Examples for numerical.mac

1

(%i1) load("/home/jvr/Downloads/numericalPackage/numerical.mac");
(%o1) */home/jvr/Downloads/numericalPackage/numerical.mac*

2

2.1

(%i2) bisect(x^3−2·sin(x), x, 0.50, 2.00, 20, 0.0005);

iter	m	ym	error
1	1.25	0.0551557612888276	0.75
2	0.875	−0.8651651294720542	0.375
3	1.0625	−0.5476869797091422	0.1875
4	1.15625	−0.2847914007983883	0.09375
5	1.203125	−0.1247986155094702	0.046875
6	1.2265625	−0.03735980652509796	0.0234375
7	1.23828125	0.00825801590073083	0.01171875
8	1.232421875	−0.01471021624269308	0.005859375
9	1.2353515625	−0.003266014170569153	0.0029296875
10	1.23681640625	0.00248601190191832	0.00146484375
11	1.236083984375	−3.924970675475148 10^{-4}	7.32421875 10^{-4}
12	1.2364501953125	0.001046133270412585	3.662109375 10^{-4}

(%o2) *bisection has converged*

2.2

(%i3) bisect(sin(x), x, 2.00, 4.00, 20, 0.0005);

iter	m	ym	error
1	3.0	0.1411200080598672	
			1.0
2	3.5	-0.3507832276896198	
			0.5
3	3.25	-0.1081951345301083	
			0.25
4	3.125	0.0165918922293479	
			0.125
5	3.1875	-0.04589122327277969	
			0.0625
6	3.15625	-0.01465682159049232	0.03125
7	3.140625	$9.676534387822795\ 10^{-4}$	0.015625
8	3.1484375	-0.006844792961296519	0.0078125
9	3.14453125	$-0.00293859 2180907726$	0.00390625
10	3.142578125	$-9.854712506993688\ 10^{-4}$	0.001953125
11	3.1416015625	$-8.908910206643689\ 10^{-6}$	$9.765625\ 10^{-4}$
12	3.14111328125	$4.793723214334506\ 10^{-4}$	$4.8828125\ 10^{-4}$

(%o3)   *bisection has converged*

2.3

(%i4) bisect(x^2−2, x, 1.00, 2.00, 20, 0.0005);

iter	m	ym	error
1	1.5	0.25	0.5
2	1.25	−0.4375	0.25
3	1.375	−0.109375	
0.125			
4	1.4375	0.06640625	
0.0625			
5	1.40625	−0.0224609375	
0.03125			
6	1.421875	0.021728515625	
0.015625			
7	1.4140625	−4.2724609375	
10^{-4}	0.0078125		
8	1.41796875		
0.0106353759765625	0.00390625		
9	1.416015625		
0.00510025024414062 5	0.001953125		
10	1.4150390625		
0.00233554840087890 6	$9.765625\ 10^{-4}$		
11	1.41455078125		
$9.539127349853516\ 10^{-4}$	$4.8828125\ 10^{-4}$		

(%o4) *bisection has converged*

(%i5) `bisect(x^3−2·x^2+x−%pi, x,  2.0,  3.0,  10,  0.0001);`

iter	m	ym	error
1	2.5	2.483407346410207	0.5
2	2.25	0.3740323464102069	0.25
3	2.125	−0.4521395285897931	0.125
4	2.1875	−0.05687585671479311	0.0625
5	2.21875	0.1540311257070818	0.03125
6	2.203125	0.04745229880278501	0.015625
7	2.1953125	−0.004991682367869288	0.0078125
8	2.19921875	0.02116015355055722	0.00390625
9	2.197265625	0.008066719276360601	0.001953125
10	2.1962890625	0.001533142169467538	$9.765625\ 10^{-4}$

(%o5) *done*

3

3.1

(%i6) `newton(x^3−2·x^2+x−3, x,  3.0, 0.0001, 0.00001, 8);`

1	2.4375	2.036865234375
2	2.213032716315109	0.2563633850614177
3	2.175554938721488	0.006463361488812325
4	2.174560100666446	$4.479068049789703\ 10^{-6}$
5	2.174559410293312	$2.156497203031904\ 10^{-12}$

(%o6) *convergence*

3.2

(%i7) newton(sin(x), x, 3.0, 0.0001, 0.00001, 8);

