EE5104/6014 ADVANCED/ADAPTIVE CONTROL SYSTEMS

Briefing Notes for CA1: Adaptive Control of a Second Order Plant with Output Feedback

CONTENTS

OBJECTIVE

- Design an adaptive control system for the following plant with a second order transfer function and uncertain parameters.
- Only the output of the plant is measurable.

$$\frac{Y(s)}{U(s)} = \frac{b_0 s + b_1}{s_2 + a_1 s + a_2}$$

- In the above transfer function, a_1, a_2, b_0, b_1 are uncertain parameters. It is only known that $b_0 < 0$, and that the plant has no zeros in the right-half of the s-plane.

CONTROL SYSTEM DESIGN

- The general input-output model of the plant in time-domain is

$$R_p(p)y = k_p Z_p(p)u \quad (1)$$

$$p \equiv \frac{d}{dt} \qquad\qquad R_p(p) = p^2 + a_1 p + a_2$$

$$Z_p(p) = p + \frac{b_1}{b_0} \qquad\qquad k_p = b_0$$

- Notice that in the above equation, $R_p(p)$ and $Z_p(p)$ are monic, and $R_p(p)$ is one-order higher than $Z_p(p)$.

CONTROL SYSTEM DESIGN
- REFERENCE MODEL

- Choose the following first order reference model.

$$H_m(s) = \frac{1}{\tau s + 1} = \frac{1/\tau}{s + 1/\tau}$$

- The above transfer function is strictly positive real (SPR). For a step reference input, the steady state offset of the model is zero and the responding speed is decided by the choice of τ.
- The pole polynomial and zero polynomial of the reference model is $R_m(s) = s + 1/\tau$ and $K_m(s) = 1/\tau$.
- The reference model in time domain is described by

$$R_m(p)y_m = K_m(p)r$$

where $R_m(p)$ is monic.

CONTROL SYSTEM DESIGN
- NON-ADAPTIVE CONTROL SYSTEM DESIGN

- Here, to first prove existence of the design, we first *ASSUME* that $R_p(p)$, k_p and $Z_p(p)$ are known.
- Design $T(p)$, which is an arbitrarily chosen stable monic polynomial of second order. $T(p)$ is the pole polynomial of the filter of the non-minimum realization of the plant.
- Determine unique $E(p)$ and $F(p)$ from the below equation.

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$$T(p)R_m(p) = R_p(p)E(p) + F(p)$$

- Notice that when $T(p)$, $R_m(p)$ and $R_p(p)$ are monic, $E(p)$ should be monic as well.

CONTROL SYSTEM DESIGN

- NON-ADAPTIVE CONTROL SYSTEM DESIGN

- Multiply $E(p)$ on both side of Eq.1 . This gives

$$E(p)R_p(p)y = k_pE(p)Z_p(p)u$$

- Substitute Eq.2 Into the above equation and we have

$$\big(T(p)R_m(p) - F(p)\big)y = k_pE(p)Z_p(p)u$$

- Therefore,

$$R_m(p)y = \frac{1}{T(p)}\big(k_pE(p)Z_p(p)u + F(p)y\big) = k_p\left(\frac{E(p)Z_p(p)}{T(p)}u + \frac{F(p)}{k_pT(p)}y\right) \quad 3$$

- Notice that $\bar{G}(p) = E(p)Z_p(p)$ and $T(p)$ are both monic second order polynomial. Thus if define $G_1(p) = \bar{G}(p) - T(p)$ and $\bar{F}(p) = F(p)/k_p$,

CONTROL SYSTEM DESIGN

- NON-ADAPTIVE CONTROL SYSTEM DESIGN

- In Eq.3 , notice that $\bar{G}(p) = E(p)Z_p(p)$ and $T(p)$ are both monic second order polynomial. Thus if define $G_1(p) = \bar{G}(p) - T(p)$ and $\bar{F}(p) = F(p)/k_p$, the equation becomes

$$R_m(p)y = k_p\left(\frac{G_1(p)}{T(p)}u + \frac{\bar{F}(p)}{T(p)}y + u\right) \qquad 4$$

- The control signal is designed in such way that

$$k_p\left(\frac{G_1(p)}{T(p)}u + \frac{\bar{F}(p)}{T(p)}y + u\right) = k_p k^* r = k_m r$$

