NATIONAL UNIVERSITY OF SINGAPORE

FACULTY OF ENGINEERING

EXAMINATION FOR

(Semester I: 2021/2022)

EE5101/ME5401 – LINEAR SYSTEMS

November 2021 - Time Allowed: 2.5 Hours

INSTRUCTIONS TO CANDIDATES:

1. Please write only your Student Number. Do not write your name.
2. This paper contains **FOUR** (4) questions and comprises **FIVE** (5) printed pages.
3. Answer all **FOUR** (4) questions.
4. Students should write the answers for each question on a new page.
5. All questions carry **EQUAL** marks. The **TOTAL** marks are 100.
6. This is an **Open BOOK** examination.

Q.1 (a) Use the Cayley-Hamilton Principle to evaluate e^{At} where

$$A = \begin{bmatrix} -2 & -1 \\ 1 & -4 \end{bmatrix}.$$

(7 marks)

(b) The following (S_1 and S_2) are two descriptions of the same physical system. Find the matrix P that transforms one description to the other. Justify your answer.

$$(S_1) \qquad \begin{cases} \dot{x} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u \\ y = \begin{bmatrix} 1 & 0 \end{bmatrix} x \end{cases}$$

$$(S_2) \qquad \begin{cases} \dot{\bar{x}} = \begin{bmatrix} 4 & 1 \\ 6 & 1 \end{bmatrix} \bar{x} + \begin{bmatrix} 2 \\ 2 \end{bmatrix} u \\ y = \begin{bmatrix} 1 & -0.5 \end{bmatrix} \bar{x} \end{cases}$$

(10 marks)

(c) Consider the system:

$$\dot{x} = \begin{bmatrix} -2 & 1 \\ -5 & 4 \end{bmatrix} x, \quad x(0) = x_0$$

Is the system asymptotically stable? Find the expression of $x(t)$ as $t \to \infty$ when $x_0 = [1 \quad 1]^T$.

(8 marks)

Q.2 (a) Given

$$\dot{x} = \begin{bmatrix} -1 & \alpha_1 & 0 \\ 0 & -1 & \alpha_2 \\ 0 & 0 & -1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} x$$

(i) Determine the values of α_1 and α_2 such that the system is controllable.

(ii) Obtain the transfer function representation of the system between the output and its input.

(12 marks)

(Continued on next page)

Q.2 (continued)

(b) Using the Lyapunov equation and determine the stability of the following system

$$\dot{x} = \begin{bmatrix} -1 & 1 \\ -5 & 1 \end{bmatrix} x$$

(8 marks)

(c) Given that $e^{At} = \boldsymbol{L}^{-1}(sI - A)^{-1}$, it follows that $[e^{At}]^{-1} = \boldsymbol{L}^{-1}(sI - A)$. Is this statement correct? Justify your answer.

(5 marks)

Q.3

(a) Consider a process modeled by

$$\frac{dx}{dt} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} x + \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} u.$$

Assume that all the state variables are accessible, design a state feedback controller that places all the three poles at -1. There are many ways to solve the pole placement problem for multiple input system. For this problem, only **FULL RANK** method can be used. In other words, the state transformation matrix T has to be constructed to transform the state space model into its controllable canonical form and the pole placement problem can be solved with the aid of its canonical form.

(15 marks)

(b) Consider a first order system,

$$\frac{dx}{dt} = x + u, x(0) = c.$$

Design an optimal controller to minimize the following performance index:

$$J = \int_0^\infty (x^2 + xu + u^2) dt.$$

(10 marks)

Q.4

(a) Find a state feedback law which decouples and stabilizes the system given by

$$\frac{dx}{dt} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} x + \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} u,$$

$$y = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} x.$$

(10 marks)

(b) Consider the following process,

$$Y(s) = \frac{1}{2s+1} U(s) + \frac{1}{s} D(s)$$

where the output $y(t)$ is affected by both the input $u(t)$ and disturbance $d(t)$. Design a unity feedback control system to meet following requirements:

(i) The dominant dynamics can be described by standard second order system with damping ratio of 0.5, and natural frequency of 1.

(ii) The feedback control system can asymptotically track a reference signal, $r = c + a \sin(t)$, $t \geq 0$, were a and c can be any constants.

(iii) The closed loop system can eliminate the effect of any step disturbance at steady state.

After you design the controller, please explain how the three requirements are met.

(15 marks)

Appendix A - Table of Laplace Transform

The following table contains some frequently used time functions x(t), and their Laplace transforms X(s).

x(t)	**X(s)**
unit impulse δ (t)	1
unit step 1(t)	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^2	$\frac{2}{s^3}$
e^{-at}	$\frac{1}{s+a}$
te^{-at}	$\frac{1}{(s+a)^2}$
$1-e^{-at}$	$\frac{a}{s(s+a)}$
$\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}$
$\cos(\omega t)$	$\frac{s}{s^2+\omega^2}$
$e^{-at}\sin(\omega t)$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at}\cos(\omega t)$	$\frac{s+a}{(s+a)^2+\omega^2}$

END OF PAPER