ORIGINAL

NATIONAL UNIVERSITY OF SINGAPORE

EXAMINATION FOR

(Semester II: 2021/2022)

EE5104 – ADAPTIVE CONTROL SYSTEMS

April/May 2022 – Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES:

1. This question paper contains **FOUR (4)** questions and comprises **THIRTEEN (13)** printed pages.
2. Answer all **FOUR (4)** questions.
3. This is a OPEN BOOK examination.
4. Note carefully that the questions **do not** carry equal marks.
5. Relevant data are provided at the end of this examination paper.
6. Total Marks: 100

Q.1 In a particular hardware set-up, it is desired to use the d.c. motor system (shown in Figure 1) to be the basis of various control system design experiments.

The d.c. motor system has the nominal dynamic model as shown in Figure 2, with the transfer function:

$$\frac{\Theta(s)}{U(s)} = \frac{K}{s(1 + s\tau)}$$

where $\Theta(s)$ is the Laplace transform of the angular position signal $\theta(t)$ and $U(s)$ is the Laplace transform of the motor drive input voltage $u(t)$. Calibration tests on the d.c. motor system, using the LabView real-time system connections of Figure 3, has yielded the data listed in Tables 1 and 2.

However, for Table 2, it is also known that the steady-state relationship between the motor drive input voltage $u(t)$ and the tachogenerator output voltage (while constant for each operation) can change in different day-to-day operations, and thus <u>cannot</u> be regarded as being known accurately. Further, simple step-response tests (which <u>cannot</u> be used as accurate calibration data) on the angular velocity has also indicated that

$$\tau \approx 190 \quad \text{milliseconds}$$

for the d.c. motor system, and that a positive-valued drive input voltage $u(t)$ results in a positive-valued angular velocity $\dot{\theta}(t)$.

For this hardware set-up of the position control servomechanism described above, it is noted that a **<u>specific</u>** situation has arisen where only the measurements of the input $u(t)$ and angular position output $\theta(t)$ are available.

Develop therefore, fully and carefully, a structure for the **<u>Control Law</u>** which will allow for globally uniformly stable adaptive control ultilizing the **<u>Reference Model</u>**

$$\frac{\Theta_m(s)}{R(s)} = \frac{1}{s^2 + 2s + 1}$$

where typically, the reference input $r(t)$ is an angular position reference/command signal where step changes are made in its value, to various different constant values, at intervals of 45 seconds or more. Include all relevant equations and detailed descriptions in developing the **<u>Control Law</u>**.

N.B.: Note particularly here that you are **<u>only</u>** required to develop fully and carefully the necessary structure for the **<u>Control Law</u>**. As already noted in class, with this appropriately developed structure, the necessary adaptive laws (even though rather complicated) are already available to ensure globally uniformly stable adaptive control. You are not required, in this case, to discuss the adaptive laws at all.

Hint: This is essentially the situation of developing the **<u>Control Law</u>** where only the measurements of the input $u(t)$ and angular position output $\theta(t)$ are available.

(15 marks)