- Therefore,

$$u = -\frac{G_1(p)}{T(p)}u - \frac{\bar{F}(p)}{T(p)}y + k^* r \qquad 5$$

CONTROL SYSTEM DESIGN
- NON-ADAPTIVE CONTROL SYSTEM DESIGN

- As has been mentioned, $T(p)$ is a second order stable monic polynomial that is designed by ourselves. Generally,

$$T(p) = p^2 + t_1 p + t_2 \qquad 6$$

- It follows here that $G_1(p), \bar{F}(p)$ are first order polynomials. Assume that they are

$$G_1(p) = g_1 p + g_2, \bar{F}(p) = f_1 p + f_2 \qquad 7$$

- The values of g_1, g_2, f_1, f_2 are known to us if we ASSUME that the original plant is known.

- Define the auxiliary signal

$$\omega_u = \frac{1}{T(p)} u$$

- From Eq.6 , we can write done

$$\begin{bmatrix} \dot{\omega_u} \\ p\dot{\omega}_u \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -t_2 & -t_1 \end{bmatrix} \begin{bmatrix} \omega_u \\ p\omega_u \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \qquad 8$$

- Then from Eq.7 , we have

$$-\frac{G_1(p)}{T(p)} u = -G_1(p)\omega_u = -g_1 p\omega_u - g_2\omega_u$$

- Similarly, define

$$\omega_y = \frac{1}{T(p)} y$$

- We have

$$\begin{bmatrix} \dot{\omega_y} \\ p\dot{\omega}_y \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -t_2 & -t_1 \end{bmatrix} \begin{bmatrix} \omega_y \\ p\omega_y \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} y$$

$$-\frac{\bar{F}(p)}{T(p)} y = -\bar{F}(p)\omega_y = -f_1 p\omega_y - f_2\omega_y$$

Thus, Eq. 5 can be written as

$$u = \bar{\theta}^{*T} \overline{\omega}$$

where $\bar{\theta}^* = [-f_2 \quad -f_1 \quad -g_2 \quad -g_1 \quad k^*]^T, \overline{\omega} = [\omega_y \quad p\omega_y \quad \omega_u \quad p\omega_u \quad r]^T$

- If the original plant is KNOWN to us, then with appropriately designed reference model and $T(p)$, $\bar{\theta}^*$ can be calculated precisely.

- $\bar{\omega}$ can be obtained from the non-minimal realization of $T(p)$ with u and y. (And of course, $\bar{\omega}_5$ is the reference signal, which is known)

- In practice, however, the original plant is UNKNOWN. In that case, the value of $\bar{\theta}^*$ is unknown.

- Instead of using $u = \bar{\theta}^{*T}\bar{\omega}$, use

$$u = \bar{\theta}^T\bar{\omega} \tag{9}$$

 where $\bar{\theta}$ is NOT a constant gain but a time-varying gain.

- If this control signal is substituted into Eq. 4 , we have

$$\begin{aligned} R_m(p)y &= k_p\left(\frac{G_1(p)}{T(p)}u + \frac{\bar{F}(p)}{T(p)}y + \bar{\theta}^T\bar{\omega}\right) \\ &= k_p\left(\frac{G_1(p)}{T(p)}u + \frac{\bar{F}(p)}{T(p)}y + \bar{\phi}^T\bar{\omega} + \bar{\theta}^{*T}\bar{\omega}\right) \\ &= K_m r + k_p\bar{\phi}^T\bar{\omega} \end{aligned} \tag{10}$$

where $\bar{\phi} = \bar{\theta} - \bar{\theta}^*$.

CONTROL SYSTEM DESIGN
- ADAPTIVE CONTROL SYSTEM DESIGN

- Subtract the reference model $R_m(p)y_m = K_m(p)r$ from Eq.10, and we can find the output error dynamics

$$R_m(p)e_1 = k_p\bar{\Phi}^T\bar{\omega}$$

where $e_1 = y - y_m$

CONTROL SYSTEM DESIGN
- ADAPTIVE CONTROL SYSTEM DESIGN

- The objective of the adaptive controller is to design such $u = \bar{\theta}^T \overline{\omega}$ that the error e_1 would converge to zero. The dynamic of $\bar{\theta}$ should be appropriately designed.

- In order to do that, we are going to use the non-minimal realization of the plant and non-minimal realization of the reference model. The control signal is designed so that the non-minimal state of the plant would converge to that of the reference model.