1   3.142546543074278 -9.538893398264409
10^{-4}
2   3.141592653300477   2.893162490762184
10^{-10}
3   3.141592653589793   1.224646799147353
10^{-16}

(%o7) *convergence*

3.3

(%i8) newton(x^2−2, x, 1.0, 0.0001, 0.00001, 8);

1   1.5   0.25
2   1.416666666666666   0.006944444444444642

3   1.414215686274509   6.007304882871267
10^{-6}
4   1.414213562374689   4.510614104447086
10^{-12}

(%o8) *convergence*

(%i9) newton(x^3−2·x^2+x−%pi, x, 3.0, 0.0001, 0.00001, 8);

1   2.446349540849362   1.975991988443487
2   2.23082941379257   0.237982391674409
3   2.196863381645069   0.005374496729482203

4   2.196060159694346   2.961172185678151
10^{-6}
5   2.196059716657127   9.00612917575927
10^{-13}

(%o9) *convergence*

4

4.1

(%i10) secant(x^3+2·x^2−3·x−1, x, −2.0, −3.0, 0.00001, 10);

0	−3.0	−1.0
1	−2.0	5.0
2	−2.833333333333333	0.8101851851851833
3	−2.907928388746803	0.046299957161572662
4	−2.912449640422374	−0.002380064066290543
5	−2.912228585591192	$6.399876401275151 \cdot 10^{-6}$

(%o10) *convergence*

4.2

(%i11) secant(sin(x), x, 1.0, 4.0, 0.00001, 10);

0	4.0	−0.7568024953079282
1	1.0	0.8414709848078965
2	2.579462454848934	0.5329898131328342
3	3.166481028136553	−0.02488580518710502
4	3.140295209333355	0.001297443892426365
5	3.141592780561861	$-1.269720678483891 \cdot 10^{-7}$

(%o11) *convergence*

4.3

(%i12) secant(x^2−2, x, 1.0, 2.0, 0.00001, 10);

0	1.0	−1.0
1	2.0	2.0
2	1.333333333333333	−0.2222222222222223
3	1.428571428571428	0.04081632653061229
4	1.413793103448275	−0.001189060642093009
5	1.41421143847487	$-6.007286838860537 \cdot 10^{-6}$

(%o12) *convergence*

5

5.1

(%i13) regula(x^3+2·x^2−3·x−1, x,   1.0, 2.0, 20, 0.0005);

iter	m	ym
1	1.1	−0.5489999999999999
2	1.151743638077285	−0.2744007202116687
3	1.176840909982786	−0.1307425288092163
4	1.188627673293828	−0.06087586326028837
5	1.194078911293239	−0.02804093844229971
6	1.196582088205247	−0.01285224023453723
7	1.197727754386817	−0.005877241523802867
8	1.198251317792008	−0.002684816279181046
9	1.198490418455614	−0.001225881007669516
10	1.198599576406594	$-5.596124951825487\ 10^{-4}$
11	1.19864940371845	$2.554366889526704\ 10^{-4}$

(%o13) *regula falsi method has converged*

5.2

(%i14) regula(sin(x), x,   2.0, 4.0, 20, 0.0005);

iter	m	ym
1	3.091528082734958	0.05004365932452196
2	3.147874957380742	−0.006282262466726139
3	3.141590357955694	$2.295634098490862\ 10^{-6}$

(%o14) *regula falsi method has converged*

5.3

(%i15) regula(x^2−2, x, 1.0, 2.0, 20, 0.0005);

iter	m	ym
1	1.333333333333333	−
0.2222222222222218		
2	1.4	−0.04000000000000026
3	1.411764705882352	−
0.006920415224913823		
4	1.413793103448275	−
0.001189060642093009		
5	1.414141414141414	−

$2.040608101208096 \ 10^{-4}$

(%o15) *regula falsi method has converged*

6

6.1

(%i18) A:matrix([1, 2, 3], [3, 5, 7], [4, 6, 9]);
luFactor(A);
luFactor(A)[1];

(%o16) $\begin{pmatrix} 1 & 2 & 3 \\ 3 & 5 & 7 \\ 4 & 6 & 9 \end{pmatrix}$

(%o17) $\left[L = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 4 & 2 & 1 \end{pmatrix}, U = \begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & -2 \\ 0 & 0 & 1 \end{pmatrix} \right]$

