ORIGINAL

NATIONAL UNIVERSITY OF SINGAPORE

EXAMINATION FOR

(Semester I: 2017/2018)

EE5104 – ADVANCED ADAPTIVE CONTROL SYSTEMS

November/December 2017 – Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES:

1. This question paper contains **FOUR (4)** questions and comprises **ELEVEN (11)** printed pages.
2. Answer all **FOUR (4)** questions.
3. This is a CLOSED BOOK examination. However, each student may bring ONE (1) A4 size help sheet.
4. Note carefully that the questions do not carry equal marks.
5. Relevant data are provided at the end of this examination paper.
6. Total Marks: 100

Q.1 For the polynomials

$$
\begin{aligned}
R_m(s) &= s^2 + a_{1m}s + a_{2m} \\
T(s) &= s^2 + t_1 s + t_2 \\
R_p(s) &= s^2 + a_1 s + a_2
\end{aligned}
$$

calculate exactly the coefficients of the resulting polynomials $E(s)$ and $F(s)$ in the polynomial identity

$$R_m(s)T(s) = R_p(s)E(s) + F(s)$$

Develop therefore, fully and carefully, how this result above would be generalized if, instead, $R_m(s)$ is of order n^*, and both $T(s)$ and $R_p(s)$ are of order n.

(12 marks)

Q.2 In a particular experimental position control servomechanism, it is desired to use the d.c. motor system, shown in Figure 1, to be the basis of the overall positioning mechanism.

The d.c. motor system has the nominal dynamic model as shown in Figure 2, with the transfer function:

$$\frac{\Theta(s)}{U(s)} = \frac{K}{s(1 + s\tau)}$$

where $\Theta(s)$ is the Laplace transform of the angular position signal $\theta(t)$ and $U(s)$ is the Laplace transform of the motor drive input voltage $u(t)$. Calibration tests on the d.c. motor system, using the LabView real-time system connections of Figure 3, has yielded the data listed in Tables 1 and 2.

However, for Table 2, it is also known that the steady-state relationship between the motor drive input voltage $u(t)$ and the tachogenerator output voltage (while constant for each operation) can change in different day-to-day operations, and thus cannot be regarded as being known accurately. Further, simple step-response tests (which cannot be used as accurate calibration data) on the angular velocity has also indicated that

$$\tau \approx 220 \quad \text{milliseconds}$$

for the d.c. motor system, and that a positive-valued drive input voltage $u(t)$ results in a positive-valued angular velocity $\dot{\theta}(t)$.

For this hardware set-up of the position control servomechanism described above, it is next noted that a **different** situation has arisen where only the measurements of the input $u(t)$ and angular position output $\theta(t)$ are available.

Question 2 continues on page 3.

Develop therefore, fully and carefully, a structure for the **Control Law** which will allow for globally uniformly stable adaptive control ultilizing the **Reference Model**

$$\frac{\Theta_m(s)}{R(s)} = \frac{1}{s^2 + 2s + 1}$$

where likewise, the reference input $r(t)$ is an angular position reference/command signal where step changes are made in its value, to various different constant values, at intervals of 45 seconds or more. Include all relevant equations and detailed descriptions in developing the **Control Law**.

N.B.: Note particularly here that you are **only** required to develop fully and carefully the necessary structure for the **Control Law**. As already noted in class, with this appropriately developed structure, the necessary adaptive laws (even though rather complicated) are already available to ensure globally uniformly stable adaptive control. You are not required, in this case, to discuss the adaptive laws at all.

Hint: This is essentially the situation of developing the **Control Law** where only the measurements of the input $u(t)$ and angular position output $\theta(t)$ are available.

(23 marks)

Calibration Results for Part 1

Potientiometer Output (in volts)	Angular Position (in degrees)
-5	-180
-4	-144
-3	-108
-2	-72
-1	-36
0	0
1	36
2	72
3	108
4	144
5	180

Table 1 shows the results for the calibration of the potientiometer

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Table 1

Question 2 continues on page 4.