- The non-minimal realization of the plant refers to the realization of the system described by Eq. 1 w.r.t. the states defined by

$$\omega = [\omega_y \quad p\omega_y \quad \omega_u \quad p\omega_u]^T$$

(Notice that ω is part of $\bar{\omega}$.)

- In order to do that, multiply $1/T(p)$ on both side of Eq. 1 . This gives

$$R_p(p)\omega_y = k_p Z_p(p)\omega_u \tag{11}$$

- Therefore,

$$(p^2 + a_1 p + a_2)\omega_y = (b_o p + b_1)\omega_u \tag{12}$$

- From Eq.12, we can find out that

$$p^2\omega_y = -a_1 p\omega_y - a_2\omega_y + b_0 p\omega_u + b_1\omega_u \tag{13}$$

- By the definition of ω_y together with Eq.11, we know that

$$y = T(p)\omega_y = \left(T(p) - R_p(p)\right)\omega_y + k_p Z_p(p)\omega_u \tag{14}$$

CONTROL SYSTEM DESIGN
- ADAPTIVE CONTROL SYSTEM DESIGN

- Use Eq. 8 ,13 ,14 , and we have

$$\begin{bmatrix} \dot{\omega}_y \\ p\dot{\omega}_y \\ \dot{\omega}_u \\ p\dot{\omega}_u \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -a_2 & -a_1 & b_1 & b_0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -t_2 & -t_1 \end{bmatrix} \begin{bmatrix} \omega_y \\ p\omega_y \\ \omega_u \\ p\omega_u \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} t_2 - a_2 & t_1 - a_1 & b_1 & b_0 \end{bmatrix} \begin{bmatrix} \omega_y \\ p\omega_y \\ \omega_u \\ p\omega_u \end{bmatrix}$$

- Rewrite it as the following form. Some of the parameters in the matrixes are unknown.

$$\dot{\omega} = A_p\omega + B_p u$$
$$y = C_p^T \omega$$

- The reference model $R_m(p)y_m = K_m(p)r$ can be realized by the non-minimal state space model as well, using the same filter $T(p)$. The realization is

$$\dot{\omega}_m = A_m\omega_m + B_m r$$
$$y_m = C_m^T\omega_m$$

- It is NOT necessary to calculate the value of A_m, B_m and C_m. Later it would be clear that we do not need to establish this system in the software because the signal ω_m is not necessary in the adaptive feedback.
- This is DIFFERENT from the state feedback adaptive control system design that is taught in first 2 lectures in the class. In the previous scenario, $e = x - x_m$ is used in the control signal dynamics.

- The control signal u is designed so that

$$\dot{\omega} = A_p\omega + B_p u$$
$$y = C_p^T\omega$$

Adaptive Control

$$\omega \rightarrow \omega_m$$

$$\dot{\omega}_m = A_m\omega_m + B_m r$$
$$y_m = C_m^T\omega_m$$

- If the above is possible, y would converge to y_m as well.
- Use the idea borrowed from state feedback adaptive control system design, we know that we can define a Lyapunov equation as follows.

$$V = e^T P e + \bar{\phi}^T \Gamma^{-1} \bar{\phi}$$

CONTROL SYSTEM DESIGN
- ADAPTIVE CONTROL SYSTEM DESIGN

- In Eq.15, P is the positive definite solution to the Algebraic Riccati equation $A_m^T P + PA_m = -Q$ and Γ is a positive definite matrix.
- The convergence of e is guaranteed when Eq.15 can fulfil all the 4 requirements (check lecture notes class 1 and 2 for details).
- One of the requirements is that $\dot{V} < 0$ for non-zero e.
- $\dot{V}$ is calculated by

$$\dot{V} = 2e^T P\left(A_m e + \frac{1}{k^*} B_m \overline{\Phi}^T \overline{\omega}\right) + 2\overline{\Phi}^T \Gamma^{-1} \dot{\overline{\Phi}} = e^T(-Q)e + 2e^T P \frac{1}{k^*} B_m \overline{\Phi}^T \overline{\omega} + 2\overline{\Phi}^T \Gamma^{-1} \dot{\overline{\Phi}} \quad 16$$

- In Eq.16, note that $P\frac{1}{|k^*|}B_m = C_m$ because $\left(A_m, \frac{1}{|k^*|}B_m, C_m^T\right)$ is STR due to that the reference model (A_m, B_m, C_m^T) is STR.
- Therefore, Eq.16 becomes

$$\begin{aligned}\dot{V} &= e^T(-Q)e + 2\text{sgn}(k^*)e^T C_m\overline{\Phi}^T\overline{\omega} + 2\overline{\Phi}^T\Gamma^{-1}\dot{\overline{\Phi}} \\ &= e^T(-Q)e + 2\text{sgn}(k^*)e_1\overline{\Phi}^T\overline{\omega} + 2\overline{\Phi}^T\Gamma^{-1}\dot{\overline{\Phi}}\end{aligned} \quad 17$$

where notice that $e^T C_m = C_m^T e = e_1$, which is a scalar and the transpose of a scalar is itself.

