EE5103

Computer Control Systems

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Part 1

The state space can be expressed as:

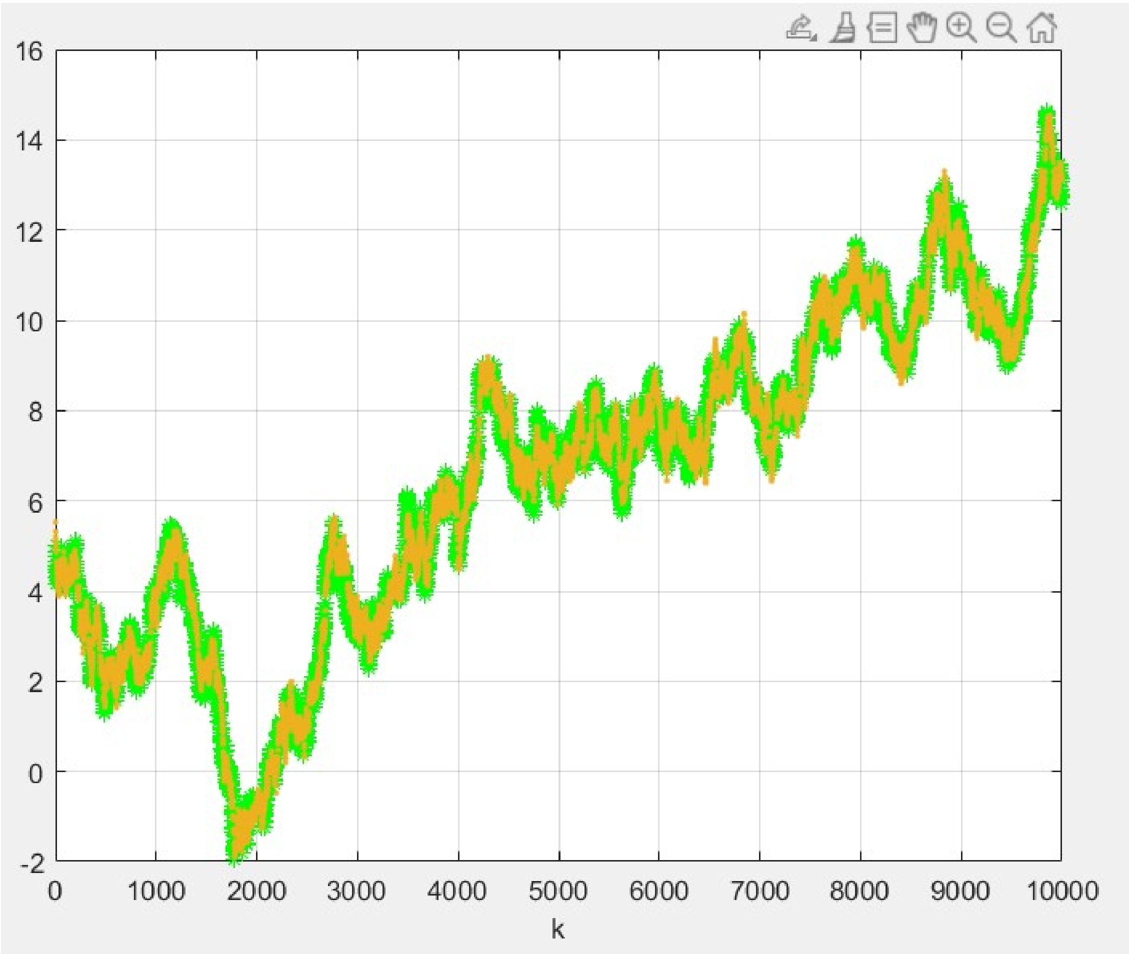
$$x(k+1) = x(k) + w(k)$$
$$y(k) = x(k) + v(k)$$

The Kalman Filter is given as:

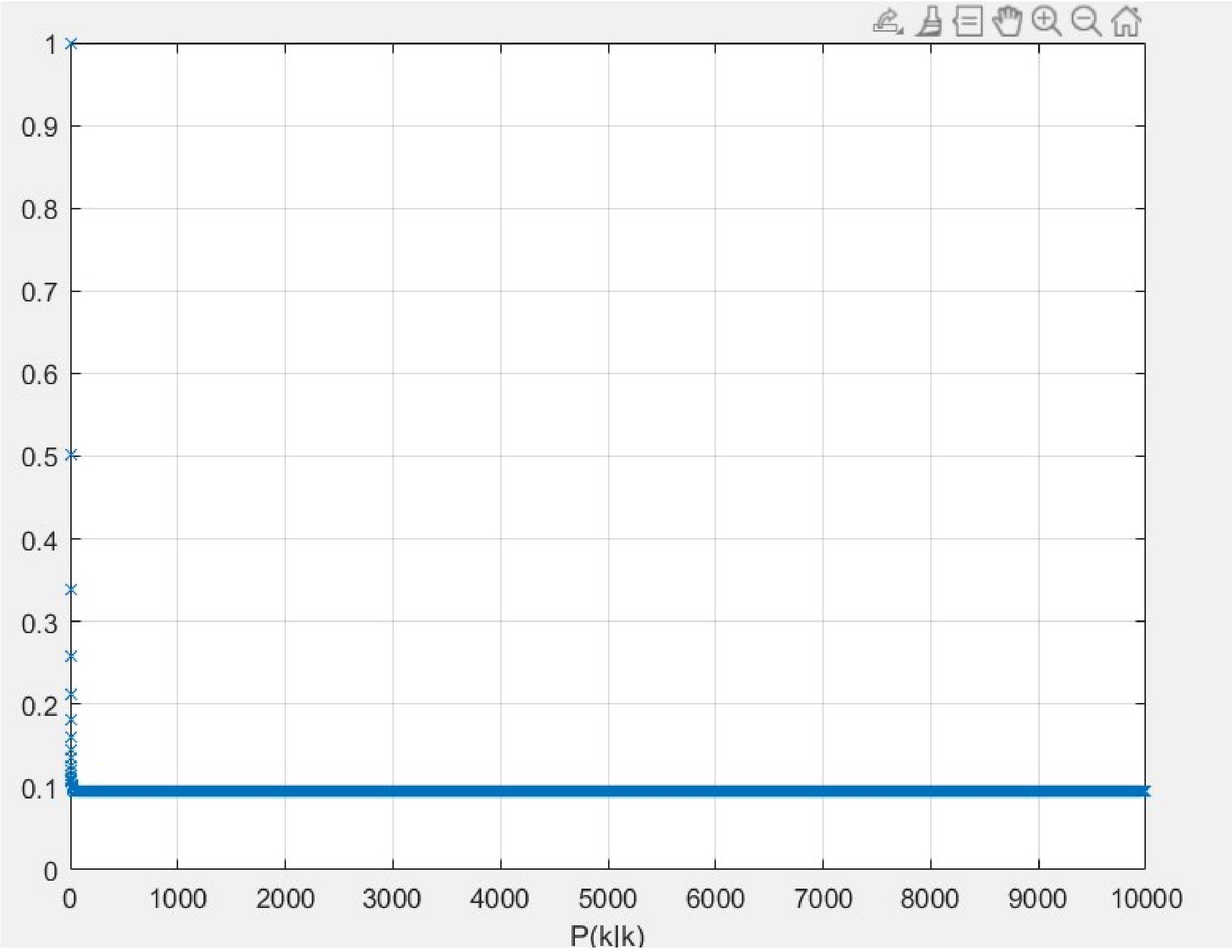
$$Kf(k) = P(k|k-1)C^T(CP(k|k-1)C^T + R_2)^{-1}$$
$$K(k) = (AP(k|k-1)C^T)(CP(k|k-1)C^T + R_2)^{-1}$$
$$\hat{x}(k|k) = \hat{x}(k|k-1) + K_f(k)(y(k) - C\hat{x}(k|k-1))$$
$$\hat{x}(k+1|k) = A\hat{x}(k|k-1) + Bu(k) + K(k)(y(k) - C\hat{x}(k|k-1))$$
$$P(k|k) = P(k|k-1) - P(k|k-1)C^T(CP(k|k-1)C^T + R_2)^{-1}CP(k|k-1)$$
$$P(k+1|k) = AP(k|k-1)A^T - K(k)(CP(k|k-1)C^T + R2)K^T(k) + R_1$$

