ORIGINAL

NATIONAL UNIVERSITY OF SINGAPORE

EXAMINATION FOR

(Semester I: 2018/2019)

EE5104 – ADAPTIVE CONTROL SYSTEMS

November/December 2018 – Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES:

1. This question paper contains **FOUR (4)** questions and comprises **FIFTEEN (15)** printed pages.
2. Answer all **FOUR (4)** questions.
3. This is a CLOSED BOOK examination. However, each student may bring ONE (1) A4 size crib sheet into the examination hall.
4. Note carefully that the questions **do not** carry equal marks.
5. Relevant data are provided at the end of this examination paper.
6. Total Marks: 100

Q.1 In a particular experimental position control servomechanism, it is desired to use the d.c. motor system, shown in Figure 1, to be the basis of the overall positioning mechanism.

The d.c. motor system has the nominal dynamic model as shown in Figure 2, with the transfer function:

$$\frac{\Theta(s)}{U(s)} = \frac{K}{s(1+s\tau)}$$

where $\Theta(s)$ is the Laplace transform of the angular position signal $\theta(t)$ and $U(s)$ is the Laplace transform of the motor drive input voltage $u(t)$. Calibration tests on the d.c. motor system, using the LabView real-time system connections of Figure 3, has yielded the data listed in Tables 1 and 2.

However, for Table 2, it is also known that the steady-state relationship between the motor drive input voltage $u(t)$ and the tachogenerator output voltage (while constant for each operation) can change in different day-to-day operations, and thus <u>cannot</u> be regarded as being known accurately. Further, simple step-response tests (which <u>cannot</u> be used as accurate calibration data) on the angular velocity has also indicated that

$$\tau \approx 230 \quad \text{milliseconds}$$

for the d.c. motor system, and that a positive-valued drive input voltage $u(t)$ results in a positive-valued angular velocity $\dot{\theta}(t)$.

Using the information above, write a suitable state-variable description of the d.c. motor system, where the state-variables are chosen as:

$$\begin{aligned} x_1(t) &= \theta(t) \\ x_2(t) &= \dot{\theta}(t) \end{aligned}$$

Your state-variable description can include unknown constants for system coefficients which are not accurately known based on the conditions described above.

In the control system to be used for the overall positioning mechanism, it is desired to use an adaptive state-feedback controller of the configuration shown in Figures 4, 5 and 6. Here, the reference input $r(t)$ is an angular position reference/command signal where step changes are made in its value, to various different constant values, at intervals of 45 seconds or more.

Based on the data furnished in the data sheet at the end of this examination paper (or otherwise), noting that both $\theta(t)$ and $\dot{\theta}(t)$ are measurable, describe fully the basis for the choices of the control law, the reference model, and the adaptive law as also shown in Figures 4, 5 and 6. Include all relevant equations and detailed descriptions.

Additionally, carefully describe also the stability and convergence properties of this adaptive state-feedback controller. Include all relevant equations and detailed descriptions.

N.B.: Relevant data is furnished at the end of this examination paper which may be helpful in your choices here. If you have other means to guide your choices, it is fine too; but the basis must be described clearly.

(20 marks)

