EE5110: Special Topics in Automation and Control
Segment C: Control Optimization

Important note:

1. **There are FIVE questions in CA. The total marks are 40. You have two options. You can either answer the first FOUR questions, or answer the last question, question FIVE, only. If you try to answer all the five questions, no bonus marks will be awarded.**
2. **Please submit the soft copy of your scripts in PDF format to the submission folder in LumiNUS. You can either handwrite or typeset your answers. The due date is 10/10/2021. Late submission is not allowed unless it is well justified.**
3. **If you want to do a research project for this segment, please contact me as soon as possible. The description of the research project of segment C is already given in the first class of this module in Week One, which is about approximate dynamic programming, the deadline is 19/11/2021.**

Q.1. Consider the first order system described by

$$\dot{y} = 2y + au, y(0) = c.$$

Determine the optimal control input u(t) to minimize the functional

$$J(y,u) = \int_0^\infty [y^2 + wu^2]dt$$

where $w > 0$, is a weight factor for the inputs. Use both calculus of variations and dynamic programming to obtain the control law, and show that the control laws obtained from the two methods are equivalent to each other from the feedback point of view.

Discuss two extreme cases where the weight factor $w \to 0$ and $w \to \infty$, and explain your findings.

(10 marks)

Q.2. Given a system described by

$$y(n+1) = 2y(n) + au(n), y(0) = c.$$

Determine the optimal control sequence u(n) using dynamic programming so that the quadratic form

$$J_N(y,u) = \sum_{n=0}^{N} (y^2(n) + wu^2(n))$$

is minimized. $w > 0$, is a weight factor for the inputs. Discuss the case when $N \to \infty$.

(10 marks)

Q.3. A lifeguard, some feet up the beach, sees a drowning swimmer diagonally ahead, some distance offshore and some distance to one side, as shown in the Figure Q.3.

The life guard can run at a certain speed and swim at a certain lesser speed. How does one find the fastest path to the drowning swimmer? This is a problem about finding the path of least time. The lifeguard travels faster on land than in water; the best path is a compromise as shown in Figure Q.3.

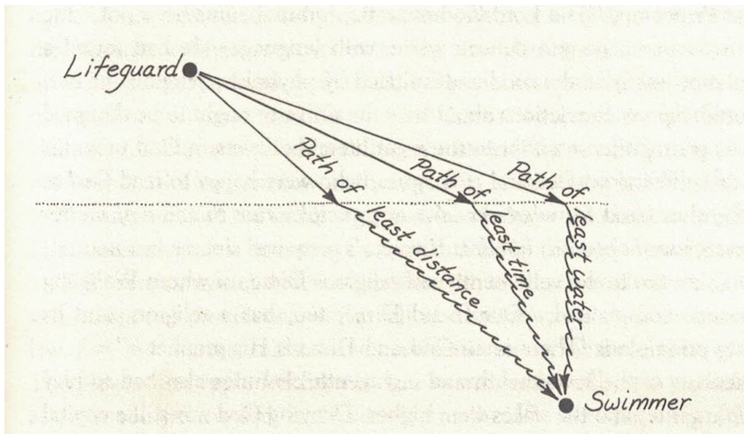

Figure Q.3.

Assume that the lifeguard's running speed on the land is 8m/s while in the water is only 2m/s. Assume that boundary between the sea and the land is the x axis. The initial position of the lifeguard in the x-y plane is assumed to be (0, 10), and the position of the drowning swimmer is (20, -20). Assume that the swimmer is drowning, in other words, the position of the swimmer is fixed in x-y plane. What is the shortest time that the lifeguard can reach the swimmer? What is the corresponding shortest path? You can use either calculus of variations or dynamic programming to solve this optimization problem.

(10 marks)

Q.4. A professor in mathematics is coming to Singapore to attend an international conference. He plans to take a one-day tour of the Singapore after the three-day conference is over. Due to time constraint, he wants to find out the best route to see all the major attractions such as Sentosa Island, Singapore Flyer, Singapore Zoo etc. He stays at Marina Bay Sands hotel. So he will leave the hotel in the morning and comes back to hotel at night. To save time, he will take taxi from one place to another. What is the best route such that the traveling time on the road is the shortest?

To plan the best path, the professor already knows the following:

1. There are total of n major attractions he wishes to visit.
2. The average traveling time from one major attraction to another (including the hotel) is known. To make it simpler, it is assumed that the average traveling time is the same for both directions for two fixed places.

Can you help the professor to find out the best route? If n is not large, you can do it with brutal force (exhaustive search) by listing out all the possible combinations. But since you are taking EE5110, you are expected to find a smarter solution with dynamic programming.

(10 marks)

Q.5. Please make up a story, just like a short novel. In this story, you **CANNOT** use any mathematical language such as “assume”, “minimize”, “maximize”, “x-y plane”, “calculus”, “programming” etc. Together with the story, you should provide a solution to show that how this story can be translated into an optimization problem, and how either calculus of variations or dynamic programming can be used to give the optimal solution.

In the solution part, you can use any mathematical languages as you wish.

When you make up the story, please note the following:

- This story cannot be a story on how to pack a luggage for traveling, how to take a tour around different cities, how to save a girl in the sea, etc. As those are already discussed in the class.
- Do not copy others’ story. I will use “google” to check the writing of your story.
- If your story is very similar to your classmates’ story, I will have a chat with all the parties involved.

If you decide to try this one, there is no need to answer the other four questions.

(40 marks)