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Consider the
s.v. system

$$\dot{x} = f(x, t); \quad x(t_0) = x_0$$

Find some

$V(x, t)$, and check for

(i) positive-definiteness

(ii) decreasence

(iii) $\dot{V}(x, t) \leq 0$

(iv) $V(x, t)$ radially unbounded

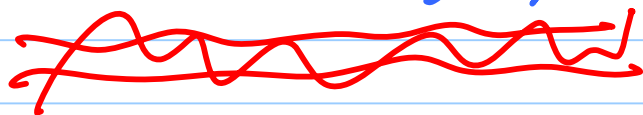
Go back and look at what we have already done:

$$\dot{e}_1 = a_m e_1 + k_p \left\{ \phi(t) y_p(t) + \psi(t) r(t) \right\}$$

$$\dot{\phi} = -\operatorname{sgn}(k_p) \gamma_1 e_1(t) y_p(t)$$

$$\dot{\psi} = -\operatorname{sgn}(k_p) \gamma_2 e_1(t) r(t)$$

$$\dot{x} = f(x, t)$$



Also, note that we used:

$$V(e, \phi, \psi) = \frac{1}{2} [e, \phi, \psi] \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{\|k_p\|}{\gamma_1} & 0 \\ 0 & 0 & \frac{\|k_p\|}{\gamma_2} \end{bmatrix} \begin{bmatrix} e \\ \phi \\ \psi \end{bmatrix}$$

$x = \begin{bmatrix} e \\ \phi \\ \psi \end{bmatrix}$

~~$V(x, t)$~~

(i) Choose

$$\frac{1}{2} \left\{ e^2 + \frac{\|k_p\|}{\gamma_1} \phi^2 + \frac{\|k_p\|}{\gamma_2} \psi^2 \right\}$$

$$\alpha(\|x\|) = \min \left\{ 1, \frac{\|k_p\|}{\gamma_1}, \frac{\|k_p\|}{\gamma_2} \right\} \|x\|^2 \frac{1}{2}$$

$$V(x, t) \geq \alpha(\|x\|)$$

(ii) Choose

$$\beta(\|x\|) = \max \left\{ \quad \right\} \|x\|^2 \frac{1}{2}$$

$$V(x, t) \leq \beta(\|x\|)$$

(iii) From "Control Law",
"Adaptive Law" etc ...

$$\dot{V}(x, t) = a_m e_1^2 \leq 0 \quad \because a_m < 0$$

(iv) Clearly choice of $\alpha(\|x\|)$
above give

$$\alpha(\|x\|) = \min \left\{ \|x\|^2 \frac{1}{2} \right\}$$

$$\|x\| \rightarrow \infty, \text{ then } \alpha(\|x\|) \rightarrow \infty$$

Adaptive Control for a Class of Systems with measurable State Variables

$$\dot{x}_p = A_p x_p + g b u$$

Diagram annotations:

- A blue arrow points from the word "scalar" to the variable u .
- An orange arrow points from the text "measurable, $x_p \in \mathbb{R}^n$ " to the term x_p .
- A green arrow points from the text " $b \in \mathbb{R}^n$ is known" to the term b .

(2.1)

We wish to achieve some
arbitrarily specified closed-loop

$$\dot{x}_m = A_m x_m + g_m b r$$

(2.2)

Clearly, a suitable approach would be to consider the state-feedback

$$u(t) = \overset{T}{\theta}_x^T x_p + \theta_r r \quad \text{--- (2.3)}$$

For the case where the system (2.1) is known, we then have =

$$\begin{aligned} \dot{x}_p &= A_p x_p + g b \left\{ \theta_x^T x_p + \theta_r r \right\} \\ &= \left\{ A_p + g b \theta_x^T \right\} x_p + \underline{\underline{g \theta_r b}} r \end{aligned}$$

Calculate $\theta_x = \theta_x^*$ so that this $\left\{ \underline{\underline{A_m}} \right\} = A_m$!!

Calculate $\theta_r = \theta_r^*$ so that

$$\underline{g\theta_r^*} = g_m$$

"Matching
Conditions"

— (2.4)

But we do not know the
system (2.1) !!

How about considering:

$$u(t) = \overset{T}{\theta_x(t)} x_p(t) + \theta_r(t) r(t)$$

— (2.5)

Control
Law

Since we are using time-varying gains, how are we specifying the time-variation?

Consider then:

$$\dot{\theta}_x(t) = -\text{sgn}(q) \Gamma^T e(t) P_b x_p(t)$$

$$\dot{\theta}_r(t) = -\text{sgn}(q) \gamma^T e(t) P_b r(t)$$

— (2.6)

Adaptive Law

where

$$e(t) \triangleq x_p(t) - x_m(t)$$

Γ is any $(n \times n)$ positive-def matrix

γ is any scalar $\gamma > 0$

and for a stable A_m matrix,
we have the matrix P , which is
the solution to \equiv (2.7)

$$A_m^T P + P A_m = -Q$$

[Lyapunov Equ.]

where Q is any $(n \times n)$ positive-def
symmetric
matrix.

The solution P always exists,
and P is also sym positive-def.

How does this work?

