

Examples for numerical.mac

1

```
(%i1) load("/home/jvr/Downloads/numericalPackage/numerical.mac");
(%o1) /home/jvr/Downloads/numericalPackage/numerical.mac
```

2

2.1

```
(%i2) bisect(t^3-2·sin(t), t, 0.50, 2.00, 20, 0.0005);
```

iter	m	ym	error
1	1.25	0.0551557612888276	
	0.75		
2	0.875	-0.8651651294720542	
	0.375		
3	1.0625	-0.5476869797091422	
	0.1875		
4	1.15625	-0.2847914007983883	
	0.09375		
5	1.203125	-	
	0.1247986155094702	0.046875	
6	1.2265625	-	
	0.03735980652509796	0.0234375	
7	1.23828125		
	0.00825801590073083	0.01171875	
8	1.232421875	-	
	0.01471021624269308	0.005859375	
9	1.2353515625	-	
	0.003266014170569153	0.0029296875	
10	1.23681640625		
	0.002486011901918328	0.00146484375	
11	1.236083984375	-	
	3.924970675475148 10 ⁻⁴	7.32421875 10 ⁻⁴	
12	1.2364501953125		
	0.001046133270412585	3.662109375 10 ⁻⁴	

```
(%o2) bisection has converged
```

2.2

→ `bisect(sin(x), x, 2.00, 4.00, 20, 0.0005);`

<i>iter</i>	<i>m</i>	<i>ym</i>	<i>error</i>
1	3.0	0.1411200080598672	
	1.0		
2	3.5	-0.3507832276896198	
	0.5		
3	3.25	-0.1081951345301083	
	0.25		
4	3.125	0.0165918922293479	
	0.125		
5	3.1875	-0.04589122327277969	
	0.0625		
6	3.15625	-	
	0.01465682159049232	0.03125	
7	3.140625	9.676534387822795	
	10^{-4}		
10	0.015625		
8	3.1484375	-	
	0.006844792961296519	0.0078125	
9	3.14453125	-	
	0.002938592180907726	0.00390625	
10	3.142578125	-	
	10^{-4}		
	9.854712506993688 10^{-4}	0.001953125	
11	3.1416015625	-	
	10^{-6}		
	8.908910206643689 10^{-6}	9.765625 10^{-4}	
12	3.14111328125		
	10^{-4}		
	4.793723214334506 10^{-4}	4.8828125 10^{-4}	

(%o3) *bisection has converged*

2.3

```
(%i4) bisect(x^2-2, x, 1.00, 2.00, 20, 0.0005);
```

<i>iter</i>	<i>m</i>	<i>ym</i>	<i>error</i>
1	1.5	0.25	0.5
2	1.25	-0.4375	0.25
3	1.375	-0.109375	
0.125			
4	1.4375	0.06640625	
0.0625			
5	1.40625	-0.0224609375	
0.03125			
6	1.421875	0.021728515625	
0.015625			
7	1.4140625	-4.2724609375	
10^{-4}			
10	0.0078125		
8	1.41796875		
0.0106353759765625		0.00390625	
9	1.416015625		
0.005100250244140625		0.001953125	
10	1.4150390625		
0.002335548400878906		$9.765625 \cdot 10^{-4}$	
11	1.41455078125		
$9.539127349853516 \cdot 10^{-4}$		$4.8828125 \cdot 10^{-4}$	

```
(%o4) bisection has converged
```

```
(%i5) bisect(x^3-2·x^2+x-%pi, x, 2.0, 3.0, 10, 0.0001);
```

<i>iter</i>	<i>m</i>	<i>ym</i>	<i>error</i>
1	2.5	2.483407346410207	
	0.5		
2	2.25	0.3740323464102069	
	0.25		
3	2.125	-0.4521395285897931	
	0.125		
4	2.1875	-0.05687585671479311	
	0.0625		
5	2.21875	0.1540311257070818	
	0.03125		
6	2.203125		
	0.04745229880278501	0.015625	
7	2.1953125	-	
	0.004991682367869288	0.0078125	
8	2.19921875		
	0.02116015355055722	0.00390625	
9	2.197265625		
	0.008066719276360601	0.001953125	
10	2.1962890625		
	0.001533142169467538	9.765625 10 ⁻⁴	

```
(%o5) done
```