CONTROL SYSTEM DESIGN
- ADAPTIVE CONTROL SYSTEM DESIGN

- Notice that in Eq.17, $\dot{\bar{\phi}}$ can be designed by the designer via choosing the dynamic of the control gain $\dot{\bar{\theta}}$.

$$\dot{\bar{\phi}} = \dot{\bar{\theta}} - \dot{\bar{\theta}}^* = \dot{\bar{\theta}}$$

- Therefore, choose Eq.18 and Eq.17 becomes Eq.19 .

$$\dot{\bar{\theta}} = -\mathrm{sgn}(k^*)\Gamma\bar{\omega}e_1 \tag{18}$$

$$\dot{V} = e^T(-Q)e \tag{19}$$

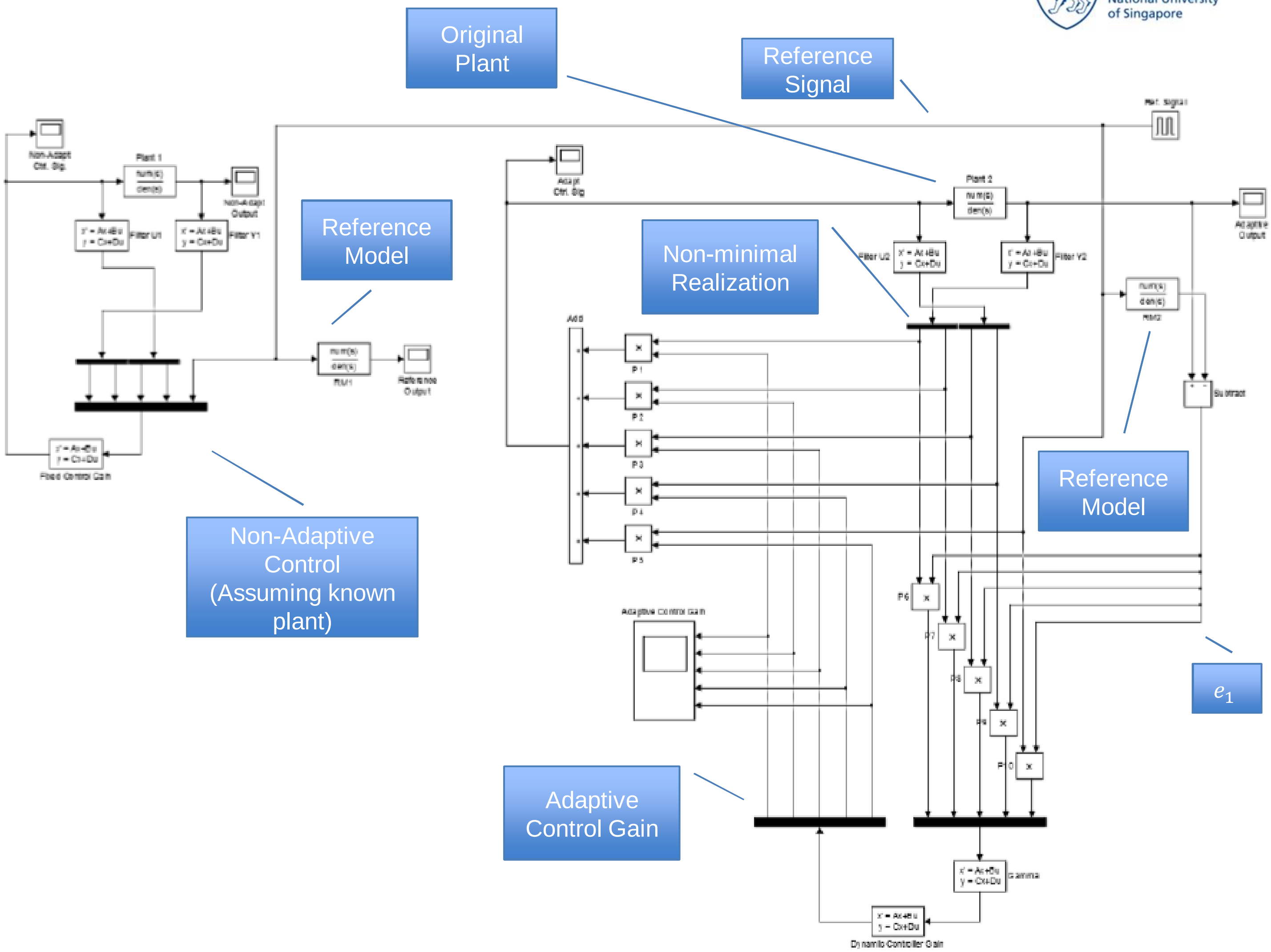
- By applying Eq.18, $\dot{V} < 0$ for non-zero e can be proved. The stability of the Lyapunov equation can be eventually proved with a few more derivations. (Check lecture notes for details.)

