Examples for numerical.mac

1

(%i1) load("/home/jvr/Downloads/numericalPackage/numerical.mac");
(%o1) */home/jvr/Downloads/numericalPackage/numerical.mac*

2

2.1

(%i2) bisect(t^3−2·sin(t), t, 0.50, 2.00, 20, 0.0005);

iter	m	ym	error
1	1.25	0.0551557612888276	
	0.75		
2	0.875	−0.8651651294720542	
	0.375		
3	1.0625	−0.5476869797091422	
	0.1875		
4	1.15625	−0.2847914007983883	
	0.09375		
5	1.203125	−0.1247986155094702	0.046875
6	1.2265625	−0.03735980652509796	0.0234375
7	1.23828125	0.00825801590073083	0.01171875
8	1.232421875	−0.01471021624269308	0.005859375
9	1.2353515625	−0.003266014170569153	0.0029296875
10	1.23681640625	0.00248601190191328	0.00146484375
11	1.236083984375	$−3.924970675475148 \cdot 10^{-4}$	$7.32421875 \cdot 10^{-4}$
12	1.2364501953125	0.001046133270412585	$3.662109375 \cdot 10^{-4}$

(%o2) *bisection has converged*

2.2

(%i3) bisect(sin(x), x, 2, 4, 20, 0.0005);

iter	m	ym	error
1	3	0.1411200080598672	1
2	3.5	-0.3507832276896198	0.5
3	3.25	-0.1081951345301083	0.25
4	3.125	0.0165918922293479	0.125
5	3.1875	-0.04589122327277969	0.0625
6	3.15625	-0.01465682159049232	0.03125
7	3.140625	$9.676534387822795 \cdot 10^{-4}$	0.015625
8	3.1484375	-0.006844792961296519	0.0078125
9	3.14453125	$-0.00293859218 0907726$	0.00390625
10	3.142578125	$-9.854712506993688 \cdot 10^{-4}$	0.001953125
11	3.1416015625	$-8.908910206643689 \cdot 10^{-6}$	$9.765625 \cdot 10^{-4}$
12	3.14111328125	$4.793723214334506 \cdot 10^{-4}$	$4.8828125 \cdot 10^{-4}$

(%o3) *bisection has converged*

2.3

(%i4) bisect(x^2−2, x, 1, 2, 20, 0.0005);

iter	m	ym	error
1	1.5	0.25	0.5
2	1.25	−0.4375	0.25
3	1.125	−0.734375	0.125
4	1.0625	−0.87109375	0.0625
5	1.03125	−0.9365234375	0.03125
6	1.015625	−0.968505859375	0.015625
7	1.0078125	−0.98431396484375	0.0078125
8	1.00390625	−0.9921722412109375	0.00390625
9	1.001953125	−0.9960899353027344	0.001953125
10	1.0009765625	−0.9980459213256836	$9.765625 \cdot 10^{-4}$
11	1.00048828125	−0.9990231990814209	$4.8828125 \cdot 10^{-4}$

(%o4) *bisection has converged*

(%i5) `bisect(x^3−2·x^2+x−%pi, x,  2,  3,  10,  0.0001);`

iter	m	ym	error
1	2.5	2.483407346410207	
	0.5		
2	2.25	0.3740323464102069	
	0.25		
3	2.125	−0.4521395285897931	
	0.125		
4	2.1875	−0.05687585671479311	
	0.0625		
5	2.21875	0.1540311257070818	
	0.03125		
6	2.203125		
0.04745229880278501		0.015625	
7	2.1953125	−	
0.004991682367869288		0.0078125	
8	2.19921875		
0.02116015355055722		0.00390625	
9	2.197265625		
0.008066719276360601		0.001953125	
10	2.1962890625		
0.00153314216946753		$9.765625 \ 10^{-4}$	

(%o5) *done*

3

3.1

(%i6) `newton(x^3−2·x^2+x−3, x,  3, 0.0001, 0.00001, 8);`

1	2.4375	2.036865234375
2	2.213032716315109	0.2563633850614177
3	2.175554938721488	0.006463361488812325
4	2.174560100666446	4.479068049789703 10^{-6}
5	2.174559410293312	2.156497203031904 10^{-12}

