

# Stability Analysis for $h^* \approx 1$

4

$$R_p(p) \underline{y(t)} = k_p \underline{Z_p(p)} \underline{u(t)}$$

From Proposition 1, the plant can be re-written as:

$$\dot{w} = \underline{u}$$

$$\underline{y} =$$

Our control law  $\bar{u}$

$$u(t) = \theta^T(t) w(t) + k(t) r(t)$$

$$= \left\{ \right\} w(t) + \left\{ \right\} r(t)$$

Thus, with this control law,  
we now have

$$\dot{w} = A_p w + b_p w(t)$$

$$y = c_p^T w$$

$$\dot{w} = A_p w + b_p \left\{ \begin{aligned} & [\theta^{*T} + \phi(t)^T] w(t) \\ & + [k^* + \phi_k(t)] r(t) \end{aligned} \right\}$$

$$= \left\{ A_p + b_p \theta^{*T} \right\} w(t) + \underline{k^* b_p} r(t)$$

$$+ b_p \phi(t)^T w(t) + b_p \phi_k(t) r(t)$$

$$y = c_p^T w$$

— (3.1)

where, by Propositions 1 & 2,  
it is implicitly possible to  
write

$$R_m(p) y_m(t) = k_m r(t)$$

as the  $2n$ -order non-minimal  
realization

$$\dot{w}_m = \quad r(t)$$

$$y_m =$$

— (3.2)

By Propositions 1 & 2, this  $w_m$  exists,  
but we do not know its values!!!

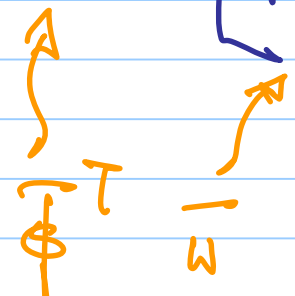
equivalent state-vector of the Reference  
Model in non-minimal realisation!!

Then, for

$$e(t) \triangleq w(t) - w_m(t)$$

$$e_1(t) \triangleq y(t) - y_m(t)$$

From (3.1) and (3.2), we have

$$\begin{aligned} \dot{e} &= A_m e + \frac{1}{k^*} b_m \begin{bmatrix} \phantom{w} \\ \phantom{r} \\ w \end{bmatrix} \\ e_1 &= c_m^T e \end{aligned}$$


Thus, we have the "error dynamical system"

$$\dot{e} = A_m e + \frac{1}{k^*} b_m \bar{\phi}^T \bar{w}$$

$$e_1 = c_m^T e$$

where

$$c_m^T \{sI - A_m\}^{-1} b_m =$$

is strictly positive-real.

Additionally, recall that the  
"adaptive law" is

$$\dot{\bar{\theta}}(t) = \dot{\bar{\phi}}(t) = -\text{sgn}(k_p) T^T \bar{w} e_1$$

Thus, now, consider the  
quadratic form

$$V(e, \bar{\phi}) = e^T P e + \bar{\phi}^T T^{-1} \bar{\phi}$$

where, from the Kalman-Yakubovich Lemma, we have

$$A_m^T P + P A_m =$$

thus,

$$\dot{V} = 2 e^T P \dot{e} + 2 \bar{\phi}^T \dot{\bar{\phi}}$$

$$= 2 e^T P \left\{ \begin{array}{l} \dot{e} \\ \dot{\bar{\phi}} \end{array} \right\}$$

$$+ 2 \bar{\phi}^T \dot{\bar{\phi}}$$

$$= -e^T Q e + 2 e^T \frac{1}{k^*} P b_m \bar{\phi}^T$$

$$+ 2 \bar{\phi}^T \dot{\bar{\phi}}$$

[Noting also that  $k_m = k_p k^*$ ]

Recall that the Kalman-Yacubovich Lemma states that for the spr transfer function

$$\frac{1}{|k^*|} \frac{k_m}{p_m(s)}$$

with state-realization  $\left. \vphantom{\frac{1}{|k^*|} \frac{k_m}{p_m(s)}} \right\}$

- $A_m^T P + P A_m = -q V^T - \varepsilon L = -Q$

- $P \frac{1}{|k^*|} b_m = c_m$  — (4.1a)  
— (4.1b)

Noting that  $k_m = k_p k_i^*$  ;  $k_m > 0$

∴  $\text{sgn}(k_p) =$

1.4

$$V = -e^T Q e$$

$$+ 2 e^T \frac{\text{sgn}(k^*)}{|k^*|} \phi_m^T \bar{\omega}$$

$$+ 2 \bar{\phi}^T \left\{ -\text{sgn}(k_p) \bar{\omega} e_1 \right\}$$

1.4

$$V = -e^T Q e$$

$$+ 2 \text{sgn}(k^*) \bar{\phi}^T \bar{\omega}$$

$$+ 2 \bar{\phi}^T \bar{\omega} e_1 \left\{ -\text{sgn}(k_p) \right\}$$



⋮

leading to  $\|\bar{w}\|$  and  $\|\bar{\tau}\|$   
bounded for all  $t \geq 0$ ,

and  $\lim_{t \rightarrow \infty} e_i(t) = 0$

