Note Title 8/15/2013

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Consider the s.v. system

$$\dot{x} = f(x, t) \; ; \quad x(t_0) = x_0$$

Find some $V(x, t)$, and check for

(i) positive-definiteness

(ii) decrescence

(iii) $\dot{V}(x, t) \leq 0$

(iv) $V(x, t)$ radially unbounded

Go back and look at what we have already done:

$$\dot{\theta} =$$

$$\dot{\phi} =$$

$$\dot{\psi} =$$

$$\dot{x} = f(x, t)$$

Also, note that we used:

$$V(e_1, \phi, \psi) = \frac{1}{2}\begin{bmatrix} e_1 & \phi & \psi \end{bmatrix} \begin{bmatrix} \quad & \quad & \quad \\ & & \\ & & \end{bmatrix} \begin{bmatrix} e_1 \\ \phi \\ \psi \end{bmatrix}$$

$$x = \begin{bmatrix} e \\ \phi \\ \psi \end{bmatrix}$$

(i) Choose

$$\alpha(\|x\|) =$$

$$V(x, t) \geq \alpha(\|x\|)$$

(ii) Choose

$$\beta(\|x\|) =$$

$$V(x,t) \leq \beta(\|x\|)$$

(iii) From "Control Law",
"Adaptive Law" etc ...

$$\dot{V}(x,t) =$$

(iv) Clearly choice of $\alpha_1(\|x\|)$
done give

Adaptive Control for a Class of Systems with measurable State Variables

scaler

$$\dot{x}_p = A_p x_p + g\, b\, u$$

$b \in \mathbb{R}^n$ is known

measurable, $x_p \in \mathbb{R}^n$

— (2.1)

We wish to achieve some suitably specified closed-loop

$$\dot{x}_m = A_m x_m + g_m b\, r$$

— (2.2)

Clearly, a suitable approach would be to consider the state-feedback

$u(t) =$

$$\qquad \text{—— (2.3)}$$

For the case where the system (2.1) is known, we then have:

$$\dot{x}_p = A_p x_p + g b \{ \qquad \}$$

$$= \{ \qquad \} x_p + \qquad b r$$

Calculate $\theta_x = \theta_x^*$ so that this $\{ \quad \} =$

Calculate $\theta_r = \theta_r^*$ so that

$g\theta_r^* =$

$\boxed{\text{"Matching Conditions"}}$ — (2.4)

But we do not know the system (2.1) !!

How about considering:

$u(t) =$

— (2.5) $\boxed{\text{Control Law}}$

Since we are using time-varying gains, how are we specifying the time-variation?

Consider then:

$$\dot{\theta}_x(t) =$$

$$\dot{\theta}_r(t) =$$

$$\text{— (2.6)} \qquad \boxed{\text{Adaptive Law}}$$

where

$$e(t) \triangleq$$

Γ is any $(n \times n)$ positive-def matrix

γ is any scalar $\gamma > 0$

and for a stable A_m matrix, we have the matrix P, which is the solution to: $\longrightarrow$ (2.7)

[Lyapunov Eqn]

where Q is <u>any</u> $(n \times n)$ symmetric positive-def matrix.

The solution P always exist, and P is also symm positive-def.

How does this work?

First, note that the "Control Law" (2.5) results in

$$\dot{x}_p = A_p x_p + g b \{ \qquad\qquad \}$$

$$\theta_x^* + \phi(t) \qquad\qquad \theta_r^* + \phi_r(t)$$

i.e.

$$\dot{x}_p(t) = \underbrace{\{ \qquad\qquad \}}_{A_m} x_p + \underbrace{\qquad b}_{g_m} r(t)$$

$$+ g b \{ \qquad\qquad \}$$

Thus, if we consider

$$e(t) \triangleq$$

$$\dot{e}(t) = \dot{x}_p(t) - \dot{x}_m(t)$$

$$\vdots$$

$$= A_m e(t) + q b \Big\{ \qquad \Big\} \quad \text{— (2.11a)}$$

$\boxed{\text{Error Signal Dynamics / Error Model}}$

Note also that we have additionally:

$$\dot{\phi}_x = \qquad \text{— (2.11b)}$$

$$\dot{\phi}_r = \qquad \text{— (2.11c)}$$

Also, define $\phi = \begin{bmatrix} \phi_x \\ \phi_r \end{bmatrix}$

Here, consider the quadratic form

$$V(e, \phi_x, \phi_r) =$$

- positive-def
- decrescent
- radially unbounded

$$\begin{bmatrix} \Gamma & 0 \\ 0 & \gamma \end{bmatrix}$$

Note then that we have:

$$\dot{V}(t) =$$

$= e^T p \{ \qquad \} + \qquad \begin{bmatrix} x_p \\ r \end{bmatrix}$

From Linear Algebra, any matrix $(n \times n)$ M can be written as:

$M = \underbrace{\qquad}_{\text{symmetric}} \qquad \underbrace{\qquad}_{\text{anti-symmetric}}$

$M_s \qquad M_{as}$

and note further that for any quadratic form:

$$x^T M x =$$

$\triangle\triangle\triangle$

Thus, this means that

$$e^T \left\{ P A_m \right\} e$$

$$=$$

$$=$$

with Q sym positive-def

from Eqn (2.7).

i.e.

$\dot{V}(t) =$

$+$

~~$\sim\sim\sim\sim$~~

$- \operatorname{sgn}(g)\, |g| \, \bar{\Gamma}^{T} e^{T} P b \begin{bmatrix} x_p \\ r \end{bmatrix}$

i.e. for the choice of our "Adaptive Law" (2.6), results in

$\dot{V}(t) =$

(a) V symm positive definite

$$\dot{V} = -\frac{1}{2} e^T Q e \leq 0$$

$\Longrightarrow$

$\vdots$

(b) $$\int_{t_0}^{t} e^T Q e \, d\tau \leq c_1$$

for all $t \geq t_0$

i.e. $\|e\|$ is "square-integrable"

(c) $\dot{e} = A_m e + gb\left\{\phi_x^T x_p + \phi_r r\right\}$

$\|\dot{e}\|$ is bounded for all $t \geq t_0$

I.e. (b) & (c) results in

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