Tutorial Question on Self-Oscillating Adaptive System

Q.3 Consider Figure Q3. The process and relay are given as

$$G_p(s) = \frac{1}{s}e^{-s}$$

$$u_2 = \begin{cases} 0.1 & \text{if } u_1 > 0 \\ -0.1 & \text{if } u_1 < 0 \end{cases}$$

(a) What is the amplitude and frequency of the oscillation in y if

$$G_c(s) = 1 \quad ?$$

(b) Design a self-oscillating adaptive system to give a gain margin of 2 and oscillation amplitude of 0.04 in y.

(c) For the system you designed in part (b), what are the amplitude and frequency of the oscillation in y if the process is changed to

$$G_p(s) = \frac{3}{s}e^{-0.5s} \quad ?$$

(d) What is the gain margin of the system in part (c) ?

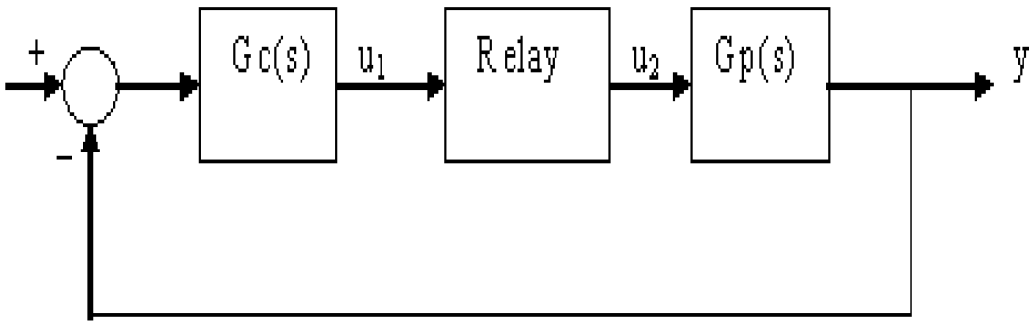

Figure Q3

Tutorial Solution on Self-Oscillating Adaptive System

Q3a

$$\begin{aligned} \angle G_p(j\omega) &= -\frac{\pi}{2} - \omega = -\pi \\ \omega &= \frac{\pi}{2} \\ d\left|G_j(j\frac{\pi}{2})\right| &= c \\ 0.1\left|\frac{1}{j\frac{\pi}{2}}\right| &= \frac{0.2}{\pi} \end{aligned}$$

Q3b
Calculate the frequency of oscillation to satisfy the amplitude of oscillation

$$\begin{aligned} 0.1\left|\frac{1}{j\omega}e^{-j\omega L}\right| &= \frac{0.1}{\omega} = 0.04 \\ \omega &= 2.5 \end{aligned}$$

Choose a lead-compensator

$$G_c(s) = K_c\frac{s+T_1}{s+T_2}$$

to satisfy the phase

$$\arctan\frac{\omega}{T_1} - \arctan\frac{\omega}{T_2} - \frac{\pi}{2} - \omega = -\pi$$

Choose

$$\begin{aligned} T_1 &= 1 \\ T_2 &= 10 \end{aligned}$$

Calculate K_c such that

$$\begin{aligned} K_c\frac{\sqrt{\omega^2+T_1^2}}{\sqrt{\omega^2+T_2^2}} &= K_c\frac{\sqrt{2.5^2+1^2}}{\sqrt{2.5^2+10^2}} = 1 \\ K_c &= 3.8 \end{aligned}$$

Q3c
When the plant change

$$\begin{aligned} \arctan\frac{\omega}{1} - \arctan\frac{\omega}{10} - \frac{\pi}{2} - 0.5\omega &= -\pi \\ \omega &= 5 \\ 0.1\left|\frac{3}{j\omega}e^{-j0.5\omega}\right| &= \frac{0.3}{5} = 0.06 \end{aligned}$$

Q3d
The gain margin is 2.