- The control strategy in Eq.18 would guarantee the convergence of ω to ω_m, thus guarantees the convergence of y to y_m.

- Note that in Eq.18, only $e_1 = y - y_m$ is used instead of using $e = \omega - \omega_m$. Therefore, it is not necessary to build the non-nominal reference model state-space realization in the control program.

SIMULATION WITH MATLAB SIMULINK

- The following Simulink program is used to verify the control effect of the design.



SIMULATION WITH MATLAB SIMULINK

- Specify the designed parameters as follows.

$$\tau = 1$$
$$T(p) = p^2 + 20P + 100$$
$$\Gamma = I$$

- Therefore, the non-minimal realization state is obtained by the following filters.

$$\begin{bmatrix} \dot{\omega}_y \\ p\dot{\omega}_y \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -100 & -20 \end{bmatrix} \begin{bmatrix} \omega_y \\ p\omega_y \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} y$$
$$\begin{bmatrix} \dot{\omega}_u \\ p\dot{\omega}_u \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -100 & -20 \end{bmatrix} \begin{bmatrix} \omega_u \\ p\omega_u \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

SIMULATION WITH MATLAB SIMULINK

- Based on some dynamic characteristics of the plant, we can have a guess of the plant as follows. This is not necessarily the same with the true plant.

$$(s^2 + 5s + 15)Y(s) = (-s - 2)U(s)$$

- The $\bar{\theta}^*$ for this supposed plant can be calculated and it is used as the initial value of $\bar{\theta}$ in the control system.

$$\bar{\theta}(0) = [-140.0 \quad 25.00 \quad 23.81 \quad -0.7619 \quad -1.000]^T$$

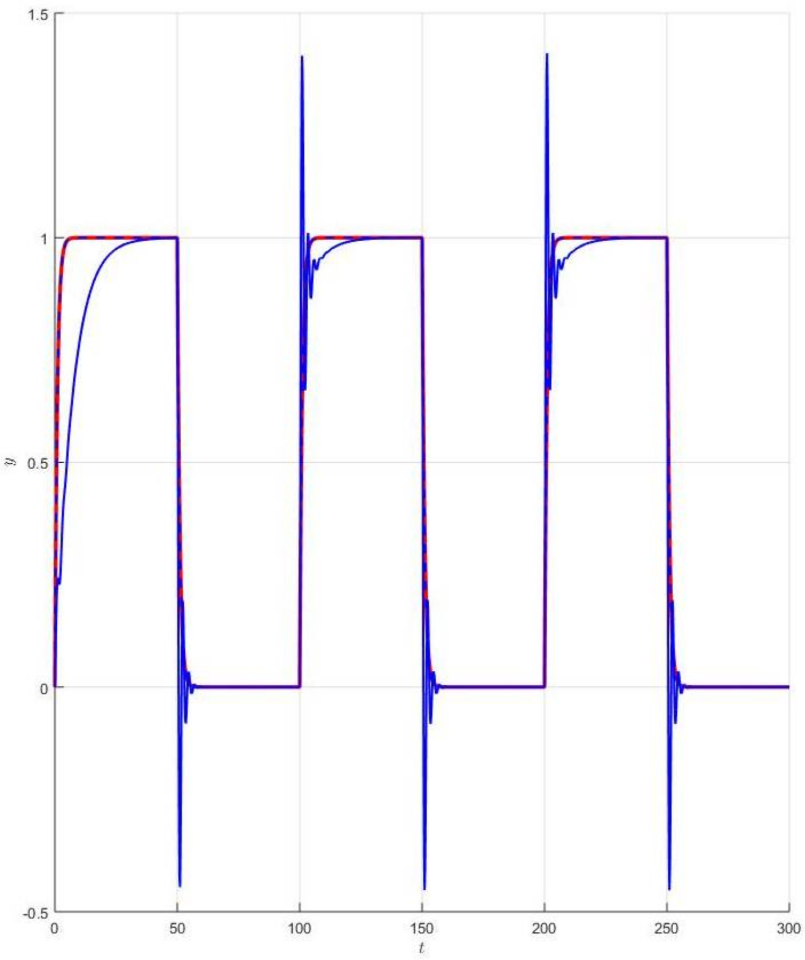
- The TRUE plant and the associated $\bar{\theta}^*$ is as follows. Note that this information is NOT used in the adaptive controller design.

