Representation of Orientation

① Rotation Matrix $\quad {}^{A}_{B}R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \longrightarrow$ 9 parameters

② Angle-set conventions — 3 parameters

X-Y-Z fixed angles (roll, pitch, yaw angles)

Z-Y-X Euler Angles

Z-Y-Z Euler angles

Start $\bar{\text{w}}$ frame coincident $\bar{\text{w}}$ known frame $\{A\}$

Rotate $\{B\}$ first about $\hat{Z}_B$ by angle α

Rotate about $\hat{Y}_B$ by angle β

Rotate about $\hat{Z}_B$ by angle γ

Total 12 fixed angle sets and 12 Euler angle sets $= 24$

$\rightarrow$ 12 unique parameterizations $\because$ duality of fixed angle $\bar{\text{w}}$ Euler angle sets

③ Equivalent angle-axis representation — 4 parameters

Start $\bar{\text{w}}$ frame coincident $\bar{\text{w}}$ known frame $\{A\}$

Rotate $\{B\}$ about the vector ${}^{A}\hat{K}$ by angle θ according to right-angle rule

④ Euler parameters — 4 parameters

In terms of equivalent axis $\hat{K} = [\,k_x \;\; k_y \;\; k_z\,]^T$ and equivalent angle θ,

$$\varepsilon_1 = k_x \sin\frac{\theta}{2}, \quad \varepsilon_2 = k_y \sin\frac{\theta}{2}, \quad \varepsilon_3 = k_z \sin\frac{\theta}{2}, \quad \varepsilon_4 = \cos\frac{\theta}{2}$$

Normality condition:

$$|\hat{K}| = 1, \quad \varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_3^2 + \varepsilon_4^2 = 1$$

ε : point on a unit hypersphere in 4D space