EE5137 Stochastic Processes: Problem Set 9
Assigned: 25/03/22, Due: 01/04/22

There are six (6) non-optional problems in this problem set.

1. Exercise 5.11 (Gallager's book)
2. Exercise 5.15 (Gallager's book)
3. Exercise 5.16 (Gallager's book)
4. Exercise 5.17 (Gallager's book)
5. Consider an i.i.d. sequence $\{X_n\}_{n\geq 1}$ with a discrete distribution that is uniform over the integers $\{1, 2, \ldots, 10\}$, i.e., $\Pr(X = i) = 1/10$, for $1 \leq i \leq 10$. Imagine that these are bonuses that are given to you by your employer each year. Let $J = \min\{n \geq 1 : X_n = 6\}$, the first time that you receive a bonus of size 6. What is the expected total (cumulative) amount of bonus received up to time J?
6. Consider a miner trapped in a room that contains three doors. Door 1 leads her to freedom after two-days' travel; door 2 returns her to her room after four-days' journey; and door 3 returns her to her room after eight-days' journey. Suppose at all times she is equally to choose any of the three doors, and let T denote the time it takes the miner to become free.
   (a) Define a sequence of independent and identically distributed random variables $X_1, X_2, \ldots$ and a stopping time J such that
   $$T = \sum_{i=1}^{J} X_i.$$
   Note: You may have to imagine that the miner continues to randomly choose doors even after she reaches safety.
   (b) Use Wald's equation to find $\mathbb{E}[T]$.
   (c) Compute $\mathbb{E}[\sum_{i=1}^{J} X_i | J = j]$ and note that it is not equal to $\mathbb{E}[\sum_{i=1}^{j} X_i]$.
   (d) Use part (c) for a second derivation of $\mathbb{E}[T]$.

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7. (Optional) Exercise 5.18 (Gallager's book)
8. (Optional) Players Jack and Jill will start with \$5 and \$10 respectively and play a game by making a series of \$1 bets until one of them loses all his/her money. We'll assume that in each bet, Jack wins with probability $p = 1/2$, Jill with probability $q = 1/2$, and tie (no money exchanged) with probability $r = 0$, so that
   $$p + q + r = 1.$$
   Let T be the number of bets made until the game ends. Calculate $\mathbb{E}[T]$ and the respective probabilities of Jack or Jill winning.