(%o18) $L = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 4 & 2 & 1 \end{pmatrix}$

7

7.1

(%i21) A:matrix([1, 2, 3], [3, 5, 7], [4, 6, 9]);
       b:matrix([1], [2], [3]);
       solve_by_lu(A,   b);

(%o19) $\begin{pmatrix} 1 & 2 & 3 \\ 3 & 5 & 7 \\ 4 & 6 & 9 \end{pmatrix}$

(%o20) $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

(%o21) $\left[z = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, x = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} \right]$

8

8.1

(%i25) A1:matrix([5,   1,   2], [−3,   9,   4], [1,   2,   −7]);
       b1:matrix([10],   [−14],   [−33]);
       start:matrix([0.0],   [0.0],   [0.0]);
       gauss_jacobi(A1,   b1,   start,   14);

$$(\%o22)\quad \begin{pmatrix} 5 & 1 & 2 \\ -3 & 9 & 4 \\ 1 & 2 & -7 \end{pmatrix}$$

$$(\%o23)\quad \begin{pmatrix} 10 \\ -14 \\ -33 \end{pmatrix}$$

$$(\%o24)\quad \begin{pmatrix} 0.0 \\ 0.0 \\ 0.0 \end{pmatrix}$$

1
————

$$\begin{pmatrix} 2.0 & -1.555555555555555 & 4.714285714285714 \end{pmatrix}$$

2
————

$$\begin{pmatrix} 0.4253968253968252 & -2.984126984126984 & 4.555555555555555 \end{pmatrix}$$

3
————

$$\begin{pmatrix} 0.7746031746031745 & -3.438447971781304 & 3.922448979591836 \end{pmatrix}$$

4
————

$$\begin{pmatrix} 1.118710002519526 & -3.040665154950868 & 3.842529604434367 \end{pmatrix}$$

5
————

$$\begin{pmatrix} 1.071121189216427 & -2.890443156686543 & 4.005339956088256 \end{pmatrix}$$

6
————

$$\begin{pmatrix} 0.9759526489020063 & -2.97866625074486 & 4.041462125120478 \end{pmatrix}$$

7
————

$$\begin{pmatrix} 0.9791484001007809 & -3.026443394863987 & 4.002660021058898 \end{pmatrix}$$

8
————

$$\begin{pmatrix} 1.004224670549238 & -3.008132764881471 & 3.989465944338972 \end{pmatrix}$$

9
————

$$\begin{pmatrix} 1.005840175240705 & -2.993909973967574 & 3.998279877255185 \end{pmatrix}$$

10
————

9

9.1

(%i29) A:matrix([5,   1,   2], [−3,   9,   4], [1,   2,   −7]);
       b:matrix([10],   [−14],   [−33]);
       start:matrix([0.0],   [0.0],   [0.0]);
       sor(A,   b,   0.9,   start,   10);

$$(\%o26)\quad \begin{pmatrix} 5 & 1 & 2 \\ -3 & 9 & 4 \\ 1 & 2 & -7 \end{pmatrix}$$

$$(\%o27)\quad \begin{pmatrix} 10 \\ -14 \\ -33 \end{pmatrix}$$

$$(\%o28)\quad \begin{pmatrix} 0.0 \\ 0.0 \\ 0.0 \end{pmatrix}$$

1
————
$$\begin{pmatrix} 1.799999999999999 & -0.8599999999999999 & 4.253142857142857 \end{pmatrix}$$

2
————
$$\begin{pmatrix} 0.6036685714285714 & -3.006156571428571 & 3.972774269387755 \end{pmatrix}$$

3
————
$$\begin{pmatrix} 0.9712763030204081 & -2.998342473991836 & 3.994010601157784 \end{pmatrix}$$

4
————
$$\begin{pmatrix} 0.9989854592037691 & -2.997742850101166 & 3.999851029130248 \end{pmatrix}$$

5
————
$$\begin{pmatrix} 0.9995458884516973 & -2.999850930126706 & 3.999965049395661 \end{pmatrix}$$

6
————
$$\begin{pmatrix} 0.9999403384855388 & -2.999989011225273 & 3.99999165985835 \end{pmatrix}$$

