

# Examples for numerical.mac

1

```
(%i1) load("/home/jvr/Downloads/numericalPackage/numerical.mac");
(%o1) /home/jvr/Downloads/numericalPackage/numerical.mac
```

2

2.1

```
(%i2) bisect(t^3-2*sin(t), t, 0.50, 2.00, 20, 0.0005);
```

iter	m	ym	error
1	1.25	0.0551557612888276	
	0.75		
2	0.875	-0.8651651294720542	
	0.375		
3	1.0625	-0.5476869797091422	
	0.1875		
4	1.15625	-0.2847914007983883	
	0.09375		
5	1.203125	-	
	0.1247986155094702	0.046875	
6	1.2265625	-	
	0.03735980652509796	0.0234375	
7	1.23828125		
	0.00825801590073083	0.01171875	
8	1.232421875	-	
	0.01471021624269308	0.005859375	
9	1.2353515625	-	
	0.003266014170569153	0.0029296875	
10	1.23681640625		
	0.002486011901918328	0.00146484375	
11	1.236083984375	-	
	3.924970675475148 10 <sup>-4</sup>	7.32421875 10 <sup>-4</sup>	
12	1.2364501953125		
	0.001046133270412585	3.662109375 10 <sup>-4</sup>	

```
(%o2) bisection has converged
```

2.2

```
(%i3) bisect(sin(x), x, 2, 4, 20, 0.0005);  
  
iter      m      ym      error  
1          3      0.1411200080598672  
      1  
2          3.5    -0.3507832276896198  
      0.5  
3          3.25   -0.1081951345301083  
      0.25  
4          3.125   0.0165918922293479  
      0.125  
5          3.1875  -0.04589122327277969  
      0.0625  
6          3.15625 -  
0.01465682159049232 0.03125  
7          3.140625 9.676534387822795  
      -4  
10         0.015625  
8          3.1484375 -  
0.006844792961296519 0.0078125  
9          3.14453125 -  
0.002938592180907726 0.00390625  
10         3.142578125 -  
      -4  
9.854712506993688 10 0.001953125  
11         3.1416015625 -  
      -6  
8.908910206643689 10 9.765625 10 -4  
12         3.14111328125  
      -4  
4.793723214334506 10 4.8828125 10 -4  
(%o3) bisection has converged
```

2.3

```
(%i4) bisect(x^2-2, x, 1, 2, 20, 0.0005);
```

<i>iter</i>	<i>m</i>	<i>ym</i>	<i>error</i>
1	1.5	0.25	0.5
2	1.25	-0.4375	0.25
3	1.125	-0.734375	
0.125			
4	1.0625	-0.87109375	
0.0625			
5	1.03125	-0.9365234375	
0.03125			
6	1.015625	-0.968505859375	
0.015625			
7	1.0078125	-0.98431396484375	
0.0078125			
8	1.00390625	-	
0.9921722412109375		0.00390625	
9	1.001953125	-	
0.9960899353027344		0.001953125	
10	1.0009765625	-	
0.9980459213256836		9.765625 10 <sup>-4</sup>	
11	1.00048828125	-	
0.9990231990814209		4.8828125 10 <sup>-4</sup>	

```
(%o4) bisection has converged
```

```
(%i5) bisect(x^3-2·x^2+x-%pi, x, 2, 3, 10, 0.0001);
```

<i>iter</i>	<i>m</i>	<i>ym</i>	<i>error</i>
1	2.5	2.483407346410207	
	0.5		
2	2.25	0.3740323464102069	
	0.25		
3	2.125	-0.4521395285897931	
	0.125		
4	2.1875	-0.05687585671479311	
	0.0625		
5	2.21875	0.1540311257070818	
	0.03125		
6	2.203125		
	0.04745229880278501	0.015625	
7	2.1953125	-	
	0.004991682367869288	0.0078125	
8	2.19921875		
	0.02116015355055722	0.00390625	
9	2.197265625		
	0.008066719276360601	0.001953125	
10	2.1962890625		
	0.001533142169467538	9.765625 10 <sup>-4</sup>	

```
(%o5) done
```

### 3

#### 3.1

```
(%i6) newton(x^3-2·x^2+x-3, x, 3, 0.0001, 0.00001, 8);
```

1	2.4375	2.036865234375
2	2.213032716315109	0.2563633850614177
3	2.175554938721488	0.006463361488812325
4	2.174560100666446	4.479068049789703
10 <sup>-6</sup>		
5	2.174559410293312	2.156497203031904
10 <sup>-12</sup>		

```
(%o6) convergence
```