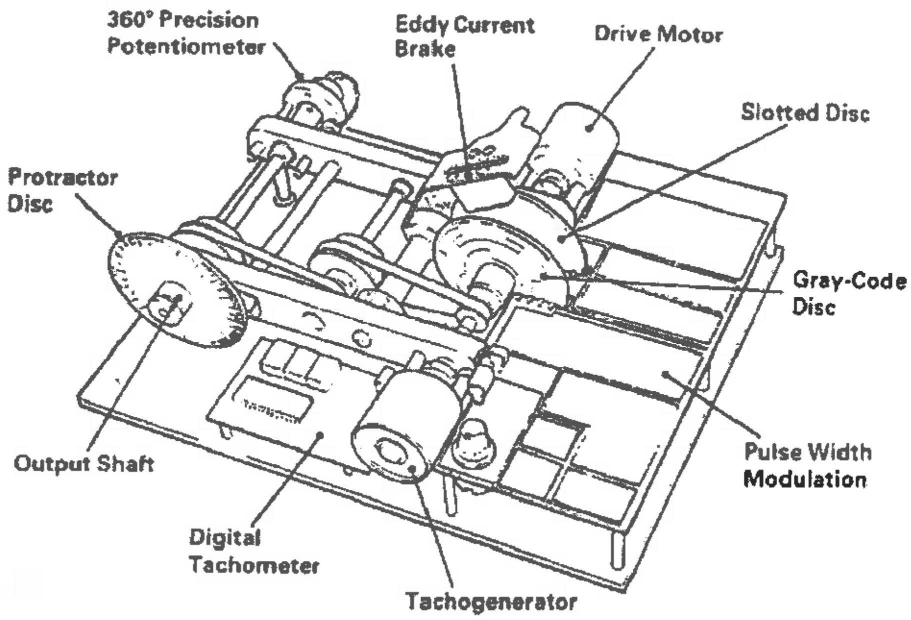

Figure 1

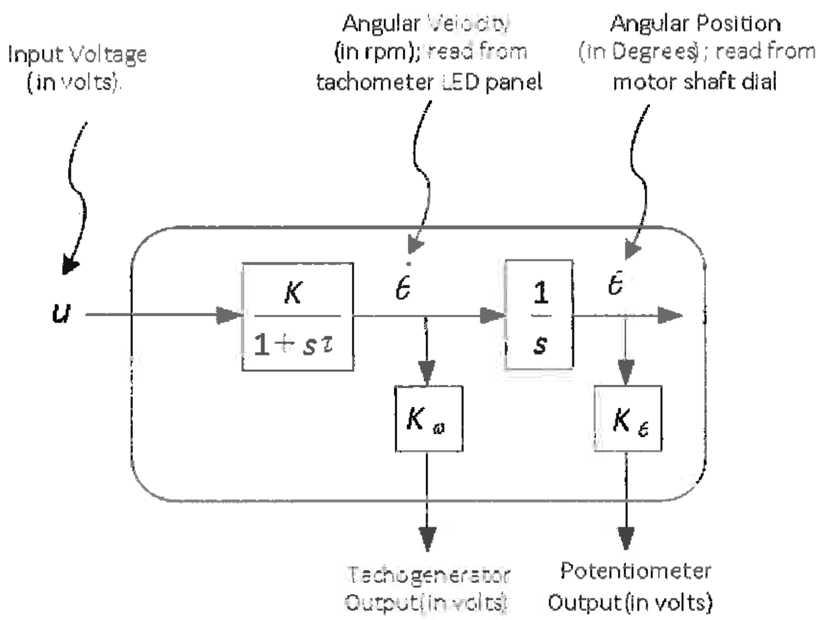

Figure 2

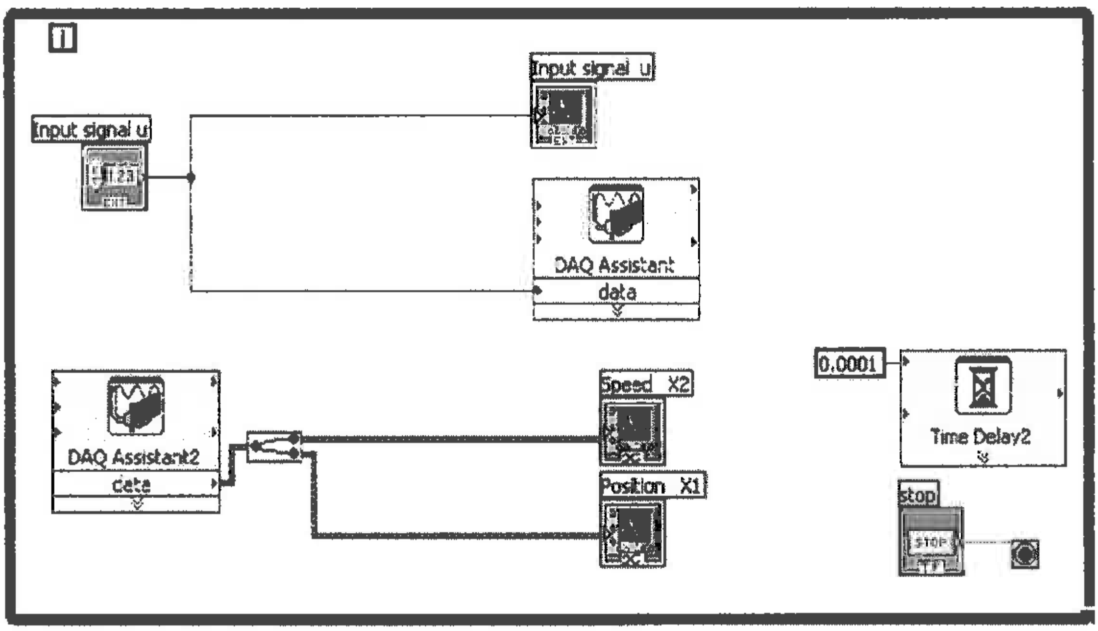

Figure 3

Calibration Results for Part 1

Potientiometer Output (in volts)	Angular Position (in degrees)
-5	-180
-4	-144
-3	-108
-2	-72
-1	-36
0	0
1	36
2	72
3	108
4	144
5	180