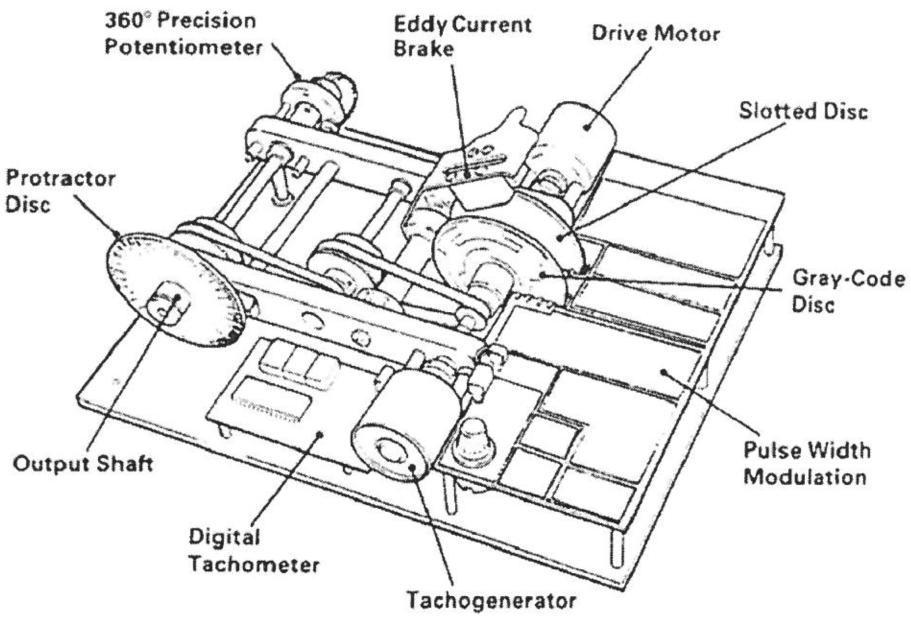


Figure 1

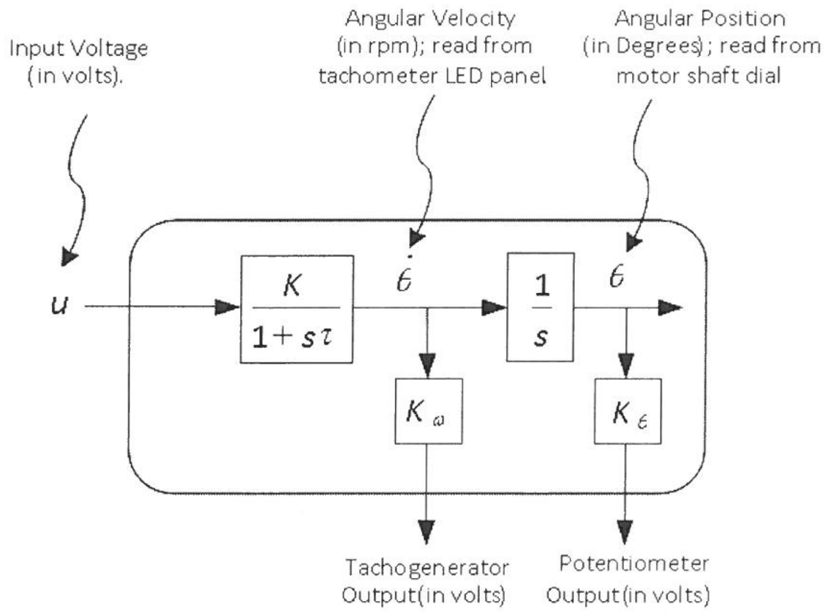


Figure 2

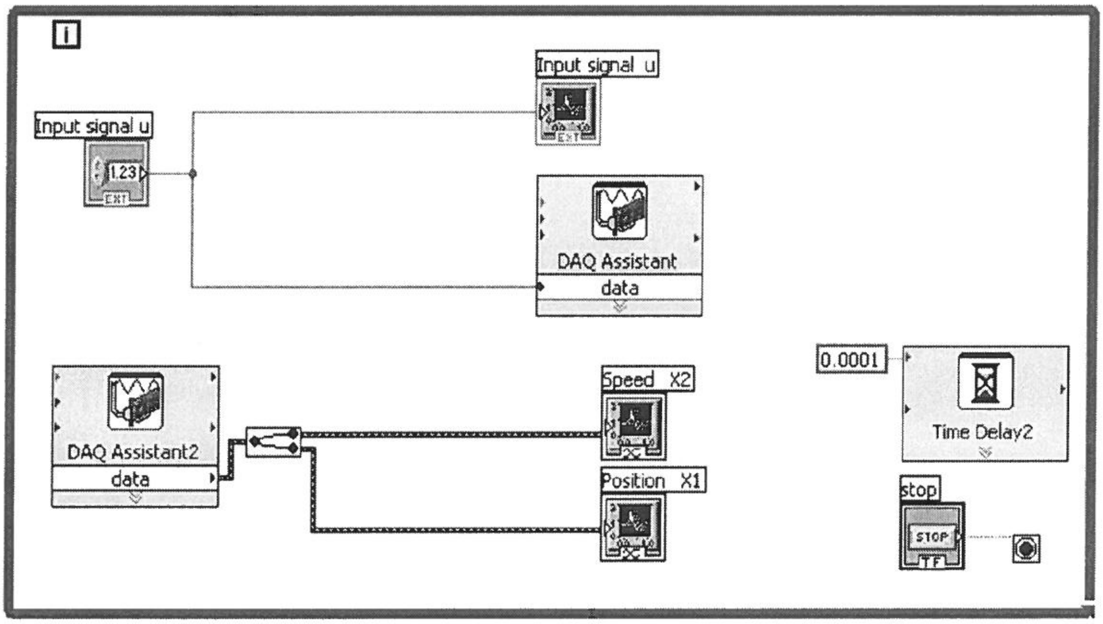

Input signal u
1.23
EXT
Input signal u
EXT
DAQ Assistant
data
DAQ Assistant2
data
Speed X2
Position X1
0.0001
Time Delay2
stop
STOP
TF