#### 3.2

```
(%i7) newton(sin(x), x, 3.0, 0.0001, 0.00001, 8);
1      3.142546543074278      -9.538893398264409
10-4
2      3.141592653300477      2.893162490762184
10-10
3      3.141592653589793      1.224646799147353
10-16
(%o7) convergence
```

### 3.3

```
(%i8) newton(x^2-2, x, 1, 0.0001, 0.00001, 8);
1      1.5      0.25
2      1.416666666666666      0.006944444444444642
3      1.414215686274509      6.007304882871267
10-6
4      1.414213562374689      4.510614104447086
10-12
(%o8) convergence
```

```
(%i9) newton(x^3-2·x^2+x-%pi, x, 3, 0.0001, 0.00001, 8);
1      2.446349540849362      1.975991988443487
2      2.230829413793257      0.237982391674409
3      2.196863381645069      0.005374496729482203
4      2.196060159694346      2.961172185678151
10-6
5      2.196059716657127      9.00612917575927
10-13
(%o9) convergence
```

## 4

### 4.1

```
(%i10) secant(x^3+2·x^2-3·x-1, x, -2, -3, 0.00001, 10);
```

0	-3	-1	
1	-2	5	
2	-2.833333333333333		0.8101851851851833
3	-2.907928388746803		0.04629957161572662
4	-2.912449640422374		-0.002380064066290543
5	-2.912228585591192		6.399876401275151 10 <sup>-6</sup>

```
(%o10) convergence
```

## 4.2

```
(%i11) secant(sin(x), x, 1, 4, 0.00001, 10);
```

0	4	-0.7568024953079282	
1	1	0.8414709848078965	
2	2.579462454848934		0.5329898131328342
3	3.166481028136553		-0.02488580518710502
4	3.140295209333355		0.001297443892426365
5	3.141592780561861		-1.269720678483891 10 <sup>-7</sup>

```
(%o11) convergence
```

## 4.3

```
(%i12) secant(x^2-2, x, 1, 2, 0.00001, 10);
```

0	1	-1	
1	2	2	
2	1.333333333333333		-0.2222222222222223
3	1.428571428571428		0.04081632653061229
4	1.413793103448275		-0.001189060642093009
5	1.41421143847487		-6.007286838860537 10 <sup>-6</sup>

```
(%o12) convergence
```

## 5

### 5.1

```
(%i13) regula(x^3+2*x^2-3*x-1, x, 1, 2, 20, 0.0005);
```

iter	m	ym
1	1.1	-0.5489999999999999
2	1.221729490022173	
	0.1436388950901026	
3	1.193880682944684	-
	0.02924075169187978	
4	1.199720665327255	
	0.006290908737020207	
5	1.198472095487697	-
	0.001337706025862806	
6	1.198737948438891	
	2.851619710524389 10 <sup>-4</sup>	

(%o13) regula falsi method has converged

## 5.2

```
(%i14) regula(sin(x), x, 2, 4, 20, 0.0005);
```

iter	m	ym
1	3.091528082734958	
	0.05004365932452196	
2	3.147874957380742	-
	0.006282262466726139	
3	3.141590357955694	
	2.295634098490862 10 <sup>-6</sup>	

(%o14) regula falsi method has converged

## 5.3

```
(%i15) regula(x^2-2, x, 1, 2, 20, 0.0005);
```

iter	m	ym
1	1.333333333333333	-
	0.2222222222222218	
2	1.428571428571428	
	0.04081632653061229	
3	1.411764705882353	-
	0.006920415224913157	
4	1.414634146341463	
	0.001189767995240842	
5	1.414141414141414	-
	2.040608101214758 10 <sup>-4</sup>	

(%o15) regula falsi method has converged

## 6

### 6.1

```
(%i18) A:matrix([1, 2, 3], [3, 5, 7], [4, 6, 9]);
luFactor(A);
luFactor(A)[1];
```

(%o16) 
$$\begin{pmatrix} 1 & 2 & 3 \\ 3 & 5 & 7 \\ 4 & 6 & 9 \end{pmatrix}$$

(%o17) 
$$\mathbf{L} = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 4 & 2 & 1 \end{pmatrix}, \mathbf{U} = \begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & -2 \\ 0 & 0 & 1 \end{pmatrix}$$

