Examples for numerical.mac

1

```
(%i1) load("/home/jvr/Downloads/numericalPackage/numerical.mac");
(%o1) /home/jvr/Downloads/numericalPackage/numerical.mac
```

2

2.1

```
(%i2) bisect(t^3−2·sin(t),  t,  0.50,  2.00,  20,  0.0005);
```

iter	*m*	*ym*	*error*
1	1.25	0.0551557612888276	0.75
2	0.875	−0.8651651294720542	0.375
3	1.0625	−0.5476869797091422	0.1875
4	1.15625	−0.2847914007983883	0.09375
5	1.203125	−0.1247986155094702	0.046875
6	1.2265625	−0.03735980652509796	0.0234375
7	1.23828125	0.00825801590073083	0.01171875
8	1.232421875	−0.01471021624269308	0.005859375
9	1.2353515625	−0.003266014170569153	0.0029296875
10	1.23681640625	0.002486011901918328	0.00146484375
11	1.236083984375	$-3.924970675475148\ 10^{-4}$	$7.32421875\ 10^{-4}$
12	1.2364501953125	0.001046133270412585	$3.662109375\ 10^{-4}$

```
(%o2) bisection has converged
```

2.2

(%i3) `bisect(sin(x), x, 2, 4, 20, 0.0005);`

iter	*m*	*ym*	*error*
1	3	0.1411200080598672	1
2	3.5	−0.3507832276896198	0.5
3	3.25	−0.1081951345301083	0.25
4	3.125	0.0165918922293479	0.125
5	3.1875	−0.04589122327277969	0.0625
6	3.15625	−0.01465682159049232	0.03125
7	3.140625	$9.676534387822795\ 10^{-4}$	0.015625
8	3.1484375	−0.006844792961296519	0.0078125
9	3.14453125	−0.002938592180907726	0.00390625
10	3.142578125	$-9.854712506993688\ 10^{-4}$	0.001953125
11	3.1416015625	$-8.908910206643689\ 10^{-6}$	$9.765625\ 10^{-4}$
12	3.14111328125	$4.793723214334506\ 10^{-4}$	$4.8828125\ 10^{-4}$

(%o3) *bisection has converged*

2.3

(%i4)
```
bisect(x^2-2, x, 1, 2, 20, 0.0005);
```

iter	*m*	*ym*	*error*
1	1.5	0.25	0.5
2	1.25	−0.4375	0.25
3	1.125	−0.734375	0.125
4	1.0625	−0.87109375	0.0625
5	1.03125	−0.9365234375	0.03125
6	1.015625	−0.968505859375	0.015625
7	1.0078125	−0.98431396484375	0.0078125
8	1.00390625	−0.9921722412109375	0.00390625
9	1.001953125	−0.9960899353027344	0.001953125
10	1.0009765625	−0.9980459213256836	$9.765625\,10^{-4}$
11	1.00048828125	−0.9990231990814209	$4.8828125\,10^{-4}$

(%o4) *bisection has converged*

(%i5) `bisect(x^3−2·x^2+x−%pi, x, 2, 3, 10, 0.0001);`

iter	*m*	*ym*	*error*
1	2.5	2.483407346410207	0.5
2	2.25	0.3740323464102069	0.25
3	2.125	−0.4521395285897931	0.125
4	2.1875	−0.05687585671479311	0.0625
5	2.21875	0.1540311257070818	0.03125
6	2.203125	0.04745229880278501	0.015625
7	2.1953125	−0.004991682367869288	0.0078125
8	2.19921875	0.02116015355055722	0.00390625
9	2.197265625	0.008066719276360601	0.001953125
10	2.1962890625	0.001533142169467538	$9.765625\,10^{-4}$

(%o5) *done*

3

3.1

(%i6) `newton(x^3−2·x^2+x−3, x, 3, 0.0001, 0.00001, 8);`

1	2.4375	2.036865234375	
2	2.213032716315109	0.2563633850614177	
3	2.175554938721488	0.006463361488812325	
4	2.174560100666446	4.479068049789703	10^{-6}
5	2.174559410293312	2.156497203031904	10^{-12}

(%o6) *convergence*

3.2

(%i7) `newton(sin(x), x, 3.0, 0.0001, 0.00001, 8);`

1	3.142546543074278	−9.538893398264409
	10^{-4}	
2	3.141592653300477	2.893162490762184
	10^{-10}	
3	3.141592653589793	1.224646799147353
	10^{-16}	

