

2

Consider the
s.v. system

$$\dot{x} = f(x, t); \quad x(t_0) = x_0$$

Find some

$V(x, t)$, and check for

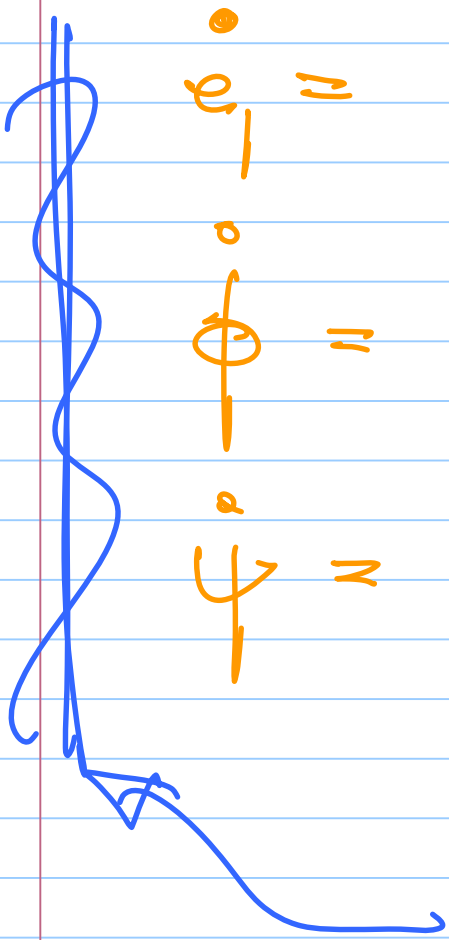
(i) positive-definiteness

(ii) decreasence

$$(iii) \quad \dot{V}(x, t) \leq 0$$

(iv) $V(x, t)$ radially unbounded

Go back and look at what we
have already done:



$$\dot{x} = f(x, t)$$

~~~~~~~~~

Also, note that we used:

$$V(e, \phi, \psi) = \sum \begin{bmatrix} e \\ \phi \\ \psi \end{bmatrix}$$

$x = \begin{bmatrix} e \\ \phi \\ \psi \end{bmatrix}$

~~\_\_\_\_\_~~

(i) Choose

$$\alpha(\|x\|) =$$

$$V(x, t) \geq \alpha(\|x\|)$$

(ii) Choose

$$\beta(\|x\|) =$$

$$V(x, t) \leq \beta(\|x\|)$$

(iii) From "Control Law",  
"Adaptive Law" etc . . .

$$\dot{V}(x, t) =$$

(iv) ~~Clearly~~ choice of  $\alpha(\|x\|)$   
above give

# Adaptive Control for a Class of Systems with measurable State Variables

$$\dot{x}_p = A_p x_p + g b u$$

Diagram annotations:

- A blue arrow points from the word "scalar" to the input  $u$ .
- A green arrow points from the text " $b \in \mathbb{R}^n$  is known" to the vector  $b$ .
- An orange arrow points from the text "measurable,  $x_p \in \mathbb{R}^n$ " to the state vector  $x_p$ .

— (2.1)

We wish to achieve some  
suitably specified closed-loop

$$\dot{x}_m = A_m x_m + g_m b r$$

— (2.2)

Clearly, a suitable approach would be to consider the state-feedback

$$u(t) =$$

— (2.3)

For the case where the system (2.1) is known, we then have =

$$\dot{x}_p = A_p x_p + g b \left\{ \right.$$

$$= \left\{ \right. x_p + b r$$

Calculate  $\hat{\theta}_x = \theta_x^*$  so that this  $\left\{ \right.$

Calculate  $\theta_r = \theta_r^*$  so that

$$\underline{\underline{g\theta_r^* =}}$$

"Matchy  
Conditions"

← (2.4)

But we do not know the  
system (2.1) !!

How about considering?

$$u(t) =$$

— (2.5)

Control  
Law

Since we are using time-varying gains, how are we specifying the time-variation?

Consider then:

•

$$\dot{D}(t) =$$

•

$$\dot{A}_r(t) =$$

— (2.6)

Adaptive Law

where

$$e(t) \triangleq$$

$\Gamma$  is any  $(n \times n)$  positive-def matrix



$\gamma$  is any scalar  $\gamma > 0$

and for a stable  $A_m$  matrix,  
we have the matrix  $P$ , which is  
the solution to  $\equiv$  (2.7)

[Lyapunov Eqn]

where  $Q$  is any  $(n \times n)$  positive-def  
symmetric  
matrix.

The solution  $P$  always exists,  
and  $P$  is also sym positive-def.

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How does this work?

First, note that the

"Control Law" (2.5) results in

$$\dot{x}_p = A_p x_p + g_b \left\{ \begin{array}{l} \theta_x^* + \phi(t) \\ \theta_r^* + \phi_r(t) \end{array} \right\}$$

$$\theta_x^* + \phi(t)$$

$$\theta_r^* + \phi_r(t)$$

is

$$\dot{x}_p(t) = \left\{ \begin{array}{l} \underline{A_m} x_p + \underline{g_m} r(t) \\ + g_b \end{array} \right\}$$

Thus, if we consider

$$e(t) \triangleq$$

$$e(t) = \hat{x}_p(t) - \hat{x}_m(t)$$

$$= A_m e(t) + g b \quad \left. \vphantom{\begin{matrix} \\ \\ \\ \end{matrix}} \right\} \text{---(2.11a)}$$

Error Signal  
Dynamics / Error Model

Note also that we have additionally:

$$\dot{\phi}_x =$$

---(2.11b)

$$\dot{\phi}_r =$$


---(2.11c)

Also, define  $\phi = \begin{bmatrix} \phi_x \\ \phi_r \end{bmatrix}$

Here, consider the quadratic form

$$V(e, \phi_x, \phi_r) =$$

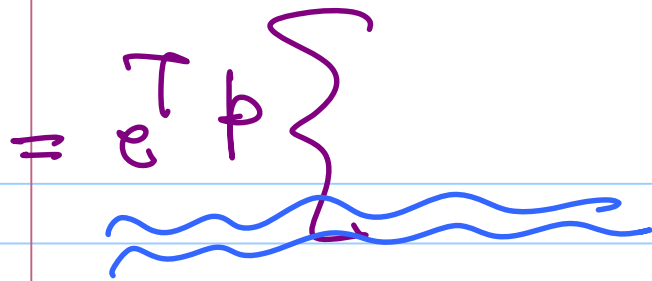
- positive-def
- decoupled
- radially unbounded

$$\begin{bmatrix} \gamma & 0 \\ 0 & \gamma \end{bmatrix}$$


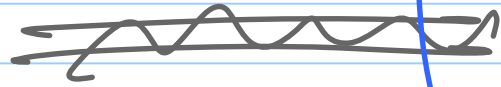
Note then that we have:

•  
 $V(t) =$



$$= e^T p$$


+



$$\begin{bmatrix} \delta_p \\ r \end{bmatrix}$$

From Linear Algebra, any  $n \times n$  matrix  $M$  can be written as:

$$M =$$

$$M_S$$

symmetric

$$M_{AS}$$

anti-symmetric

and note further that for any  
quadratic form =

$$x^T M x =$$

$\Delta \Delta \Delta$

---

Thus, this means that

$$e^T \{ P A_m \} e$$

=

=

with  $\mathcal{Q}$  symm positive-def  
from Eqn (2.7).

$\hat{y}_d$

$$\dot{V}(t) =$$

+



$$- \text{sgn}(y) |y| \overline{\Gamma}^T e^T P b \begin{bmatrix} x_p \\ r \end{bmatrix}$$

$\hat{y}_d$  for the choice of our

"Adaptive Law" (2.6),

results in

$$\dot{V}(t) =$$

(a)  $V$  sym positive definite

$$\dot{V} = -\frac{1}{2} e^T Q e \leq 0$$



(b)  $\int_{t_0}^t e^T Q e \, d\tau \leq C_1$

for all  $t \geq t_0$

is,  $\|e\|$  is 'square-integrable'



$$(c) \quad \ddot{e} = A_m e + g_b \left\{ \phi_x^T x_p + \phi_r^T \right\}$$

Well  $\ddot{e}$  bounded for all  $t \geq t_0$

I.e. (b) & (c) results in

~~##~~