Figure 3

Calibration Results for Part 1

Potientiometer Output (in volts)	Angular Position (in degrees)
-5	-180
-4	-144
-3	-108
-2	-72
-1	-36
0	0
1	36
2	72
3	108
4	144
5	180

Table 1 shows the results for the calibration of the potientiometer

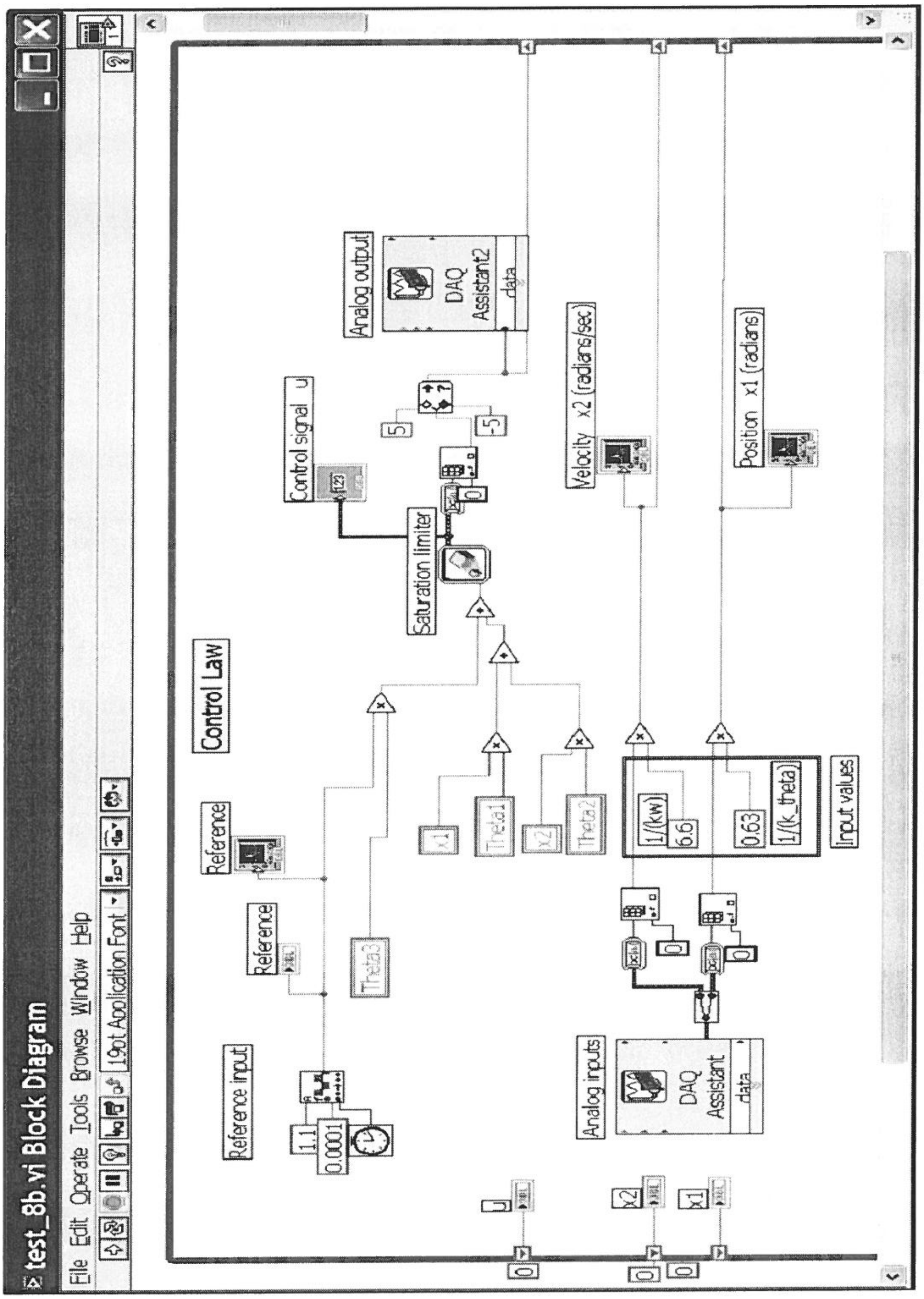
Table 1

Calibration Results for Part 1

Input Voltage (volts)	Tachogenerator Output (volts)	Angular Velocity (rpm)	Angular Velocity (rad/sec)
-5	-4.03	-301	-31.52
-4	-3.17	-237	-24.82
-3	-2.3	-172	-18.01
-2	-1.45	-108	-11.31
-1	-0.6	-45	-4.71
0	0	0	0
1	0.62	48	5.03
2	1.48	111	11.62