(%o6) *convergence*

3.2

(%i7) newton(sin(x), x,   3.0, 0.0001, 0.00001, 8);

1	3.142546543074278	$-9.538893398264409 \cdot 10^{-4}$
2	3.141592653300477	$2.893162490762184 \cdot 10^{-10}$
3	3.141592653589793	$1.224646799147353 \cdot 10^{-16}$

(%o7) *convergence*

3.3

(%i8) newton(x^2−2, x,   1, 0.0001, 0.00001, 8);

1	1.5	0.25
2	1.416666666666666	0.006944444444444642
3	1.414215686274509	$6.007304882871267 \cdot 10^{-6}$
4	1.414213562374689	$4.510614104447086 \cdot 10^{-12}$

(%o8) *convergence*

(%i9) newton(x^3−2·x^2+x−%pi, x,   3, 0.0001, 0.00001, 8);

1	2.446349540849362	1.975991988443487
2	2.23082941379325	0.237982391674409
3	2.196863381645069	0.005374496729482203
4	2.196060159694346	$2.961172185678151 \cdot 10^{-6}$
5	2.196059716657127	$9.00612917575927 \cdot 10^{-13}$

(%o9) *convergence*

4

4.1

(%i10) secant(x^3+2·x^2−3·x−1, x, −2, −3, 0.00001, 10);

0	−3	−1
1	−2	5
2	−2.833333333333333	0.8101851851851833
3	−2.907928388746803	0.046299957161572662
4	−2.912449640422374	−0.002380064066290543
5	−2.912228585591192	$6.399876401275151 \, 10^{-6}$

(%o10) *convergence*

4.2

(%i11) secant(sin(x), x, 1, 4, 0.00001, 10);

0	4	−0.7568024953079282
1	1	0.8414709848078965
2	2.579462454848934	0.5329898131328342
3	3.166481028136553	−0.02488580518710502
4	3.140295209333355	0.001297443892426365
5	3.141592780561861	$−1.269720678483891 \, 10^{-7}$

(%o11) *convergence*

4.3

(%i12) secant(x^2−2, x, 1, 2, 0.00001, 10);

0	1	−1
1	2	2
2	1.333333333333333	−0.2222222222222223
3	1.428571428571428	0.04081632653061229
4	1.413793103448275	−0.001189060642093009
5	1.41421143847487	$−6.007286838860537 \, 10^{-6}$

(%o12) *convergence*

5

5.1

(%i13) regula(x^3+2·x^2−3·x−1, x,   1,   2, 20, 0.0005);

iter	m	ym
1	1.1	−0.5489999999999999
2	1.221729490022173	0.1436388950901026
3	1.193880682944684	−0.02924075169187978
4	1.199720665327255	0.006290908737020207
5	1.198472095487697	−0.001337706025862806
6	1.198737948438891	$2.851619710524389 \cdot 10^{-4}$

(%o13) *regula falsi method has converged*

5.2

(%i14) regula(sin(x), x,   2,   4, 20, 0.0005);

iter	m	ym
1	3.091528082734958	0.05004365932452196
2	3.147874957380742	−0.006282262466726139
3	3.141590357955694	$2.295634098490862 \cdot 10^{-6}$

(%o14) *regula falsi method has converged*

5.3

(%i15) regula(x^2−2, x,   1,   2, 20, 0.0005);

iter	m	ym
1	1.333333333333333	−0.2222222222222218
2	1.428571428571428	0.04081632653061229
3	1.411764705882353	−0.006920415224913157
4	1.414634146341463	0.00118976799524084 2
5	1.414141414141414	$−2.040608101214758 \cdot 10^{-4}$

(%o15) *regula falsi method has converged*

6

6.1

```
(%i18) A:matrix([1,  2,  3], [3,  5,  7],  [4,  6,  9]);
       luFactor(A);
       luFactor(A)[1];
```

$$(\%o16) \quad \begin{pmatrix} 1 & 2 & 3 \\ 3 & 5 & 7 \\ 4 & 6 & 9 \end{pmatrix}$$

$$(\%o17) \quad \left[L = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 4 & 2 & 1 \end{pmatrix}, U = \begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & -2 \\ 0 & 0 & 1 \end{pmatrix} \right]$$

$$(\%o18) \quad L = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 4 & 2 & 1 \end{pmatrix}$$