The initial condition is $x(0) = 5$. Assume that $P(0|-1) = 10^5$, we can get:

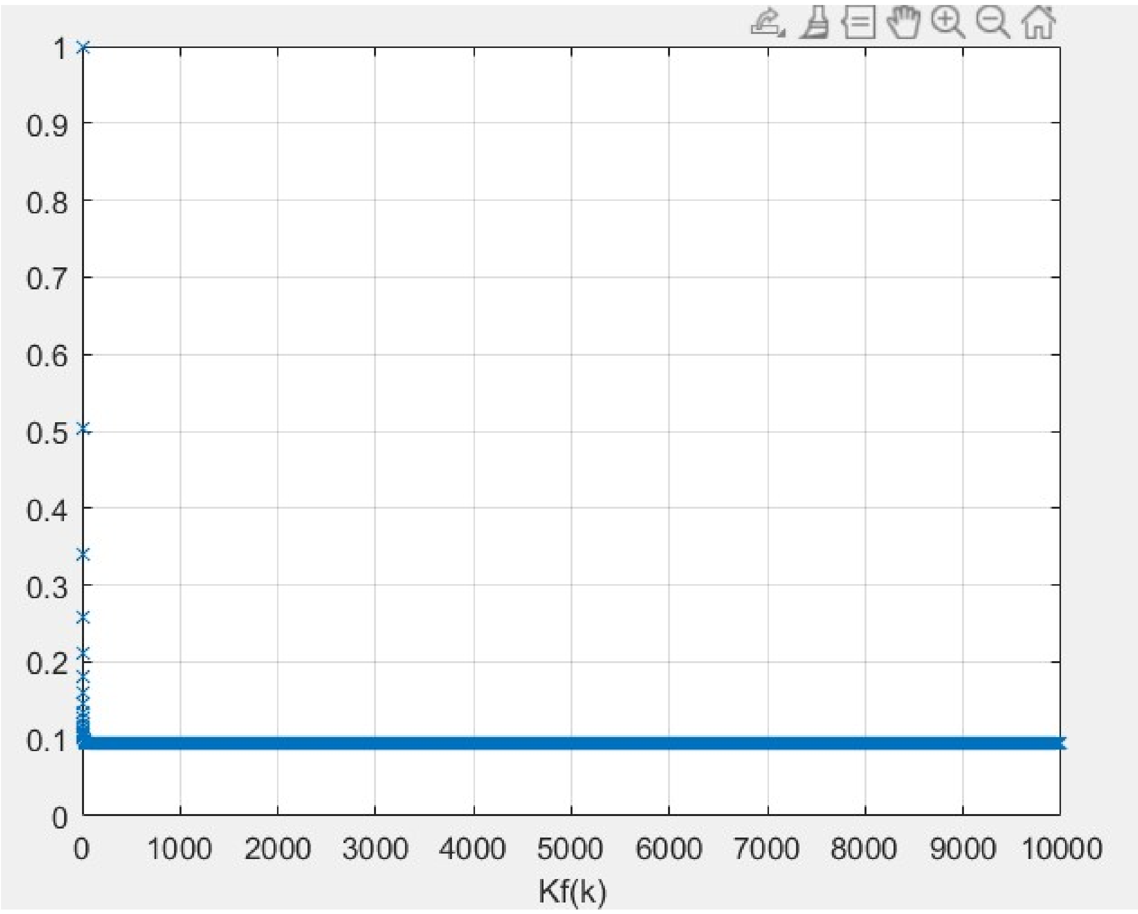
Graph 1, green line: $x(k)$; yellow line: $\hat{x}(k|k)$



