EE5103/ME5403 Computer Control Systems: Homework #2
(Due date: 3/10/2021)

Q1. (10 Marks)
A DC motor can be described by a second-order model with one integrator and one time constant (see figure below). The input is the voltage to the motor and the output is the shaft position. The time constant is due to the mechanical parts of the system, and the dynamics due to the electrical parts are neglected. A normalized model of the process is then given by

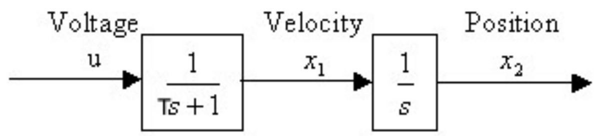

Figure 1: Normalized model of a DC motor

$$Y(s) = \frac{1}{s(s+1)}U(s)$$

Introduce the velocity and the position of the motor shaft as states (see above figure). The state-space model of the motor is then given by

$$\frac{dx}{dt} = \begin{pmatrix} -1 & 0 \\ 1 & 0 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u$$

$$y = \begin{pmatrix} 0 & 1 \end{pmatrix} x$$

a) Assuming the sampling period is h, determine the deadbeat controller for the normalized motor shown above.

(5 Marks)

b) Assume that $x(0) = \begin{bmatrix} 1 & 0.5 \end{bmatrix}^T$. Determine the sampling period h such that the control signal is less than one in magnitude. It can be assumed that the maximum value of $u(k)$ is at $k = 0$.

(5 Marks)

Q2. (10 Marks)
Given the discrete-time system

$$x(k+1) = \begin{pmatrix} 0.5 & 1 \\ 0.5 & 0.8 \end{pmatrix} x(k) + \begin{pmatrix} 0.2 \\ 0.1 \end{pmatrix} u(k) + \begin{pmatrix} 1 \\ 0 \end{pmatrix} v(k)$$

$$y(k) = \begin{pmatrix} 1 & 0 \end{pmatrix} x(k)$$

where v is a constant disturbance. Determine controllers such that the influence of v can be eliminated in steady state in each of the following cases.

a) The state and v can be measured. (4 Marks)

b) The state can be measured. (3 Marks)

c) Only the output can be measured. (3 Marks)

Q3. (20 Marks)
Consider a vehicle, which has a weight m = 1000 kg. Assuming the average friction coefficient b= 100, design a position control system such that the vehicle can move 100 m in 10.0 s with an overshoot less than 10%.
Let y denote the displacement of the vehicle, then the dynamics of the vehicle can be described by the following equation

$$m\ddot{y}(t) + b\dot{y}(t) = u(t)$$

where m=1000, and b=100. Assume that the sampling period is 0.1s.

Design a two-degree-of-freedom controller such that the transfer function from the command signal $u_c(k)$ to the output $y(k)$ follows the reference model.

The two-degree-of-freedom controller consists of two parts, the feed-forward controller and the feedback controller, in the form of

$$u(k) = u_{ff}(k) + u_{fb}(k)$$

where $u_{ff}(k)$ is the feed-forward control signal and $u_{fb}(k)$ is the feedback control signal.

a) From the performance specification, determine the desired damping ratio and the natural frequency, and then the reference model in the continuous-time. Convert the reference model from the continuous-time to discrete time, which is the reference model $H_m(z)$ to be used in the design of two-degree-of-freedom controller.

(4 Marks)

b) Define the state variable $x(t) = (y(t), \dot{y}(t))^T$, find out the state-space model of the sampled system.

(4 Marks)

c) Determine L in the state feedback controller

$$u_{fb}(k) = -Lx\,(k)$$

(4 Marks)

d) Determine the transfer function for the feed-forward controller, $H_{ff}(z)$, such that

$$U_{ff}(z) = H_{ff}(z)U_c(z)$$

(4 Marks)

e) Assume that only the displacement, y, is measurable, is it still possible to use the two-degree-of-freedom controller to meet the performance specification? Justify your answer.

(4 Marks)