7

7.1

```
(%i21) A:matrix([1,  2,  3], [3,  5,  7],  [4,  6,  9]);
       b:matrix([1], [2], [3]);
       solve_by_lu(A,    b);
```

$$(\%o19) \quad \begin{pmatrix} 1 & 2 & 3 \\ 3 & 5 & 7 \\ 4 & 6 & 9 \end{pmatrix}$$

$$(\%o20) \quad \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$(\%o21) \quad \left[z = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, x = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} \right]$$

8

8.1

(%i25) A1:matrix([5,   1,   2], [−3,   9,   4],[1,   2,   −7]);
       b1:matrix([10],   [−14],   [−33]);
       start:matrix([0],   [0],   [0]);
       gauss_jacobi(A1,   b1,   start,   14);

(%o22) $\begin{pmatrix} 5 & 1 & 2 \\ -3 & 9 & 4 \\ 1 & 2 & -7 \end{pmatrix}$

(%o23) $\begin{pmatrix} 10 \\ -14 \\ -33 \end{pmatrix}$

(%o24) $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

1
−−−−
$\begin{pmatrix} 2.0 & -1.555555555555555 & 4.714285714285714 \end{pmatrix}$

2
−−−−
$\begin{pmatrix} 0.4253968253968252 & -2.984126984126984 & 4.555555555555555 \end{pmatrix}$

3
−−−−
$\begin{pmatrix} 0.7746031746031745 & -3.438447971781304 & 3.922448979591836 \end{pmatrix}$

4
−−−−
$\begin{pmatrix} 1.118710002519526 & -3.040665154950868 & 3.842529604434367 \end{pmatrix}$

5
−−−−
$\begin{pmatrix} 1.071121189216427 & -2.890443156686543 & 4.005339956088256 \end{pmatrix}$

6
−−−−
$\begin{pmatrix} 0.9759526489020063 & -2.97866625074486 & 4.041462125120478 \end{pmatrix}$

7
−−−−
$\begin{pmatrix} 0.9791484001007809 & -3.026443394863987 & 4.002660021058898 \end{pmatrix}$

8
−−−−
$\begin{pmatrix} 1.004224670549238 & -3.008132764881471 & 3.989465944338972 \end{pmatrix}$

9
−−−−
$\begin{pmatrix} 1.005840175240705 & -2.993909973967574 & 3.998279877255185 \end{pmatrix}$

10
−−−−

9

9.1

(%i29) A1:matrix([5, 1, 2], [−3, 9, 4],[1, 2, −7]);
b1:matrix([10], [−14], [−33]);
start:matrix([0], [0], [0]);
sor(A1, b1, 0.9, start, 10);

(%o26) $\begin{pmatrix} 5 & 1 & 2 \\ -3 & 9 & 4 \\ 1 & 2 & -7 \end{pmatrix}$

(%o27) $\begin{pmatrix} 10 \\ -14 \\ -33 \end{pmatrix}$

(%o28) $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

1
————
$\begin{pmatrix} 1.799999999999999 & -0.8599999999999999 & 4.253142857142857 \end{pmatrix}$

2
————
$\begin{pmatrix} 0.6036685714285714 & -3.006156571428571 & 3.972774269387755 \end{pmatrix}$

3
————
$\begin{pmatrix} 0.9712763030204081 & -2.998342473991836 & 3.994010601157784 \end{pmatrix}$

4
————
$\begin{pmatrix} 0.9989854592037691 & -2.997742850101166 & 3.999851029130248 \end{pmatrix}$

5
————
$\begin{pmatrix} 0.9995458884516973 & -2.999850930126706 & 3.999965049395661 \end{pmatrix}$

6
————
$\begin{pmatrix} 0.9999403384855388 & -2.999989011225273 & 3.99999165985835 \end{pmatrix}$

7
————
$\begin{pmatrix} 0.9999950583200967 & -2.999997047569838 & 3.999999289823317 \end{pmatrix}$