3	2.33	175	18.33
4	3.2	239	25.03
5	4.06	303	31.73

Table 2 shows the results for the calibration of the tachogenerator

Table 2


test_8b.vi Block Diagram
File Edit Operate Tools Browse Window Help
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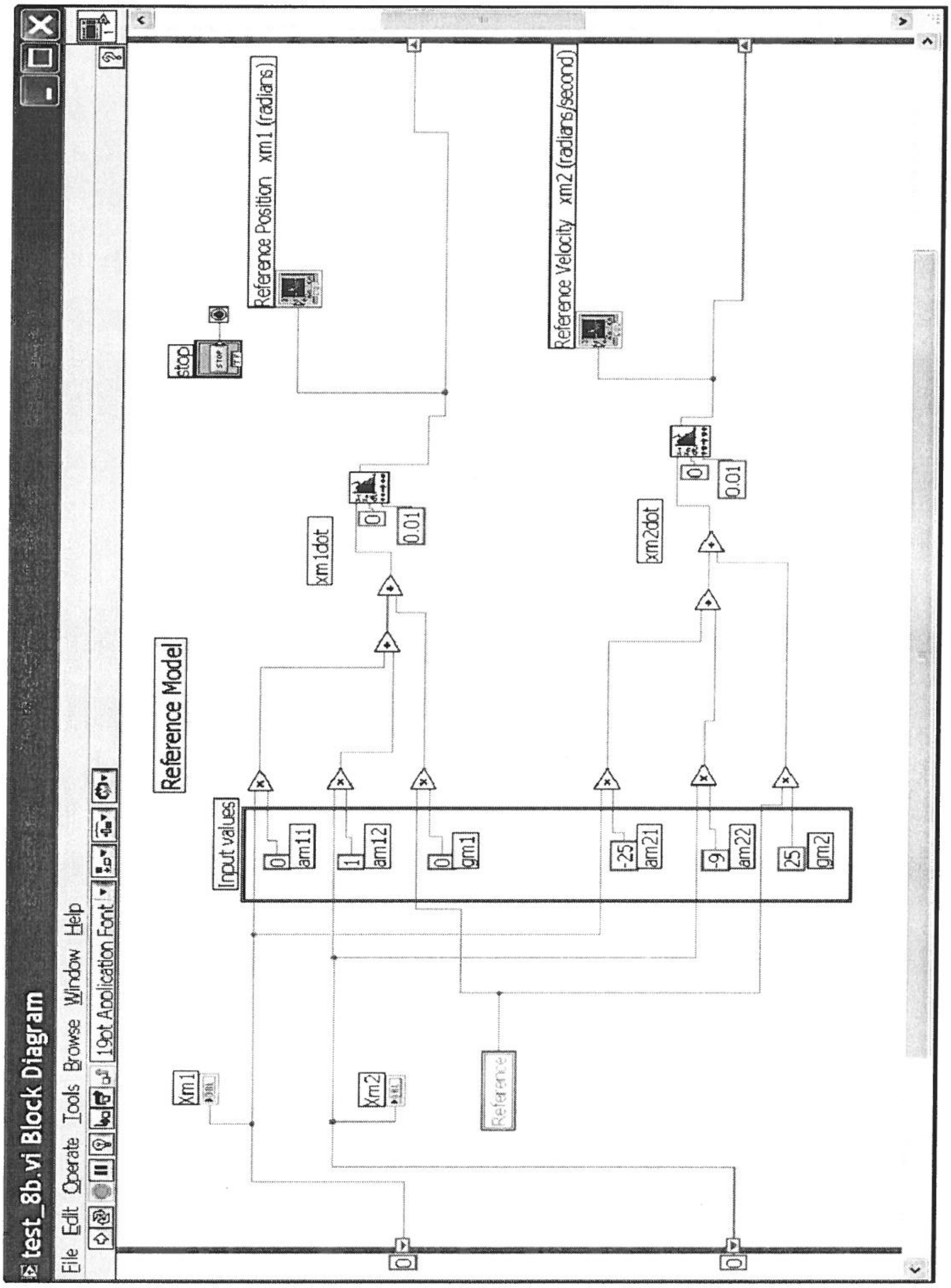
Control Law
Reference input
Reference
Reference
Theta3
x1
Theta1
x2
Theta2
Saturation limiter
Control signal u
5
-5
Analog output
DAQ Assistant2
data
Velocity x2 (radians/sec)
Position x1 (radians)
Analog inputs
DAQ Assistant
data
1/(kw)
6.6
0.63
1/(k_theta)
Input values
1.1
0.0001

