ORIGINAL

NATIONAL UNIVERSITY OF SINGAPORE

EXAMINATION FOR

(Semester I: 2018/2019)

EE6104 – ADAPTIVE CONTROL SYSTEMS (ADVANCED)

November/December 2018 – Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES:

1. This question paper contains **FOUR (4)** questions and comprises **THIRTEEN (13)** printed pages.
2. Answer all **FOUR (4)** questions.
3. This is a CLOSED BOOK examination. However, each student may bring ONE (1) A4 size crib sheet into the examination hall.
4. Note carefully that the questions **do not** carry equal marks.
5. Relevant data are provided at the end of this examination paper.
6. Total Marks: 100

Q.1 In developing an adaptive controller when only the input and output signals are measurable, a development involving the polynomials

$$\begin{aligned} R_m(s) &= s^2 + a_{1m}s + a_{2m} \\ T(s) &= s^2 + t_1 s + t_2 \\ R_p(s) &= s^2 + a_1 s + a_2 \end{aligned}$$

is essential. Thus here, for the above polynomials, calculate exactly the coefficients of the resulting polynomials $E(s)$ and $F(s)$ in the polynomial identity

$$R_m(s)T(s) = R_p(s)E(s) + F(s)$$

Develop therefore, fully and carefully, how this result above would be generalized if, instead, $R_m(s)$ is of order $n^* \leq n$, and both $T(s)$ and $R_p(s)$ are of order n.

(12 marks)

Q.2 It is desired to use the d.c. motor system, shown in Figure 1, to be the basis of the overall positioning mechanism in a particular experimental position control servomechanism.

The d.c. motor system has the nominal dynamic model as shown in Figure 2, with the transfer function:

$$\frac{\Theta(s)}{U(s)} = \frac{K}{s(1 + s\tau)}$$

where $\Theta(s)$ is the Laplace transform of the angular position signal $\theta(t)$ and $U(s)$ is the Laplace transform of the motor drive input voltage $u(t)$. Calibration tests on the d.c. motor system, using the LabView real-time system connections of Figure 3, has yielded the data listed in Tables 1 and 2.

However, for Table 2, it is also known that the steady-state relationship between the motor drive input voltage $u(t)$ and the tachogenerator output voltage (while constant for each operation) can change in different day-to-day operations, and thus cannot be regarded as being known accurately. Further, simple step-response tests (which cannot be used as accurate calibration data) on the angular velocity has also indicated that

$$\tau \approx 220 \quad \text{milliseconds}$$

for the d.c. motor system, and that a positive-valued drive input voltage $u(t)$ results in a positive-valued angular velocity $\dot{\theta}(t)$.

For this hardware set-up of the position control servomechanism described above, it is next noted that a **different** situation has arisen where only the measurements of the input $u(t)$ and angular position output $\theta(t)$ are available.

Develop therefore, fully and carefully, a structure for the **Control Law** which will allow for globally uniformly stable adaptive control ultilizing the **Reference Model**

$$\frac{\Theta_m(s)}{R(s)} = \frac{1}{s^2 + 2s + 1}$$

where likewise, the reference input $r(t)$ is an angular position reference/command signal where step changes are made in its value, to various different constant values, at intervals of 45 seconds or more. Include all relevant equations and detailed descriptions in developing the **Control Law**.

N.B.: Note particularly here that you are **only** required to develop fully and carefully the necessary structure for the **Control Law**. As already noted in class, with this appropriately developed structure, the necessary adaptive laws (even though rather complicated) are already available to ensure globally uniformly stable adaptive control. You are not required, in this case, to discuss the adaptive laws at all.

Hint: This is essentially the situation of developing the **Control Law** where only the measurements of the input $u(t)$ and angular position output $\theta(t)$ are available.

(11 marks)

For the situation as described above, it should happen that the development of the **Control Law** should require the user-choice of a stable filter polynomial typically denoted as $T(s)$.

Here, carefully describe your user-choice of this $T(s)$ filter polynomial for the situation as described in Question 1, explaining carefully all your numerical considerations. Include all relevant equations and detailed descriptions.

Further, show (carefully and in detail) that the signals that you generate using suitable sets of this stable filter polynomial $T(s)$ actually constitute the signals of a state-vector in a non-minimal realization of

$$\frac{\Theta(s)}{U(s)} = \frac{K}{s(1+s\tau)}$$

Include all relevant equations and detailed descriptions.