(%o7) *convergence*

3.3

(%i8) `newton(x^2−2, x, 1, 0.0001, 0.00001, 8);`

1	1.5	0.25
2	1.416666666666666	0.006944444444444642
3	1.414215686274509	6.007304882871267
	10^{-6}	
4	1.414213562374689	4.510614104447086
	10^{-12}	

(%o8) *convergence*

(%i9) `newton(x^3−2·x^2+x−%pi, x, 3, 0.0001, 0.00001, 8);`

1	2.446349540849362	1.975991988443487
2	2.230829413793257	0.237982391674409
3	2.196863381645069	0.005374496729482203
4	2.196060159694346	2.961172185678151
	10^{-6}	
5	2.196059716657127	9.00612917575927
	10^{-13}	

(%o9) *convergence*

4

4.1

(%i10) `secant(x^3+2·x^2−3·x−1, x, −2, −3, 0.00001, 10);`

0	−3	−1
1	−2	5
2	−2.833333333333333	0.8101851851851833
3	−2.907928388746803	0.04629957161572662
4	−2.912449640422374	−0.002380064066290543
5	−2.912228585591192	$6.399876401275151\,10^{-6}$

(%o10) *convergence*

4.2

(%i11) `secant(sin(x), x, 1, 4, 0.00001, 10);`

0	4	−0.7568024953079282
1	1	0.8414709848078965
2	2.579462454848934	0.5329898131328342
3	3.166481028136553	−0.02488580518710502
4	3.140295209333355	0.001297443892426365
5	3.141592780561861	$-1.269720678483891\,10^{-7}$

(%o11) *convergence*

4.3

(%i12) `secant(x^2−2, x, 1, 2, 0.00001, 10);`

0	1	−1
1	2	2
2	1.333333333333333	−0.2222222222222223
3	1.428571428571428	0.04081632653061229
4	1.413793103448275	−0.001189060642093009
5	1.41421143847487	$-6.007286838860537\,10^{-6}$

(%o12) *convergence*

5

5.1

(%i13) `regula(x^3+2·x^2−3·x−1, x, 1, 2, 20, 0.0005);`

iter	*m*	*ym*
1	1.1	−0.5489999999999999
2	1.221729490022173	0.1436388950901026
3	1.193880682944684	−0.02924075169187978
4	1.199720665327255	0.006290908737020207
5	1.198472095487697	−0.001337706025862806
6	1.198737948438891	$2.851619710524389\,10^{-4}$

(%o13) *regula falsi method has converged*

5.2

(%i14) `regula(sin(x), x, 2, 4, 20, 0.0005);`

iter	*m*	*ym*
1	3.091528082734958	0.05004365932452196
2	3.147874957380742	−0.006282262466726139
3	3.141590357955694	$2.295634098490862\,10^{-6}$

(%o14) *regula falsi method has converged*

5.3

(%i15) `regula(x^2−2, x, 1, 2, 20, 0.0005);`

iter	*m*	*ym*
1	1.333333333333333	−0.2222222222222218
2	1.428571428571428	0.04081632653061229
3	1.411764705882353	−0.006920415224913157
4	1.414634146341463	0.001189767995240842
5	1.414141414141414	$-2.040608101214758\,10^{-4}$

(%o15) *regula falsi method has converged*

6

6.1

```
(%i18) A:matrix([1,  2,  3], [3,  5,  7],  [4,  6,  9]);
       luFactor(A);
       luFactor(A)[1];
```

$$(\%o16)\quad \begin{pmatrix} 1 & 2 & 3 \\ 3 & 5 & 7 \\ 4 & 6 & 9 \end{pmatrix}$$

$$(\%o17)\quad \left[L=\begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 4 & 2 & 1 \end{pmatrix},U=\begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & -2 \\ 0 & 0 & 1 \end{pmatrix}\right]$$

$$(\%o18)\quad L=\begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 4 & 2 & 1 \end{pmatrix}$$

7

7.1

```
(%i21) A:matrix([1,  2,  3], [3,  5,  7],  [4,  6,  9]);
       b:matrix([1], [2], [3]);
       solve_by_lu(A,   b);
```

$$(\%o19)\quad \begin{pmatrix} 1 & 2 & 3 \\ 3 & 5 & 7 \\ 4 & 6 & 9 \end{pmatrix}$$

$$(\%o20)\quad \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$(\%o21)\quad \left[z=\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix},x=\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}\right]$$