$$(s^2 + 0.29s + 7.2)Y(s) = (-0.21s - 1)U(s)$$

$$\bar{\theta}^* = [-233.9 \quad 508.5 \quad 1.381 \quad -5.472 \quad -4.762]^T$$

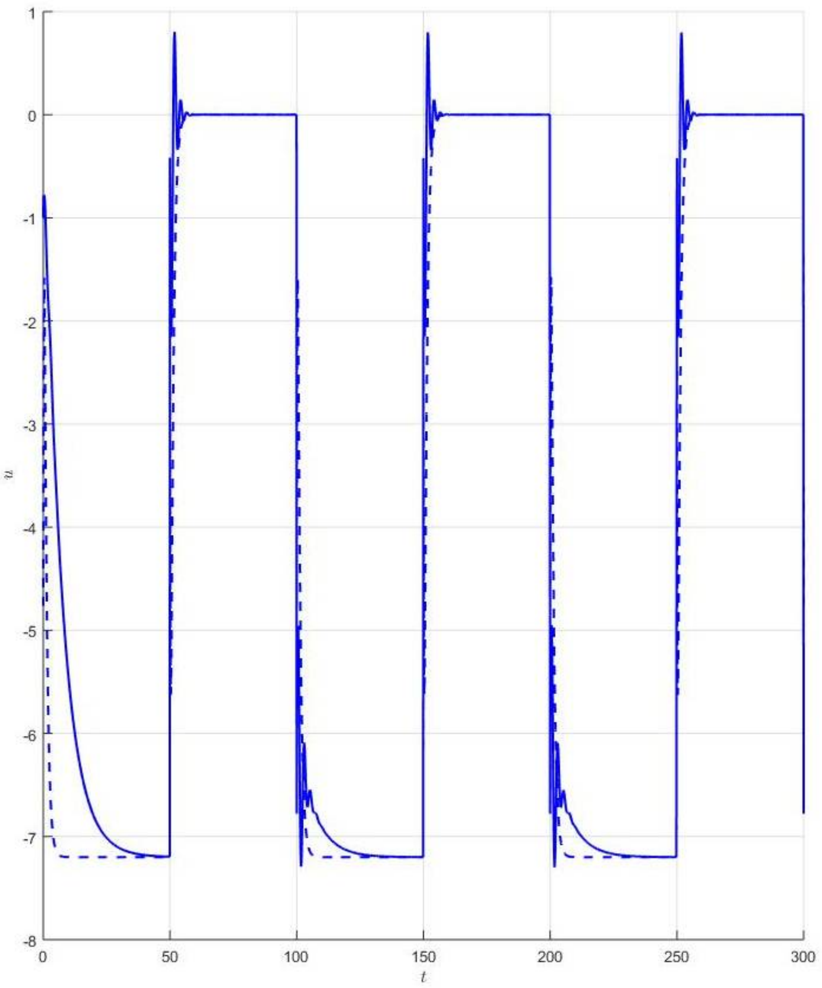
SIMULATION WITH MATLAB SIMULINK

- This figure shows the *output* of the reference model (red solid line), the *output* of the non-adaptive control system where the plant parameters are assumed KNOWN (blue dashed line) and the *output* of the adaptive control system.



