National University of Singapore
Faculty of Engineering

ME5402/EE5016R Advanced Robotics

Exercise 1

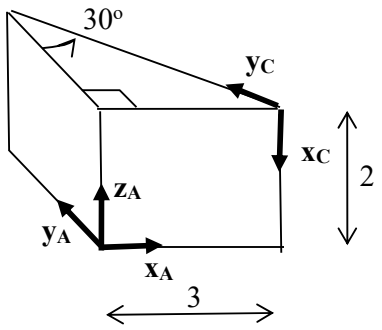
1. Define kinematics, workspace, and trajectory.

2. Define 'frame' and "degrees of freedom".

3. A vector ${}^{A}\mathbf{P}$ is rotated about $\mathbf{z_A}$ by θ degrees and is subsequently rotated about $\mathbf{x_A}$ by ϕ degrees. Give the rotation matrix that accomplishes these rotations in the given order.

4. A frame {B} is located initially coincident with a frame {A}. We rotate {B} about $\mathbf{z_B}$ by θ degrees, and then we rotate the resulting frame about $\mathbf{x_B}$ by ϕ degrees. Give the rotation matrix that will change the descriptions of vectors from ${}^{B}\mathbf{P}$ to ${}^{A}\mathbf{P}$.

5. Referring to the following figure, determine the homogeneous transformation matrix that describes frame {C} in frame {A}. Also determine the homogeneous transformation matrix that describes frame {A} in frame {C}.

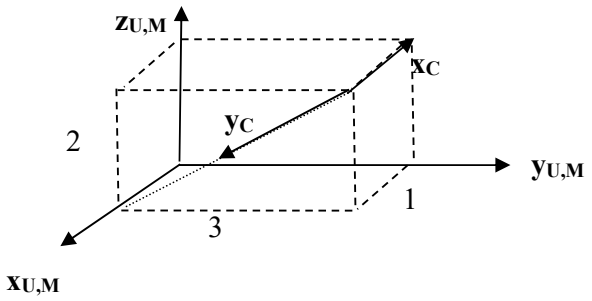


6. Frames {M} and {C} are attached rigidly to a cuboid as shown in the following figure. Frame U is fixed and serves as the universal frame of reference. The cube undergoes the following motions in the indicated sequence:

1) Rotation about the z axis of frame {C} by 30^o , then
2) Translation of (1,2,3) along frame {C}, then
3) Rotation about the x axis of frame {M} by 45^o, and then
4) Rotation about the y axis of frame {U} by 60^o.

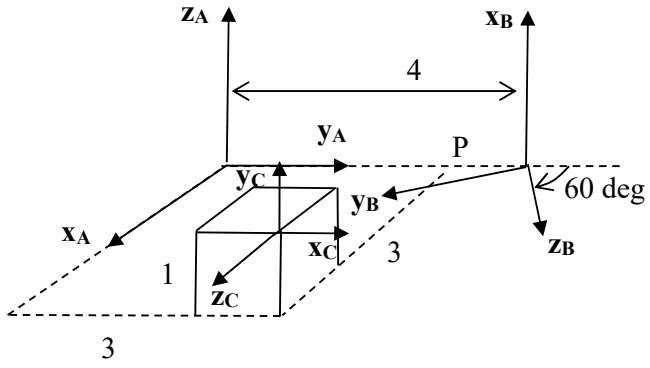
Let ${}^{U}_{C_i}T$ and ${}^{U}_{M_i}T$ be the 4x4 homogeneous transformation matrices that describe the position and orientation of frame C and M, respectively, in U after motion *i*.

Find ${}^{U}_{C_1}T$, ${}^{U}_{C_2}T$, ${}^{U}_{C_3}T$, ${}^{U}_{C_4}T$, ${}^{U}_{M_4}T$.



7. Frame {C} is firmly attached to a corner of the rigid cube with $\mathbf{z_c}$ parallel to $\mathbf{x_A}$ and $\mathbf{y_C}$ parallel to $\mathbf{z_A}$, as shown in the figure below. Frame {B} is located at a fixed position and orientation with respect to Frame {A} with $\mathbf{x_B}$ parallel to $\mathbf{z_A}$ and the angle 60 degrees represents a rotation about $\mathbf{x_B}$. The following ordered sequence of motions is applied to the cube:
   I) rotation about $\mathbf{y_B}$ by 45 degrees, followed by
   II) rotation about $\mathbf{x_C}$ by 30 degrees.

   Find the new position and orientation of Frame {C} expressed in Frame {A}.



8. Frame {B} is initially coincident to frame {A} in the following figure. Frame {B} is then rotated 30 degrees about the vector described by the directed line segment from P to Q (following the right-hand rule). Determine the position and orientation of the new frame {B} with respect to frame {A}. Express your answer in the form of a homogeneous transformation matrix.