(12 marks)

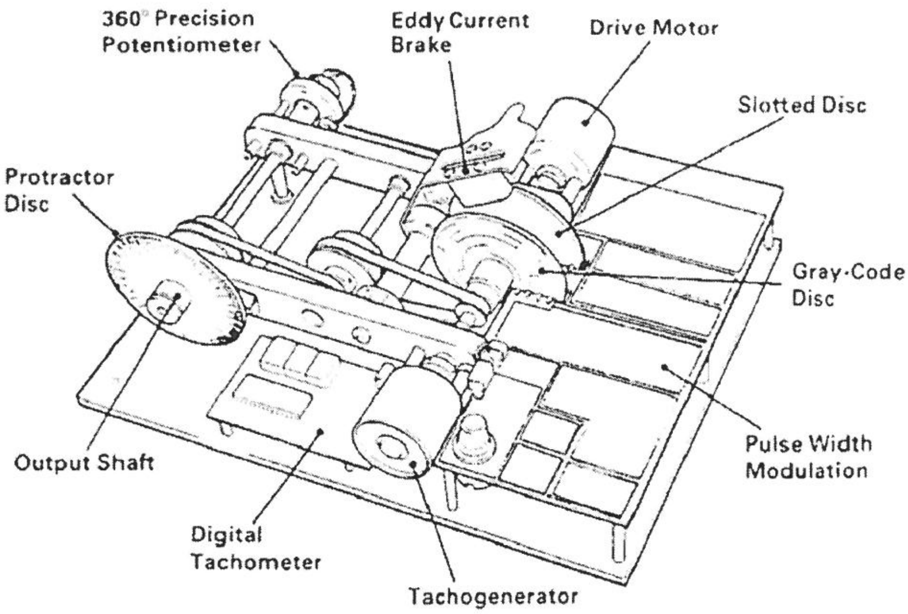

Figure 1

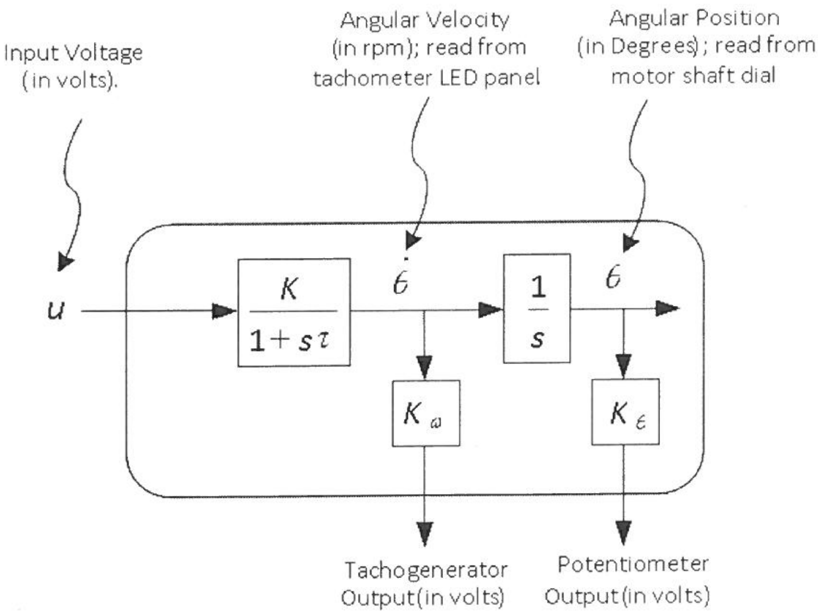

Figure 2

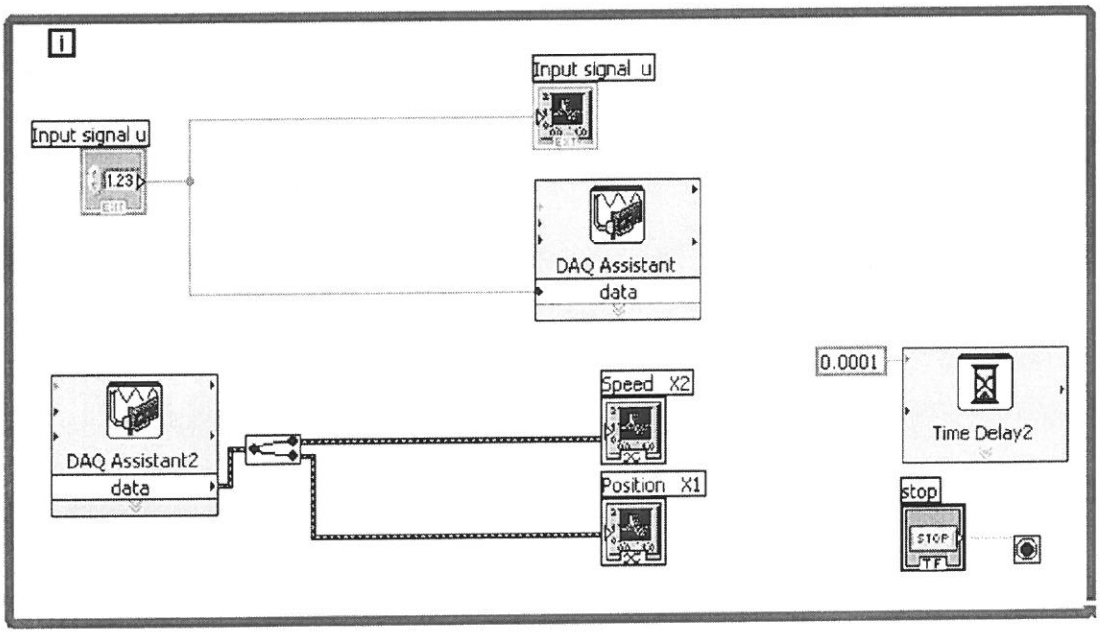
Input signal u
Input signal u
1.23
DAQ Assistant
data
0.0001
Time Delay2
DAQ Assistant2
data
Speed X2
Position X1
stop
STOP
TF

Figure 3

Calibration Results for Part 1

Potientiometer Output (in volts)	Angular Position (in degrees)
-5	-180
-4	-144
-3	-108
-2	-72
-1	-36
0	0
1	36
2	72
3	108
4	144
5	180

Table 1 shows the results for the calibration of the potientiometer

Table 1

Calibration Results for Part 1

Input Voltage (volts)	Tachogenerator Output (volts)	Angular Velocity (rpm)	Angular Velocity (rad/sec)
-5	-4.03	-301	-31.52
-4	-3.17	-237	-24.82
-3	-2.3	-172	-18.01
-2	-1.45	-108	-11.31
-1	-0.6	-45	-4.71
0	0	0	0
1	0.62	48	5.03
2	1.48	111	11.62
3	2.33	175	18.33
4	3.2	239	25.03
5	4.06	303	31.73

Table 2 shows the results for the calibration of the tachogenerator

Table 2

Q.3 Consider the process

$$
\begin{aligned}
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} &= \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} u \\
y &= \begin{bmatrix} 0 & 1 \end{bmatrix}
\end{aligned}
$$

