National University of Singapore

Department of Mechanical Engineering

ME5401/MCH5201/EE5101 Linear System 2021/2022

Tutorial 2

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1. Consider the system defined by the differential equation

$$I\ddot{\theta} + b\dot{\theta} + k\theta = H\omega\cos\theta \, .$$

Write the state space representation of the system if the input is $\omega(t)$ and the output is $\theta(t)$. Linearize the state equation about the equilibrium point $\theta_0 = 0, \dot{\theta}_0 = 0$ and $\omega_0 = 0$.

2. Let $A = \begin{bmatrix} 0 & 0 & -2 \\ 0 & 1 & 0 \\ 1 & 0 & 3 \end{bmatrix}$. Find e^{At} using the method of Caley-Hamilton theorem.

3. Find the solution for the state-space system given by

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x(t) \text{ with } x(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

using the system modal expansion method.

4. Verify controllability and observability for the following systems.

(i) $A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$

(ii) $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}$

(iii) $A = \begin{bmatrix} -1 & 0 & 0 \\ -1 & -4 & 4 \\ -1 & -2 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 1 & -1 \end{bmatrix}$.

5. Show that for any real nonsingular square matrix, $\left[A^{-1}\right]^T = \left[A^T\right]^{-1}$.

6. Show that the SISO system $\{A,b\}$ with

$$A = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 1 \\ 0 & 0 & \lambda \end{bmatrix}$$

cannot be controllable for whatever value of b.

7. Consider the system

$$\dot{x} = \begin{bmatrix} \lambda & 0 \\ 0 & \bar{\lambda} \end{bmatrix} x + \begin{bmatrix} b_1 \\ \bar{b}_1 \end{bmatrix} u$$
$$y = \begin{bmatrix} c_1 & \bar{c}_1 \end{bmatrix} x$$

where the overbar refers to the complex conjugate. Verify that the equation can be transformed into

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -\lambda\bar{\lambda} & \lambda + \bar{\lambda} \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$
$$y = \begin{bmatrix} -2\,\mathrm{Re}(\bar{\lambda} b_1 c_1) & 2\,\mathrm{Re}(b_1 c_1) \end{bmatrix} x$$

using the transformation $x = Q\bar{x}$ with

$$Q = \begin{bmatrix} -\bar{\lambda} b_1 & b_1 \\ -\lambda \bar{b}_1 & \bar{b}_1 \end{bmatrix}.$$