8

8.1

(%i25)
```
A1:matrix([5,  1,  2], [−3,  9,  4], [1,  2,  −7]);
b1:matrix([10],  [−14],  [−33]);
start:matrix([0],  [0],  [0]);
gauss_jacobi(A1,  b1,  start,  14);
```

(%o22)
$$\begin{pmatrix} 5 & 1 & 2 \\ -3 & 9 & 4 \\ 1 & 2 & -7 \end{pmatrix}$$

(%o23)
$$\begin{pmatrix} 10 \\ -14 \\ -33 \end{pmatrix}$$

(%o24)
$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

1

$$\begin{pmatrix} 2.0 & -1.555555555555555 & 4.714285714285714 \end{pmatrix}$$

2

$$\begin{pmatrix} 0.4253968253968252 & -2.984126984126984 & 4.555555555555555 \end{pmatrix}$$

3

$$\begin{pmatrix} 0.7746031746031745 & -3.438447971781304 & 3.922448979591836 \end{pmatrix}$$

4

$$\begin{pmatrix} 1.118710002519526 & -3.040665154950868 & 3.842529604434367 \end{pmatrix}$$

5

$$\begin{pmatrix} 1.071121189216427 & -2.890443156686543 & 4.005339956088256 \end{pmatrix}$$

6

$$\begin{pmatrix} 0.9759526489020063 & -2.97866625074486 & 4.041462125120478 \end{pmatrix}$$

7

$$\begin{pmatrix} 0.9791484001007809 & -3.026443394863987 & 4.002660021058898 \end{pmatrix}$$

8

$$\begin{pmatrix} 1.004224670549238 & -3.008132764881471 & 3.989465944338972 \end{pmatrix}$$

9

$$\begin{pmatrix} 1.005840175240705 & -2.993909973967574 & 3.998279877255185 \end{pmatrix}$$

10

9

9.1

(%i29)
```
A1:matrix([5,  1,  2], [−3,  9,  4], [1,  2,  −7]);
b1:matrix([10],  [−14],  [−33]);
start:matrix([0],  [0],  [0]);
sor(A1,  b1,  0.9,  start,  10);
```

(%o26)
$$\begin{pmatrix} 5 & 1 & 2 \\ -3 & 9 & 4 \\ 1 & 2 & -7 \end{pmatrix}$$

(%o27)
$$\begin{pmatrix} 10 \\ -14 \\ -33 \end{pmatrix}$$

(%o28)
$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

1

$$\begin{pmatrix} 1.799999999999999 & -0.8599999999999999 & 4.253142857142857 \end{pmatrix}$$

2

$$\begin{pmatrix} 0.6036685714285714 & -3.006156571428571 & 3.972774269387755 \end{pmatrix}$$

3

$$\begin{pmatrix} 0.9712763030204081 & -2.998342473991836 & 3.994010601157784 \end{pmatrix}$$

4

$$\begin{pmatrix} 0.9989854592037691 & -2.997742850101166 & 3.999851029130248 \end{pmatrix}$$

5

$$\begin{pmatrix} 0.9995458884516973 & -2.999850930126706 & 3.999965049395661 \end{pmatrix}$$

6

$$\begin{pmatrix} 0.9999403384855388 & -2.999989011225273 & 3.99999165985835 \end{pmatrix}$$

7

$$\begin{pmatrix} 0.9999950583200967 & -2.999997047569838 & 3.999999289823317 \end{pmatrix}$$

8

$$\begin{pmatrix} 0.9999992300581864 & -2.999999651668854 & 3.999999919560678 \end{pmatrix}$$

9

$$\begin{pmatrix} 0.9999998892643681 & -2.999999966211847 & 3.999999986407011 \end{pmatrix}$$

10

10

10.1

(%i33)
```
A1:matrix([5,  1,  2], [−3,  9,  4], [1,  2,  −7]);
b1:matrix([10],  [−14],  [−33]);
start:matrix([0],  [0],  [0]);
gauss_seidel(A1,  b1,  start,  10);
```