Calibration Results for Part 1

Input Voltage (volts)	Tachogenerator Output (volts)	Angular Velocity (rpm)	Angular Velocity (rad/sec)
-5	-4.03	-301	-31.52
-4	-3.17	-237	-24.82
-3	-2.3	-172	-18.01
-2	-1.45	-108	-11.31
-1	-0.6	-45	-4.71
0	0	0	0
1	0.62	48	5.03
2	1.48	111	11.62
3	2.33	175	18.33
4	3.2	239	25.03
5	4.06	303	31.73

Table 2 shows the results for the calibration of the tachogenerator

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Table 2

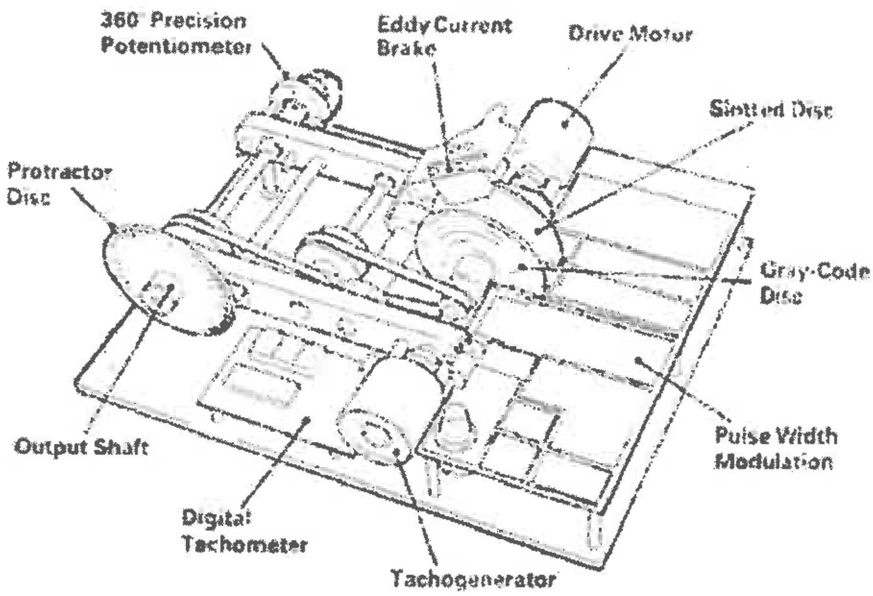

Figure 1

Question 2 continues on page 5.

