Practical 5

Plotting a family of solutions of
the given third order
differential equations:

1 $y''' - 5y'' + 8y' - 4y = 0$

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(%i22) eqn: diff(y(x),x,3)- 5* diff(y(x),x,2) + 8* diff(y(x),x) - 4* y(x)=0;
       sol: desolve(eqn,y(x));
       sol1: ev(sol, y(0)=a, diff(y(x),x)=b, diff(y(x),x,2)=c);
       p1: ev(sol1,a=1,b=2,c=3);
       p2: ev(sol1,a=1,b=3,c=2);
       p3: ev(sol1,a=2,b=1,c=3);
       p4: ev(sol1,a=2,b=3,c=1);
       p5: ev(sol1,a=3,b=2,c=1);
       wxplot2d([rhs(p1),rhs(p2),rhs(p3),rhs(p4),rhs(p5)], [x,2,4],
                [legend,"p1","p2","p3","p4","p5"]);
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(%o14) $\dfrac{d^3}{dx^3} y(x) - 5\left(\dfrac{d^2}{dx^2} y(x)\right) + 8\left(\dfrac{d}{dx} y(x)\right) - 4\,y(x) = 0$

(%o15) $y(x) = \%e^x\left(\left.\dfrac{d^2}{dx^2} y(x)\right|_{x=0} - 4\left(\left.\dfrac{d}{dx} y(x)\right|_{x=0}\right) + 4\,y(0)\right) + x$

$\%e^{2x}\left(\left.\dfrac{d^2}{dx^2} y(x)\right|_{x=0}\right) + \%e^{2x}$

$\left(\left.-\dfrac{d^2}{dx^2} y(x)\right|_{x=0} + 4\left(\left.\dfrac{d}{dx} y(x)\right|_{x=0}\right) - 3\,y(0)\right) - 3\,x\,\%e^{2x}$

$\left(\left.\dfrac{d}{dx} y(x)\right|_{x=0}\right) + 2\,y(0)\,x\,\%e^{2x}$

(%o16) $y(x) = c\,x\,\%e^{2x} - 3\,b\,x\,\%e^{2x} + 2\,a\,x\,\%e^{2x} + (-c + 4\,b - 3\,a)$

$\%e^{2x} + (c - 4\,b + 4\,a)\,\%e^x$

(%o17) $y(x) = -x\,\%e^{2x} + 2\,\%e^{2x} - \%e^x$

(%o18) $y(x) = -5\,x\,\%e^{2x} + 7\,\%e^{2x} - 6\,\%e^x$

(%o19) $y(x) = 4\,x\,\%e^{2x} - 5\,\%e^{2x} + 7\,\%e^x$

(%o20) $y(x) = -4\,x\,\%e^{2x} + 5\,\%e^{2x} - 3\,\%e^x$

(%o21) $y(x) = x\,\%e^{2x} - 2\,\%e^{2x} + 5\,\%e^x$

(%t22)
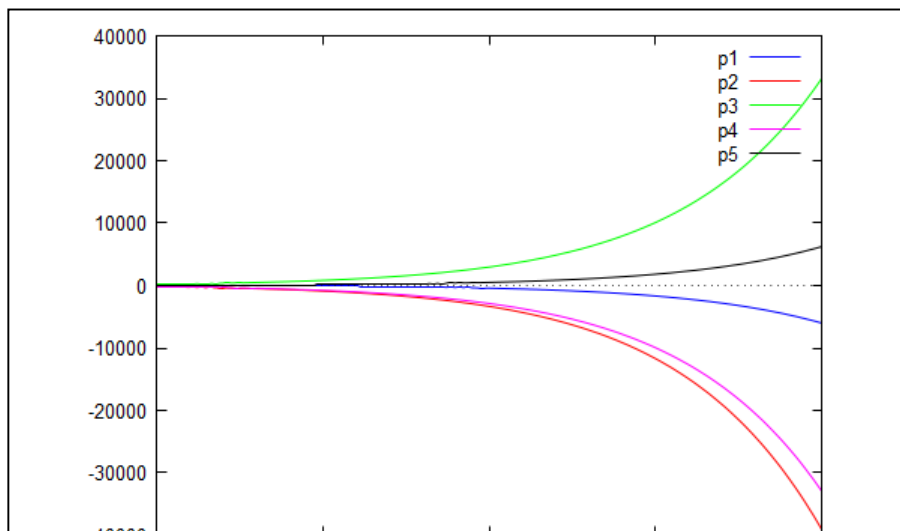
$$2 \qquad y''' - 5y'' - 22\,y' + 56y = 0;\ y(0)=1,\ y'(0)=-2,\ y''(0)=-4$$