8
————
$\begin{pmatrix} 0.9999992300581864 & -2.99999965166854 & 3.999999919560678 \end{pmatrix}$

9
————
$\begin{pmatrix} 0.9999998892643681 & -2.99999996211847 & 3.999999986407011 \end{pmatrix}$

10
————

10

10.1

(%i33) A1:matrix([5, 1, 2], [−3, 9, 4],[1, 2, −7]);
b1:matrix([10], [−14], [−33]);
start:matrix([0], [0], [0]);
gauss_seidel(A1, b1, start, 10);

(%o30)
$$\begin{pmatrix} 5 & 1 & 2 \\ -3 & 9 & 4 \\ 1 & 2 & -7 \end{pmatrix}$$

(%o31)
$$\begin{pmatrix} 10 \\ -14 \\ -33 \end{pmatrix}$$

(%o32)
$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

1
————
$$\begin{pmatrix} 2.0 & -0.8888888888888887 & 4.746031746031746 \end{pmatrix}$$

2
————
$$\begin{pmatrix} 0.2793650793650792 & -3.571781305114638 & 3.7336860670194 \end{pmatrix}$$

3
————
$$\begin{pmatrix} 1.220881834215167 & -2.808010973936899 & 4.086408555191624 \end{pmatrix}$$

4
————
$$\begin{pmatrix} 0.9270387727107303 & -3.062724211403811 & 3.971655764271872 \end{pmatrix}$$

5
————
$$\begin{pmatrix} 1.023882536572013 & -2.979441716374605 & 4.0092855862604 \end{pmatrix}$$

6
————
$$\begin{pmatrix} 0.992174108770761 & -3.006735557636591 & 3.996957570499654 \end{pmatrix}$$

7
————
$$\begin{pmatrix} 1.002564083327456 & -2.997793114668471 & 4.000996836284359 \end{pmatrix}$$

8
————
$$\begin{pmatrix} 0.9991598884199506 & -3.000723075541953 & 3.999673391048006 \end{pmatrix}$$

9
————
$$\begin{pmatrix} 1.000275258689188 & -2.999763087569384 & 4.000107011935774 \end{pmatrix}$$

10
————

11

(%i36) xval:[0, 1,   −1, 2,   −2]$
      yval:[−5, −3, −15, 39, −9]$
      LP(xval,   yval,   x);

(%o36) $1.625\,(x-1)\,x\,(x+1)\,(x+2)+0.5\,(x-2)\,x\,(x+1)\,(x+2)$
        $+1.25\,(1-x)\,(x-2)\,(x+1)\,(x+2)+2.5\,(x-2)\,(x-1)\,x\,(x+2)-$
        $0.375\,(x-2)\,(x-1)\,x\,(x+1)$

(%i37) LP(xval,   yval,   3);
(%o37) 241.0

12

12.1

(%i38) dd_table([−1, 0, 1, 2, −2, 3], [5, 1,   1,   11, 5,   35]);
(%o38)

$$\begin{vmatrix} -1 & 5 & -4 & 2 & 1 & -0.08333333333333326 & -1.7347234 \\ 0 & 1 & 0 & 5 & 1.083333333333333 & -0.08333333333333333 & \\ 1 & 1 & 10 & 2.833333333333333 & 0.8333333333333333 & A_{3,6} & \\ 2 & 11 & 1.5 & 4.5 & A_{4,5} & A_{4,6} & \\ -2 & 5 & 6 & A_{5,4} & A_{5,5} & A_{5,6} & \\ 3 & 35 & A_{6,3} & A_{6,4} & A_{6,5} & A_{6,6} & \end{vmatrix}$$

12.2

(%i41) xval:[−1, 0, 1, 2,   −2,   3]$
      yval:[5, 1,   1,   11, 5,   35]$
      NP(xval,   yval,   x);

(%o41) $-1.734723475976807\,10^{-17}\,(x-2)\,(x-1)\,x\,(x+1)$
        $(x+2)-0.08333333333333326\,(x-2)\,(x-1)\,x\,(x+1)+(x-1)\,x$
        $(x+1)+2\,x\,(x+1)-4\,(x+1)+5$