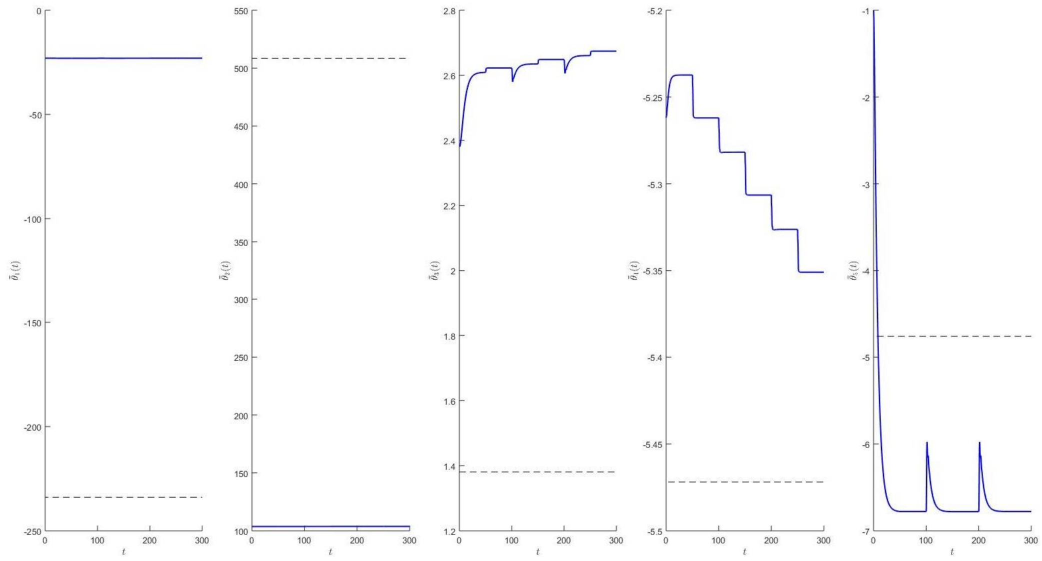
SIMULATION WITH MATLAB SIMULINK

- This figure shows the *control signal* of the non-adaptive control system (blue dashed line) and that of the adaptive control system (blue solid line).



SIMULATION WITH MATLAB SIMULINK

- The following figure shows the adaptive control gain $\bar{\theta}$ in the first 200 seconds (blue solid line) and the ideal control gain $\bar{\theta}^*$ for the true plant (black dashed line).



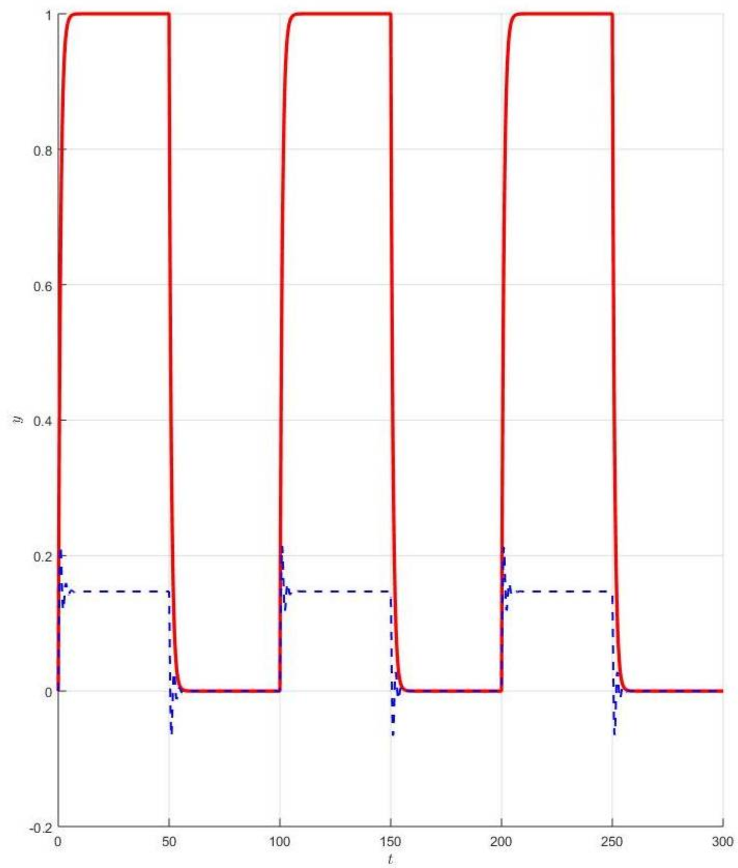
SIMULATION WITH MATLAB SIMULINK

- Note that previously for the non-adaptive control systems, it is assumed that the true plant is KNOWN (so that we can compare $\bar{\theta}$ with $\bar{\theta}^*$). The guess of the plant is only used to decide the initial value $\bar{\theta}(0)$ in the adaptive controller.