3

3.1

```
(%i6) newton(x^3-2·x^2+x-3, x, 3.0, 0.0001, 0.00001, 8);
```

1	2.4375	2.036865234375
2	2.213032716315109	0.2563633850614177
3	2.175554938721488	0.006463361488812325
4	2.174560100666446	4.479068049789703
10 ⁻⁶		
5	2.174559410293312	2.156497203031904
10 ⁻¹²		

```
(%o6) convergence
```

3.2

```
(%i7) newton(sin(x), x, 3.0, 0.0001, 0.00001, 8);
1      3.142546543074278      -9.538893398264409
10-4
2      3.141592653300477      2.893162490762184
10-10
3      3.141592653589793      1.224646799147353
10-16
(%o7) convergence
```

3.3

```
(%i8) newton(x^2-2, x, 1.0, 0.0001, 0.00001, 8);
1      1.5      0.25
2      1.416666666666666      0.006944444444444642
3      1.414215686274509      6.007304882871267
10-6
4      1.414213562374689      4.510614104447086
10-12
(%o8) convergence
```

```
(%i9) newton(x^3-2·x^2+x-%pi, x, 3.0, 0.0001, 0.00001, 8);
1      2.446349540849362      1.975991988443487
2      2.230829413793257      0.237982391674409
3      2.196863381645069      0.005374496729482203
4      2.196060159694346      2.961172185678151
10-6
5      2.196059716657127      9.00612917575927
10-13
(%o9) convergence
```

4

4.1

```
(%i10) secant(x^3+2·x^2-3·x-1, x, -2.0, -3.0, 0.00001, 10);
```

0	-3.0	-1.0	
1	-2.0	5.0	
2	-2.833333333333333	0.8101851851851833	
3	-2.907928388746803	0.04629957161572662	
4	-2.912449640422374	-0.002380064066290543	
5	-2.912228585591192	6.399876401275151	10^{-6}

```
(%o10) convergence
```

4.2

```
(%i11) secant(sin(x), x, 1.0, 4.0, 0.00001, 10);
```

0	4.0	-0.7568024953079282	
1	1.0	0.8414709848078965	
2	2.579462454848934	0.5329898131328342	
3	3.166481028136553	-0.02488580518710502	
4	3.140295209333355	0.001297443892426365	
5	3.141592780561861	-1.269720678483891	10^{-7}

```
(%o11) convergence
```

4.3

```
(%i12) secant(x^2-2, x, 1.0, 2.0, 0.00001, 10);
```

0	1.0	-1.0	
1	2.0	2.0	
2	1.333333333333333	-0.2222222222222223	
3	1.428571428571428	0.04081632653061229	
4	1.413793103448275	-0.001189060642093009	
5	1.41421143847487	-6.007286838860537	10^{-6}

```
(%o12) convergence
```

5

5.1

```
(%i13) regula(x^3+2·x^2-3·x-1, x, 1.0, 2.0, 20, 0.0005);
```

<i>iter</i>	<i>m</i>	<i>ym</i>
1	1.1	-0.5489999999999999
2	1.151743638077285	—
	0.2744007202116687	
3	1.176840909982786	—
	0.1307425288092163	
4	1.188627673293828	—
	0.06087586326028837	
5	1.194078911293239	—
	0.02804093844229971	
6	1.196582088205247	—
	0.01285224023453723	
7	1.197727754386817	—
	0.005877241523802867	
8	1.198251317792008	—
	0.002684816279181046	
9	1.198490418455614	—
	0.001225881007669516	
10	1.198599576406594	—
	5.596124951825487 10 ⁻⁴	
11	1.19864940371845	—
	2.554366889526704 10 ⁻⁴	

```
(%o13) regula falsi method has converged
```

5.2

```
(%i14) regula(sin(x), x, 2.0, 4.0, 20, 0.0005);
```

<i>iter</i>	<i>m</i>	<i>ym</i>
1	3.091528082734958	
	0.05004365932452196	
2	3.147874957380742	—
	0.006282262466726139	
3	3.141590357955694	
	2.295634098490862 10 ⁻⁶	

```
(%o14) regula falsi method has converged
```

5.3

```
(%i15) regula(x^2-2, x, 1.0, 2.0, 20, 0.0005);
```

iter	m	ym
1	1.3333333333333333	—
	0.22222222222222218	
2	1.4	—0.040000000000000026
3	1.411764705882352	—
	0.006920415224913823	
4	1.413793103448275	—
	0.001189060642093009	
5	1.414141414141414	—
	2.040608101208096 10 ⁻⁴	

```
(%o15) regula falsi method has converged
```