Next, note that the

"Control Law" (2.5) results in

$$\dot{x}_p = A_p x_p + g_b \left\{ \theta_x^T(t) x_p(t) + \theta_r(t) r(t) \right\}$$

$$\theta_x^* + \phi(t)$$

$$\theta_r^* + \phi_r(t)$$

is

$$\dot{x}_p(t) = \underbrace{\left\{ A_p + g_b \theta_x^{*T} \right\}}_{A_m} x_p + \underbrace{g \theta_r^*}_{g_m} r(t) + g_b \left\{ \phi_x^T(t) x_p(t) + \phi_r(t) r(t) \right\}$$

Thus, if we consider

$$e(t) \triangleq x_p(t) - x_m(t)$$

$$\dot{e}(t) = \dot{x}_p(t) - \dot{x}_m(t)$$

$$= A_m e(t) + g \left\{ \phi_x^T x_p + \phi_r^T r \right\} \quad \text{--- (2.11a)}$$

Error Signal
Dynamics / Error Model

Note also that we have alternatively:

$$\dot{\phi}_x = \dot{\phi}_x = -\text{sgn}(g) \Gamma e^T \phi_b x_p \quad \text{--- (2.11b)}$$

$$\dot{\phi}_r = \dot{\phi}_r = -\text{sgn}(g) \Gamma e^T \phi_b r \quad \text{--- (2.11c)}$$

Also, define $\phi = \begin{bmatrix} \phi_x \\ \phi_r \end{bmatrix}$

Here, consider the quadratic form

$$V(e, \phi_x, \phi_r) = \frac{1}{2} e^T P e + \frac{1}{2} |g| \phi^T \overline{T}^{-1} \phi$$

- positive-def
- decoupled
- radially unbounded

$$\begin{bmatrix} T & 0 \\ 0 & \gamma \end{bmatrix} = \overline{T}$$

Note then that we have:

$$V(t) = \frac{1}{2} \left\{ 2 e^T P e \right\} + \frac{1}{2} \left\{ 2 |g| \phi^T \overline{T}^{-1} \phi \right\}$$

~~XXXXXXXXXX~~

$$= \underbrace{e^\top p \left\{ A_m e + g \delta \phi^\top \bar{x} \right\}}_{\text{wavy line}} + \cancel{\text{wavy line with red X's}}$$

$\left[\begin{array}{c} x_p \\ r \end{array} \right]$

From Linear Algebra, any $n \times n$ matrix M can be written as:

$$M = \underbrace{\frac{1}{2} \{ M + M^T \}}_{\text{symmetric}} + \underbrace{\frac{1}{2} \{ M - M^T \}}_{\text{anti-symmetric}}$$

$M_s = M_s^T$

$M_{as} = -M_{as}^T$

and note further that for any
quadratic form =

$$x^T M x = \dots = x^T M_S x$$

△△△

Thus, this means that

$$e^T \{ P A_m \} e$$

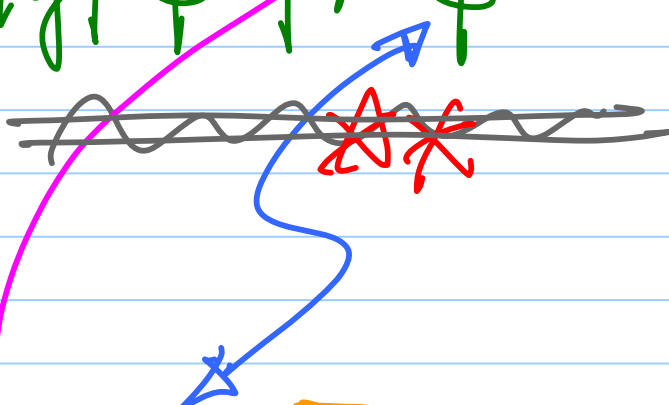
$$= e^T \left\{ \frac{1}{2} \left[P A_m + (P A_m)^T \right] \right\} e$$

$$= \frac{1}{2} e^T \left\{ \underline{P A_m + A_m^T P} \right\} e \quad \text{!!} \quad = -Q$$

with Q symm positive-def

from Eqn (2.7).

$$\dot{V}(t) = -\frac{1}{2} e^T Q e + e^T \phi g b \phi^T \bar{x} + |g| \phi^T \bar{T}^{-1} \dot{\phi}$$



$$- \text{sgn}(g) \bar{T}^{-1} e^T \phi b \begin{bmatrix} x_p \\ r \end{bmatrix} \quad \bar{x}$$

for the choice of our
 "Adaptive Law" (2.6),
 results in

$$\dot{V}(t) = -\frac{1}{2} e^T Q e$$

(a) V sym positive definite

$$\dot{V} = -\frac{1}{2} e^T Q e \leq 0$$

$$\Rightarrow \|e(t)\|, \|\phi_x(t)\|, \|\phi_r(t)\|$$

are bounded for all
 $t \geq t_0$

...

$$(b) \int_{t_0}^t e^T Q e \, d\tau \leq C_1$$

for all $t \geq t_0$

i.e. $\|e\|$ is 'square-integrable'

$$(c) \quad \dot{e} = A_m e + g_b \left\{ \phi_x^T x_p + \phi_r^T \right\}$$

$\|e\|$ is bounded for all $t \geq t_0$

I.e. (b) & (c) results in

$$\lim_{t \rightarrow \infty} \|e(t)\| = \|x_p(t) - x_m(t)\| \rightarrow 0$$

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