7
————
$$\begin{pmatrix} 0.9999950583200967 & -2.999997047569838 & 3.999999289823317 \end{pmatrix}$$

8
————
$$\begin{pmatrix} 0.9999992300581864 & -2.99999965166854 & 3.999999919560678 \end{pmatrix}$$

9
————
$$\begin{pmatrix} 0.9999998892643681 & -2.99999996211847 & 3.999999986407011 \end{pmatrix}$$

10
————

10

10.1

(%i33) A1:matrix([5, 1, 2], [−3, 9, 4], [1, 2, −7]);
       b1:matrix([10], [−14], [−33]);
       start:matrix([0.0], [0.0], [0.0]);
       gauss_seidel(A1, b1, start, 10);

(%o30) $\begin{pmatrix} 5 & 1 & 2 \\ -3 & 9 & 4 \\ 1 & 2 & -7 \end{pmatrix}$

(%o31) $\begin{pmatrix} 10 \\ -14 \\ -33 \end{pmatrix}$

(%o32) $\begin{pmatrix} 0.0 \\ 0.0 \\ 0.0 \end{pmatrix}$

1
————
$\begin{pmatrix} 2.0 & -0.8888888888888887 & 4.746031746031746 \end{pmatrix}$

2
————
$\begin{pmatrix} 0.2793650793650792 & -3.571781305114638 & 3.7336860670194 \end{pmatrix}$

3
————
$\begin{pmatrix} 1.220881834215167 & -2.808010973936899 & 4.086408555191624 \end{pmatrix}$

4
————
$\begin{pmatrix} 0.9270387727107303 & -3.062724211403811 & 3.971655764271872 \end{pmatrix}$

5
————
$\begin{pmatrix} 1.023882536572013 & -2.979441716374605 & 4.0092855862604 \end{pmatrix}$

6
————
$\begin{pmatrix} 0.992174108770761 & -3.006735557636591 & 3.996957570499654 \end{pmatrix}$

7
————
$\begin{pmatrix} 1.002564083327456 & -2.997793114668471 & 4.000996836284359 \end{pmatrix}$

8
————
$\begin{pmatrix} 0.9991598884199506 & -3.000723075541953 & 3.999673391048006 \end{pmatrix}$

9
————
$\begin{pmatrix} 1.000275258689188 & -2.999763087569384 & 4.000107011935774 \end{pmatrix}$

10
————

11

(%i34) define(f(x),   LP([0, 1,   −1,   2,   −2],   [−5, −3, −15, 39, −9],   x));

(%o34) $f(x):=1.625\,(x-1)\,x\,(x+1)\,(x+2)+0.5\,(x-2)\,x\,(x+1)$
$(x+2)+1.25\,(1-x)\,(x-2)\,(x+1)\,(x+2)+2.5\,(x-2)\,(x-1)\,x$
$(x+2)-0.375\,(x-2)\,(x-1)\,x\,(x+1)$

(%i35) f(3);

(%o35) 241.0

(%i36) LP([0, 1,   −1,   2,   −2],   [−5, −3, −15, 39, −9], 3);

(%o36) 241.0

12

12.1

(%i37) dd_table([−1, 0, 1, 2, −2, 3], [5, 1,   1,   11, 5, 35]);

(%o37)

$$\begin{vmatrix} -1 & 5 & -4 & 2 & 1 & -0.08333333333333326 & -1.7347234 \\ 0 & 1 & 0 & 5 & 1.083333333333333 & -0.08333333333333333 & \\ 1 & 1 & 10 & 2.833333333333333 & 0.8333333333333333 & A_{3,6} & \\ 2 & 11 & 1.5 & 4.5 & A_{4,5} & A_{4,6} & \\ -2 & 5 & 6 & A_{5,4} & A_{5,5} & A_{5,6} & \\ 3 & 35 & A_{6,3} & A_{6,4} & A_{6,5} & A_{6,6} & \end{vmatrix}$$

12.2

(%i38) NP([−1, 0, 1, 2,   −2,   3], [5, 1,   1,   11, 5,   35],   x);

(%o38) $-1.734723475976807\,10^{-17}\,(x-2)\,(x-1)\,x\,(x+1)$
$(x+2)-0.08333333333333326\,(x-2)\,(x-1)\,x\,(x+1)+(x-1)\,x$
$(x+1)+2\,x\,(x+1)-4\,(x+1)+5$