(%o30)
$$\begin{pmatrix} 5 & 1 & 2 \\ -3 & 9 & 4 \\ 1 & 2 & -7 \end{pmatrix}$$

(%o31)
$$\begin{pmatrix} 10 \\ -14 \\ -33 \end{pmatrix}$$

(%o32)
$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

1

————

(2.0 −0.8888888888888887 4.746031746031746)

2

————

(0.2793650793650792 −3.571781305114638 3.7336860670194)

3

————

(1.220881834215167 −2.808010973936899 4.086408555191624)

4

————

(0.9270387727107303 −3.062724211403811 3.971655764271872)

5

————

(1.023882536572013 −2.979441716374605 4.0092855862604)

6

————

(0.992174108770761 −3.006735557636591 3.996957570499654)

7

————

(1.002564083327456 −2.997793114668471 4.000996836284359)

8

————

(0.9991598884199506 −3.000723075541953 3.999673391048006)

9

————

(1.000275258689188 −2.999763087569384 4.000107011935774)

10

————

11

```
(%i36) xval:[0, 1, −1, 2, −2]$
       yval:[−5, −3, −15, 39, −9]$
       LP(xval, yval, x);
```

(%o36) $1.625\,(x-1)\,x\,(x+1)\,(x+2)+0.5\,(x-2)\,x\,(x+1)\,(x+2)+1.25\,(1-x)\,(x-2)\,(x+1)\,(x+2)+2.5\,(x-2)\,(x-1)\,x\,(x+2)-0.375\,(x-2)\,(x-1)\,x\,(x+1)$

```
(%i37) LP(xval, yval, 3);
```

(%o37) 241.0

12

12.1

```
(%i38) dd_table([−1, 0, 1, 2, −2, 3], [5, 1, 1, 11, 5, 35]);
```

(%o38)

$$\begin{pmatrix} -1 & 5 & -4 & 2 & 1 & -0.0833333333333326 & -1.734723 \\ 0 & 1 & 0 & 5 & 1.083333333333333 & -0.0833333333333333 & \\ 1 & 1 & 10 & 2.833333333333333 & 0.8333333333333333 & A_{3,6} & \\ 2 & 11 & 1.5 & 4.5 & A_{4,5} & A_{4,6} & \\ -2 & 5 & 6 & A_{5,4} & A_{5,5} & A_{5,6} & \\ 3 & 35 & A_{6,3} & A_{6,4} & A_{6,5} & A_{6,6} & \end{pmatrix}$$

12.2

```
(%i41) xval:[−1, 0, 1, 2, −2, 3]$
       yval:[5, 1, 1, 11, 5, 35]$
       NP(xval, yval, x);
```

(%o41) $-1.734723475976807\,10^{-17}\,(x-2)\,(x-1)\,x\,(x+1)\,(x+2)-0.0833333333333326\,(x-2)\,(x-1)\,x\,(x+1)+(x-1)\,x\,(x+1)+2\,x\,(x+1)-4\,(x+1)+5$

```
(%i42) NP(xval, yval, 1.5);
```

(%o42) 4.453125

13

```
(%i43) ratprint:false;
(%o43) false
```

13.1

```
(%i44) T(1/s,    s,    1.0,    2.0);
(%o44) 0.75

(%i45) compare(g,    x,    a,    b):=block(

          [t,   v,    e],
          numer:true,
          local(f),
          define(f(x),    g),
          t:T(f(x),   x,    a,    b),
          v:integrate(f(x),    x,    a,   b),
          e:abs(v−t),
          print(t, "   ",   v,  "      ", e)
        );

(%o45) compare(g,x,a,b):=block([t,v,e],numer:true,local(f),define(f(x),g),t:T(f
```

```
(%i46) compare(1/x,    x,   1,    2);
       0.75     0.6931471805599453       0.05685281944005471
(%o46) 0.05685281944005471
```

13.2

```
(%i47) for k:2 next 2·k thru 150 do block(
                temp:Tc(sin(x),    x,   0,   %pi,   k),
                print(k, "      ", temp)
                );
       2        1.570796326794896
       4        1.896118897937039
       8        1.97423160194555
       16        1.993570343772339
       32        1.998393360970144
       64        1.999598388640037
       128        1.999899600184203
(%o47) done
```

14

14.1

```
(%i48) S(1/u,   u,   1.0,   2.0);
(%o48) 0.6944444444444443
```

14.2

```
(%i49) for k:2 next 2·k thru 150 do block(
                temp:Sc(sin(t),   t,   0,   %pi,   k),
                print(k, "      ", temp)
                );
       2        2.094395102393195
       4        2.00455975498442
       8        2.000269169948387
       16        2.000016591047935
       32        2.000001033369412
       64        2.000000064530001
       128        2.000000004032257
(%o49) done
```

