Note Title 8/12/2011

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Consider the s.v. system

$$\dot{x} = f(x, t) \,; \qquad x(t_0) = x_0$$

Find some

$V(x, t)$, and check for

(i) positive-definiteness

(ii) decrescence

(iii) $\dot{V}(x, t) \leq 0$

(iv) $V(x, t)$ radially unbounded

Go back and look at what we have already done:

$$\dot{e}_1 = a_m e_1 + k_p \left\{ \phi(t) y_p(t) + \psi(t) r(t) \right\}$$

$$\dot{\phi} = -\mathrm{sgn}(k_p)\, \gamma_1\, e_1(t)\, y_p(t)$$

$$\dot{\psi} = -\mathrm{sgn}(k_p)\, \gamma_2\, e_1(t)\, r(t)$$

$$\dot{x} = f(x, t)$$

Also, note that we used:

$$V(e_1, \phi, \psi) = \frac{1}{2}\begin{bmatrix} e_1 & \phi & \psi \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{|k_\phi|}{\gamma_1} & 0 \\ 0 & 0 & \frac{|k_\phi|}{\gamma_2} \end{bmatrix} \begin{bmatrix} e_1 \\ \phi \\ \psi \end{bmatrix}$$

$$x = \begin{bmatrix} e \\ \phi \\ \psi \end{bmatrix}$$

$$= \frac{1}{2}\left\{ e_1^2 + \frac{|k_\phi|}{\gamma_1}\phi^2 + \frac{|k_\phi|}{\gamma_2}\psi^2 \right\}$$

(i) Choose

$$\alpha(\|x\|) = \min\left\{1, \frac{|k_\phi|}{\gamma_1}, \frac{|k_\phi|}{\gamma_2}\right\} \|x\|^2 \frac{1}{2}$$

$$V(x,t) \geq \alpha(\|x\|)$$

(ii) Choose

$$\beta(\|x\|) = \max\left\{ \quad \right\} \|x\|^2 \frac{1}{2}$$

$$V(x,t) \le \beta(\|x\|)$$

(iii) From "Control Law", "Adaptive Law" etc ...

$$\dot{V}(x,t) = a_m e_1^2 \le 0$$

$\because a_m < 0$

(iv) ~~Clearly~~ choice of $\alpha(\|x\|)$ above give

$$\alpha(\|x\|) = \min\left\{ \quad \right\} \|x\|^2 \frac{1}{2}$$

$\|x\| \to \infty$, then $\alpha(\|x\|) \to \infty$

Adaptive Control for a Class of Systems with measurable State Variables

scalar

$$\dot{x}_p = A_p x_p + g b u$$

$b \in \mathbb{R}^n$ is known

measurable, $x_p \in \mathbb{R}^n$

$$\text{——— (2.1)}$$

We wish to achieve some suitably specified closed-loop

$$\boxed{\dot{x}_m = A_m x_m + g_m b r} \qquad \text{——— (2.2)}$$

Clearly, a suitable approach would be to consider the state-feedback

$$u(t) = \theta_x^T x_p + \theta_r r \qquad (2.3)$$

For the case where the system (2.1) is known, we then have:

$$\dot{x}_p = A_p x_p + g b \left\{ \theta_x^T x_p + \theta_r r \right\}$$

$$= \left\{ A_p + g b \theta_x^T \right\} x_p + g \theta_r b r$$

Calculate $\theta_x = \theta_x^*$ so that this $\{ \quad \} = A_m$!!

Calculate $\theta_r = \theta_r^*$ so that

$$g\theta_r^* = g_m$$

$$\boxed{\text{"Matching Conditions"}} \quad \text{— (2.4)}$$

But we do not know the system (2.1) !!

How about considering:

$$u(t) = \theta_x^T(t)\, x_p(t) + \theta_r(t)\, r(t) \quad \text{— (2.5)} \quad \boxed{\text{Control Law}}$$

Since we are using time-varying gains, how are we specifying the time-variation?

Consider them:

$$\dot{\theta}_x(t) = -\operatorname{sgn}(g)\,\Gamma\, e(t)^T P b\, x_p(t)$$

$$\dot{\theta}_r(t) = -\operatorname{sgn}(g)\,\gamma\, e^T(t) P b\, r(t)$$

— (2.6) $\boxed{\text{Adaptive Law}}$

where

$$e(t) \triangleq x_p(t) - x_m(t)$$

Γ is any $(n \times n)$ positive-def matrix

γ is any scalar $\gamma > 0$

and for a stable A_m matrix,
we have the matrix P, which is
the solution to : (2.7)

$$A_m^T P + P A_m = -Q$$

[Lyapunov Eqn]

where Q is <u>any</u> $(n \times n)$ symmetric positive-def
matrix.