The initial conditions are $x_1(0) = 0$, $x_2(0) = 1$ and the sliding line is given as $\sigma = x_1 + x_2 = 0$

a) Design a sliding mode controller such that it takes $t_\sigma = 1$ for the states to reach the sliding line.

(7 marks)

b) For $0 \leq t < t_\sigma$, find $x_1(t)$ and $x_2(t)$.

(8 marks)

c) For $t \geq t_\sigma$, find $x_1(t)$ and $x_2(t)$.

(8 marks)

d) Sketch $u(t)$ for $0 \leq t \leq 2$.

(12 marks)

Q.4 Consider the single-input single-output process

$$
A(q^{-1})y(k) = B(q^{-1})u(k) + C(q^{-1})e(k)
$$

where

$$
\begin{aligned}
A(q^{-1}) &= 1 + aq^{-1} \\
B(q^{-1}) &= bq^{-1} \\
C(q^{-1}) &= 1 + cq^{-1}
\end{aligned}
$$

and $k = 0, 1, \ldots, N$. The input, output and Gaussian independent random variable with standard deviation σ are given by $u(k)$, $y(k)$ and $e(k)$ respectively. In state-space form

$$
\begin{aligned}
x(k+1) &= -cx(k) + bu(k) + (c-a)y(k) \\
y(k) &= x(k) + e(k)
\end{aligned}
$$

For simplicity, let $u(k) = 0$.

Question 4 continues next page.

a) Express $x(1)$ and $x(2)$ in terms of $x(0)$

(6 marks)

b) Write the vector $E = \begin{bmatrix} e(0) & e(1) & e(2) \end{bmatrix}^T$ in the following form.

$$Z = \Phi x(0) + E$$

where Z and Φ are column vectors. Give the elements in Z and Φ.

