Chapter 4

$\boxed{\text{EDF}}$ (4.1) $T_1 = (e_1 = 10, p_1 = 20)$
$T_2 = (5, 50)$
$T_3 = (10, 35)$

Verify the utilization due to the 3 tasks;

$$\sum_{i=1}^{3} \frac{e_i}{p_i} = \frac{10}{20} + \frac{5}{50} + \frac{10}{35} = 0.89 < 1 \Rightarrow \text{EDF schedulable,}$$

(4.2) $\boxed{\text{RMA}}$ $T_1 = (20, 100)$
$T_2 = (30, 150)$
$T_3 = (60, 200)$

first compute utilization (nec. condition)

$$\sum_{\forall i} u_i = \frac{20}{100} + \frac{30}{150} + \frac{60}{200} = 0.7 < 1$$

Let us check sufficient condition (LL condition)

$$\Rightarrow \quad 3 \cdot (2^{1/3} - 1) = 0.78.$$

Total utilization is < 1 & it is $0.7 < 0.78 \Rightarrow$ RMA schedulability is satisfied. Task set is RMA schedulable.

(4.3) $T_1 = (20, 100)$ $T_2 = (30, 150)$, $T_3 = (90, 200)$

<u>Step1:</u> $\sum_{\forall i} u_i = \frac{20}{100} + \frac{30}{150} + \frac{90}{200} = 0.85$

(2) Now let's check LL condition:

$$\sum_{i=1}^{3} U_i \underbrace{\leq 0.78}_{\text{bound for 3 tasks}} \Rightarrow \text{violates } (0.85 > 0.78)$$

$\Rightarrow$ task set is not RMA schedulable.

$\boxed{\text{Use the theorem}}$: A set of tasks is RMA schedulable under any task phasings, iff all the tasks meet their respective first deadlines under zero phasing.

For T_1: $e_1 < p_i$ $\Rightarrow$ it would meet its 1st deadline & it does not have any hi-priority task.

For T_2: T_1 is its higher priority task & considering zero phasing it would occur once before the deadline of T_2.

$\Rightarrow$ $(e_1 + e_2) < p_2$ holds since $20 + 30 = 50$ msec < 150 msec

$\Rightarrow$ T_2 meets its first deadline

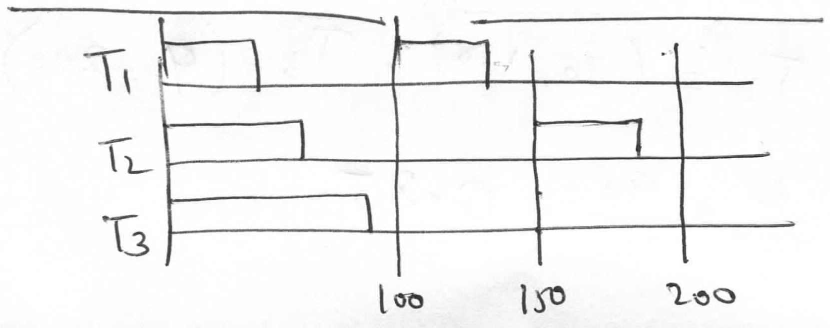
For T_3: $(\underbrace{2 \cdot e_1 + 2 e_2}) + e_3) < p_3$ holds since

$2 \cdot 20 + 2 \cdot 30 + 90 = 190$ msec < 200 msec

T_2 & T_1 occurs twice within the deadline (first) of T_3

$\Rightarrow$ T_3 meets its first deadline

$\Rightarrow$ task set is RMA schedulable



(4.4) Formal expression for schedulability criterion.

Let $\{T_1, \cdots, T_i\}$ be the ordered task set s.t,

$$\underbrace{pri(T_j)}_{\text{priority of task } T_j} > pri(T_{j+1}), \; j=1, .., i-1.$$

Let task T_i arrives at $t=0$. We need to determine the exact # of times that T_1 occurs within a single instance of T_i. This is $\left\lceil \frac{p_i}{p_1} \right\rceil$. Then the total execution time due to T_1 before the deadline of T_i is $\left\lceil \frac{p_i}{p_1} \right\rceil \times e_1$. Generalizing this, the time for which T_i has to wait due to its all higher pri. tasks can be expressed as,

$$\sum_{k=1}^{i-1} \left\lceil \frac{p_i}{p_k} \right\rceil * e_k \quad \text{——— (1)}$$

Then T_i will meet its deadline <u>iff</u>

$$e_i + \sum_{k=1}^{i-1} \left\lceil p_i / p_k \right\rceil * e_k \leq p_i \quad \text{——— (2)}$$

execution time of kth task.