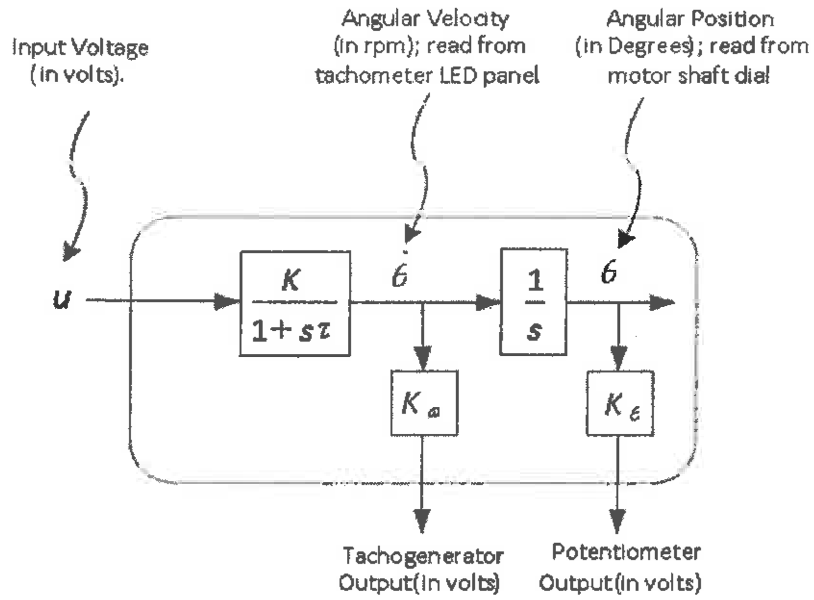

Figure 2

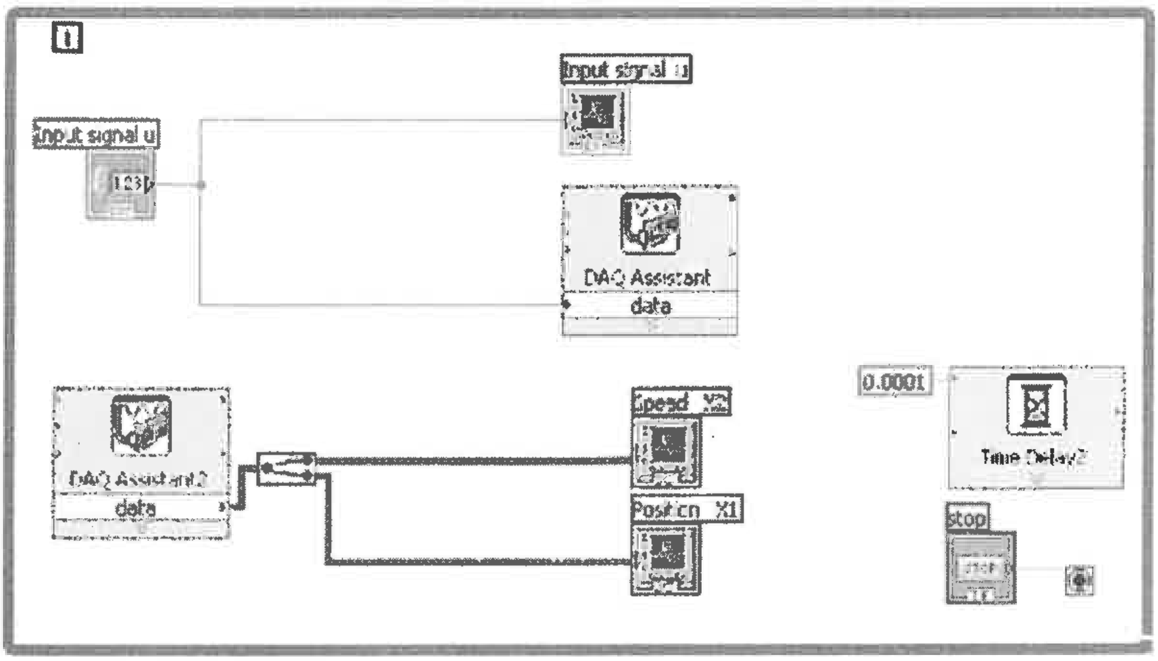

Figure 3

Q.3a Consider the system

$$y(k) = -ay(k-1) + bu(k-1) + e(k)$$

where $e(k)$ is a zero-mean Gaussian independent random variable. In an experiment to estimate a and b, the following data were calculated:

$$\begin{aligned} \sum y^2(k) &= 30 & \sum u^2(k) &= 50 & & \\ \sum y(k+1)y(k) &= 1 & \sum y(k)u(k) &= 20 & \sum y(k+1)u(k) &= 36 \end{aligned}$$

All sums are from $k = 1$ to $k = 999$. Determine the least-squares estimat of a and b.

(15 marks)

Q.3b Consider the system

$$y(k) = b_0u(k) + b_1u(k-1) + e(k)$$

where $e(k)$ is a Gaussian independent random variable of standard devaiation 1. The parameters b_1 and b_2 are estimated using the method of least squares. Let

$$u(k) = \begin{cases} 0 & \text{for } k = -1,\ 0 \\ 1 & \text{for } k \geq 1 \end{cases}$$

Find the covariance of the estimates for $k \to \infty$.

(20 marks)

Q.4 An ideal relay of amplitude 1 is connected to a plant in a feedback loop. The transfer function of the plant is given by

$$\frac{Y(s)}{U(s)} = \frac{1-s}{s(s+1)}$$

where $U(s)$ and $Y(s)$ are the input and output of the plant.

(a) Using the describing function approximation, find the amplitude and period of the resulting limit cycles.

(10 marks)

(b) Find the exact period of the limit cycles given the zero-order hold equivalent of the continuous-time plant

$$H_p(z) = \frac{(h - 1 + e^{-h})z + (1 - e^{-h} - he^{-h})}{z^2 - (1 + e^{-h})z + e^{-h}} - \frac{1 - e^{-h}}{z - e^{-h}}$$

(10 marks)

(c) Determine the ultimate gain and ultimate period if the relay is replaced by a proportional controller.