15

```
(%i50) eulerm(1+(x/t),   t,   x,   1.0,   1.0,   6,   10);
       1.0 ---- 1.0
       1.5 ---- 2.0
       2.0 ---- 3.166666666666666
       2.5 ---- 4.458333333333333
       3.0 ---- 5.85
       3.5 ---- 7.324999999999999
       4.0 ---- 8.87142857142857
       4.5 ---- 10.48035714285714
       5.0 ---- 12.14484126984126
       5.5 ---- 13.85932539682539
       6.0 ---- 15.61926406926406
(%o50) done
```

16

```
(%i51) eulermod(1+(x/t),  t,  x,  1.0,  1.0,  6,  10);
       1.0 ----  1.0
       1.5 ----  2.1
       2.0 ----  3.371428571428571
       2.5 ----  4.76984126984127
       3.0 ----  6.26926406926407
       3.5 ----  7.852602952602953
       4.0 ----  9.507736707736708
       4.5 ----  11.22561556090967
       5.0 ----  12.99922196826221
       5.5 ----  14.82295368889796
       6.0 ----  16.69223406377801
(%o51) done
```

17

```
(%i52) heun(1+(x/t),  t,  x,  1.0,  1.0,  6,  10);
       1.0 ----  1.0
       1.5 ----  2.083333333333333
       2.0 ----  3.340277777777777
       2.5 ----  4.725347222222221
       3.0 ----  6.212083333333332
       3.5 ----  7.78314484126984
       4.0 ----  9.426272675736959
       4.5 ----  11.13233453798185
       5.0 ----  12.89426059775761
       5.5 ----  14.70641393026065
       6.0 ----  16.56419398452677
(%o52) done
```

18

```
(%i53) rk2(1+(x/t),  t,  x,  1.0,  1.0,  6,  10);
       1.0  ----  1.0
       1.5  ----  2.09375
       2.0  ----  3.359848484848484
       2.5  ----  4.753382034632034
       3.0  ----  6.248176088617265
       3.5  ----  7.827038770053475
       4.0  ----  9.477795861427574
       4.5  ----  11.19136649795217
       5.0  ----  12.96071373335682
       5.5  ----  14.7802226066925
       6.0  ----  16.64530777872948
(%o53) done
```

18.1

(%i57)
```
A1:matrix([5,  1,  2], [−3,  9,  4], [1,  2,  −7]);
b1:matrix([10],  [−14],  [−33]);
start:matrix([0],  [0],  [0]);
gauss_jacobi(A1,  b1,  start,  14);
```

(%o54)
$$\begin{pmatrix} 5 & 1 & 2 \\ -3 & 9 & 4 \\ 1 & 2 & -7 \end{pmatrix}$$

(%o55)
$$\begin{pmatrix} 10 \\ -14 \\ -33 \end{pmatrix}$$

(%o56)
$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

1

$$\begin{pmatrix} 2.0 & -1.555555555555555 & 4.714285714285714 \end{pmatrix}$$

2

$$\begin{pmatrix} 0.4253968253968252 & -2.984126984126984 & 4.555555555555555 \end{pmatrix}$$

3

$$\begin{pmatrix} 0.7746031746031745 & -3.438447971781304 & 3.922448979591836 \end{pmatrix}$$

4

$$\begin{pmatrix} 1.118710002519526 & -3.040665154950868 & 3.842529604434367 \end{pmatrix}$$

5

$$\begin{pmatrix} 1.071121189216427 & -2.890443156686543 & 4.005339956088256 \end{pmatrix}$$

6

$$\begin{pmatrix} 0.9759526489020063 & -2.97866625074486 & 4.041462125120478 \end{pmatrix}$$

7

$$\begin{pmatrix} 0.9791484001007809 & -3.026443394863987 & 4.002660021058898 \end{pmatrix}$$

8

$$\begin{pmatrix} 1.004224670549238 & -3.008132764881471 & 3.989465944338972 \end{pmatrix}$$

9

$$\begin{pmatrix} 1.005840175240705 & -2.993909973967574 & 3.998279877255185 \end{pmatrix}$$

10