- If the plant is UNKNOWN to the non-adaptive control systems and the guess of the system is used to calculate the fixed gain, the non-adaptive control system output would be (see next page)

SIMULATION WITH MATLAB SIMULINK

- From the figure, clearly we can see that for the non-adaptive control system, if the plant is unknown, the output of the system cannot follow the reference model with only a guess of the plant parameters.



SIMULATION WITH MATLAB SIMULINK

- There is much more to be investigated using the Simulink program. With different controller design, it can be observed from the program that how the system would respond differently.

QUESTIONS FOR EE5104

- Required:
  - Write down the algorithm for an adaptive controller for the given plant: a) The asymptotic closed loop attained should be reasonably fast had have no steady-state offset for step changes in set point command signals; b) The output of the system as well as the control signal should be bounded in the adaptation process.
  - Verify the effectiveness of the control system design with simulations. The reference signal is a square wave of an appropriately chosen period. Show plots of signals such as y, y_m, $\bar{\theta}$.

QUESTIONS FOR EE5104

- Required:
  - Investigate the effects of different choice of $T(p)$ and Γ to the closed loop system.
  - For a particular design of the controller, investigate the specific case there the reference signal is a single sinusoid. Noting especially the output tracking error and the adapted controller gains.

$$r = 10\sin(2.0t)$$

  - Include all your program code / block diagram into the report with proper comment.

QUESTIONS FOR EE5104

- Suggested:
  - Try different initial guess $\bar{\theta}$. Choose some of $\bar{\theta}$ that is closed to $\bar{\theta}^*$ as well as some that is far from it. Compare and comment on the closed loop system performance.
  - For a particular design of the controller, use sinusoid signal as the reference signal and generally increase the frequency of the signal. Discuss the closed loop system performance.

QUESTIONS FOR EE6104

- Required:
  - Write down the algorithm for an adaptive controller for the given plant: a) The asymptotic closed loop attained should be reasonably fast had have no steady-state offset for step changes in set point command signals; b) The output of the system as well as the control signal should be bounded in the adaptation process.
  - Verify the effectiveness of the control system design with simulations. The reference signal is a square wave of an appropriately chosen period. Show plots of signals such as y, y_m, $\bar{\theta}$.

QUESTIONS FOR EE6104

- Required:
  - Investigate the effects of different choice of $T(p)$ and Γ to the closed loop system.
  - For a particular design of the controller, investigate the specific case there the reference signal is a single sinusoid. Noting especially the output tracking error and the adapted controller gains.

$$r = 10\sin(2.0t)$$

Compare this case with: a) The reference signal is a square wave of comparable period and amplitude; b) The reference signal is a sum of 5 or more sinusoidal signals of different but comparable periods and amplitude. Discuss your observations.

QUESTIONS FOR EE6104

- Required:
  - Run additional simulations to investigate the performance of your controller when the parameters of the plant is suddenly changed in the middle of the experiment. See whether the controller would adapt to the new plant and discuss your observations.
  - Include all your program code / block diagram into the report with proper comment.

QUESTIONS FOR EE6104

- Suggested:
  - Try different initial guess $\bar{\theta}$. Choose some of $\bar{\theta}$ that is closed to $\bar{\theta}^*$ as well as some that is far from it. Compare and comment on the closed loop system performance.
  - Observe the change of $\bar{\theta}$ in a long period of time where there are many step changes in the reference signal. See whether there is any patterns.

QUESTIONS FOR EE6104

- Suggested:
  - Compare the closed loop system output of the following reference signals.

    $$r_1 = \sin(1.0t), r_2 = \sin(2.0t), r_3 = r_1 + r_2$$

    Test whether the system satisfies superposition and explain why.
  - For a particular design of the controller, use sinusoid signal as the reference signal and generally increase the frequency of the signal. Discuss the closed loop system performance. Think of a way to describe the bandwidth of the system.

KEY REFERENCES

- Lecture Notes of EE5104/6104 Advanced/Adaptive Control Systems
- CA1 Experiment Requirement Sheet for EE5104 Y2015/2016 S1
- CA1 Experiment Requirement Sheet for EE6104 Y2015/2016 S1

THANK YOU