NATIONAL UNIVERSITY OF SINGAPORE

DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING

ONLINE EXAMINATION

Matriculation No.:	**A0232935A**
Module Code:	**EE5103**
Number of pages in this PDF file (including this cover page and Declaration Form): i.e. 2+no. of answer pages	**11**

INSTRUCTIONS TO CANDIDATES

1. Follow the instructions for online examination and invigilation.

2. Write your answers on A4 size paper with black or dark blue ink.

3. Write the question number at the top left corner of each page. Start the answer to each question on a new page. Indicate the part, e.g. "(a)", on the left margin.

4. At the end of the exam:
   a) scan or take photographs of your answers (make sure your writing and/or drawings can be seen clearly);
   b) enter your matriculation number, module code and the total number of pages (including the cover and declaration pages, i.e. 2+number of answer pages) on the cover page;
   c) merge the completed cover page, signed declaration form and your answers into a single PDF file named `<matric_no>-<module code>.pdf` (e.g. `A1234567R-EExxxx.pdf`)
   d) open the PDF file to ensure that it has been generated without error and the contents are correct;
   e) upload your PDF file into the stated LumiNUS exam submission folder within the stipulated deadline. Late submissions will not be accepted.

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Question	Mark	Remarks
TOTAL		

Q.1

(a) $H(z) = \frac{Y(z)}{U(z)} = C(zI-\phi)^{-1}\cdot\Gamma = \frac{\alpha z+1}{z^2-\alpha z-1}$

We can use Jury's stability criterion

$a_1 = -\alpha; \quad a_2 = -1$

$$\begin{cases} 1-\alpha_2^2 > 0 \\ \frac{1-\alpha_2}{1+\alpha_2}\left[(1+\alpha_2)^2-\alpha_1^2\right] > 0 \end{cases} \Rightarrow \phi$$

No mater what α is, the open loop system is unstable.

(b) The desired poles are z^2, so $\Delta_c(z) = z^2$

$W_c = [\Gamma \quad \phi\Gamma] = \begin{bmatrix} 1 & 1 \\ 1 & 1+\alpha \end{bmatrix}$

$L = [0 \quad 1]\cdot W_c^{-1}\Delta_c(\phi)$

$= [0 \quad 1]\begin{bmatrix} 1 & 1 \\ 1 & 1+\alpha \end{bmatrix}^{-1}\begin{bmatrix} 0 & 1 \\ 1 & \alpha \end{bmatrix}^2$

$= [1-\frac{1}{\alpha} \quad \alpha-1+\frac{1}{\alpha}]$

The controller can be expressed as:

$u(k) = -Lx(k) = [\frac{1}{\alpha}-1 \quad 1-\frac{1}{\alpha}-\alpha]\cdot x(k)$

(c) The observer can be expresed as:

$\hat{x}(k+1) = \phi\hat{x}(k) + \Gamma u(k) + K[y(k)-\hat{y}(k)]$

$\hat{y}(k) = C\hat{x}(k)$

$W_o = \begin{bmatrix} C \\ C\phi \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad f(z) = z^2$

$K = f(\phi)\cdot W_o^{-1}\cdot[0 \quad 1]^T$

$= \begin{bmatrix} 0 & 1 \\ 1 & \alpha \end{bmatrix}^2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^{-1}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha \\ 1+\alpha^2 \end{bmatrix}$

The controller can be expressed as:

$u(k) = -L\cdot\hat{x}(k)$, where L is from Q.1 (b)

Q. 1

(d) Let $z(k) = \begin{bmatrix} x(k) \\ w(k) \end{bmatrix}$

$$z(k+1) = \phi_z z(k) + \Gamma_z u(k)$$
$$y(k) = C_z z(k)$$

$$\phi_z = \begin{bmatrix} \phi & \phi_{xw} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & \alpha & 1 \\ 0 & 0 & 1 \end{bmatrix}, \quad \Gamma_z = \begin{bmatrix} \Gamma \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad C_z = [C \quad 0] = [1 \quad 0 \quad 0].$$