Graph 2



Graph 3



Calculate variables

Bias: $\frac{1}{N+1}\sum_{k=0}^{N}(x(k) - \hat{x}(k|k) = -0.00249$

Variance: $\frac{1}{N+1}\sum_{k=0}^{N}(x(k) - \hat{x}(k|k)^2 = 0.0986$

Part 2

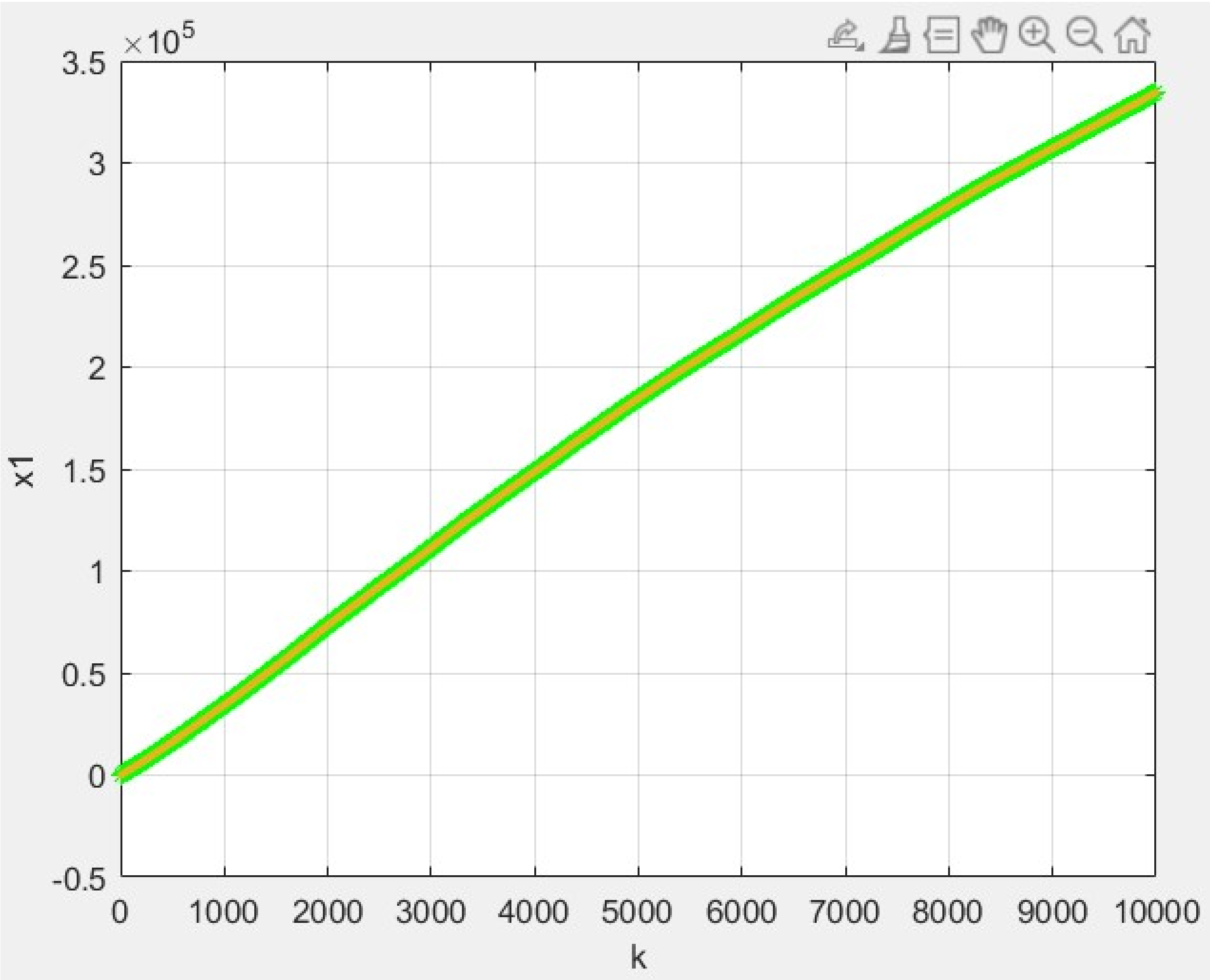
The true state position $x_1(k)$ and velocity $x_2(k)$ of a moving target are given by the following equation:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} \frac{1}{2}T^2 \\ T \end{bmatrix} w(k)$$
$$y(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + v(k)$$

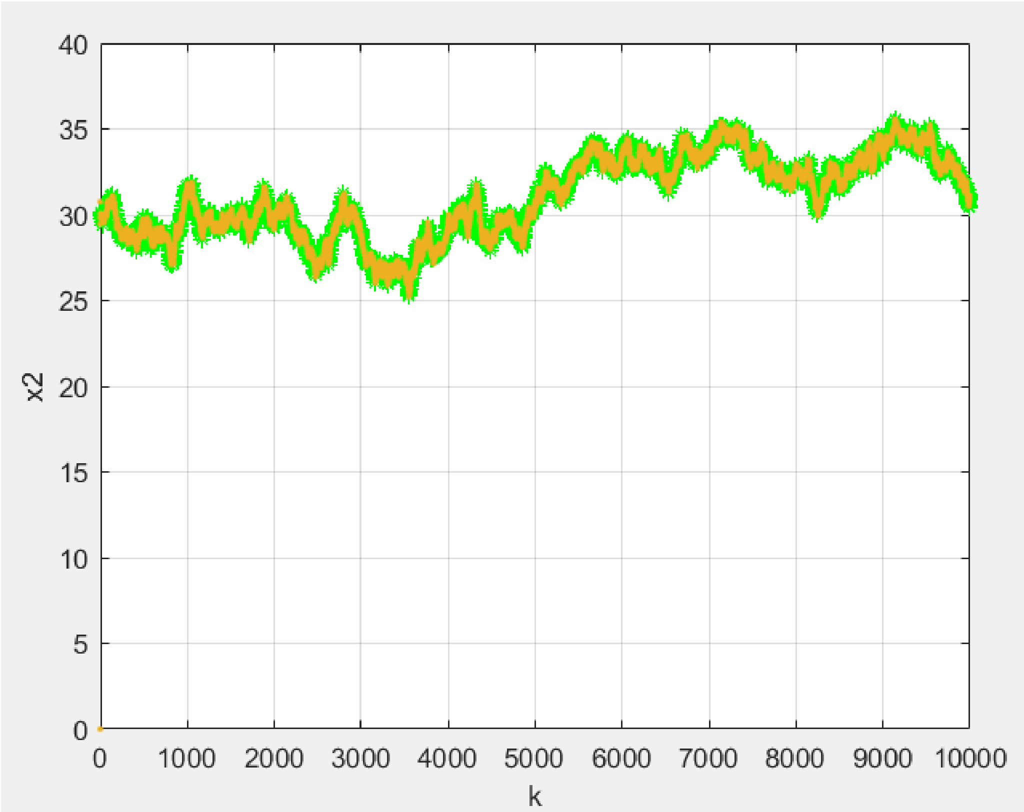
The initial contidions are: $x(k) = \begin{bmatrix} 0 \\ 30 \end{bmatrix}$, we can assume that: $P(0|-1) = 10^5 \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