(%o18) 
$$\mathbf{L} = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 4 & 2 & 1 \end{pmatrix}$$

## 7

### 7.1

```
(%i21) A:matrix([1, 2, 3], [3, 5, 7], [4, 6, 9]);
b:matrix([1], [2], [3]);
solve_by_lu(A, b);
```

(%o19) 
$$\begin{pmatrix} 1 & 2 & 3 \\ 3 & 5 & 7 \\ 4 & 6 & 9 \end{pmatrix}$$

(%o20) 
$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

(%o21) 
$$\mathbf{Z} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \mathbf{x} = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$$

## 8

### 8.1



```
(%i25) A1:matrix([5, 1, 2], [-3, 9, 4], [1, 2, -7]);
      b1:matrix([10], [-14], [-33]);
      start:matrix([0], [0], [0]);
      gauss_jacobi(A1, b1, start, 14);
```

```
(%o22) 
$$\begin{pmatrix} 5 & 1 & 2 \\ -3 & 9 & 4 \\ 1 & 2 & -7 \end{pmatrix}$$

```

```
(%o23) 
$$\begin{pmatrix} 10 \\ -14 \\ -33 \end{pmatrix}$$

```

```
(%o24) 
$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

```

```
1
```

```
-----
```

```
(2.0 -1.5555555555555555 4.714285714285714)
```

```
2
```

```
-----
```

```
(0.4253968253968252 -2.984126984126984 4.555555555555555)
```

```
3
```

```
-----
```

```
(0.7746031746031745 -3.438447971781304 3.922448979591836)
```

```
4
```

```
-----
```

```
(1.118710002519526 -3.040665154950868 3.842529604434367)
```

```
5
```

```
-----
```

```
(1.071121189216427 -2.890443156686543 4.005339956088256)
```

```
6
```

```
-----
```

```
(0.9759526489020063 -2.97866625074486 4.041462125120478)
```

```
7
```

```
-----
```

```
(0.9791484001007809 -3.026443394863987 4.002660021058898)
```

```
8
```

```
-----
```

```
(1.004224670549238 -3.008132764881471 3.989465944338972)
```

```
9
```

```
-----
```

```
(1.005840175240705 -2.993909973967574 3.998279877255185)
```

```
10
```

```
-----
```

## 9

### 9.1

```
(%i29) A1:matrix([5, 1, 2], [-3, 9, 4], [1, 2, -7]);
      b1:matrix([10], [-14], [-33]);
      start:matrix([0], [0], [0]);
      sor(A1, b1, 0.9, start, 10);
```

```
(%o26) 
$$\begin{pmatrix} 5 & 1 & 2 \\ -3 & 9 & 4 \\ 1 & 2 & -7 \end{pmatrix}$$

```

```
(%o27) 
$$\begin{pmatrix} 10 \\ -14 \\ -33 \end{pmatrix}$$

```

```
(%o28) 
$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

```

```
1
```

```
-----
```

```
(1.7999999999999999 -0.8599999999999999 4.253142857142857)
```

```
2
```

```
-----
```

```
(0.6036685714285714 -3.006156571428571 3.972774269387755)
```

```
3
```

```
-----
```

```
(0.9712763030204081 -2.998342473991836 3.994010601157784)
```

```
4
```

```
-----
```

```
(0.9989854592037691 -2.997742850101166 3.999851029130248)
```

```
5
```

```
-----
```

```
(0.9995458884516973 -2.999850930126706 3.999965049395661)
```

```
6
```

```
-----
```

```
(0.9999403384855388 -2.999989011225273 3.99999165985835)
```

```
7
```

```
-----
```

```
(0.9999950583200967 -2.999997047569838 3.999999289823317)
```

```
8
```

```
-----
```

```
(0.9999992300581864 -2.999999651668854 3.999999919560678)
```

```
9
```

```
-----
```

```
(0.9999998892643681 -2.999999966211847 3.999999986407011)
```

```
10
```

```
-----
```

**10****10.1**

```
(%i33) A1:matrix([5, 1, 2], [-3, 9, 4], [1, 2, -7]);
      b1:matrix([10], [-14], [-33]);
      start:matrix([0], [0], [0]);
      gauss_seidel(A1, b1, start, 10);
```

```
(%o30) 
$$\begin{pmatrix} 5 & 1 & 2 \\ -3 & 9 & 4 \\ 1 & 2 & -7 \end{pmatrix}$$