The observer can be expressed as:

$$\hat{z}(k+1) = \phi_z \hat{z}(k) + \Gamma_z u(k) + \cancel{K[y(k)]} \; K[y(k) - \hat{y}(k)]$$
$$\hat{y}(k) = C_z \hat{z}(k)$$

$$W_o = \begin{bmatrix} C_z \\ C_z \phi_z \\ C_z \phi_z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The charateristic polynomial is $f(z) = z^3$.

$$K = f(\phi_z) W_o^{-1} [0 \quad 0 \quad 1]^T$$
$$= \begin{bmatrix} 0 & 1 & 0 \\ 1 & \alpha & 1 \\ 0 & 0 & 1 \end{bmatrix}^3 \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$
$$= \begin{bmatrix} \alpha + 1 \\ \alpha^2 + \alpha + 2 \\ 1 \end{bmatrix}.$$

Then, because we want eliminate the disturbance.

$$u(k) = -[L \quad L_w] \begin{bmatrix} x(k) \\ w(k) \end{bmatrix} = -L x(k) - L_w w(k)$$

$$H_w(z) = \frac{Y(z)}{W(z)} = C[zI - (\phi - \Gamma L)]^{-1} (\phi_{xw} - \Gamma L_w).$$

When, $H_w(1) = 0$, the disturbance can be eliminated.

$\Rightarrow L_w =$

$$u(k) = -L x(k) - L_w w(k)$$

Q. 1

(e) $\cancel{x(k)=}$ Let $\phi = \begin{bmatrix} 0 & 1 \\ 1 & \alpha \end{bmatrix}$, $\Gamma = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

$$x(k) = \phi x(k-2) + \Gamma u(k-4) \quad ①$$

$$x(k-2) = \phi x(k-4) + \Gamma u(k-6) \quad ②$$

take ② into ①.

$$x(k) = \phi[\phi x(k-4) + \Gamma u(k-6)] + \Gamma u(k-4)$$

$$= \phi^2 x(k-4) + \phi \cdot \Gamma u(k-6) + \Gamma u(k-4).$$

~~So:~~ $k = 1, 2, \cdots$

$\cancel{x(k) = x(2k) = \phi^k x(0) + \phi^{k-1}\Gamma u(k-2) + \cdots + \phi \Gamma u(2k-4)}$

So:

$$x(2k) = \phi^k x(0) + \phi^{k-1} \cdot \Gamma u(-2) + \cdots + \Gamma u(2k-4)$$

$\cancel{x(2k+1) = \phi^k x(0) +}$

$$x(2k-1) = \phi^k \Gamma u(-3) + \phi^{k-1} \Gamma u(-1) + \cdots + \Gamma u(2k-5)$$

Q. 2

(a) $q\cdot y(k) = q^{-1}y(k) + 2u(k) + q^{-1}\cdot u(k) + q\cdot v(k) + v(k)$

$(q^2-1)y(k) = (2q+1)u(k) + (q^2+q)\cdot v(k)$

Open loop T.F. from u to y.

$$\cancel{H(z)}\ G(z) = \frac{Y(z)}{U(z)} = \frac{2z+1}{z^2-1}$$

T.F from v to y:

$$G_v(z) = \frac{Y(z)}{V(z)} = \frac{z^2+z}{z^2-1}.$$

(b) $U(z) = \frac{T(z)}{R(z)} U_c(z) - \frac{S(z)}{R(z)} \cdot Y(z)$

$A_m(z) = z^2$.

We want to reject ~~v(k)~~ disturbance,

so, $H_v(1)=0$, $R(z)$ contain : $z-1$.

From Q.2 (a) we know

$A(z) = z^2-1$, $B(z) = 2z+1$, $B_v(z) = z^2+z$.