Figure 4


test_8b.vi Block Diagram
File Edit Operate Tools Browse Window Help
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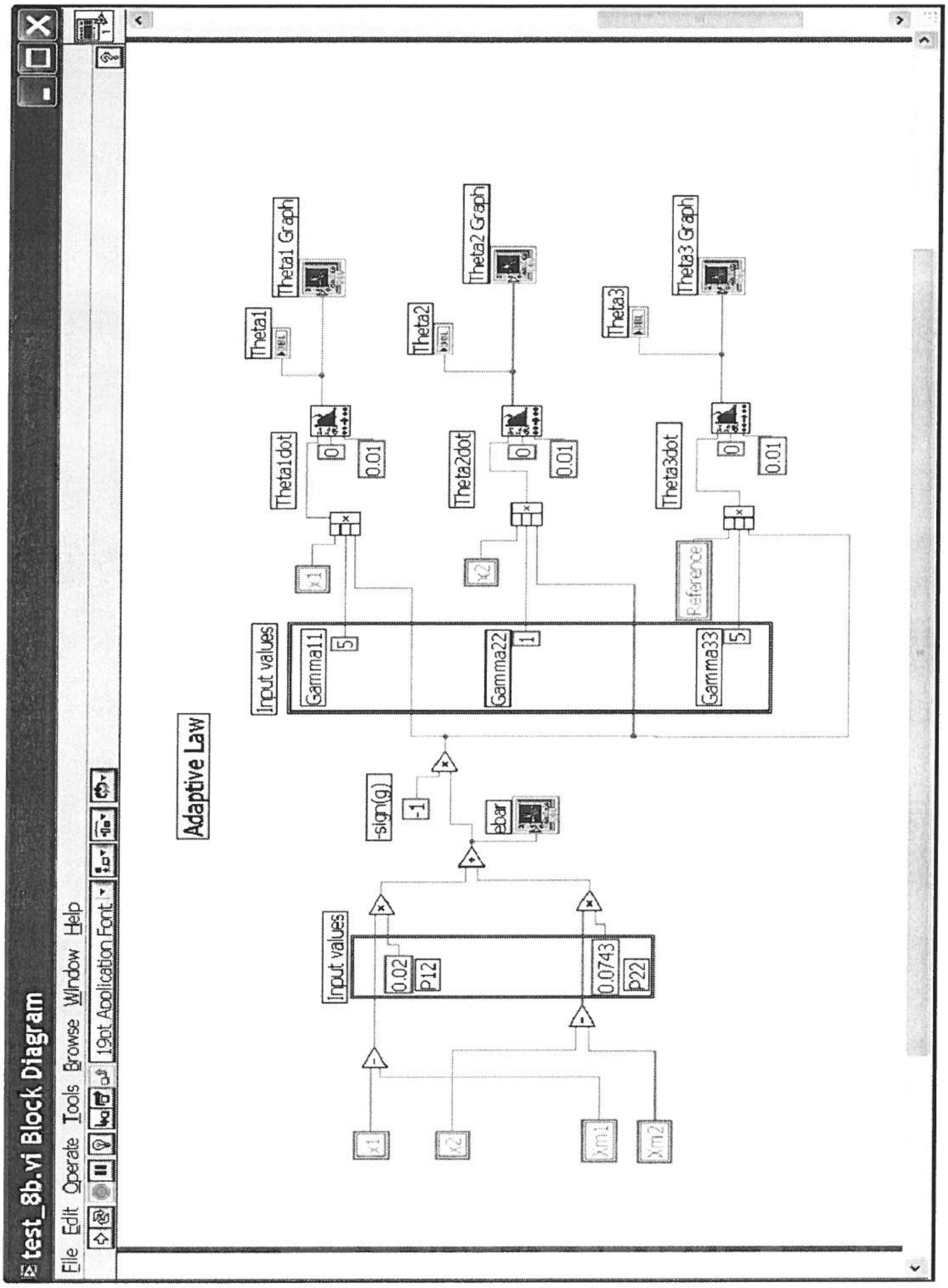
Reference Model
Input values
0 am11
1 am12
0 gm1
-25 am21
-9 am22
25 gm2
xm1dot
xm2dot
0
0.01
stop
Reference Position xm1 (radians)
Reference Velocity xm2 (radians/second)
Xm1
Xm2
Reference