6

6.1

```
(%i18) A:matrix([1, 2, 3], [3, 5, 7], [4, 6, 9]);
luFactor(A);
luFactor(A)[1];
```

```
(%o16) 
$$\begin{pmatrix} 1 & 2 & 3 \\ 3 & 5 & 7 \\ 4 & 6 & 9 \end{pmatrix}$$

```

```
(%o17) 
$$[L = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 4 & 2 & 1 \end{pmatrix}, U = \begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & -2 \\ 0 & 0 & 1 \end{pmatrix}]$$

```

```
(%o18) 
$$L = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 4 & 2 & 1 \end{pmatrix}$$

```

7

7.1


```
(%i21) A:matrix([1, 2, 3], [3, 5, 7], [4, 6, 9]);
      b:matrix([1], [2], [3]);
      solve_by_lu(A, b);
```

```
(%o19) 
$$\begin{pmatrix} 1 & 2 & 3 \\ 3 & 5 & 7 \\ 4 & 6 & 9 \end{pmatrix}$$

```

```
(%o20) 
$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

```

```
(%o21) 
$$\mathbf{L} \mathbf{z} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \mathbf{x} = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} \mathbf{I}$$

```

8

8.1

```
(%i25) A1:matrix([5, 1, 2], [-3, 9, 4], [1, 2, -7]);
      b1:matrix([10], [-14], [-33]);
      start:matrix([0.0], [0.0], [0.0]);
      gauss_jacobi(A1, b1, start, 14);
```

```
(%o22) 
$$\begin{pmatrix} 5 & 1 & 2 \\ -3 & 9 & 4 \\ 1 & 2 & -7 \end{pmatrix}$$

```

```
(%o23) 
$$\begin{pmatrix} 10 \\ -14 \\ -33 \end{pmatrix}$$

```

```
(%o24) 
$$\begin{pmatrix} 0.0 \\ 0.0 \\ 0.0 \end{pmatrix}$$

```

```
1
```

```
-----
```

```
(2.0 -1.5555555555555555 4.714285714285714)
```

```
2
```

```
-----
```

```
(0.4253968253968252 -2.984126984126984 4.555555555555555)
```

```
3
```

```
-----
```

```
(0.7746031746031745 -3.438447971781304 3.922448979591836)
```

```
4
```

```
-----
```

```
(1.118710002519526 -3.040665154950868 3.842529604434367)
```

```
5
```

```
-----
```

```
(1.071121189216427 -2.890443156686543 4.005339956088256)
```

```
6
```

```
-----
```

```
(0.9759526489020063 -2.97866625074486 4.041462125120478)
```

```
7
```

```
-----
```

```
(0.9791484001007809 -3.026443394863987 4.002660021058898)
```

```
8
```

```
-----
```

```
(1.004224670549238 -3.008132764881471 3.989465944338972)
```

```
9
```

```
-----
```

```
(1.005840175240705 -2.993909973967574 3.998279877255185)
```

```
10
```

```
-----
```

9

9.1

```
(%i29) A:matrix([5, 1, 2], [-3, 9, 4], [1, 2, -7]);
      b:matrix([10], [-14], [-33]);
      start:matrix([0.0], [0.0], [0.0]);
      sor(A, b, 0.9, start, 10);
```

```
(%o26) 
$$\begin{pmatrix} 5 & 1 & 2 \\ -3 & 9 & 4 \\ 1 & 2 & -7 \end{pmatrix}$$

```

```
(%o27) 
$$\begin{pmatrix} 10 \\ -14 \\ -33 \end{pmatrix}$$

```

```
(%o28) 
$$\begin{pmatrix} 0.0 \\ 0.0 \\ 0.0 \end{pmatrix}$$

```

```
1
```

```
-----
```

```
(1.7999999999999999 -0.8599999999999999 4.253142857142857)
```

```
2
```

```
-----
```

```
(0.6036685714285714 -3.006156571428571 3.972774269387755)
```

```
3
```

```
-----
```

```
(0.9712763030204081 -2.998342473991836 3.994010601157784)
```

```
4
```

```
-----
```

```
(0.9989854592037691 -2.997742850101166 3.999851029130248)
```

```
5
```

```
-----
```

```
(0.9995458884516973 -2.999850930126706 3.999965049395661)
```

```
6
```

```
-----
```

```
(0.9999403384855388 -2.999989011225273 3.99999165985835)
```

```
7
```

```
-----
```

```
(0.9999950583200967 -2.999997047569838 3.999999289823317)
```

```
8
```

```
-----
```

```
(0.9999992300581864 -2.999999651668854 3.999999919560678)
```

```
9
```

```
-----
```

```
(0.9999998892643681 -2.999999966211847 3.999999986407011)
```

```
10
```

```
-----
```