(6 marks)

c) Using batch least-squares, find $\hat{x}(0)$ that minimizes the least-squares objective function.

$$J = \frac{1}{2} E^T E$$

(6 marks)

d) Given $a = c = -1$, $y(0) = 0$, $y(1) = 1$, $y(2) = 2$, and using the recursive least-squares algorithm, find $\hat{x}(0)$ at every iteration from $k = 0$ to $k = 2$. Initialize $\hat{x}(0) = 0$ and covariance $P = 100$.

(6 marks)

e) Given $a = c = -1$, $y(0) = 0$, $y(1) = 1$, find the least-squares estimate $\hat{x}(1)$.

(6 marks)

– End of Questions –

DATA SHEET:

0. Prototype Response Tables

	k	Pole Locations for $\omega_0 = 1\ rad/s$[a]
ITAE	1	$s+1$
	2	$s + 0.7071 \pm 0.7071j$[b]
	3	$(s+0.7081)(s+0.5210 \pm 1.068j)$
	4	$(s+0.4240 \pm 1.2630j)(s+0.6260 \pm 0.4141j)$
	5	$(s+0.8955)(s+0.3764 \pm 1.2920j)(s+0.5758 \pm 0.5339j)$
Bessel	1	$s+1$
	2	$s + 0.8660 \pm 0.5000j$[b]
	3	$(s+0.9420)(s+0.7455 \pm 0.7112j)$
	4	$(s+0.6573 \pm 0.8302j)(s+0.9047 \pm 0.2711j)$
	5	$(s+0.9264)(s+0.5906 \pm 0.9072j)(s+0.8516 \pm 0.4427j)$

[a] Pole locations for other values of ω_0 can be obtained by substituting s/ω_0 for s.
[b] The factors $(s+a+bj)(s+a-bj)$ are written as $(s+a \pm bj)$ to conserve space.

1. The Lyapunov Equation states that given any $n \times n$ stability matrix A_m, for every symmetric positive definite matrix Q, there exists a unique symmetric positive definite matrix P that is the solution to the equation

$$A_m^\top P + PA_m = -Q.$$

In addition, the error system dynamics (with $\mathbf{e} \in \mathbf{R}^n$ and Γ an $n \times n$ symmetric positive-definite matrix) given by

$$\begin{aligned} \dot{\mathbf{e}}(t) &= A_m \mathbf{e}(t) + g\mathbf{b}\phi(t)^\top \mathbf{x}(t) \\ \dot{\phi}(t) &= -\mathrm{sgn}(g)\Gamma \mathrm{e}(t)^\top P\mathbf{b}\,\mathbf{x}(t) \end{aligned}$$

has the properties that $\|\mathbf{e}(t)\|$ and $\|\phi(t)\|$ are bounded, and if it should also be known that $\|\mathbf{x}(t)\|$ is bounded, then additionally

$$\lim_{t\to\infty} \mathbf{e}(t) = 0$$

2. For the triple

$$\begin{aligned} A_m &= \begin{bmatrix} 0 & 1 & 0 \\ a_1 & a_2 & a_3 \\ 1 & 0 & 0 \end{bmatrix} \\ b_m &= \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \\ c_m &= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \end{aligned}$$

the equivalent transfer function is

$$c_m^\top[sI - A_m]^{-1}b_m = \frac{-a_3}{s^3 - a_2s^2 - a_1s - a_3}$$

3. For

$$A_m = \begin{bmatrix} 0 & 1 & 0 \\ -21 & -12 & -10 \\ 1 & 0 & 0 \end{bmatrix}$$

and

$$Q = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

the solution to

$$A_m^\top P + PA_m = -Q.$$

is

$$P = \begin{bmatrix} 9.30 & 0.38 & 5.40 \\ 0.38 & 0.24 & 0.25 \\ 5.40 & 0.25 & 9.01 \end{bmatrix}$$

and the eigenvalues of P are $\lambda = 14.57, 3.76, 0.22$.

4. For

$$A_m = \begin{bmatrix} 0 & 1 & 0 \\ -11 & -7 & -5 \\ 1 & 0 & 0 \end{bmatrix}$$

and

$$Q = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

the solution to

$$A_m^\top P + PA_m = -Q.$$

is

$$P = \begin{bmatrix} 9.92 & 0.76 & 5.83 \\ 0.76 & 0.47 & 0.50 \\ 5.83 & 0.50 & 9.28 \end{bmatrix}$$

and the eigenvalues of P are $\lambda = 15.49, 3.77, 0.40$.

