Note Title 10/4/2010

# Stability properties of :

7 in Class

**Adaptive Law**

$$\hat{y}(t) = \hat{\theta}(t)^T w(t-d)$$

$$e_1(t) = \hat{y}(t) - y(t)$$

$$\Delta\hat{\theta}(t) \triangleq \hat{\theta}(t+1) - \hat{\theta}(t)$$

$$= \frac{-\gamma\, w(t-d)\, e_1(t)}{1 + \| w(t-d) \|^2}$$

with $0 < \gamma < 2$

Note that from earlier (7.21), we had:

$$y(t) = \theta^{*T} w(t-d)$$

Thus:

$$e_1(t) = \hat{y}(t) - y(t)$$

$$= \tilde{\theta}^T(t)\, w(t-d)$$

Note also that

$$\Delta\hat{\theta}(t) = \Delta\tilde{\theta}(t)$$

where $\tilde{\theta}(t) \triangleq \hat{\theta}(t) - \theta^*$

We can consider the quadratic form:

$$V(t) = \|\tilde{\theta}(t)\|^2$$

$$= \tilde{\theta}(t)^T \tilde{\theta}(t)$$

Look at:

$$\Delta V(t) = V(t+1) - V(t)$$

$$= \|\tilde{\theta}(t+1)\|^2 - \|\tilde{\theta}(t)\|^2$$

$$= \left\{ \tilde{\theta}(t+1) + \tilde{\theta}(t) \right\}^T \left\{ \tilde{\theta}(t+1) - \tilde{\theta}(t) \right\}$$

$$= \left\{ 2\tilde{\theta}(t) + \Delta\tilde{\theta}(t) \right\}^T \Delta\tilde{\theta}(t)$$

$2\tilde{\theta}(t) + \Delta\tilde{\theta}(t)$

$$= 2\tilde{\theta}(t)^T \Delta\tilde{\theta}(t) + \|\Delta\tilde{\theta}\|^2 \qquad \text{--- (**)}$$

Note now that:

$$2\tilde{\theta}(t)^T \Delta\tilde{\theta}(t)$$

$$= 2\tilde{\theta}(t)^T \left\{ \frac{-\gamma\, w(t-d)\, e_1(t)}{1 + \|w(t-d)\|^2} \right\}$$

$$= \frac{-2\gamma\, \underbrace{\tilde{\theta}(t)^T w(t-d)}_{e_1(t)}\, e_1(t)}{1 + \|w\|^2}$$

$$= \frac{-2\gamma\, e_1^2(t)}{1+\|w\|^2}$$

Also need to look at:

$$\|\Delta\tilde{\theta}(t)\|^2$$

$$= \frac{\gamma^2 \, \|w(t-d_1)\|^2 \, e_1^2(t)}{\{1+\|w\|^2\}\{1+\|w\|^2\}}$$

$\leq 1$

$$\leq \frac{\gamma^2 e_1^2(t)}{1+\|w\|^2}$$

Then, the Equation (**) becomes:

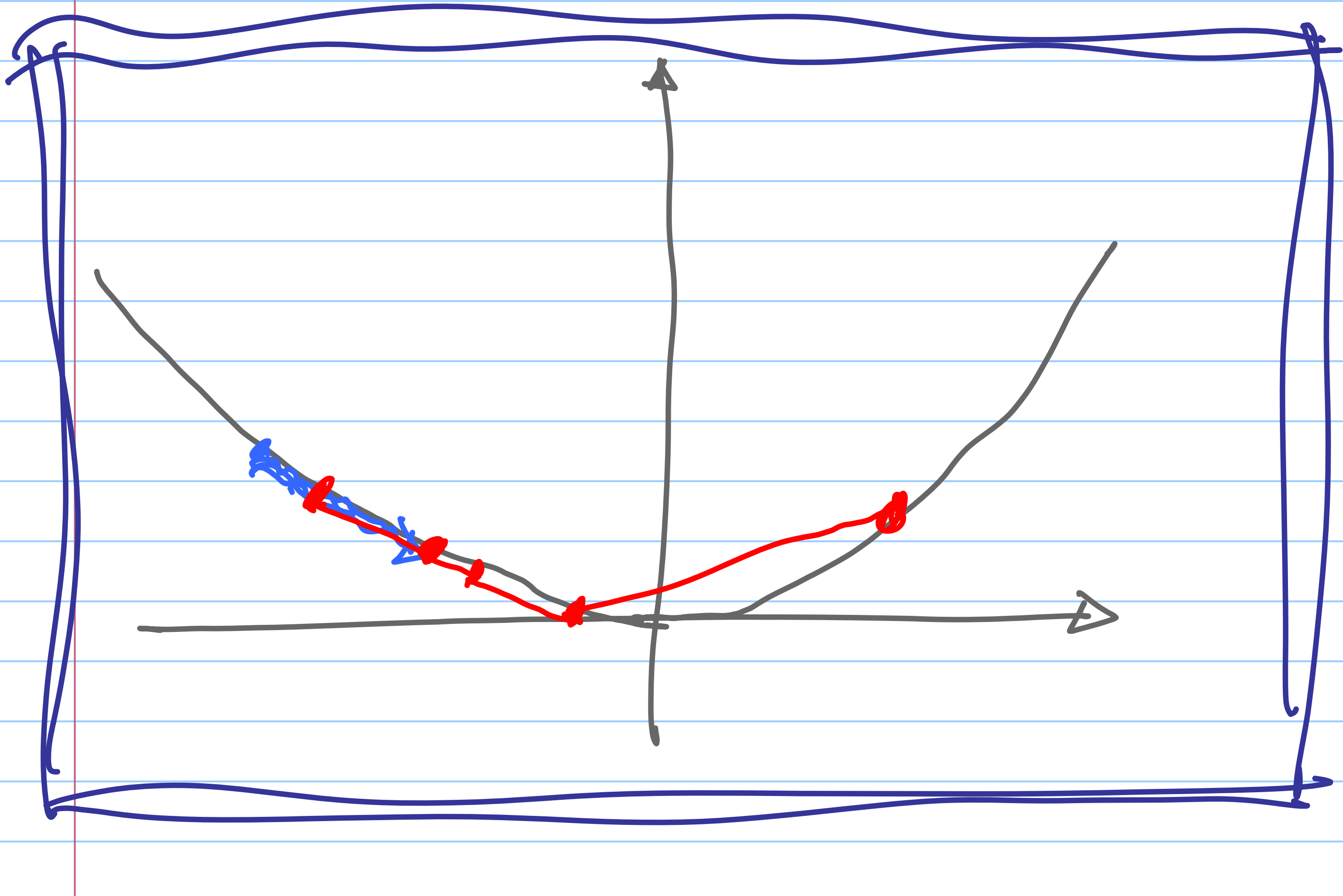
$$\Delta V(t) = \|\tilde{\theta}(t+1)\|^2 - \|\tilde{\theta}(t)\|^2$$

$$\leq \frac{-2\gamma\, e_1(t)^2}{1+\|w\|^2} + \frac{\gamma^2\, e_1(t)^2}{1+\|w\|^2}$$

$$= \frac{-\gamma(2-\gamma)\, e_1^2(t)}{1+\|w\|^2}$$

$$\leq 0 \quad \text{if } 0 < \gamma < 2$$

$\Delta\Delta$



# Some illustrative pictures on the "Key Technical Lemma"

$\{s_t\}$ $\{\sigma_t\}$


(I) $\|\sigma_t\| \le c_1 + c_2 \max\limits_{0\le k\le t}\{|s_k|\}$

(II) $z_t = \dfrac{s_t^2}{\alpha_1 + \alpha_2\|\sigma_t\|^2} \longrightarrow 0$

$\{s_t\}$
$t$