

EE5137 Semester 1 2018/9: Quiz 2 (Total 30 points)

Name: _____

Matriculation Number: _____

Score: _____

You have 75 mins for this quiz. There are FOUR (4) printed pages. You're allowed 1 sheet of handwritten notes. Please provide *careful explanations* for all your solutions.

1. [Covariances in the Poisson Process] (10 points)

Given two random variables X and Y , their *covariance* is defined as

$$\text{Cov}(X, Y) = \mathbb{E}[XY] - \mathbb{E}[X]\mathbb{E}[Y]$$

Answer the following questions.

- (i) (2 points) Show that if X and Y are independent, then $\text{Cov}(X, Y) = 0$.

- (ii) (8 points) $N(t)$ is a Poisson counting process with intensity/rate λ . Find the covariance between $N(3) - N(1)$ and $N(4) - N(2)$.

2. [Poisson Arrivals] (10 points)

Suppose customers arrive to a shop according to a Poisson process with intensity/rate of $\lambda = 8$ per hour.

- (i) (2 points) If X is an $\text{Exp}(\lambda)$ random variable, what is the variance of X ?
- (ii) (3 points) What is the variance of the time when the *fourth* (4^{th}) customer arrives?
- (iii) (5 points) Assume that 25% of the customers are men and 75% percent are women (and that whether each customer is a man/woman is independent of every other customer). What is the expected time when the *fifth* (5^{th}) woman arrives?

3. [Travelers and Poisson Processes] (10 points)

Suppose that travelers arrive at a train depot according to a Poisson process with rate λ . The train departs at time t . We would like to compute the expected sum of the waiting times of travelers arriving in the interval $(0, t)$, i.e., we want to find the number

$$\alpha := \mathbb{E} \left[\sum_{i=1}^{N(t)} (t - S_i) \right]$$

where S_i are the arrival epochs of the travelers.

(i) (6 points) Find a simple expression in terms of n and t for

$$\mathbb{E} \left[\sum_{i=1}^{N(t)} (t - S_i) \mid N(t) = n \right].$$

That is, we condition on the event $\{N(t) = n\}$.

(ii) (4 points) Hence, find

$$\alpha := \mathbb{E} \left[\sum_{i=1}^{N(t)} (t - S_i) \right].$$