(10 marks)

– End of Paper –

DATA SHEET:

0. Prototype Response Tables

	k	Pole Locations for $\omega_0 = 1\ rad/s$[a]
ITAE	1	$s+1$
	2	$s+0.7071 \pm 0.7071j$[b]
	3	$(s+0.7081)(s+0.5210 \pm 1.068j)$
	4	$(s+0.4240 \pm 1.2630j)(s+0.6260 \pm 0.4141j)$
	5	$(s+0.8955)(s+0.3764 \pm 1.2920j)(s+0.5758 \pm 0.5339j)$
Bessel	1	$s+1$
	2	$s+0.8660 \pm 0.5000j$[b]
	3	$(s+0.9420)(s+0.7455 \pm 0.7112j)$
	4	$(s+0.6573 \pm 0.8302j)(s+0.9047 \pm 0.2711j)$
	5	$(s+0.9264)(s+0.5906 \pm 0.9072j)(s+0.8516 \pm 0.4427j)$

[a] Pole locations for other values of ω_0 can be obtained by substituting s/ω_0 for s.
[b] The factors $(s+a+bj)(s+a-bj)$ are written as $(s+a\pm bj)$ to conserve space.

1. The Lyapunov Equation states that given any $n\times n$ stability matrix A_m, for every symmetric positive definite matrix Q, there exists a unique symmetric positive definite matrix P that is the solution to the equation

$$A_m^\top P + PA_m = -Q.$$

In addition, the error system dynamics (with $\mathbf{e} \in \mathbf{R}^n$ and Γ an $n \times n$ symmetric positive-definite matrix) given by

$$\begin{aligned} \dot{\mathbf{e}}(t) &= A_m\mathbf{e}(t) + g\mathbf{b}\phi(t)^\top \mathbf{x}(t) \\ \dot{\phi}(t) &= -\mathrm{sgn}(g)\Gamma \mathbf{e}(t)^\top P\mathbf{b}\,\mathbf{x}(t) \end{aligned}$$

has the properties that $\|\mathbf{e}(t)\|$ and $\|\phi(t)\|$ are bounded, and if it should also be known that $\|\mathbf{x}(t)\|$ is bounded, then additionally

$$\lim_{t\to\infty} \mathbf{e}(t) = 0$$

2. For the triple

$$\begin{aligned} A_m &= \begin{bmatrix} 0 & 1 & 0 \\ a_1 & a_2 & a_3 \\ 1 & 0 & 0 \end{bmatrix} \\ b_m &= \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \\ c_m &= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \end{aligned}$$

the equivalent transfer function is

$$c_m^\top [sI - A_m]^{-1} b_m = \frac{-a_3}{s^3 - a_2 s^2 - a_1 s - a_3}$$

3. For

$$A_m = \begin{bmatrix} 0 & 1 & 0 \\ -21 & -12 & -10 \\ 1 & 0 & 0 \end{bmatrix}$$

and

$$Q = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

the solution to

$$A_m^\top P + P A_m = -Q.$$

is

$$P = \begin{bmatrix} 9.30 & 0.38 & 5.40 \\ 0.38 & 0.24 & 0.25 \\ 5.40 & 0.25 & 9.01 \end{bmatrix}$$

and the eigenvalues of P are $\lambda = 14.57, 3.76, 0.22$.

4. For

$$A_m = \begin{bmatrix} 0 & 1 & 0 \\ -11 & -7 & -5 \\ 1 & 0 & 0 \end{bmatrix}$$

and

$$Q = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

the solution to

$$A_m^\top P + P A_m = -Q.$$

is

$$P = \begin{bmatrix} 9.92 & 0.76 & 5.83 \\ 0.76 & 0.47 & 0.50 \\ 5.83 & 0.50 & 9.28 \end{bmatrix}$$

and the eigenvalues of P are $\lambda = 15.49, 3.77, 0.40$.