Simulate for $k = 0, 1, \ldots, N$ where $N = 10,000$

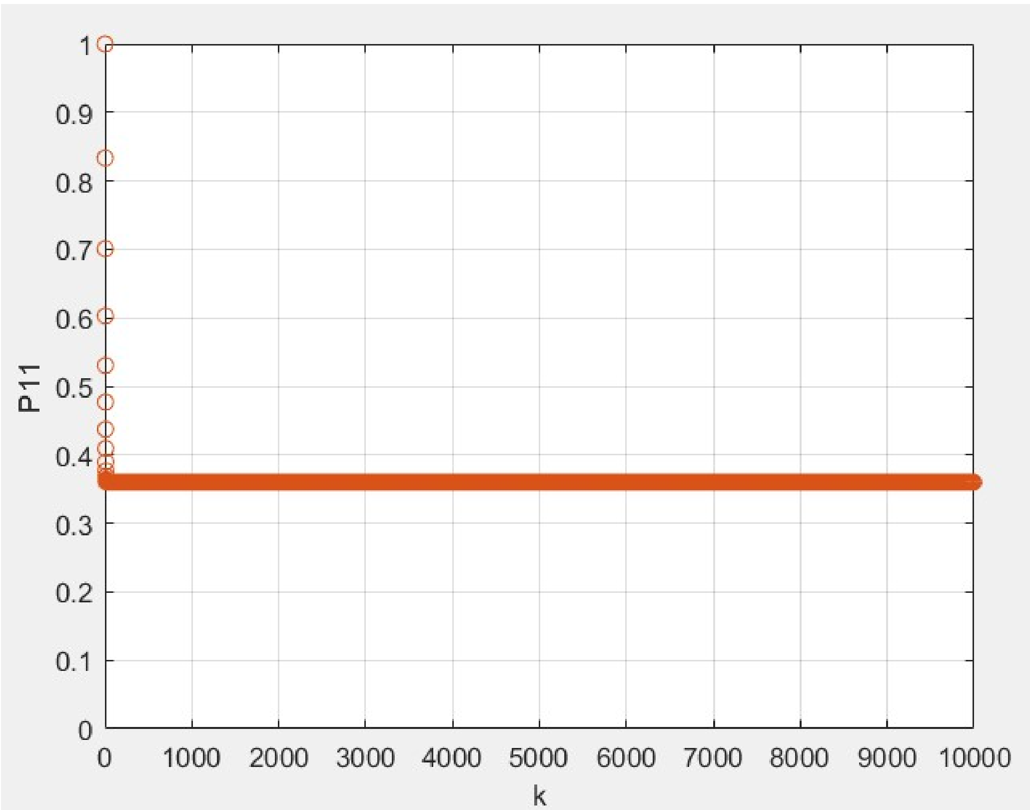
Graph 4, $x_1(k)$: green; $\hat{x}_1(k|k)$: yellow



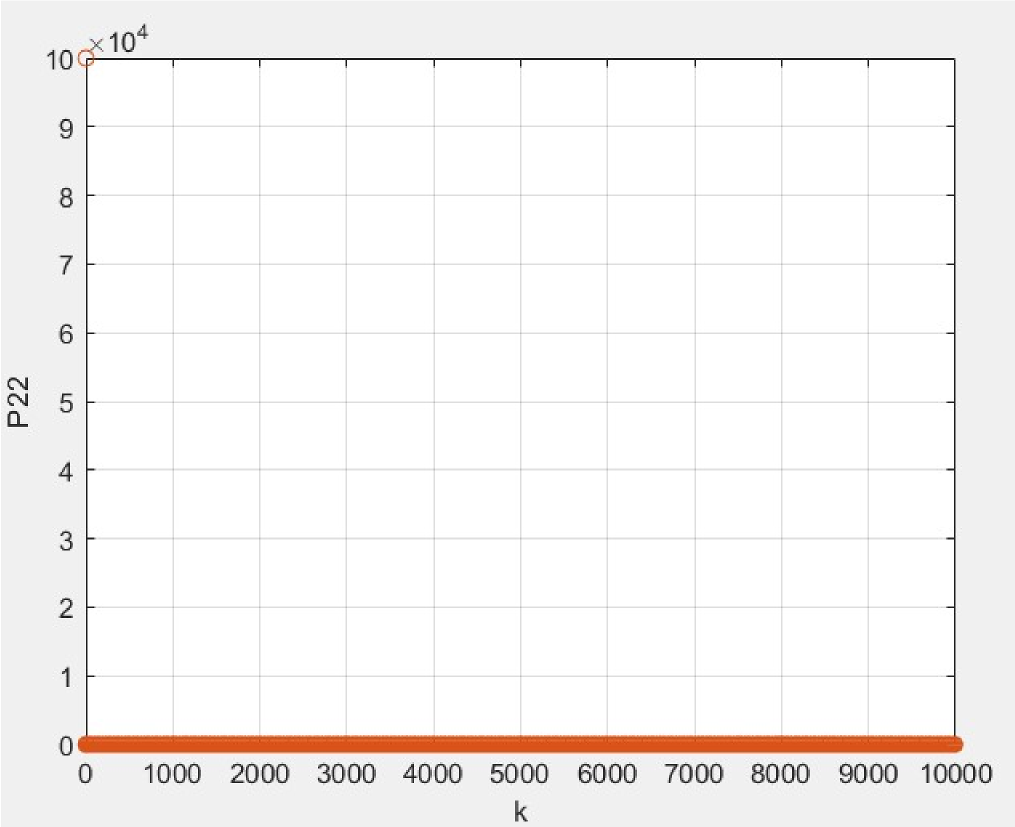
Graph 5, $x_2(k)$**: green;** $\hat{x}_2(k|k)$**: yellow**