Table 1 shows the results for the calibration of the potientiometer

Table 1

Calibration Results for Part 1

Input Voltage (volts)	Tachogenerator Output (volts)	Angular Velocity (rpm)	Angular Velocity (rad/sec)
-5	-4.03	-301	-31.52
-4	-3.17	-237	-24.82
-3	-2.3	-172	-18.01
-2	-1.45	-108	-11.31
-1	-0.6	-45	-4.71
0	0	0	0
1	0.62	48	5.03
2	1.48	111	11.62
3	2.33	175	18.33
4	3.2	239	25.03
5	4.06	303	31.73

Table 2 shows the results for the calibration of the tachogenerator

Table 2

Q2 For the type of adaptive controller considered in Question 1, show in detail (with full supporting analyses) that the signals generated, when taken together, constitute the state-variables of a non-minimal realization of the system

$$\frac{\Theta(s)}{U(s)} = \frac{K}{s(1+s\tau)}$$

Additionally, since the appropriate controller structure developed in Question 1 above utilizes these signals, it is thus actually a situation of state-feedback with this non-minimal state realization. Under these circumstances, thus show (with full supporting analyses) the location of all the $2n = 4$ closed-loop poles for the "perfect" case when the "perfect" control-gains are applied to the controller structure.

(20 marks)

Q.3 Consider the following constant-velocity state-space model that can be used in tracking problem

$$\begin{aligned} x(k+1) &= Ax(k) \\ y(k) &= Cx(k) + e(k) \end{aligned} \tag{1}$$

where

$$\begin{aligned} x(k) &= \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} \\ A &= \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \\ C &= \begin{bmatrix} 1 & 0 \end{bmatrix} \end{aligned}$$

The position and velocity of the target are given by $x_1(k)$ and $x_2(k)$ respectively. The sampling interval is 1 second and $e(k)$ is a zero-mean independent Gaussian random variable. The velocity is constant for all $k = 1, 2, 3, \ldots, N$ as Equation (1) gives $x_2(k+1) = x_2(k)$. Equation (1) also gives $x_1(k+1) = x_1(k) + x_2(k)$ i.e. the next position is given by the current position plus the product, current velocity $\times$ 1 (time interval).

For this question, although numerical values of A and C are given, you can leave your answers in term of A and C. There is no need to substitute them with their numerical values.

a) By iterating from the initial condition $x(1)$, express $y(N)$ in terms of A, C, $x(1)$ and $e(N)$.

(10 marks)

Question 3 continues next page.

b) Consider the least-squares cost function

$$J = \frac{1}{2}\sum_{k=1}^{N} e(k)^2$$

By differentiating J with respect to $x(1)$, obtain the least-squares estimate of the initial condition, $\hat{x}(1)$.

(15 marks)

c) By defining the least-squares estimate of $\hat{x}(1)$ at the N^{th} recursion as $\hat{\theta}(N)$ and expressing the covariance matrix $P(N)$ and vector $\phi(N)$ in terms of A and C, write down the recursive least-squares algorithm for $\hat{\theta}(N)$.

(15 marks)

Q.4 The input, $u(t)$, and output, $y(t)$, of the plant

$$G_p(s) = \frac{Y(s)}{U(s)} = \frac{K}{(sT+1)^3}$$

connected to a relay in a negative feedback loop are shown in Figures Q4.

a) Find the ultimate gain, K_u, and ultimate period, T_u.

(10 marks)

b) Find K and T.

(10 marks)

c) Sketch the input, $u(t)$, and output, $y(t)$, if the amplitude of the relay and the gain, K, of the plant are halved.

(5 marks)

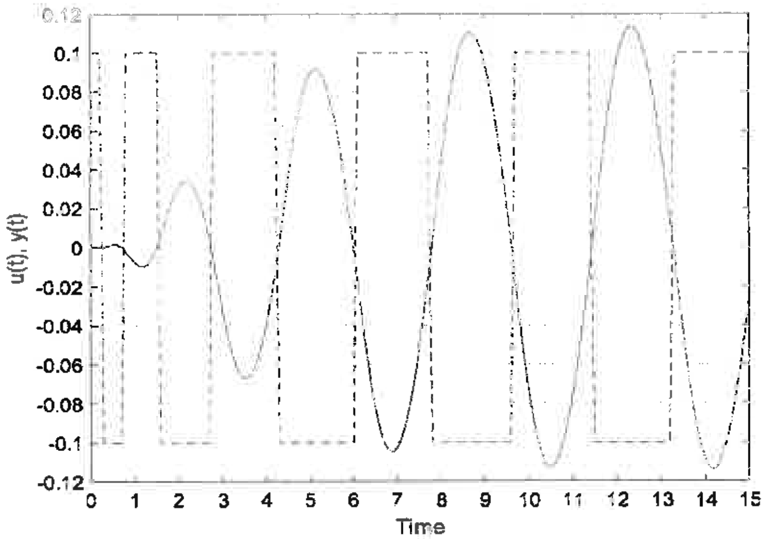

Figure Q4: The plant input, $u(t)$, and output, $y(t)$, are given by the dashed-line and solid-line respectively.

