Practical 3

Solving Cauchy Problem for first
order partial differential
equation
and hence plotting the integral
surface with initial curve.

1 u_x - u_y =1 ; u(x,0)=x^2

(%i5)
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eqn1: 'diff(y,x)=-1;
sol1: ode2(eqn1,y,x);
sol1: subst([%c= c1],sol1);
solve(sol1,c1);
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(%o2) $\frac{d}{d\,x}\,y=-1$

(%o3) $y=\%c-x$

(%o4) $y=c1-x$

(%o5) $[c1=y+x]$

(%i9)
```
eqn2: 'diff(u,x)= 1;
sol2: ode2(eqn2,u,x);
sol2: subst([%c= c2],sol2);
solve(sol2,c2);
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(%o6) $\frac{d}{d\,x}\,u=1$

(%o7) $u=x+\%c$

(%o8) $u=x+c2$

(%o9) $[c2=u-x]$

Therefore, the general solution of
the PDE is given by:
u-x = f(x+y)

(%i10)
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solve(u-x=f(x+y),u);
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(%o10) $[u=f\,(y+x)+x]$

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(%i13) u(x,y):= f(y+x) +x;
       u(x,0)=x^2;
       solve(%,f(x));
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(%o11) $u(x,y):=f(y+x)+x$

(%o12) $f(x)+x=x^2$

(%o13) $[f(x)=x^2-x]$

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(%i16) f(x):= x^2 -x;
       'u(x,y)= u(x,y);
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(%o15) $f(x):=x^2-x$

(%o16) $u(x,y)=(y+x)^2-y$

2 x u_x + u_t =t ;
u(x,0)= x^2

3 x u_x + y u_y = x
e^(-y); u=0 on y=x^2

4 u_x + x u_y =0;
u(0,y)= sin(y)