REMARK

Corrections and Errors in John Ivie's Some MACSYMA Programs for Solving Recurrence Equations [John Ivie, *ACM Trans. Math. Softw.* 4, 1 (March 1978), 24–33]

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The programs written by John Ivie for solving difference equations that appeared in "Some MACSYMA Programs for Solving Recurrence Relations" have some serious errors as follows.

At the end of the routine VARC1 the command B: SUBSTPART(N, B, 4) should be changed to B: RATSUBST(N, PART(B, 4), B); otherwise, for the only example of the use of VARC1 given in the paper one obtains

$$u(n) = n! \sum_{i3=0}^{n} \frac{1}{(i2 - i3)!} \tag{d11}$$

instead of the correct result

$$u(n) = n! \sum_{i6=0}^{n} \frac{1}{(n - i6)!}. \tag{d15}$$

The routine CHAR that solves an equation with constant coefficients accepts as the right-hand side of the equation a constant to a polynomial power. This can in general only be solved for polynomials of degree 1. The routine returns an incorrect answer for higher degree polynomials. For example, for the difference equation

$$u(n + 1) - u(n) = 2^{3n^2-n}$$
$$u(0) = \tfrac{1}{7}$$

the following incorrect result is returned by the routine CHAR:

$$u(n) = \frac{2^{3n^2-n}}{4\ 2^{6n} - 1} - \frac{4}{21}. \tag{d13}$$

The routine POLYINN, which checks for a constant to a polynomial power, only looks at the first two factors of the expression it receives and therefore returns *true* for expressions like $3^n e^n 2^{2^n}$. On the other hand, it returns *false* for expresions like $2\ 3^n$.

The routine VARC1, when finding the differential equation (on the exponential generating function) from the difference equation, performs the wrong transformation on each term of the difference equations (both left- and right-hand sides). The exponential generating function of $u(n)$ called $Y(x)$ is defined as

$$Y(x) = \sum_{n=0}^{\infty} u(n) \frac{x^n}{n!}$$

and its derivatives as

$$\frac{\partial^j}{\partial x^j} Y(x) = \sum_{n=0}^{\infty} u(n+j) \frac{x^n}{n!}.$$

The transformation that VARC1 applies is

$$n^j u(n+i) \rightarrow x^j \frac{\partial^{i+j}}{\partial x^{i+j}} Y(x)$$

and the one it should apply is

$$n(n-1) \cdots (n-j+1) u(n+i) \rightarrow x^j \frac{\partial^{i+j}}{\partial x^{i+j}} Y(x).$$

Also, in the VARC1 routine, the line that checks that the coefficients in the equations are polynomials in n has a *return* statement inside a loop, so when the error is detected, the routine does not exit as it should.