– End of Questions –

DATA SHEET:

0. Prototype Response Tables

	k	Pole Locations for $\omega_0 = 1\ rad/s^a$
ITAE	1	$s+1$
	2	$s+0.7071 \pm 0.7071j^b$
	3	$(s+0.7081)(s+0.5210 \pm 1.068j)$
	4	$(s+0.4240 \pm 1.2630j)(s+0.6260 \pm 0.4141j)$
	5	$(s+0.8955)(s+0.3764 \pm 1.2920j)(s+0.5758 \pm 0.5339j)$
Bessel	1	$s+1$
	2	$s+0.8660 \pm 0.5000j^b$
	3	$(s+0.9420)(s+0.7455 \pm 0.7112j)$
	4	$(s+0.6573 \pm 0.8302j)(s+0.9047 \pm 0.2711j)$
	5	$(s+0.9264)(s+0.5906 \pm 0.9072j)(s+0.8516 \pm 0.4427j)$

[a] Pole locations for other values of ω_0 can be obtained by substituting s/ω_0 for s.
[b] The factors $(s+a+bj)(s+a-bj)$ are written as $(s+a \pm bj)$ to conserve space.

1. The Lyapunov Equation states that given any $n \times n$ stability matrix A_m, for every symmetric positive definite matrix Q, there exists a unique symmetric positive definite matrix P that is the solution to the equation

$$A_m^\top P + PA_m = -Q.$$

In addition, the error system dynamics (with $\mathbf{e} \in \mathbf{R}^n$ and Γ an $n \times n$ symmetric positive-definite matrix) given by

$$\begin{aligned} \dot{\mathbf{e}}(t) &= A_m \mathbf{e}(t) + g\mathbf{b}\phi(t)^\top \mathbf{x}(t) \\ \dot{\phi}(t) &= -\mathrm{sgn}(g)\Gamma \mathrm{e}(t)^\top P\mathbf{b}\,\mathbf{x}(t) \end{aligned}$$

has the properties that $\|\mathbf{e}(t)\|$ and $\|\phi(t)\|$ are bounded, and if it should also be known that $\|\mathbf{x}(t)\|$ is bounded, then additionally

$$\lim_{t\to\infty} \mathbf{e}(t) = 0$$

2. For the triple

$$\begin{aligned} A_m &= \begin{bmatrix} 0 & 1 & 0 \\ a_1 & a_2 & a_3 \\ 1 & 0 & 0 \end{bmatrix} \\ b_m &= \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \\ c_m &= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \end{aligned}$$

the equivalent transfer function is

$$c_m^\top[sI - A_m]^{-1}b_m = \frac{-a_3}{s^3 - a_2 s^2 - a_1 s - a_3}$$

3. For

$$A_m = \begin{bmatrix} 0 & 1 & 0 \\ -21 & -12 & -10 \\ 1 & 0 & 0 \end{bmatrix}$$

and

$$Q = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

the solution to

$$A_m^\top P + P A_m = -Q.$$

is

$$P = \begin{bmatrix} 9.30 & 0.38 & 5.40 \\ 0.38 & 0.24 & 0.25 \\ 5.40 & 0.25 & 9.01 \end{bmatrix}$$

and the eigenvalues of P are $\lambda = 14.57, 3.76, 0.22$.

4. For

$$A_m = \begin{bmatrix} 0 & 1 & 0 \\ -11 & -7 & -5 \\ 1 & 0 & 0 \end{bmatrix}$$

and

$$Q = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

the solution to

$$A_m^\top P + P A_m = -Q.$$

is

$$P = \begin{bmatrix} 9.92 & 0.76 & 5.83 \\ 0.76 & 0.47 & 0.50 \\ 5.83 & 0.50 & 9.28 \end{bmatrix}$$

and the eigenvalues of P are $\lambda = 15.49, 3.77, 0.40$.

5. For

$$A_m = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}$$

and

$$Q = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

the solution to

$$A_m^\top P + P A_m = -Q.$$

is

$$P = \begin{bmatrix} 7.50 & 2.50 \\ 2.50 & 2.50 \end{bmatrix}$$

and the eigenvalues of P are $\lambda = 8.54, 1.46$.