10

10.1

```
(%i33) A1:matrix([5, 1, 2], [-3, 9, 4], [1, 2, -7]);
      b1:matrix([10], [-14], [-33]);
      start:matrix([0.0], [0.0], [0.0]);
      gauss_seidel(A1, b1, start, 10);
```

```
(%o30) 
$$\begin{pmatrix} 5 & 1 & 2 \\ -3 & 9 & 4 \\ 1 & 2 & -7 \end{pmatrix}$$

```

```
(%o31) 
$$\begin{pmatrix} 10 \\ -14 \\ -33 \end{pmatrix}$$

```

```
(%o32) 
$$\begin{pmatrix} 0.0 \\ 0.0 \\ 0.0 \end{pmatrix}$$

```

```
1
```

```
-----
```

```

$$\begin{pmatrix} 2.0 & -0.8888888888888887 & 4.746031746031746 \end{pmatrix}$$

```

```
2
```

```
-----
```

```

$$\begin{pmatrix} 0.2793650793650792 & -3.571781305114638 & 3.7336860670194 \end{pmatrix}$$

```

```
3
```

```
-----
```

```

$$\begin{pmatrix} 1.220881834215167 & -2.808010973936899 & 4.086408555191624 \end{pmatrix}$$

```

```
4
```

```
-----
```

```

$$\begin{pmatrix} 0.9270387727107303 & -3.062724211403811 & 3.971655764271872 \end{pmatrix}$$

```

```
5
```

```
-----
```

```

$$\begin{pmatrix} 1.023882536572013 & -2.979441716374605 & 4.0092855862604 \end{pmatrix}$$

```

```
6
```

```
-----
```

```

$$\begin{pmatrix} 0.992174108770761 & -3.006735557636591 & 3.996957570499654 \end{pmatrix}$$

```

```
7
```

```
-----
```

```

$$\begin{pmatrix} 1.002564083327456 & -2.997793114668471 & 4.000996836284359 \end{pmatrix}$$

```

```
8
```

```
-----
```

```

$$\begin{pmatrix} 0.9991598884199506 & -3.000723075541953 & 3.999673391048006 \end{pmatrix}$$

```

```
9
```

```
-----
```

```

$$\begin{pmatrix} 1.000275258689188 & -2.999763087569384 & 4.000107011935774 \end{pmatrix}$$

```

```
10
```

```
-----
```

11

```
(%i34) define(f(x), LP([0, 1, -1, 2, -2], [-5, -3, -15, 39, -9], x));
```

```
(%o34) f(x):=1.625 (x-1) x (x+1) (x+2)+0.5 (x-2) x (x+1)
      (x+2)+1.25 (1-x) (x-2) (x+1) (x+2)+2.5 (x-2) (x-1) x
      (x+2)-0.375 (x-2) (x-1) x (x+1)
```

```
(%i35) f(3);
```

```
(%o35) 241.0
```

```
(%i36) LP([0, 1, -1, 2, -2], [-5, -3, -15, 39, -9], 3);
```

```
(%o36) 241.0
```

12

12.1

```
(%i37) dd_table([-1, 0, 1, 2, -2, 3], [5, 1, 1, 11, 5, 35]);
```

```
(%o37)
```

-1	5	-4	2	1	-0.08333333333333326	-1.734723
0	1	0	5	1.0833333333333333	-0.08333333333333333	
1	1	10	2.8333333333333333	0.8333333333333333	$A_{3,6}$	
2	11	1.5	4.5	$A_{4,5}$	$A_{4,6}$	
-2	5	6	$A_{5,4}$	$A_{5,5}$	$A_{5,6}$	
3	35	$A_{6,3}$	$A_{6,4}$	$A_{6,5}$	$A_{6,6}$	