Because $B(z)$ is stable, ~~Because B(z) is stable~~, let $A_{cl}(z) = A_o(z)\cdot A_m(z) = z^3 B(z)$

$R(z) = (z-1)(r_0 + r_1 z)$, $S(z) = s_0 + s_1 z + s_2 z^2$

$A_{cl}(z) = A(z)R(z) + B(z)S(z)$

~~$z^3 B(z) = A(z)R(z$~~

$z^3\cdot B(z) = (z^2-1)(z-1)(r_0+r_1 z) + B(z)(s_0+s_1 z+s_2 z^2)$

$$\Rightarrow \begin{cases} r_0 = 1 \\ r_1 = 2 \\ s_0 = -1 \\ s_1 = 1 \\ s_2 = 1 \end{cases} \Rightarrow S_z = z^2+z-1,\quad R(z) = (z-1)(2z+1)$$

Close T.F:

$H_{cl}(z) = \frac{B_z T(z)}{A_{cl}(z)} = \frac{T(z)}{z^3}$. We want $y(k)$ follow $\frac{1}{z^2}$

so, $T(z) = z$.

Q.2

(c) Yes.

Q.3

(a) $K_f(2) = P(2|1)\cdot C^T[C\cdot P(2|1)\cdot C^T + R_2]^{-1}$

$$= \begin{bmatrix} 0.5454 \\ 0.8376 \end{bmatrix}$$

$$K(2) = (A P(2|1) C^T)(CP(2|1) C^T + R_2)^{-1} = \begin{bmatrix} 0.4363 \\ 1.274 \end{bmatrix}.$$

(b) $\cancel{\hat{x}(2|2) = A}$

$$\hat{x}(2|2) = \hat{x}(2|1) + K_f(2)(y(2) - Cx(2|1))$$

$$= \begin{bmatrix} 0.8 \\ 1.8 \end{bmatrix} + \begin{bmatrix} 0.5454 \\ 0.8376 \end{bmatrix}\left(2 - [0 \quad 1]\begin{bmatrix} 0.8 \\ 1.8 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 0.909 \\ 1.9675 \end{bmatrix}$$

because $w(k)$ and $v(k)$ are zero-mean.

$y(\cancel{2) = Cx(2)}$, $\cancel{x(2) =}$

$E[x(2) \cancel{= E[A x(1)] = E[A^2 x(0)}]$

$E[x(2) - \hat{x}(2|2)] = E[A^2 x(0) - \hat{x}(2|2)]$

$$E\{[x(2) - \hat{x}(2|2)][x(2) - \hat{x}(2|2)]^T\} = P(2|2) = \cancel{\frac{P(2|1)\ R_2}{CP(2|1)\ C^T + R_2}} =$$

$$= P(2|1) - P(2|1)C^T(CP(2|1)\ C^T + R_2)^{-1}\ CP(2|1)$$

$$= \begin{bmatrix} 1.3636 & 0.2727 \\ 0.2727 & 0.4188 \end{bmatrix}$$

Q.3 (c)

$$\hat{x}(3|2) = A\hat{x}(2|1) + K(2)[y(2) - C\hat{x}(2|1)]$$

$$= \begin{bmatrix} 0.7272 \\ 2.6948 \end{bmatrix}$$

$$E\{[x(3) - \hat{x}(3|2)][x(3) - \hat{x}(3|2)]^T\} = P(3|2) =$$

$$= A P(2|1) - K(2)[CP(2|1)C^T + R_2]K^T(2) + R_1$$

$$= \begin{bmatrix} 2.2377 & -0.368 \\ 1.792 & -1.075 \end{bmatrix}$$

(d). $A_m(z) = [z - (0.32 + j0.29)][z - (0.32 - j0.29)] = z^2 - 0.64z + 0.1865$.

$W_o = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0.8 & 1 \end{bmatrix}$.

$K_{ob} = A_m(A) \cdot W_o^{-1} \cdot [0 \quad 1]^T =$

$= (A^2 + 0.64A + 0.1865I)(W_o^{-1} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$= \begin{bmatrix} 1.6731 \\ 2.44 \end{bmatrix}$.