6. For

$$A_m = \begin{bmatrix} 0 & 1 \\ -2 & -2 \end{bmatrix}$$

and

$$Q = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

the solution to

$$A_m^\top P + PA_m = -Q.$$

is

$$P = \begin{bmatrix} 6.25 & 1.25 \\ 1.25 & 1.875 \end{bmatrix}$$

and the eigenvalues of P are $\lambda = 6.58, 1.54$.

7. For

$$A_m = \begin{bmatrix} 0 & 1 & 0 \\ -3,600 & -120 & -32,000 \\ 1 & 0 & 0 \end{bmatrix}$$

and

$$Q = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

the solution to

$$A_m^\top P + PA_m = -Q.$$

is

$$P = \begin{bmatrix} 105.1 & 0.2 & 720.2 \\ 0.2 & 0.0225 & 0.0 \\ 720.2 & 0.0 & 6,424.4 \end{bmatrix}$$

and the eigenvalues of P are $\lambda = 6,505.4; 24.1; 0.021$.

8. For

$$A_m = \begin{bmatrix} 0 & 1 \\ -400 & -40 \end{bmatrix}$$

and

$$Q = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

the solution to

$$A_m^\top P + PA_m = -Q.$$

is

$$P = \begin{bmatrix} 25.31 & 0.0063 \\ 0.0063 & 0.0627 \end{bmatrix}$$

and the eigenvalues of P are $\lambda = 25.31, 0.0627$.

6. For

$$A_m = \begin{bmatrix} 0 & 1 \\ -400 & -20 \end{bmatrix}$$

and

$$Q = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

the solution to

$$A_m^\top P + P A_m = -Q.$$

is

$$P = \begin{bmatrix} 50.25 & 0.006 \\ 0.006 & 0.125 \end{bmatrix}$$

and the eigenvalues of P are $\lambda = 50.25, 0.125$.

7. The standard discrete-time gradient estimator is

$$\begin{aligned} \hat{y}(j) &= \hat{\theta}(j)^\top \omega(j) \\ e_1(j) &= \hat{y}(j) - y(j) \\ \hat{\theta}(j+1) &= \hat{\theta}(j) - \frac{\omega(j) e_1(j)}{1 + \|\omega(j)\|^2} \end{aligned}$$

It is applicable to the process

$$y(j) = \theta^{*\top} \omega(j)$$

Laplace Transform Table

Laplace Transform, F(s)	Time Function, f(t)
1	$\delta(t)$ (unit impulse)
$\frac{1}{s}$	$u(t)$ (unit step)
$\frac{1}{s^2}$	t
$\frac{1}{s^n}$	$\frac{t^{n-1}}{(n-1)!}$ (n = positive integer)
$\frac{1}{s+a}$	e^{-at}
$\frac{1}{(s+a)^2}$	te^{-at}
$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$
$\frac{1}{s(s+a)^2}$	$\frac{1}{a^2} - \left(\frac{1}{a^2} + \frac{1}{a}t\right) e^{-at}$
$\frac{1}{s^2(s+a)^2}$	$\left(\frac{2}{a^3} + \frac{1}{a^2}t\right) e^{-at} - \frac{2}{a^3} + \frac{1}{a^2}t$
$\frac{1}{(s+a)^n}$	$\frac{1}{(n-1)!}t^{n-1}e^{-at}$ (n = positive integer)
$\frac{1}{(s+a)(s+b)}$	$\frac{e^{-at}-e^{-bt}}{b-a}$
$\frac{1}{s(s+a)(s+b)}$	$\frac{1}{ab}\left[1 + \frac{1}{a-b}(be^{-at} - ae^{-bt})\right]$
$\frac{\omega}{s^2+\omega^2}$	$\sin \omega t$
$\frac{s}{s^2+\omega^2}$	$\cos \omega t$
$\frac{\omega}{(s+a)^2+\omega^2}$	$e^{-at} \sin \omega t$
$\frac{s+a}{(s+a)^2+\omega^2}$	$e^{-at} \cos \omega t$
$\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin \omega_n \sqrt{1-\zeta^2}t$
$\frac{s}{s^2+2\zeta\omega_n s+\omega_n^2}$	$-\frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1-\zeta^2}t - \phi)$ $\phi = \tan^{-1} \frac{\sqrt{1-\zeta^2}}{\zeta}$
$\frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$	$1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1-\zeta^2}t + \phi)$ $\phi = \tan^{-1} \frac{\sqrt{1-\zeta^2}}{\zeta}$

– End of Paper –