Figure 5


test_8b.vi Block Diagram
File Edit Operate Tools Browse Window Help
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Adaptive Law
Input values
Gamma11
5
Gamma22
1
Gamma33
5
Reference
Theta1dot
Theta2dot
Theta3dot
0
0.01
Theta1
Theta2
Theta3
Theta1 Graph
Theta2 Graph
Theta3 Graph
-sign(g)
-1
ebar
Input values
0.02
P12
0.0743
P22
x1
x2
Xm1
Xm2

Figure 6

Q.2 For the same hardware set-up of the position control servomechanism described in Question 1, it is next noted that a **different** situation has arisen where only the measurements of the input $u(t)$ and angular position output $\theta(t)$ are available.

Develop next, fully and carefully, a structure for the **Control Law** which will allow for globally uniformly stable adaptive control ultilizing the **Reference Model**

$$\frac{\Theta_m(s)}{R(s)} = \frac{1}{s^2 + 2s + 1}$$

where likewise as in the situation of Question 1, the reference input $r(t)$ is an angular position reference/command signal where step changes are made in its value, to various different constant values, at intervals of 45 seconds or more. Include all relevant equations and detailed descriptions.

N.B.: Note particularly here that you are **only** required to develop fully and carefully the necessary structure for the **Control Law**. As already noted in class, with this appropriately developed structure, the necessary adaptive laws (even though rather complicated) are already available to ensure globally uniformly stable adaptive control. You are not required, in this case, to discuss the adaptive laws at all.

Hint: This is essentially the situation of developing the **Control Law** where only the measurements of the input $u(t)$ and angular position output $\theta(t)$ are available.

(15 marks)

Q.3 Consider the process

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

The initial conditions are $x_1(0) = 0$, $x_2(0) = 1$ and the sliding line is given as $\sigma = x_1 + x_2 = 0$

a) Design a sliding mode controller such that it takes $t_\sigma = 1$ for the states to reach the sliding line.

(10 marks)

b) For $0 \le t < t_\sigma$, find $x_1(t)$ and $x_2(t)$.

(12 marks)

c) For $t \ge t_\sigma$, find $x_1(t)$ and $x_2(t)$.

(13 marks)

Q.4 Consider the single-input single-output process

$$A(q^{-1})y(k) = B(q^{-1})u(k) + C(q^{-1})e(k)$$

where

$$\begin{aligned} A(q^{-1}) &= 1 + aq^{-1} \\ B(q^{-1}) &= bq^{-1} \\ C(q^{-1}) &= 1 + cq^{-1} \end{aligned}$$

and $k = 0, 1, \ldots, N$. The input, output and Gaussian independent random variable with standard deviation σ are given by $u(k)$, $y(k)$ and $e(k)$ respectively. In state-space form

$$\begin{aligned} x(k+1) &= -cx(k) + bu(k) + (c - a)y(k) \\ y(k) &= x(k) + e(k) \end{aligned}$$

For simplicity, let $u(k) = 0$.

a) Express $x(1)$ and $x(2)$ in terms of $x(0)$

(7 marks)

b) Write the vector $E = \left[\begin{array}{ccc} e(0) & e(1) & e(2) \end{array} \right]^T$ in the following form.

$$Z = \Phi x(0) + E$$

where Z and Φ are column vectors. Give the elements in Z and Φ.