12.2

```
(%i38) NP([-1, 0, 1, 2, -2, 3], [5, 1, 1, 11, 5, 35], x);
```

```
(%o38) -1.734723475976807 10-17 (x-2) (x-1) x (x+1)
      (x+2)-0.08333333333333326 (x-2) (x-1) x (x+1)+(x-1) x
      (x+1)+2 x (x+1)-4 (x+1)+5
```

```
(%i39) NP([-1, 0, 1, 2, -2, 3], [5, 1, 1, 11, 5, 35], 1.5);
```

```
(%o39) 4.453125
```

13

```
(%i40) ratprint:false;
(%o40) false
```

13.1

```
(%i41) T(1/s, s, 1.0, 2.0);
(%o41) 0.75

(%i42) foo(g, x, a, b):=block(
```

```
    [t, v, e],
    numer:true,
    local(f),
    define(f(x), g),
    t:T(f(x), x, a, b),
    v:integrate(f(x), x, a, b),
    e:abs(v-t),
    print(t, " ", v, " ", e)
);
```

```
(%o42) foo(g,x,a,b):=block([t,v,e],numer:true,local(f),define(f(x),g),t:T(f(x),x,a,b),v:=integrate(f(x),x,a,b),e:=abs(v-t),print(t," ",v," ",e))
```

```
(%i43) foo(1/x, x, 1, 2);
0.75 0.6931471805599453 0.05685281944005471
(%o43) 0.05685281944005471
```

13.2

```
(%i44) for k:2 next 2*k thru 150 do block(
    temp:Tc(sin(x), x, 0, %pi, k),
    print(k, " ", temp)
);

2 1.570796326794896
4 1.896118897937039
8 1.97423160194555
16 1.993570343772339
32 1.998393360970144
64 1.999598388640037
128 1.999899600184203
(%o44) done
```


14.1

```
(%i45) S(1/x, x, 1.0, 2.0);
(%o45) 0.6944444444444443
```

14.2

```
(%i46) for k:2 next 2*k thru 150 do block(
    temp:Sc(sin(x), x, 0, %pi, k),
    print(k, " ", temp)
);
2      2.094395102393195
4      2.00455975498442
8      2.000269169948387
16     2.000016591047935
32     2.000001033369412
64     2.000000064530001
128    2.000000004032257
(%o46) done
```

15

```
(%i47) eulerm(1+(x/t), t, x, 1.0, 1.0, 6, 10);
1.0 ---- 1.0
1.5 ---- 2.0
2.0 ---- 3.166666666666666
2.5 ---- 4.458333333333333
3.0 ---- 5.85
3.5 ---- 7.324999999999999
4.0 ---- 8.87142857142857
4.5 ---- 10.48035714285714
5.0 ---- 12.14484126984126
5.5 ---- 13.85932539682539
6.0 ---- 15.61926406926406
(%o47) done
```

16

```
(%i48) eulermode(1+(x/t), t, x, 1.0, 1.0, 6, 10);  
1.0 ----- 1.0  
1.5 ----- 2.1  
2.0 ----- 3.371428571428571  
2.5 ----- 4.76984126984127  
3.0 ----- 6.26926406926407  
3.5 ----- 7.852602952602953  
4.0 ----- 9.507736707736708  
4.5 ----- 11.22561556090967  
5.0 ----- 12.99922196826221  
5.5 ----- 14.82295368889796  
6.0 ----- 16.69223406377801  
(%o48) done
```

17

```
(%i49) heun(1+(x/t), t, x, 1.0, 1.0, 6, 10);  
1.0 ----- 1.0  
1.5 ----- 2.083333333333333  
2.0 ----- 3.340277777777777  
2.5 ----- 4.725347222222221  
3.0 ----- 6.212083333333332  
3.5 ----- 7.78314484126984  
4.0 ----- 9.426272675736959  
4.5 ----- 11.13233453798185  
5.0 ----- 12.89426059775761  
5.5 ----- 14.70641393026065  
6.0 ----- 16.56419398452677  
(%o49) done
```

18

```
(%i50) rk2(1+(x/t), t, x, 1.0, 1.0, 6, 10);  
1.0 ---- 1.0  
1.5 ---- 2.09375  
2.0 ---- 3.359848484848484  
2.5 ---- 4.753382034632034  
3.0 ---- 6.248176088617265  
3.5 ---- 7.827038770053475  
4.0 ---- 9.477795861427574  
4.5 ---- 11.19136649795217  
5.0 ---- 12.96071373335682  
5.5 ---- 14.7802226066925  
6.0 ---- 16.64530777872948  
(%o50) done
```