Eqn (2) is the generalized form, and we assumed $p_i = d_i$. If $p_i < d_i$, then (2) can be rewritten as,

$$e_i + \sum_{k=1}^{i-1} \left\lceil \frac{d_i}{p_k} \right\rceil * e_k \leq d_i \quad \text{—— (3)}$$

<u>Note</u>: We also assumed zero phasing, which is the worst case. It may be possible that (3) fails yet the task set may be schedulable & this can occur when tasks have non-zero phasings.

(4.5) $\sum_{i=1}^{3} u_i = \frac{22}{100} + \frac{32}{150} + \frac{92}{200} = \underline{\underline{0.893}}$

$0.893 > 0.78 \Rightarrow$ not RMA schedulable as per LL criteria.

<u>Lehoczky's test</u>: (remember to use zero phasing)

T_1: $22 < 100 \Rightarrow T_1$ meets its deadline

T_2: $(2 * 22 + 32) < 150 \Rightarrow T_2$ meets its deadline

T_3: $2 * 22 + 2 * 32 + 90 < 200 \Rightarrow T_3$ is also schedulable.

Hence the given task set is schedulable.

We know that,

4.b $$e_i + bt_i + \sum_{k=1}^{i-1} \left\lceil \frac{p_i}{p_k} \right\rceil * p_k \le p_i$$

$$\Rightarrow e_i + b_i + \sum_{k=1}^{i-1} \min(e_k, b_k)$$

$$+ \sum_{k=1}^{i-1} \left\lceil \frac{p_i}{p_k} \right\rceil * p_k \le p_i$$

If $p_i < d_i$ then replace p_i with d_i in the above expression.

(4.7) $\boxed{\text{Harmonic Tasks}}$

Task set $\{T_1, T_2, \cdots, T_n\}$ s.t for any i, j, $p_i < p_j$ whenever $i < j$. A task meets its deadline if

$$e_i + \sum_{k=1}^{i-1} \left\lceil p_i/p_k \right\rceil \times e_k \le p_i$$ [Ex 4.4 Equation (2)]

Since the task set is harmonically releated, let

$$p_i = m \cdot p_k \text{ for some } m.$$

$$\Rightarrow \left\lceil \frac{p_i}{p_k} \right\rceil = \left(\frac{p_i}{p_k} \right).$$

(b)

Thus,

$$e_i + \sum_{k=1}^{i-1} \frac{p_i}{p_k} \cdot e_k \le p_i$$

For $T_i = T_n$ we can write,

$$e_n + \sum_{k=1}^{n-1} \frac{p_n}{p_k} * e_k \le p_n$$

$\div$ both sides by p_n,

$$\left(\frac{e_n}{p_n}\right) + \sum_{k=1}^{n-1} \frac{e_k}{p_k} \le 1$$

$$\Rightarrow \sum_{i=1}^{n} U_i \le 1$$

(4.8) $\boxed{\text{DMA}}$

Use Lehoczky's condition,

$T_1 : 10 < 35 \Rightarrow T_1$ would meet its deadline

$T_2 : 10 + 15 \not< 20 \Rightarrow T_2$ will miss its first deadline

$\Rightarrow$ not <u>RMA</u> schedulable.

Under DMA, $Pr(T_2) > Pr(T_1) > Pr(T_3)$

T_2 : $15 < 20 \Rightarrow T_2$ is schedulable

T_1 : $(15 + 10) < 35 \Rightarrow T_1$ is schedulable

40 + 30 + 20 = 90

T_3 : $(\cancel{20} + \cancel{10} + \cancel{15}) < 200 \Rightarrow T_3$ is also schedulable.

$\Rightarrow$ given task set is DMA schedulable.