(8 marks)

c) Using batch least-squares, find $\hat{x}(0)$ that minimizes the least-squares objective function.

$$J = \frac{1}{2}E^T E$$

(7 marks)

d) Given $a = c = -1$, $y(0) = 0$, $y(1) = 1$, $y(2) = 2$, and using the recursive least-squares algorithm, find $\hat{x}(0)$ at every iteration from $k = 0$ to $k = 2$. Initialize $\hat{x}(0) = 0$ and covariance $P = 100$.

(8 marks)

– End of Questions –

DATA SHEET:

0. Prototype Response Tables

	k	Pole Locations for $\omega_0 = 1\ rad/s^a$
ITAE	1	$s+1$
	2	$s+0.7071 \pm 0.7071j^b$
	3	$(s+0.7081)(s+0.5210 \pm 1.068j)$
	4	$(s+0.4240 \pm 1.2630j)(s+0.6260 \pm 0.4141j)$
	5	$(s+0.8955)(s+0.3764 \pm 1.2920j)(s+0.5758 \pm 0.5339j)$
Bessel	1	$s+1$
	2	$s+0.8660 \pm 0.5000j^b$
	3	$(s+0.9420)(s+0.7455 \pm 0.7112j)$
	4	$(s+0.6573 \pm 0.8302j)(s+0.9047 \pm 0.2711j)$
	5	$(s+0.9264)(s+0.5906 \pm 0.9072j)(s+0.8516 \pm 0.4427j)$

[a] Pole locations for other values of ω_0 can be obtained by substituting s/ω_0 for s.
[b] The factors $(s+a+bj)(s+a-bj)$ are written as $(s+a \pm bj)$ to conserve space.

1. The Lyapunov Equation states that given any $n \times n$ stability matrix A_m, for every symmetric positive definite matrix Q, there exists a unique symmetric positive definite matrix P that is the solution to the equation

$$A_m^\top P + P A_m = -Q.$$

In addition, the error system dynamics (with $\mathbf{e} \in \mathbf{R}^n$ and Γ an $n \times n$ symmetric positive-definite matrix) given by

$$\begin{aligned} \dot{\mathbf{e}}(t) &= A_m \mathbf{e}(t) + g\mathbf{b}\phi(t)^\top \mathbf{x}(t) \\ \dot{\phi}(t) &= -\mathrm{sgn}(g)\Gamma \mathrm{e}(t)^\top P\mathbf{b}\, \mathbf{x}(t) \end{aligned}$$

has the properties that $\|\mathbf{e}(t)\|$ and $\|\phi(t)\|$ are bounded, and if it should also be known that $\|\mathbf{x}(t)\|$ is bounded, then additionally

$$\lim_{t\to\infty} \mathbf{e}(t) = 0$$

2. For the triple

$$\begin{aligned} A_m &= \begin{bmatrix} 0 & 1 & 0 \\ a_1 & a_2 & a_3 \\ 1 & 0 & 0 \end{bmatrix} \\ b_m &= \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \\ c_m &= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \end{aligned}$$

the equivalent transfer function is

$$c_m^\top [sI - A_m]^{-1} b_m = \frac{-a_3}{s^3 - a_2 s^2 - a_1 s - a_3}$$

3. For

$$A_m = \begin{bmatrix} 0 & 1 & 0 \\ -21 & -12 & -10 \\ 1 & 0 & 0 \end{bmatrix}$$

and

$$Q = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

the solution to

$$A_m^\top P + P A_m = -Q.$$

is

$$P = \begin{bmatrix} 9.30 & 0.38 & 5.40 \\ 0.38 & 0.24 & 0.25 \\ 5.40 & 0.25 & 9.01 \end{bmatrix}$$

and the eigenvalues of P are $\lambda = 14.57, 3.76, 0.22$.

4. For

$$A_m = \begin{bmatrix} 0 & 1 & 0 \\ -11 & -7 & -5 \\ 1 & 0 & 0 \end{bmatrix}$$

and

$$Q = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

the solution to

$$A_m^\top P + P A_m = -Q.$$

is

$$P = \begin{bmatrix} 9.92 & 0.76 & 5.83 \\ 0.76 & 0.47 & 0.50 \\ 5.83 & 0.50 & 9.28 \end{bmatrix}$$

and the eigenvalues of P are $\lambda = 15.49, 3.77, 0.40$.