(%i25) eqn: diff(y(x),x,3)- 5* diff(y(x),x,2) - 22* diff(y(x),x) + 56* y(x)=0
        sol: desolve(eqn,y(x));
        sol1: ev(sol, y(0)=1, diff(y(x),x)=-2, diff(y(x),x,2)=-4);

(%o23) $\dfrac{d^3}{dx^3}\,y(x) - 5\left(\dfrac{d^2}{dx^2}\,y(x)\right) - 22\left(\dfrac{d}{dx}\,y(x)\right) + 56\,y(x) = 0$

(%o24) $y(x) = \dfrac{\%e^{7x}\left(\left.\dfrac{d^2}{dx^2}y(x)\right|_{x=0} + 2\left(\left.\dfrac{d}{dx}y(x)\right|_{x=0}\right) - 8\,y(0)\right)}{55} -$

$\dfrac{\%e^{2x}\left(\left.\dfrac{d^2}{dx^2}y(x)\right|_{x=0} - 3\left(\left.\dfrac{d}{dx}y(x)\right|_{x=0}\right) - 28\,y(0)\right)}{30} +$

$\dfrac{\%e^{-4x}\left(\left.\dfrac{d^2}{dx^2}y(x)\right|_{x=0} - 9\left(\left.\dfrac{d}{dx}y(x)\right|_{x=0}\right) + 14\,y(0)\right)}{66}$

(%o25) $y(x) = -\dfrac{16\,\%e^{7x}}{55} + \dfrac{13\,\%e^{2x}}{15} + \dfrac{14\,\%e^{-4x}}{33}$

Another way of doing this:

(%i26) kill(all);

(%o0)   done

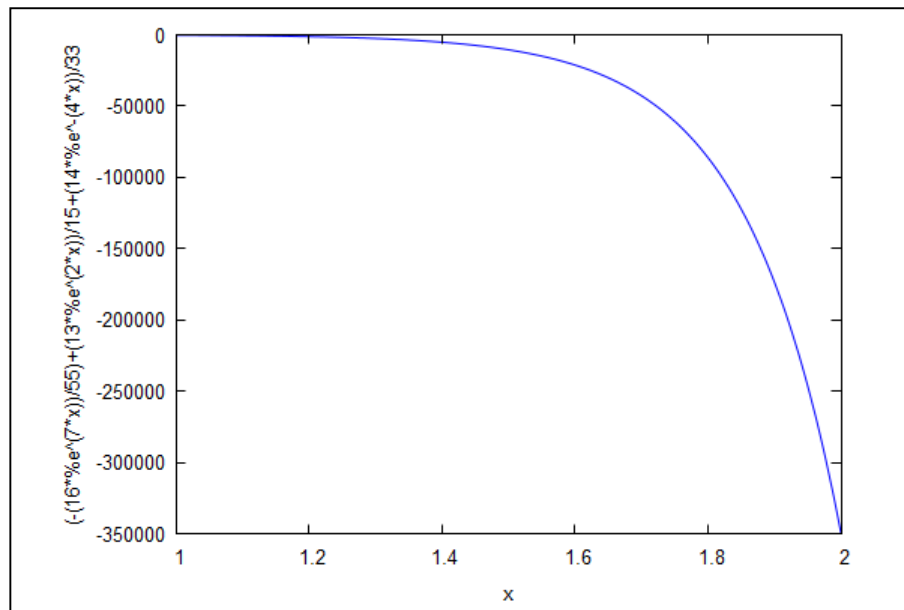(%i21) eqn: diff(y(x),x,3)- 5* diff(y(x),x,2) - 22* diff(y(x),x) + 56* y(x)=0

atvalue(y(x),x=0,1) $
atvalue(diff(y(x),x),x=0,-2)$
atvalue(diff(y(x),x,2),x=0,-4)$

sol: desolve(eqn,y(x));
wxplot2d(rhs(sol), [x,1,2]);

(%o16) $\dfrac{d^3}{d x^3} y(x) - 5\left(\dfrac{d^2}{d x^2} y(x)\right) - 22\left(\dfrac{d}{d x} y(x)\right) + 56\, y(x) = 0$

(%o20) $y(x) = -\dfrac{16\, \%e^{7x}}{55} + \dfrac{13\, \%e^{2x}}{15} + \dfrac{14\, \%e^{-4x}}{33}$

(%t21)

(%o21)

3   Exercise: y''' -12 y'' + 48 y' -64y = 12 - 32 exp(-8x) + 2 exp(4x)