```

```
(%o31) 
$$\begin{pmatrix} 10 \\ -14 \\ -33 \end{pmatrix}$$

```

```
(%o32) 
$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

```

```
1
```

```
-----
```

```
(2.0 -0.8888888888888887 4.746031746031746)
```

```
2
```

```
-----
```

```
(0.2793650793650792 -3.571781305114638 3.7336860670194)
```

```
3
```

```
-----
```

```
(1.220881834215167 -2.808010973936899 4.086408555191624)
```

```
4
```

```
-----
```

```
(0.9270387727107303 -3.062724211403811 3.971655764271872)
```

```
5
```

```
-----
```

```
(1.023882536572013 -2.979441716374605 4.0092855862604)
```

```
6
```

```
-----
```

```
(0.992174108770761 -3.006735557636591 3.996957570499654)
```

```
7
```

```
-----
```

```
(1.002564083327456 -2.997793114668471 4.000996836284359)
```

```
8
```

```
-----
```

```
(0.9991598884199506 -3.000723075541953 3.999673391048006)
```

```
9
```

```
-----
```

```
(1.000275258689188 -2.999763087569384 4.000107011935774)
```

```
10
```

```
-----
```

# 11

```
(%i36) xval:[0, 1, -1, 2, -2]$
      yval:[-5, -3, -15, 39, -9]$
      LP(xval, yval, x);
(%o36) 1.625 (x-1) x (x+1) (x+2) + 0.5 (x-2) x (x+1) (x+2)
      + 1.25 (1-x) (x-2) (x+1) (x+2) + 2.5 (x-2) (x-1) x (x+2) -
      0.375 (x-2) (x-1) x (x+1)

(%i37) LP(xval, yval, 3);
(%o37) 241.0
```

# 12

## 12.1

```
(%i38) dd_table([-1, 0, 1, 2, -2, 3], [5, 1, 1, 11, 5, 35]);
(%o38)
```

-1	5	-4	2	1	-0.08333333333333326	-1.734723475976807
0	1	0	5	1.0833333333333333	-0.08333333333333333	
1	1	10	2.8333333333333333	0.8333333333333333	$A_{3,6}$	
2	11	1.5	4.5	$A_{4,5}$	$A_{4,6}$	
-2	5	6	$A_{5,4}$	$A_{5,5}$	$A_{5,6}$	
3	35	$A_{6,3}$	$A_{6,4}$	$A_{6,5}$	$A_{6,6}$	

## 12.2

```
(%i41) xval:[-1, 0, 1, 2, -2, 3]$
      yval:[5, 1, 1, 11, 5, 35]$
      NP(xval, yval, x);
(%o41) -1.734723475976807 10-17 (x-2) (x-1) x (x+1)
      (x+2) - 0.08333333333333326 (x-2) (x-1) x (x+1) + (x-1) x
      (x+1) + 2 x (x+1) - 4 (x+1) + 5

(%i42) NP(xval, yval, 1.5);
(%o42) 4.453125
```

# 13

```
(%i43) ratprint:false;
(%o43) false
```

## 13.1

```
(%i58) T(1/s, s, 1.0, 2.0);
(%o58) 0.75
```

```
(%i61) compare_value(g, x, a, b):=block(
    [t, v, e],
    numer:true,
    local(f),

    define(f(x), g),
    t:T(f(x), x, a, b),
    v:integrate(f(x), x, a, b),
    e:abs(v-t),
    print(t, " ", v, " ", e)
);
```

```
(%o61) compare_value(g,x,a,b):=block([t,v,e],numer:true,local(f),define(f(x),g)
```

```
(%i62) compare_value(1/x, x, 1, 2);
      0.75    0.6931471805599453    0.05685281944005471
(%o62) 0.05685281944005471
```

## 13.2

```
(%i47) for k:2 next 2*k thru 150 do block(
    temp:Tc(sin(x), x, 0, %pi, k),
    print(k, " ", temp)
);
2      1.570796326794896
4      1.896118897937039
8      1.97423160194555
16     1.993570343772339
32     1.998393360970144
64     1.999598388640037
128    1.999899600184203
(%o47) done
```