The solution P always exist,
and P is also symm positive-def.

How does this work?

First, note that the "Control Law" (2.5) results in

$$\dot{x}_p = A_p x_p + g b \left\{ \theta_x^T(t)\, x_p(t) + \theta_r(t)\, r(t) \right\}$$

$$\theta_x^* + \phi(t) \qquad\qquad \theta_r^* + \phi_r(t)$$

i.e.

$$\dot{x}_p(t) = \underbrace{\left\{ A_p + g b\, \theta_x^{*T} \right\}}_{A_m} x_p + \underbrace{g\, \theta_r^*}_{g_m} b\, r(t)$$

$$+ \, g b \left\{ \phi_x^T(t)\, x_p(t) + \phi_r(t)\, r(t) \right\}$$

Thus, if we consider

$$e(t) \triangleq x_p(t) - x_m(t)$$

$$\dot{e}(t) = \dot{x}_p(t) - \dot{x}_m(t)$$

$$= A_m e(t) + g b \left\{ \phi_x^T x_p + \phi_r r \right\} \quad \text{— (2.11a)}$$

Error Signal Dynamics / Error Model

Note also that we have additionally:

$$\dot{\theta}_x = \dot{\phi}_x = -\text{sgn}(g) \Gamma e^T P b\, x_p \quad \text{— (2.11b)}$$

$$\dot{\theta}_r = \dot{\phi}_r = -\text{sgn}(g) \gamma\, e^T P b\, r \quad \text{— (2.11c)}$$

Also, define $\phi = \begin{bmatrix} \phi_x \\ \phi_r \end{bmatrix}$

Here, consider the quadratic form

$$V(e, \phi_{x_0}, \phi_r) = \frac{1}{2} e^T P e + \frac{1}{2} |g| \phi^T \bar{\Gamma}^{-1} \phi$$

- positive-def
- decrescent
- radially unbounded

$$\begin{bmatrix} \Gamma & 0 \\ 0 & \gamma \end{bmatrix} = \bar{\Gamma}$$

Note then that we have:

$$\dot{V}(t) = \frac{1}{2}\left\{ 2 e^T P \dot{e} \right\} + \frac{1}{2}\left\{ 2 |g| \phi^T \bar{\Gamma}^{-1} \dot{\phi} \right\}$$

$$= e^T p \left\{ A_m e + q\,\delta\phi^T \bar{x} \right\}$$

$$\bar{x} = \begin{bmatrix} x_p \\ r \end{bmatrix}$$

$$+$$

From Linear Algebra, any matrix $(n \times n)$ M can be written as:

$$M = \underbrace{\frac{1}{2}\left\{ M + M^T \right\}}_{\text{symmetric}} + \underbrace{\frac{1}{2}\left\{ M - M^T \right\}}_{\text{anti-symmetric}}$$

$$M_s = M_s^T \qquad M_{as} = -M_{as}^T$$

and note further that for any quadratic form:

$$x^T M x = \ldots = x^T M_s x$$

$\Delta\Delta\Delta$

Thus, this means that

$$e^T \{ P A_m \} e$$

$$= e^T \left\{ \frac{1}{2} \left[P A_m + (P A_m)^T \right] \right\} e$$

$$= \frac{1}{2} e^T \{ \underbrace{P A_m + A_m^T P}_{= -Q} \} e \quad !!$$

with Q sym positive-def

from Eqn (2.7).

i.e.

$$\dot{V}(t) = -\frac{1}{2} e^T Q e + e^T P g b \phi^T \bar{x}$$

$$+ |g| \phi^T \Gamma^{-1} \dot{\phi}$$

$$- \operatorname{sgn}(g) \, \Gamma e^T P b \underbrace{\begin{bmatrix} x_p \\ r \end{bmatrix}}_{\bar{x}}$$

ii) for the choice of our "Adaptive Law" (2.6), results in

$$\dot{V}(t) = -\frac{1}{2} e^T Q e$$

(a) V sym positive definite

$\dot{V} = -\frac{1}{2} e^T Q e \leq 0$

$\Rightarrow \|e(t)\|, \|\Phi_x(t)\|, \|\Phi_r(t)\|$

are bounded for all

$t \geq t_0$

$\vdots$

(b) $\int_{t_0}^{t} e^T Q e \, d\tau \leq c_1$

for all $t \geq t_0$

i.e. $\|e\|$ is 'square-integrable'

(d) $\dot{e} = A_m e + gb\left\{\phi_x^T x_p + \phi_r r\right\}$

$\|\dot{e}\|$ is bounded for all $t \geq t_0$

I.e. (b) & (c) results in

$$\lim_{t\to\infty} \|e(t)\| = \|x_p(t) - x_m(t)\| \longrightarrow 0$$

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