Note Title 8/22/2013

Stability Analysis for $n^* = 1$

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$$R_p(p)\, y(t) = k_p Z_p(p)\, u(t)$$

From Proposition 1, the plant can be re-written as:

$$\dot{w} = \qquad u$$

$$y =$$

Our control law is

$$u(t) = \theta^T(t)\, w(t) + k(t)\, r(t)$$

$$= \Big\{ \qquad\qquad \Big\} w(t)$$

$$+ \Big\{ \qquad\qquad \Big\} r(t)$$

Thus, with this control law, we now have

$$\dot{w} = A_p w + b_p \, w(t)$$

$$y = c_p^T w$$

i.e.

$$\dot{w} = A_p w + b_p \left\{ \begin{array}{l} \left[\theta^{*T} + \phi(t)^T \right] w(t) \\ + \left[k^* + \phi_k(t) \right] r(t) \end{array} \right\}$$

$$= \underline{\left\{ A_p + b_p \theta^{*T} \right\}} w(t) + \underline{k^* b_p} \, r(t)$$

$$+ b_p \phi(t)^T w(t) + b_p \phi_k(t) \, r(t)$$

$$y = \underline{c_p^T} w \qquad \text{— (3.1)}$$

where, by Propositions 1 & 2, it is implicitly possible to write

$$R_m(p)\, y_m(t) = k_m\, r(t)$$

as the 2n-order non-minimal realization

$$\dot{w}_m = \qquad\qquad r(t)$$

$$y_m = \qquad\qquad \longrightarrow (3.2)$$

By Propositions 1 & 2, this w_m exists, but we do not know its values !!!

equivalent state-vector of the Reference Model in non-minimal realisation !!

Then, for

$$e(t) \triangleq w(t) - w_m(t)$$

$$e_1(t) \triangleq y(t) - y_m(t)$$

From (3.1) and (3.2), we have

$$\dot{e} = A_m e + \frac{1}{k^*} b_m \underbrace{[\qquad]}_{\bar{\phi}^T} \underbrace{\begin{bmatrix} w \\ r \end{bmatrix}}_{\bar{w}}$$

$$e_1 = c_m^T e$$

i.e. we have the "error dynamical system"

$$\dot{e} = A_m e + \frac{1}{k^*} b_m \phi^T \bar{w}$$

$$e_1 = c_m^T e$$

where

$$c_m^T \{sI - A_m\}^{-1} b_m =$$

is strictly positive-real.

Additionally, recall that the "adaptive law" is

$$\dot{\bar{\theta}}(t) = \dot{\bar{\Phi}}(t) = -\operatorname{sgn}(k_p)\Gamma \bar{w} e_1$$

Thus, now, consider the quadratic form

$$V(e, \bar{\Phi}) = e^T P e + \bar{\Phi} \Gamma^{-1} \bar{\Phi}$$

where, from the Kalman-Yakubovich Lemma, we have

$$A_m^T P + P A_m =$$

Thus,

$$\dot{V} = 2 e^T P \dot{e} + 2 \tilde{\phi} \Gamma^{-1} \dot{\tilde{\phi}}$$

$$= 2 e^T P \left\{ \qquad \qquad \qquad \qquad \right\}$$

$$+ 2 \tilde{\phi} \Gamma^{-1} \dot{\tilde{\phi}}$$

$$= -e^T Q e + 2 e^T \frac{1}{k^*} P b_m \tilde{\phi}^T w$$

$$+ 2 \tilde{\phi} \Gamma^{-1} \dot{\tilde{\phi}}$$

[Noting also that $k_m = k_p k^*$]

Recall that the Kalman-Yacubovich Lemma states that for the spr transfer function

$$\frac{1}{|k^*|} \frac{k_m}{R_m(s)}$$

with state-realization $\{\quad\}$

- $A_m^T P + P A_m = -q q^T - \varepsilon L = -Q$ — (4.1a)
- $P \frac{1}{|k^*|} b_m = c_m$ — (4.1b)

Noting that $k_m = k_p k^*$; $k_m > 0$

i.e. $\mathrm{sgn}(k_p) =$

$$\dot{V} = -e^T Q e + 2 e^T \frac{\mathrm{sgn}(k^*)}{|k^*|} \phi\, \phi_m \bar{\phi}^T \bar{w} + 2 \bar{\phi}^T \Gamma^{-1} \left\{ -\mathrm{sgn}(k_p) \Gamma \bar{w} e_1 \right\}$$

$$\dot{V} = -e^T Q e + 2 \qquad \mathrm{sgn}(k^*) \, \bar{\phi}^T \bar{w} + 2 \bar{\phi}^T \bar{w} e_1 \left\{ -\mathrm{sgn}(k_p) \right\}$$

⋮

leading to $\|\tilde{w}\|$ and $\|\tilde{\theta}\|$

bounded for all $t \geq 0$,

and $\lim_{t \to \infty} e_1(t) = 0$

$\Delta\Delta\Delta$