NATIONAL UNIVERSITY OF SINGAPORE

FACULTY OF ENGINEERING

EXAMINATION FOR

(Semester I: 2020/2021)

EE5101/ME5401 – LINEAR SYSTEMS

November 2020 - Time Allowed: 2.5 Hours

INSTRUCTIONS TO CANDIDATES:

1. This paper contains **FOUR** (4) questions and comprises **FIVE** (5) printed pages.
2. Answer all **FOUR** (4) questions.
3. All questions carry **EQUAL** marks. The **TOTAL** marks are 100.
4. This is an **OPEN BOOK** examination.

Q.1 (a) Consider the system in the state space form where the A, b and c matrices are

$$A = \begin{bmatrix} 3 & 1 \\ 0 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad c = [\,1 \quad 1\,]$$

(i) Compute the characteristic polynomial of the matrix A. Hence, or otherwise, find the representation of A and b under a coordinate change in the form of $x = Q\bar{x}$ where $Q = [b \quad Ab]$.

(8 Marks)

(ii) Find the transfer function representation of the above system.

(5 Marks)

(b) Given the system

$$\dot{x} = \begin{bmatrix} -1 & 1 \\ -1 & -1 \end{bmatrix} x$$

Find the state-transition matrix e^{At} using the Caley-Hamilton Principle.

(12 Marks)

Q.2 (a) Given a n^{th} order Single-Input-Single-Output system in the usual form of $\{A,b,c\}$ where b is a column vector and c is a row vector. Suppose A is a diagonal matrix,

(i) Suppose one of the elements of b is zero, is the system controllable? Hence, or otherwise, determine the structure of b for the system to be controllable.

(ii) What is the structure of c in order for the system to be observable?

Justify all your answers.

(12 marks)

(b) Determine the stability of

$$\dot{x} = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$
$$y = [0 \quad 1]x$$

using the Lyapunov Equation. State clearly the type of stability and explain your answers. Discuss any observation made.

(13 marks)

Q.3. Consider a process given by

$$\frac{dx}{dt} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 1 & 0 \end{bmatrix} u.$$

$$\mathrm{y} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} x$$

Design Specification. It is desired that all the three poles are placed at -1.

(a) Assume that all the state variables are accessible, design a state feedback controller to place the poles to the desired ones. There are many ways to solve the pole placement problem for multiple input system. For this problem, only **FULL RANK** method can be used. In other words, the state transformation matrix T has to be constructed to transform the state space model into its controllable canonical form and the pole placement problem can be solved with the aid of its canonical form.

(15 marks)

(b) Assume only the output signal can be measured. Design a reduced-order observer to estimate the state variables.

(10 marks)

Q. 4 Let a MIMO plant be described by the transfer function matrix,

$$G(s) = \begin{bmatrix} \frac{1}{(s+1)(s+2)} & \frac{1}{s+1} \\ \frac{1}{s+2} & \frac{-1}{s+2} \end{bmatrix}.$$

(a) Design a controller $\mathrm{K}_d(s)$ in unity output feedback configuration to decouple the system. At this step, $\mathrm{K}_d(s)$ is not required to be proper and the closed loop is not required to be stable.

(8 marks)

(b) Use servo control mechanism to design the stabilizer $\mathrm{K}_s(s)$ such that the overall controller will be ,

$$\mathrm{K}(s) = K_d(s)K_s(s).$$

The design of the unity feedback control system should meet following requirements:

(i) The overall controller K(s) must be proper.

(ii) The dominant dynamics for both outputs can be described by standard second order system with damping ratio of 0.5, and natural frequency of 1.

(iii) The first output is required to track a constant value of 1, and the second output is required to track a constant value of 10.

(iv) The closed loop system can eliminate the effect of any unknown constant disturbance.

(17 marks)

<u>Appendix A</u> - Table of Laplace Transform

The following table contains some frequently used time functions x(t), and their Laplace transforms X(s).

x(t)	**X(s)**
unit impulse δ (t)	1
unit step 1(t)	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^2	$\frac{2}{s^3}$
e^{-at}	$\frac{1}{s+a}$
te^{-at}	$\frac{1}{(s+a)^2}$
$1-e^{-at}$	$\frac{a}{s(s+a)}$
$\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}$
$\cos(\omega t)$	$\frac{s}{s^2+\omega^2}$
$e^{-at}\sin(\omega t)$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at}\cos(\omega t)$	$\frac{s+a}{(s+a)^2+\omega^2}$

END OF PAPER