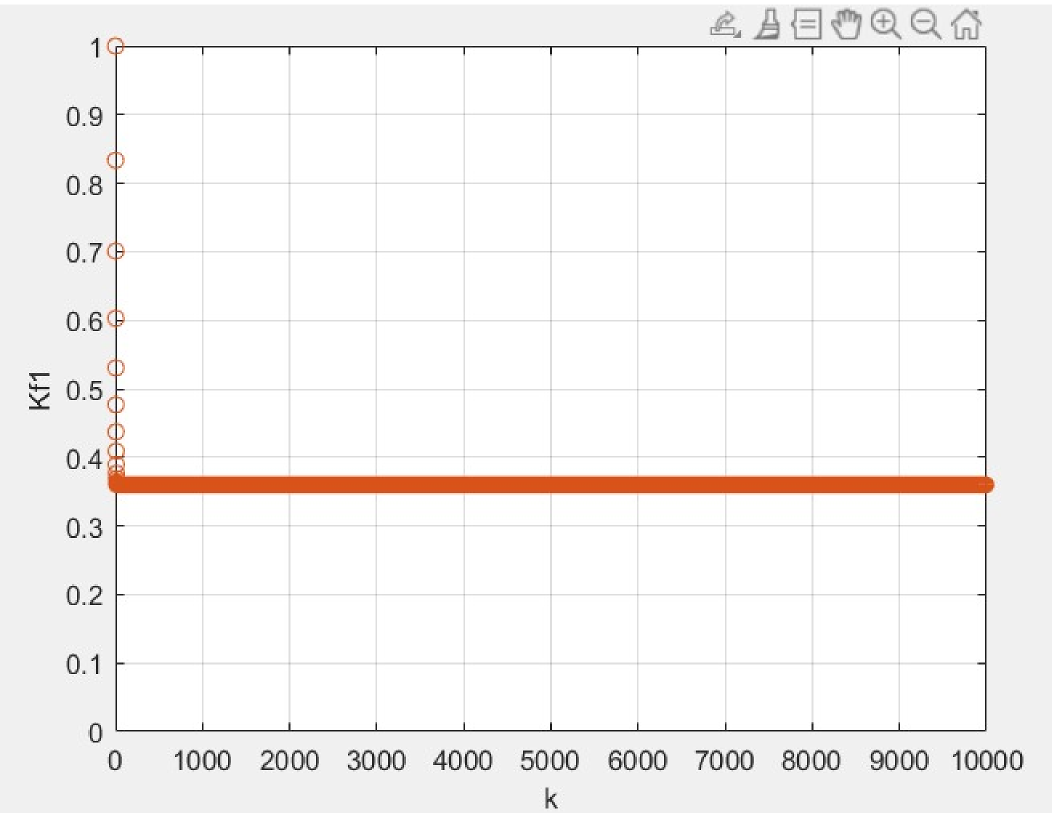
Graph 6: $P_{11}(k|k)$



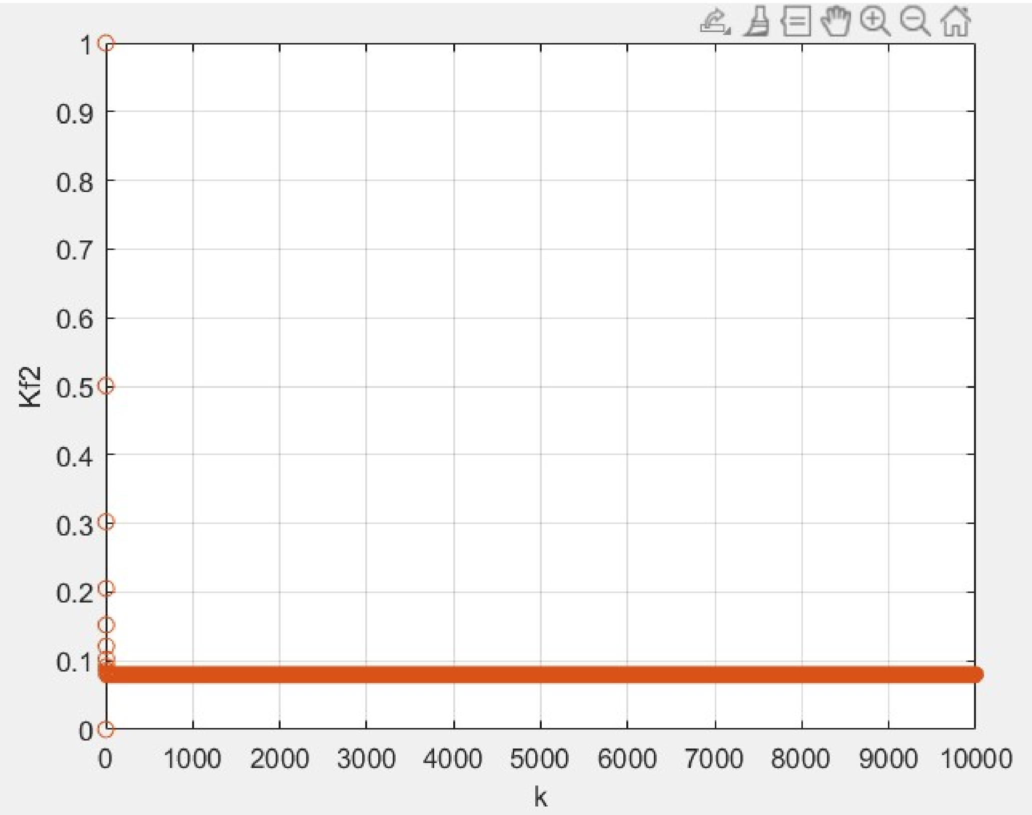
Graph 7: $P_{22}(k|k)$



Graph 8: $K_{f1}(k)$



Graph 9: $K_{f2}(k)$



Calculate variables

The biases

$$\frac{1}{N+1}\sum_{k=0}^{N}(x_1(k)-\hat{x}_1(k|k)=-0.0042$$

$$\frac{1}{N+1}\sum_{k=0}^{N}(x_2(k)-\hat{x}_2(k|k)=0.0014$$

The variance:

$$\frac{1}{N+1}\sum_{k=0}^{N}(x_1(k)-\hat{x}_1(k|k)^2=0.3452$$

$$\frac{1}{N+1}\sum_{k=0}^{N}(x_2(k)-\hat{x}_2(k|k)^2=0.1290$$

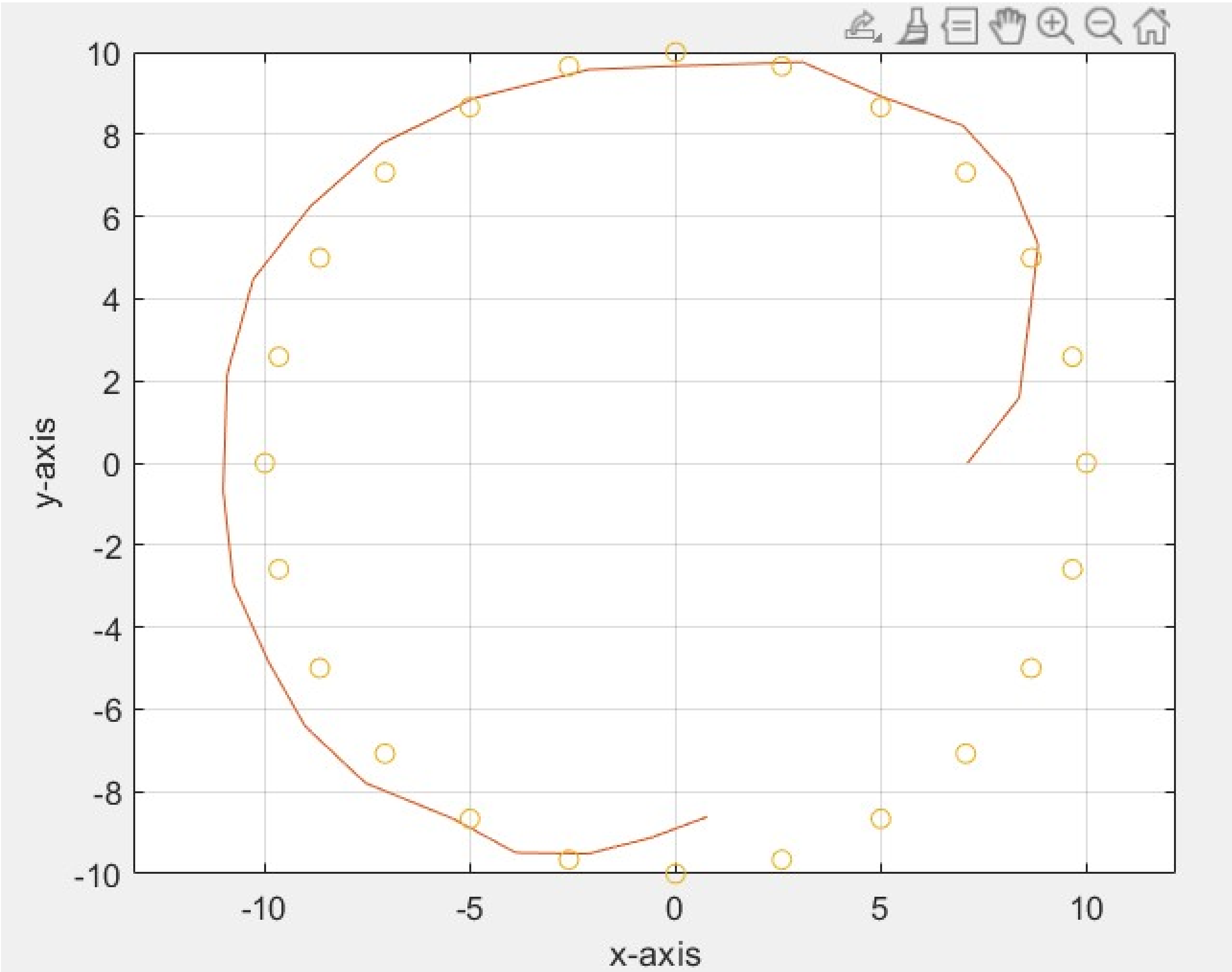
Part 3

The target moving in a circle, we can decompose it into y and z, caculate the speed and position separately. We can assume that the state $x(k)$ is: $x(k) = \begin{bmatrix} y(k) \\ \dot{y}(k) \\ z(k) \\ \dot{z}(k) \end{bmatrix}$, $\dot{y}(k)$ and $\dot{z}(k)$ are the speed in the y and z directions speed. In this case, we can use constant speed module. The whole state-space can be expressed as:

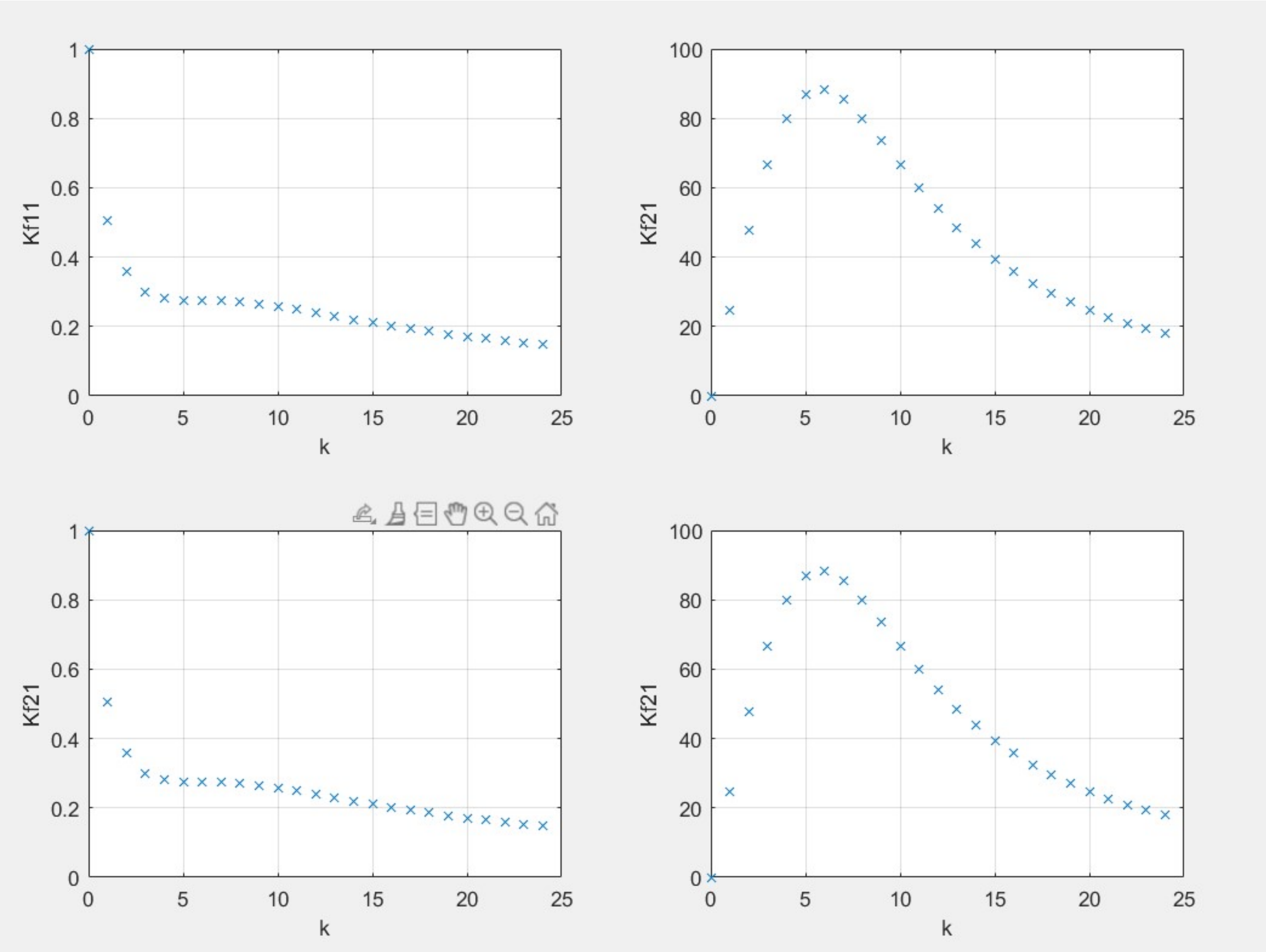
$$x(k+1) = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix} x(k) + \begin{bmatrix} \frac{T^2}{2} \\ T \\ \frac{T^2}{2} \\ T \end{bmatrix} w(k)$$

$$y(k) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} x(k) + v(k)$$

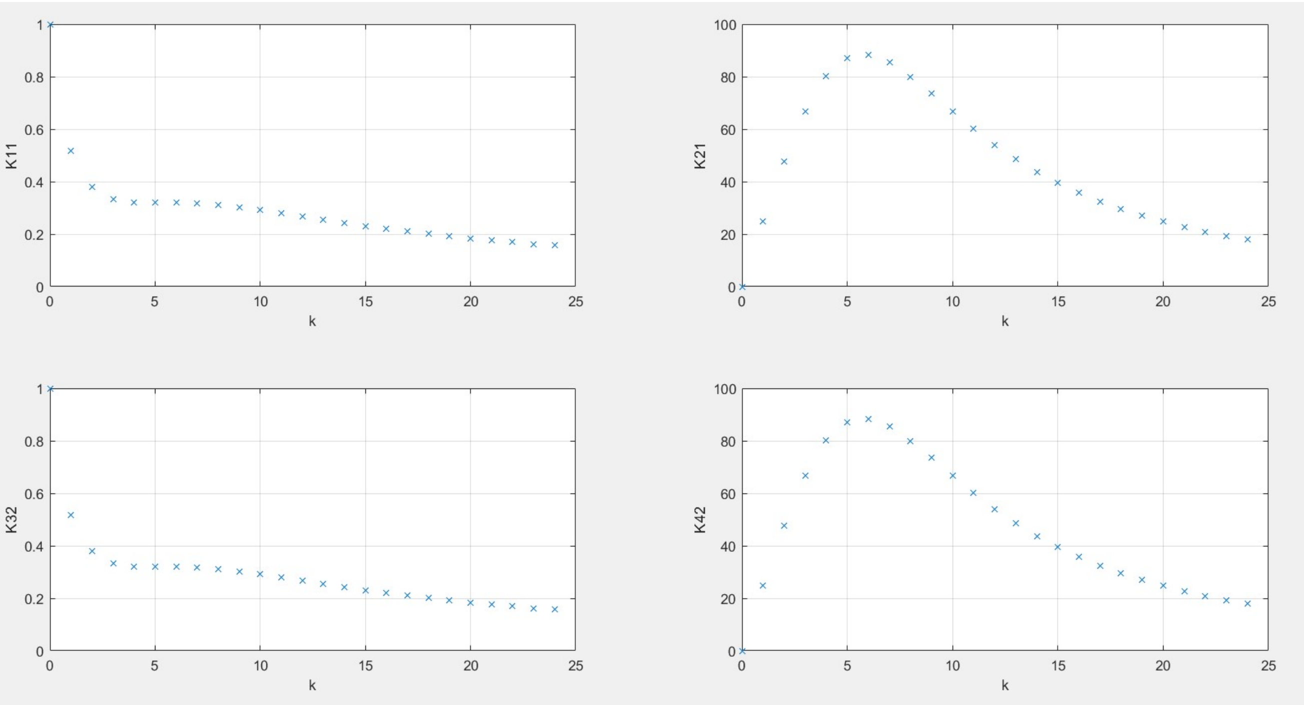
We can assume that, the sampling period $T = 0.0005$. The output of Kalman filter are as follows:



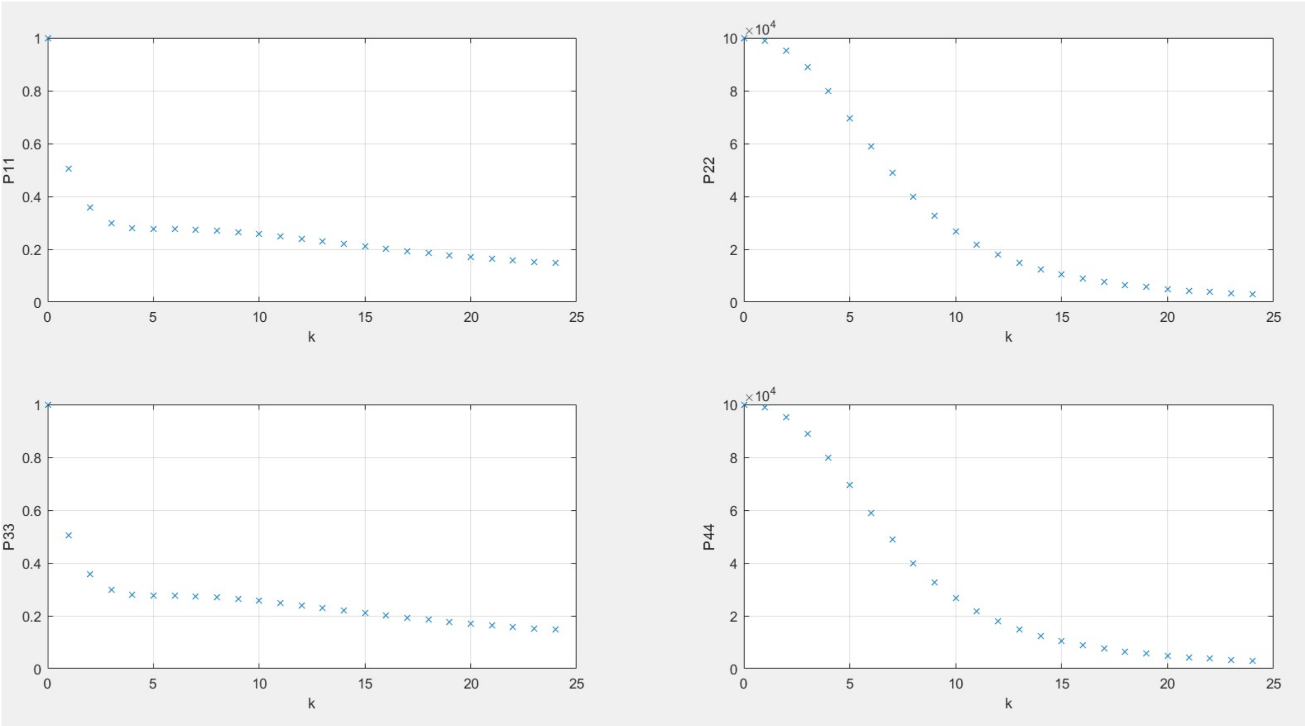
Graph 10: $K_f(k)$



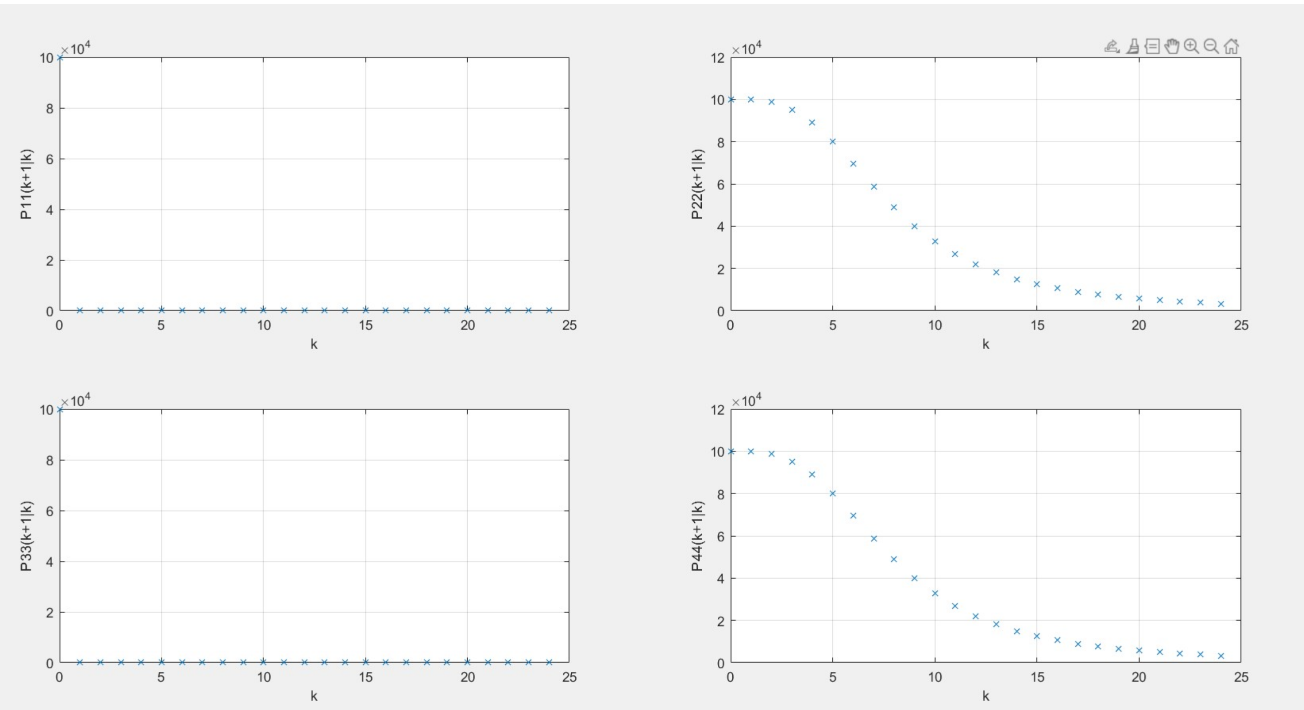
Graph 11: $K(k)$



Graph 12: $P(k|k)$



Graph 13: P(k+1|k)



Calculate variables

Biases:

$$\frac{1}{N+1}\sum_{k=0}^{N}\{10cos\frac{2\pi k}{N} - \hat{x}_y(k|k)\} = 2.207$$

$$\frac{1}{N+1}\sum_{k=0}^{N}\{10sin\frac{2\pi k}{N} - \hat{x}_z(k|k)\} = -0.9531$$

Variances

$$\frac{1}{N+1}\sum_{k=0}^{N}\{10cos\frac{2\pi k}{N}-\hat{x}_y(k|k)\}^2=38.43$$

$$\frac{1}{N+1}\sum_{k=0}^{N}\{10sin\frac{2\pi k}{N}-\hat{x}_z(k|k)\}^2=17.65$$