Note Title 9/11/2013

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Adaptive Systems

Error Dynamical Models

Error Dynamical Model I

$\dot{e} =$ — Ia

$k_p \in \mathbb{R}^1$ with known sign $\left\{ e \in \mathbb{R}^n; \ \phi, w \in \mathbb{R}^{n_1} \right.$

$\dot{\phi} =$ — Ib

where $\Gamma \in \mathbb{R}^{n_1 \times n_1} > 0$

sym p-d.

$A_m \in \mathbb{R}^{n \times n}$ is a stability matrix

and P is the sym p-d. soln to

where Q is any suitably chosen sym p-d. matrix.

$\Delta\Delta$

For the system (Ia) & (Ib), we have ~~the~~ result ~~that~~:

- $\|e\|$, $\|\Phi\|$ are bounded for all $t \geq t_0$

• If $\|w\|$ is bounded, we also have

$$\lim_{t\to\infty} \|e\| = 0$$

$\triangle$

Error Dynamical Model II

IIa

$e_1(t) =$

$W_m(s)$ is a strictly positive-real transfer function in s.

$p \triangleq \frac{d}{dt}$

$e_1(t) \in \mathbb{R}^1$; $k_p \in \mathbb{R}^1$ with $\mathrm{sgn}(k_p)$ known

$\phi, w \in \mathbb{R}^{n}$,

$$\dot{\phi} =$$

— II b

△△

For the system II a and II b,

we have the result that

$\|\phi\|$ is bounded for all $t \geq t_0$,

and if $\|w\|$ is bounded, then

we also have:

$$\lim_{t \to \infty} |e_1(t)| = 0$$

△△

Remark II :

$a_m < 0$

The system :

$$\dot{e}_1 =$$

$$\begin{bmatrix} \dot{\phi} \\ \vdots \\ \dot{\phi}_{n-1} \end{bmatrix} =$$

is a special case of the above!

$\Delta\Delta$

Error Dynamical Model (I)

$e_1(t) =$ (Ia)

$e_1 \in \mathbb{R}^1$; $\phi, w \in \mathbb{R}^{n_1}$

$k_p \in \mathbb{R}^1$; $\mathrm{sgn}(k_p)$ known

$\dot{\phi} =$ (Ib)

$\Delta\Delta$

For the system (Ia) and (Ib),
we have the result that

$\|\phi\|$ is bounded for all $t > t_0$

△△

<u>**Remark 0 :**</u>

If, in addition, the signal $w(t)$ is "persistently exciting", then we also have:

But the condition of $w(t)$ being "persistently exciting" is a rather stringent & difficult one.

△△

<u>Aside:</u>

A signal $w(t) \in \mathbb{R}^{n_1}$ is p.e. iff there exists $\delta, \epsilon > 0$ s.t.

$$\int_t^{t+\delta} w(\tau)\, w^T \, d\tau \geq \epsilon$$

Stability Analysis of Error Dynamical Model ①

for all $\|x\| = 1$ with $x \in \mathbb{R}^{n_1}$; and all $t > t_0$.

Consider the quadratic form

$$V(t) =$$

then

$$\dot{V}(t) = 2\phi^T \dot{\phi}$$

$$= 2\phi^T \left\{ \qquad \right\}$$

$$=$$

$$= -2\,\mathrm{sgn}(k_p)\frac{\mathrm{sgn}(k_p)}{|k_p|} e_1^2$$

$$\leq 0$$

$\Delta\Delta$

⋮

Cases of stability analysis

for $n^* > 1$ case