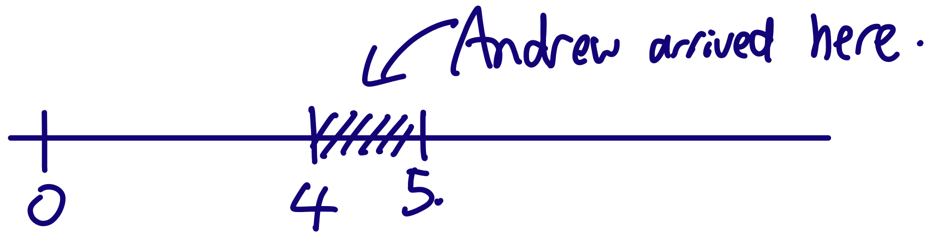



$\lambda_M = 3$

Men shop for a time unif $[0,1]$.

Let $S$ be the time Andrew arrived.

Let $W$ be the time that Andrew spent shopping. $S \perp\!\!\!\perp W$.

We want $\Pr(S+W>5 \mid S\in[4,5])$

$$\Pr(S\in[4,5]) = \int_4^5 3e^{-3x}\,dx$$

$$= \left[-e^{-3x}\right]_4^5$$

$$= e^{-12}-e^{-15}.$$

Consider

$\Pr(S+W>5 \cap S\in[4,5])$

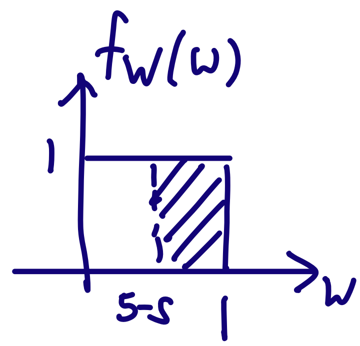
$$= \int_0^{\infty} Pr(S+W>5 \cap S\in[4,5] \mid S=s)\, f_S(s)\, ds.$$

$$= \int_4^5 Pr(S+W>5 \cap S\in[4,5] \mid S=s)\, f_S(s)\, ds.$$

$$= \int_4^5 Pr(W>5-s \cap S\in[4,5] \mid S=s)\, f_S(s)\, ds.$$

$$= \int_4^5 Pr(W>5-s \mid S=s)\, f_S(s)\, ds$$

$$= \int_4^5 Pr(W>5-s)\, f_S(s)\, ds$$




$$= \int_4^5 (s-4)\, 3e^{-3s}\, ds.$$

$$= \int_4^5 3se^{-3s}\, ds - 12\int_4^5 e^{-3s}\, ds$$

$$1-(5-s)$$
$$= s-4$$

$$= \left[\cancel{3}s \frac{e^{-3s}}{\cancel{-3}}\right]_4^5 - \int_4^5 \cancel{3}\frac{e^{-3s}}{\cancel{-3}}\, ds$$
$$-12\int_4^5 e^{-3s}\, ds$$

$$= 4e^{-12} - 5e^{-15} + \int_4^5 e^{-3s}\, ds - 12\int_4^5 e^{-3s}\, ds$$

$$= 4e^{-12} - 5e^{-15} - 11\int_4^5 e^{-2s}\, ds$$

$$= 4e^{-12} - 5e^{-15} - 11\left[\frac{e^{-3s}}{-3}\right]_4^5$$

$$= 4e^{-12} - 5e^{-15} - 11\left(\frac{e^{-12}}{3} - \frac{e^{-15}}{3}\right)$$

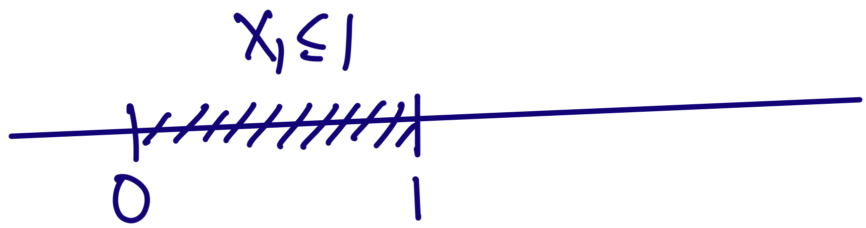
$$= \frac{1}{3}e^{-12} - \frac{4}{3}e^{-15}$$

$$\frac{\frac{1}{3}e^{-12} - \frac{4}{3}e^{-15}}{e^{-12} - e^{-15}}$$

$$= \frac{1}{3}\left(\frac{1-4e^{-3}}{1-e^{-3}}\right)$$

$Pr(S > 1-w \mid S \le 1)$

$$f_{S|S\le 1}(s) = \begin{cases} \dfrac{f_S(s)}{Pr(S \le 1)} & 0 \le s \le 1 \\ 0 & \text{else.} \end{cases}$$




$$\{S_1 \le 1\} \Longleftrightarrow \{N(1) \ge 1\}$$

$$= \begin{cases} \dfrac{3e^{-3s}}{Pr(S \le 1)} & 0 \le s \le 1 \\ 0 & \text{else} \end{cases}$$

$Pr(S > 1-w \mid S \le 1)$

$$= \int_{1-w}^{1} \frac{3e^{-3s}}{Pr(S \le 1)}\, ds.$$

$$= \frac{1}{Pr(S \le 1)} \left[ -e^{-3s} \right]_{1-w}^{1}$$

$$= \frac{e^{-3(1-w)} - e^{-3}}{1-e^{-3}}$$

$$= \frac{e^{-3+3w} - e^{-3}}{1-e^{-3}}$$

$$\int_0^1 \frac{e^{-3+3w} - e^{-3}}{1-e^{-3}} dw$$

$$= \int_0^1 \frac{e^{-3}(e^{3w}-1)}{1-e^{-3}} dw$$

$$= \frac{e^{-3}}{1-e^{-3}} \left[ \frac{e^{3w}}{3} - w \right]_0^1$$

$$= \frac{e^{-3}}{1-e^{-3}} \left( \frac{e^3}{3} - 1 - \frac{1}{3} \right)$$

$$= \frac{e^{-3}}{1-e^{-3}} \left( \frac{e^3}{3} - \frac{4}{3} \right)$$

$$= \frac{1}{3} \frac{1}{1-e^{-3}} \left( 1 - 4e^{-3} \right).$$