5. For

$$A_m = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}$$

and

$$Q = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

the solution to

$$A_m^\top P + P A_m = -Q.$$

is

$$P = \begin{bmatrix} 7.50 & 2.50 \\ 2.50 & 2.50 \end{bmatrix}$$

and the eigenvalues of P are $\lambda = 8.54, 1.46$.

6. For

$$A_m = \begin{bmatrix} 0 & 1 \\ -2 & -2 \end{bmatrix}$$

and

$$Q = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

the solution to

$$A_m^T P + P A_m = -Q.$$

is

$$P = \begin{bmatrix} 6.25 & 1.25 \\ 1.25 & 1.875 \end{bmatrix}$$

and the eigenvalues of P are $\lambda = 6.58, 1.54$.

7. For

$$A_m = \begin{bmatrix} 0 & 1 & 0 \\ -3,600 & -120 & -32,000 \\ 1 & 0 & 0 \end{bmatrix}$$

and

$$Q = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

the solution to

$$A_m^T P + P A_m = -Q.$$

is

$$P = \begin{bmatrix} 105.1 & 0.2 & 720.2 \\ 0.2 & 0.0225 & 0.0 \\ 720.2 & 0.0 & 6,424.4 \end{bmatrix}$$

and the eigenvalues of P are $\lambda = 6,505.4; 24.1; 0.021$.

8. For

$$A_m = \begin{bmatrix} 0 & 1 \\ -400 & -40 \end{bmatrix}$$

and

$$Q = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

the solution to

$$A_m^T P + P A_m = -Q.$$

is

$$P = \begin{bmatrix} 25.31 & 0.0063 \\ 0.0063 & 0.0627 \end{bmatrix}$$

and the eigenvalues of P are $\lambda = 25.31, 0.0627$.

6. For

$$A_m = \begin{bmatrix} 0 & 1 \\ -400 & -20 \end{bmatrix}$$

and

$$Q = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

the solution to

$$A_m^\top P + PA_m = -Q.$$

is

$$P = \begin{bmatrix} 50.25 & 0.006 \\ 0.006 & 0.125 \end{bmatrix}$$

and the eigenvalues of P are $\lambda = 50.25, 0.125$.

7. The standard discrete-time gradient estimator is

$$\begin{aligned} \hat{y}(j) &= \hat{\theta}(j)^\top \omega(j) \\ e_1(j) &= \hat{y}(j) - y(j) \\ \hat{\theta}(j+1) &= \hat{\theta}(j) - \frac{\omega(j)e_1(j)}{1 + \|\omega(j)\|^2} \end{aligned}$$

It is applicable to the process

$$y(j) = \theta^{*\top} \omega(j)$$

Laplace Transform Table

Laplace Transform, F(s)	Time Function, f(t)
1	$\delta(t)$ (unit impulse)
$\frac{1}{s}$	$1(t)$ (unit step)
$\frac{1}{s^2}$	t
$\frac{1}{s^n}$	$\frac{t^{n-1}}{(n-1)!}$ (n = positive integer)
$\frac{1}{s+a}$	e^{-at}
$\frac{1}{(s+a)^2}$	te^{-at}
$\frac{1}{(s+a)^n}$	$\frac{1}{(n-1)!}t^{n-1}e^{-at}$ (n = positive integer)
$\frac{1}{(s+a)(s+b)}$	$\frac{e^{-at}-e^{-bt}}{b-a}$
$\frac{1}{s(s+a)(s+b)}$	$\frac{1}{ab}\left[1+\frac{1}{a-b}(be^{-at}-ae^{-bt})\right]$
$\frac{\omega}{s^2+\omega^2}$	$\sin\omega t$
$\frac{s}{s^2+\omega^2}$	$\cos\omega t$
$\frac{\omega}{(s+a)^2+\omega^2}$	$e^{-at}\sin\omega t$
$\frac{s+a}{(s+a)^2+\omega^2}$	$e^{-at}\cos\omega t$
$\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\frac{\omega_n}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin\omega_n\sqrt{1-\zeta^2}t$
$\frac{s}{s^2+2\zeta\omega_n s+\omega_n^2}$	$-\frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}t-\phi)$ $\phi=\tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}$
$\frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$	$1-\frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}t+\phi)$ $\phi=\tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}$