Q.4

(a) $\cancel{\frac{Y(z)}{U(z)} = z}$ Apply z-trans.

$$z X_p(z) = a_p X_p(z) + (1-a_p) U(z)$$

$$X_p(z) = \frac{1-a_p}{z-a} U(z)$$

$$Y(z) = X_p(z) = \frac{1-a_p}{z-a} U(z)$$

So, open loop T.F.

$$\frac{Y(z)}{U(z)} = \frac{1-a_p}{z-a}$$

(b) Let $x(k) = \begin{Bmatrix} \Delta X_p(k) \\ y(k) \end{Bmatrix}$.

$$X_p(k+1) - X_p(k) = a_p[X_p(k) - X_p(k-1)] + (1-a_p)\cdot[u(k) - u(k-1)]$$

$$\Delta X_p(k+1) = a_p \Delta X_p(k) + (1-a_p)\Delta u(k)$$

$$y(k+1) - y(k) = X_p(k+1) - X_p(k) = \Delta X_p(k+1)$$

$$y(k+1) = y(k) + a_p \Delta X_p(k) + (1-a_p)\Delta u(k).$$

So: $A = \begin{bmatrix} a_p & 0 \\ a_p & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1-a_p \\ 1-a_p \end{bmatrix}$, $C = [0 \quad 1]$

$$x(k) = \begin{Bmatrix} \Delta X_p(x) \\ y(k) \end{Bmatrix} = \begin{Bmatrix} X_p(k) - X_p(k-1) \\ y(k) \end{Bmatrix}.$$

(c) $\bar{R}_s = \overbrace{[1 \quad 1 \ldots 1]}^{N_p = n} \cdot r(k)$

$$\bar{F} = \begin{bmatrix} CA \\ CA^2 \\ \vdots \\ CA^{N_p} \end{bmatrix} = \begin{bmatrix} a_p & 1 \\ a_p^2 + a_p & 1 \\ \vdots & \\ a_p^n + a_p^{n-1} + \cdots + a_p & 1 \end{bmatrix}$$

$$\bar{R} = r_w \cdot I_{N_C \times N_C} = 0.$$

$$\phi = \begin{bmatrix} CB \\ CAB \\ \vdots \\ CA^{n-1}B \end{bmatrix} = \begin{bmatrix} (1-a_p)(a_p+1) \\ (1-a_p)(a_p^2 + a_p + 1) \\ \vdots \\ (1-a_p)(a_p^n + a_p^{n-1} + \cdots + a_p + 1) \end{bmatrix}$$

Q.4

(d) $\lim_{n\to\infty} \frac{1}{n} \phi^T \phi$

$$= \lim_{n\to\infty} \frac{1}{n} \cdot [(1-a_p)(a_p+1), \; \cdots \; (1-a_p)(a_p^n + a_p^{n-1} + \cdots + 1)] \cdot \begin{bmatrix} (1-a_p)(a_p+1) \\ \vdots \\ (1-a_p)(a_p^n + a_n^{n-1} + \cdots + 1) \end{bmatrix}$$

$$= \cancel{\lim_{n\to\infty} \frac{1}{n} (1-a_p)(1-a_p)(}$$

(e) $\lim_{n\to\infty} \frac{1}{n} \phi^T \bar{R}_s$

(f) $\lim_{n\to\infty} \phi^T F$

(g) $K_r = [1 \; 0 \; \ldots] (\phi^T \phi + \bar{R})^{-1} \phi^T \bar{R}_s$

$K_{mpc} = [1 \; 0 \; \ldots] (\phi^T \phi + \bar{R})^{-1} \phi^T F$.

Exam Declaration Form

Please read sections A, B and C below. Sign and submit this declaration form together with your answers.

A. Academic, Professional and Personal Integrity

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Signature: 刘伟豪.　　Date: 27 Nov 2021

Matric. No.: A0232935J