(%i42) NP(xval, yval, 1.5);
(%o42) 4.453125

13

(%i43) ratprint:false;
(%o43) **false**

13.1

(%i44) T(1/s,   s,   1.0,   2.0);
(%o44) 0.75

(%i45) compare(g,   x,   a,   b):=block(

        [t, v,   e],
        numer:true,
        local(f),
        define(f(x),   g),
        t:T(f(x),   x,   a,   b),
        v:integrate(f(x),   x,   a, b),
        e:abs(v−t),
        print(t, "   ",   v, "   ", e)
    );

(%o45) $\mathrm{compare}\boldsymbol{(g,x,a,b){:=}}\mathrm{block}\left(\boldsymbol{[t,v,e]},numer\boldsymbol{{:}true},\mathrm{local}\boldsymbol{(f)},\mathrm{define}\boldsymbol{(f(x),g)},t\boldsymbol{{:}}\mathrm{T(f}\right.$

(%i46) compare(1/x,   x,   1,   2);
       0.75   0.6931471805599453   0.05685281944005471
(%o46) 0.05685281944005471

13.2

(%i47) for k:2 next 2·k thru 150 do block(
            temp:Tc(sin(x),   x,   0,   %pi,   k),
            print(k, "   ", temp)
            );
       2   1.570796326794896
       4   1.896118897937039
       8   1.97423160194555
       16   1.993570343772339
       32   1.998393360970144
       64   1.999598388640037
       128   1.999899600184203
(%o47) **done**

14

14.1

(%i48) S(1/u,   u,   1.0,   2.0);
(%o48) 0.6944444444444443

14.2

(%i49) for k:2 next 2·k thru 150 do block(
            temp:Sc(sin(t),   t,   0,   %pi,   k),
            print(k, "   ", temp)
            );
        2   2.094395102393195
        4   2.00455975498442
        8   2.0000269169948387
        16   2.000016591047935
        32   2.000001033369412
        64   2.000000064530001
        128   2.000000004032257
(%o49) *done*

15

(%i50) eulerm(1+(x/t),   t,   x,   1.0,   1.0,   6,   10);
        1.0 ———— 1.0
        1.5 ———— 2.0
        2.0 ———— 3.166666666666666
        2.5 ———— 4.458333333333333
        3.0 ———— 5.85
        3.5 ———— 7.324999999999999
        4.0 ———— 8.87142857142857
        4.5 ———— 10.48035714285714
        5.0 ———— 12.14484126984126
        5.5 ———— 13.85932539682539
        6.0 ———— 15.61926406926406
(%o50) *done*

16

(%i51) eulermod(1+(x/t), t, x, 1.0, 1.0, 6, 10);

```
1.0 ———— 1.0
1.5 ———— 2.1
2.0 ———— 3.371428571428571
2.5 ———— 4.76984126984127
3.0 ———— 6.26926406926407
3.5 ———— 7.852602952602953
4.0 ———— 9.507736707736708
4.5 ———— 11.22561556090967
5.0 ———— 12.99922196826221
5.5 ———— 14.82295368889796
6.0 ———— 16.69223406377801
```

(%o51) *done*

17

(%i52) heun(1+(x/t), t, x, 1.0, 1.0, 6, 10);

```
1.0 ———— 1.0
1.5 ———— 2.083333333333333
2.0 ———— 3.340277777777777
2.5 ———— 4.725347222222221
3.0 ———— 6.212083333333332
3.5 ———— 7.78314484126984
4.0 ———— 9.426272675736959
4.5 ———— 11.13233453798185
5.0 ———— 12.89426059775761
5.5 ———— 14.70641393026065
6.0 ———— 16.56419398452677
```

(%o52) *done*

18

(%i53) rk2(1+(x/t), t,   x,   1.0,   1.0, 6,   10);

```
       1.0 ———— 1.0
       1.5 ———— 2.09375
       2.0 ———— 3.359848484848484
       2.5 ———— 4.753382034632034
       3.0 ———— 6.248176088617265
       3.5 ———— 7.827038770053475
       4.0 ———— 9.477795861427574
       4.5 ———— 11.19136649795217
       5.0 ———— 12.96071373335682
       5.5 ———— 14.7802226066925
       6.0 ———— 16.64530777872948
```

(%o53) *done*