5. For

$$A_m = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}$$

and

$$Q = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

the solution to

$$A_m^\top P + P A_m = -Q.$$

is

$$P = \begin{bmatrix} 7.50 & 2.50 \\ 2.50 & 2.50 \end{bmatrix}$$

and the eigenvalues of P are $\lambda = 8.54, 1.46$.

6. For

$$A_m = \begin{bmatrix} 0 & 1 \\ -2 & -2 \end{bmatrix}$$

and

$$Q = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

the solution to

$$A_m^\top P + P A_m = -Q.$$

is

$$P = \begin{bmatrix} 6.25 & 1.25 \\ 1.25 & 1.875 \end{bmatrix}$$

and the eigenvalues of P are $\lambda = 6.58, 1.54$.

7. For

$$A_m = \begin{bmatrix} 0 & 1 & 0 \\ -3,600 & -120 & -32,000 \\ 1 & 0 & 0 \end{bmatrix}$$

and

$$Q = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

the solution to

$$A_m^\top P + P A_m = -Q.$$

is

$$P = \begin{bmatrix} 105.1 & 0.2 & 720.2 \\ 0.2 & 0.0225 & 0.0 \\ 720.2 & 0.0 & 6,424.4 \end{bmatrix}$$

and the eigenvalues of P are $\lambda = 6,505.4; 24.1; 0.021$.

8. For

$$A_m = \begin{bmatrix} 0 & 1 \\ -400 & -40 \end{bmatrix}$$

and

$$Q = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

the solution to

$$A_m^\top P + P A_m = -Q.$$

is

$$P = \begin{bmatrix} 25.31 & 0.0063 \\ 0.0063 & 0.0627 \end{bmatrix}$$

and the eigenvalues of P are $\lambda = 25.31, 0.0627$.

6. For

$$A_m = \begin{bmatrix} 0 & 1 \\ -400 & -20 \end{bmatrix}$$

and

$$Q = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

the solution to

$$A_m^\top P + P A_m = -Q.$$

is

$$P = \begin{bmatrix} 50.25 & 0.006 \\ 0.006 & 0.125 \end{bmatrix}$$

and the eigenvalues of P are $\lambda = 50.25, 0.125$.

7. The standard discrete-time gradient estimator is

$$\begin{aligned} \hat{y}(j) &= \hat{\theta}(j)^\top \omega(j) \\ e_1(j) &= \hat{y}(j) - y(j) \\ \hat{\theta}(j+1) &= \hat{\theta}(j) - \frac{\omega(j) e_1(j)}{1 + \|\omega(j)\|^2} \end{aligned}$$

It is applicable to the process

$$y(j) = \theta^{*\top} \omega(j)$$

Laplace Transform Table

Laplace Transform, F(s)	Time Function, f(t)
1	$\delta(t)$ (unit impulse)
$\frac{1}{s}$	$1(t)$ (unit step)
$\frac{1}{s^2}$	t
$\frac{1}{s^n}$	$\frac{t^{n-1}}{(n-1)!}$ (n = positive integer)
$\frac{1}{s+a}$	e^{-at}
$\frac{1}{(s+a)^2}$	te^{-at}
$\frac{1}{(s+a)^n}$	$\frac{1}{(n-1)!}t^{n-1}e^{-at}$ (n = positive integer)
$\frac{1}{(s+a)(s+b)}$	$\frac{e^{-at}-e^{-bt}}{b-a}$
$\frac{1}{s(s+a)(s+b)}$	$\frac{1}{ab}\left[1+\frac{1}{a-b}(be^{-at}-ae^{-bt})\right]$
$\frac{\omega}{s^2+\omega^2}$	$\sin\omega t$
$\frac{s}{s^2+\omega^2}$	$\cos\omega t$
$\frac{\omega}{(s+a)^2+\omega^2}$	$e^{-at}\sin\omega t$
$\frac{s+a}{(s+a)^2+\omega^2}$	$e^{-at}\cos\omega t$
$\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\frac{\omega_n}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin\omega_n\sqrt{1-\zeta^2}t$
$\frac{s}{s^2+2\zeta\omega_n s+\omega_n^2}$	$-\frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}t-\phi)$ $\phi=\tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}$
$\frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$	$1-\frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}t+\phi)$ $\phi=\tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}$

– End of Paper –