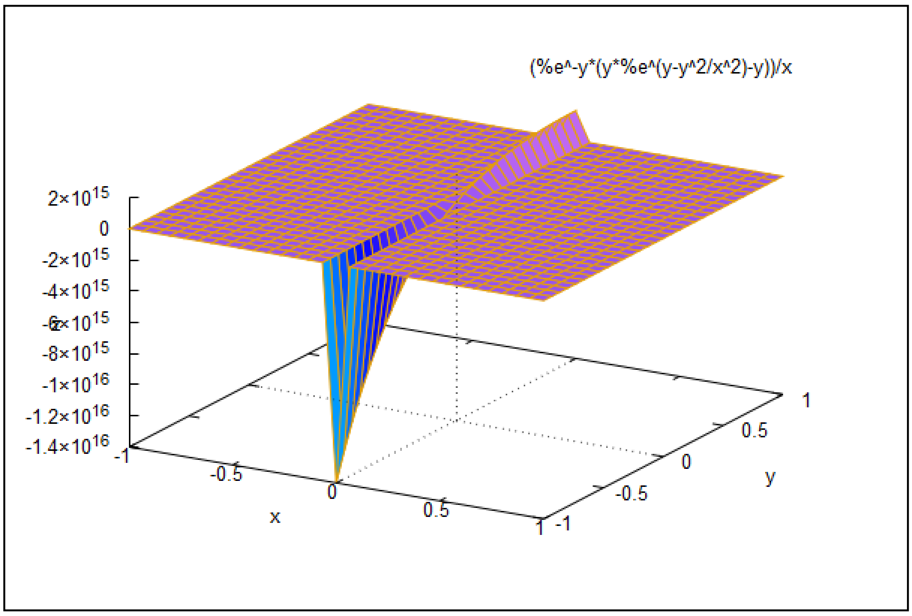
(%i14)
```
f(x):= x* exp(-(x^2));
'u(x,y)=u(x,y);
```

(%o13) $$f(x):=x\exp(-x^2)$$

(%o14) $$u(x,y)=\frac{\%e^{-y}\left(y\,\%e^{y-\frac{y^2}{x^2}}-y\right)}{x}$$

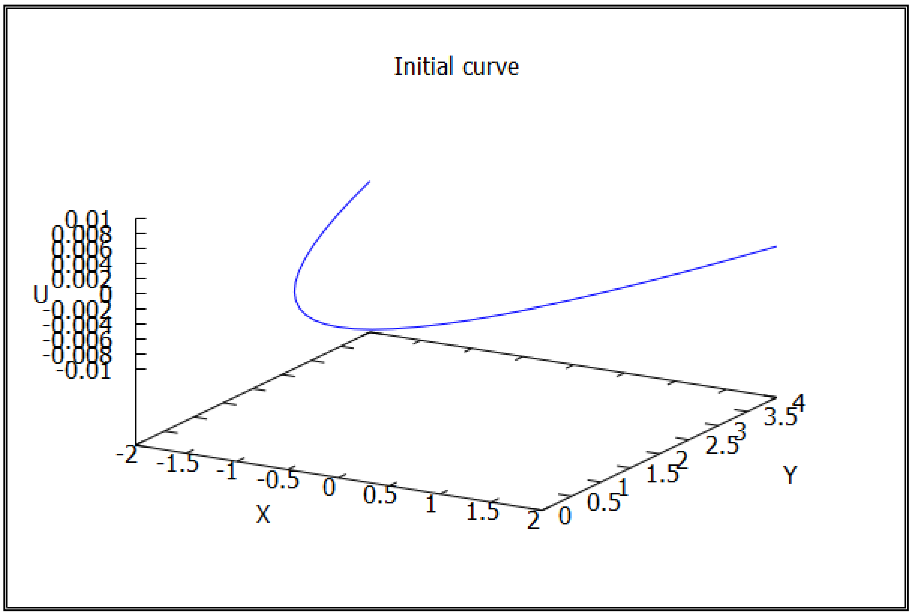
```
/*plotting integral surface*/
wxplot3d(u(x,y),[x,-1,1],[y,-1,1]);

/*plotting initial curve*/
wxdraw3d(xlabel="X", ylabel="Y", zlabel="U",
        title= "Initial curve",
        parametric(s,s^2,0,s, -2,2)
         );

/*plotting integral surface with initial curve*/
wxdraw3d(xlabel="X", ylabel="Y", zlabel="U",
        title= "Integral surface with initial curve",
        color= green, explicit(exp(-y)*(y*exp(y- (y^2/x^2))-y)/x,
         x,-1,1,y,-1,1),
        color= blue, parametric(s,s^2,0,s, -2,2)
         );
```

(%t21)

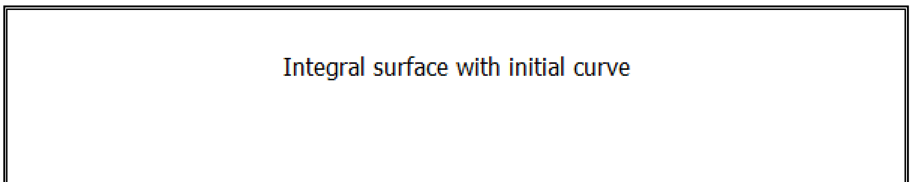


(%o21)

(%t22)



(%o22)



4 u_x + x u_y =0; u(0,y)= sin(y) (Exercise)