## 14

### 14.1

```
(%i48) S(1/u, u, 1.0, 2.0);
(%o48) 0.69444444444444443
```

### 14.2

```
(%i49) for k:2 next 2·k thru 150 do block(
      temp:Sc(sin(t), t, 0, %pi, k),
      print(k, " ", temp)
    );
2      2.094395102393195
4      2.00455975498442
8      2.000269169948387
16     2.000016591047935
32     2.000001033369412
64     2.000000064530001
128    2.000000004032257
(%o49) done
```

## 15

```
(%i50) eulerm(1+(x/t), t, x, 1.0, 1.0, 6, 10);
1.0 ---- 1.0
1.5 ---- 2.0
2.0 ---- 3.166666666666666
2.5 ---- 4.458333333333333
3.0 ---- 5.85
3.5 ---- 7.324999999999999
4.0 ---- 8.87142857142857
4.5 ---- 10.48035714285714
5.0 ---- 12.14484126984126
5.5 ---- 13.85932539682539
6.0 ---- 15.61926406926406
(%o50) done
```

## 16



```
(%i51) eulermod(1+(x/t), t, x, 1.0, 1.0, 6, 10);  
1.0 ----- 1.0  
1.5 ----- 2.1  
2.0 ----- 3.371428571428571  
2.5 ----- 4.76984126984127  
3.0 ----- 6.26926406926407  
3.5 ----- 7.852602952602953  
4.0 ----- 9.507736707736708  
4.5 ----- 11.22561556090967  
5.0 ----- 12.99922196826221  
5.5 ----- 14.82295368889796  
6.0 ----- 16.69223406377801  
(%o51) done
```

## 17

```
(%i52) heun(1+(x/t), t, x, 1.0, 1.0, 6, 10);  
1.0 ----- 1.0  
1.5 ----- 2.083333333333333  
2.0 ----- 3.340277777777777  
2.5 ----- 4.725347222222221  
3.0 ----- 6.212083333333332  
3.5 ----- 7.78314484126984  
4.0 ----- 9.426272675736959  
4.5 ----- 11.13233453798185  
5.0 ----- 12.89426059775761  
5.5 ----- 14.70641393026065  
6.0 ----- 16.56419398452677  
(%o52) done
```

## 18

```
(%i53) rk2(1+(x/t), t, x, 1.0, 1.0, 6, 10);  
1.0 ---- 1.0  
1.5 ---- 2.09375  
2.0 ---- 3.359848484848484  
2.5 ---- 4.753382034632034  
3.0 ---- 6.248176088617265  
3.5 ---- 7.827038770053475  
4.0 ---- 9.477795861427574  
4.5 ---- 11.19136649795217  
5.0 ---- 12.96071373335682  
5.5 ---- 14.7802226066925  
6.0 ---- 16.64530777872948  
(%o53) done
```

## 18.1

```
(%i57) A1:matrix([5, 1, 2], [-3, 9, 4], [1, 2, -7]);
      b1:matrix([10], [-14], [-33]);
      start:matrix([0], [0], [0]);
      gauss_jacobi(A1, b1, start, 14);
```

```
(%o54) 
$$\begin{pmatrix} 5 & 1 & 2 \\ -3 & 9 & 4 \\ 1 & 2 & -7 \end{pmatrix}$$

```

```
(%o55) 
$$\begin{pmatrix} 10 \\ -14 \\ -33 \end{pmatrix}$$

```

```
(%o56) 
$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

```

```
1
```

```
-----
```

```
(2.0 -1.5555555555555555 4.714285714285714)
```

```
2
```

```
-----
```

```
(0.4253968253968252 -2.984126984126984 4.555555555555555)
```

```
3
```

```
-----
```

```
(0.7746031746031745 -3.438447971781304 3.922448979591836)
```

```
4
```

```
-----
```

```
(1.118710002519526 -3.040665154950868 3.842529604434367)
```

```
5
```

```
-----
```

```
(1.071121189216427 -2.890443156686543 4.005339956088256)
```

```
6
```

```
-----
```

```
(0.9759526489020063 -2.97866625074486 4.041462125120478)
```

```
7
```

```
-----
```

```
(0.9791484001007809 -3.026443394863987 4.002660021058898)
```

```
8
```

```
-----
```

```
(1.004224670549238 -3.008132764881471 3.989465944338972)
```

```
9
```

```
-----
```

```
(1.005840175240705 -2.993909973967574 3.998279877255185)
```

```
10
```

```
-----
```