5. For

$$A_m = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}$$

and

$$Q = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

the solution to

$$A_m^\top P + PA_m = -Q.$$

is

$$P = \begin{bmatrix} 7.50 & 2.50 \\ 2.50 & 2.50 \end{bmatrix}$$

and the eigenvalues of P are $\lambda = 8.54, 1.46$.

6. For

$$A_m = \begin{bmatrix} 0 & 1 \\ -2 & -2 \end{bmatrix}$$

and

$$Q = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

the solution to

$$A_m^\top P + PA_m = -Q.$$

is

$$P = \begin{bmatrix} 6.25 & 1.25 \\ 1.25 & 1.875 \end{bmatrix}$$

and the eigenvalues of P are $\lambda = 6.58, 1.54$.

7. For

$$A_m = \begin{bmatrix} 0 & 1 & 0 \\ -3,600 & -120 & -32,000 \\ 1 & 0 & 0 \end{bmatrix}$$

and

$$Q = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

the solution to

$$A_m^\top P + PA_m = -Q.$$

is

$$P = \begin{bmatrix} 105.1 & 0.2 & 720.2 \\ 0.2 & 0.0225 & 0.0 \\ 720.2 & 0.0 & 6,424.4 \end{bmatrix}$$

and the eigenvalues of P are $\lambda = 6,505.4; 24.1; 0.021$.

8. For

$$A_m = \begin{bmatrix} 0 & 1 \\ -400 & -40 \end{bmatrix}$$

and

$$Q = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

the solution to

$$A_m^\top P + PA_m = -Q.$$

is

$$P = \begin{bmatrix} 25.31 & 0.0063 \\ 0.0063 & 0.0627 \end{bmatrix}$$

and the eigenvalues of P are $\lambda = 25.31, 0.0627$.

6. For

$$A_m = \begin{bmatrix} 0 & 1 \\ -400 & -20 \end{bmatrix}$$

and

$$Q = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

the solution to

$$A_m^\top P + PA_m = -Q.$$

is

$$P = \begin{bmatrix} 50.25 & 0.006 \\ 0.006 & 0.125 \end{bmatrix}$$

and the eigenvalues of P are $\lambda = 50.25, 0.125$.

7. The standard discrete-time gradient estimator is

$$\begin{aligned} \hat{y}(j) &= \hat{\theta}(j)^\top \omega(j) \\ e_1(j) &= \hat{y}(j) - y(j) \\ \hat{\theta}(j+1) &= \hat{\theta}(j) - \frac{\omega(j)e_1(j)}{1 + \|\omega(j)\|^2} \end{aligned}$$

It is applicable to the process

$$y(j) = \theta^{*\top} \omega(j)$$

Laplace Transform Table

Laplace Transform, F(s)	Time Function, f(t)
1	$\delta(t)$ (unit impulse)
$\frac{1}{s}$	$1(t)$ (unit step)
$\frac{1}{s^2}$	t
$\frac{1}{s^n}$	$\frac{t^{n-1}}{(n-1)!}$ (n = positive integer)
$\frac{1}{s+a}$	e^{-at}
$\frac{1}{(s+a)^2}$	te^{-at}
$\frac{1}{(s+a)^n}$	$\frac{1}{(n-1)!}t^{n-1}e^{-at}$ (n = positive integer)
$\frac{1}{(s+a)(s+b)}$	$\frac{e^{-at}-e^{-bt}}{b-a}$
$\frac{1}{s(s+a)(s+b)}$	$\frac{1}{ab}\left[1+\frac{1}{a-b}(be^{-at}-ae^{-bt})\right]$
$\frac{\omega}{s^2+\omega^2}$	$\sin\omega t$
$\frac{s}{s^2+\omega^2}$	$\cos\omega t$
$\frac{\omega}{(s+a)^2+\omega^2}$	$e^{-at}\sin\omega t$
$\frac{s+a}{(s+a)^2+\omega^2}$	$e^{-at}\cos\omega t$
$\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\frac{\omega_n}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin\omega_n\sqrt{1-\zeta^2}t$
$\frac{s}{s^2+2\zeta\omega_n s+\omega_n^2}$	$-\frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}t-\phi)$ $\phi=\tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}$
$\frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$	$1-\frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}t+\phi)$ $\phi=\tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}$

– End of Paper –