(%i39) NP([−1, 0, 1, 2,   −2,   3], [5, 1,   1,   11, 5,   35], 1.5);

(%o39) 4.453125

13

(%i40) ratprint:false;
(%o40) **false**

13.1

(%i41) T(1/x,   1.0,   2.0);
(%o41) 0.75

(%i42) foo(g,   x,   a,   b):=block(

        [t, v,   e],
        numer:true,
        local(f),
        define(f(x),   g),
        t:T(f(x),   a,   b),
        v:integrate(f(x),   x,   a, b),
        e:abs(v−t),
        print(t, "   ", v, "   ", e)
      );

(%o42) foo**(g,x,a,b):=**block$\left([t,v,e],numer:true,\text{local}(f),\text{define}(f(x),g),t:T(f(x),a\right.$

(%i43) foo(1/x,   x,   1,   2);
        0.75   0.6931471805599453   0.05685281944005471
(%o43) 0.05685281944005471

13.2

(%i44) for k:2 next 2·k thru 150 do block(
            temp:Tc(sin(x),   0,   %pi,   k),
            print(k, "   ", temp)
            );
        2   1.570796326794896
        4   1.896118897937039
        8   1.97423160194555
        16   1.993570343772339
        32   1.998393360970144
        64   1.999598388640037
        128   1.999899600184203
(%o44) *done*

14

14.1

(%i45) S(1/x,   1.0,   2.0);
(%o45) 0.6944444444444443

14.2

(%i46) for k:2 next 2·k thru 150 do block(
            temp:Sc(sin(x),   0,   %pi,   k),
            print(k, "   ", temp)
            );
        2   2.094395102393195
        4   2.00455975498442
        8   2.0000269169948387
        16   2.000016591047935
        32   2.000001033369412
        64   2.000000064530001
        128   2.000000004032257
(%o46) *done*

15

(%i47) eulerm(1+(x/t),   1.0,   1.0,   6,   10);
        1.0 −−−− 1.0
        1.5 −−−− 2.0
        2.0 −−−− 3.166666666666666
        2.5 −−−− 4.458333333333333
        3.0 −−−− 5.85
        3.5 −−−− 7.324999999999999
        4.0 −−−− 8.87142857142857
        4.5 −−−− 10.48035714285714
        5.0 −−−− 12.14484126984126
        5.5 −−−− 13.85932539682539
        6.0 −−−− 15.61926406926406
(%o47) *done*

16

(%i48) eulermod(1+(x/t), 1.0,   1.0, 6, 10);

        1.0 ———— 1.0
        1.5 ———— 2.1
        2.0 ———— 3.371428571428571
        2.5 ———— 4.76984126984127
        3.0 ———— 6.26926406926407
        3.5 ———— 7.852602952602953
        4.0 ———— 9.507736707736708
        4.5 ———— 11.22561556090967
        5.0 ———— 12.99922196826221
        5.5 ———— 14.82295368889796
        6.0 ———— 16.69223406377801

(%o48) *done*

17

(%i49) heun(1+(x/t), 1.0,   1.0, 6, 10);

        1.0 ———— 1.0
        1.5 ———— 2.083333333333333
        2.0 ———— 3.340277777777777
        2.5 ———— 4.725347222222221
        3.0 ———— 6.212083333333332
        3.5 ———— 7.78314484126984
        4.0 ———— 9.426272675736959
        4.5 ———— 11.13233453798185
        5.0 ———— 12.89426059775761
        5.5 ———— 14.70641393026065
        6.0 ———— 16.56419398452677

(%o49) *done*

18

(%i50) rk2(1+(x/t), 1.0,   1.0, 6, 10);

        1.0 ———— 1.0
        1.5 ———— 2.09375
        2.0 ———— 3.359848484848484
        2.5 ———— 4.753382034632034
        3.0 ———— 6.248176088617265
        3.5 ———— 7.827038770053475
        4.0 ———— 9.477795861427574
        4.5 ———— 11.19136649795217
        5.0 ———— 12.96071373335682
        5.5 ———— 14.7802226066925
        6.0 ———— 16.64530777872948

(%o50) *done*