National University of Singapore
Faculty of Engineering

ME5402/EE5016R Advanced Robotics

Exercise 3

1. Consider the manipulator as shown in Figure 1. The base frame {0} and end frame {3} are given as shown. Adopt the D-H notation to assign frame {1} and frame {2}. Find the Jacobian matrix of the following manipulator whose joint displacements are θ_1, θ_2 and θ_3. Perform a singularity analysis to identify the singular configuration of the manipulator by considering the linear part (first three rows) of the Jacobian matrix.

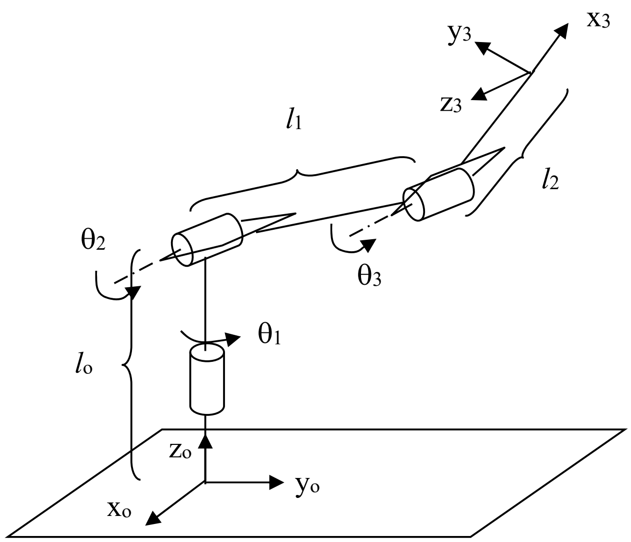


Figure 1

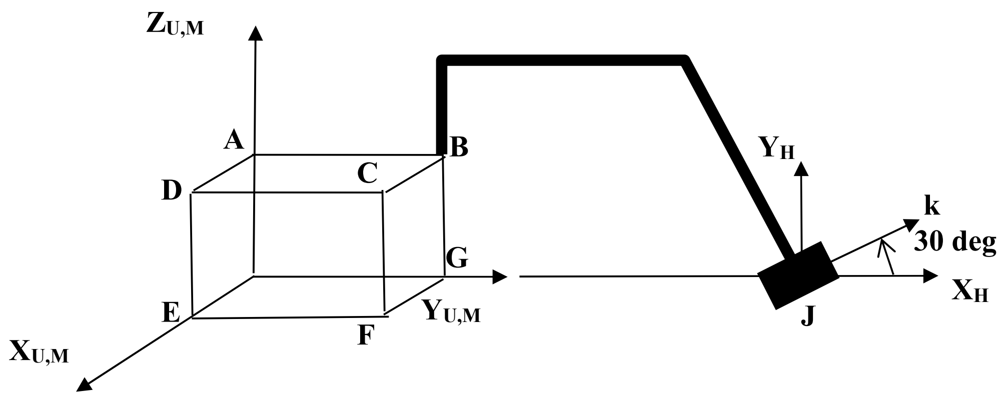
2. Fig. 2 shows a connecting (bent) rod (BJ) rigidly connecting the rotational joint J to the cuboid (ABCDEFGO). Let
   a) Frame U be fixed and serve as the universe frame of reference.
   b) Frame H be translating with respect to Frame U.
   c) Frame H is attached rigidly to the rotational joint J.
   d) Axis **k** be fixed rigidly onto Frame H.
   e) Frame M be fixed rigidly onto the cuboid.
   f) The cuboid (ABCDEFGO), bent connecting rod (BJ), and joint J be all connected as one rigid assembly.
   g) Joint J be a rotational joint that rotates the cuboid-connecting rod rigid assembly about axis **k**. This assembly is rotating about axis **k**.

At a certain instant of time, the origin of Frame H (joint J) is translating at a velocity of ${}^{U}u_H = [\ 1\ 2\ 3]^T$ m/s, and the cuboid-connecting rod assembly is rotating about axis **k** at an angular velocity of 10 rad/sec. At this same instant of time the assembly and frames are at the configuration indicated in Fig. 2.

At the same instant of time, determine:

i. The angular velocity of the cuboid-connecting rod assembly with respect to Frame U.

ii. The translational velocity of point C (on the cuboid) with respect to Frame U.



length of line segments: AD=CB=1 DE=CF=2 OG=EF=3 OJ=10

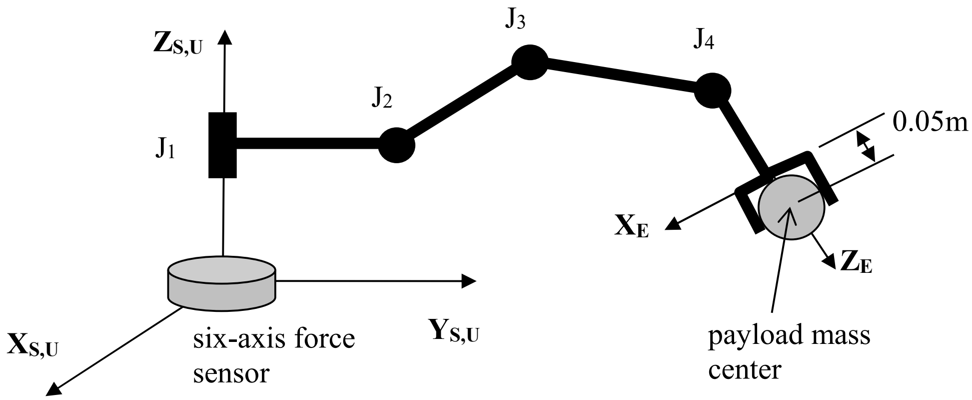
Figure 2

$$\text{Ans: } \mathbf{u} = \begin{bmatrix} 53.32 \\ 7 \\ -5.66 \end{bmatrix} m/s;\ \omega = \begin{bmatrix} 0 \\ 0.866 \\ 5 \end{bmatrix} rad/s$$

3. Fig. 3 shows a 4-axis robot with all rotational joints. The first joint rotates about a vertical axis while the next three joints rotate about a horizontal axis parallel to the xy plane of Frame U. Frame E is attached to the robot end-effector. A six-axis force-torque sensor provides 3 force and 3 torque readings along and about the x, y, and z axes of the sensor frame S. Frame S is coincident to the fixed frame of reference U. If the robot end-effector is carrying a payload of 20 kg when it is at a configuration indicated by

$$ {}_E^U\mathbf{T} = \begin{bmatrix} {}_E^UR & {}^U\mathbf{p}_E \\ \mathbf{0} & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1.0 & 0 & 10 \\ -0.866 & 0 & 0.5 & 5 \\ -0.5 & 0 & -0.866 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Determine the six readings of the six-axis force-torque sensor. Assume that the robot is weightless. The gravitational force is pointing downward along the negative **Z** axis direction of Frame **U**.



$$\text{Ans:}\quad F = \begin{bmatrix} 0 \\ 0 \\ -196 \end{bmatrix} N \qquad\qquad T = \begin{bmatrix} -986 \\ 1962 \\ 0 \end{bmatrix} Nm$$

4. There are three moving frames A, B, and C. At a certain time instant, they are at:

$$ {}^{A}_{B}\mathbf{T} = \begin{bmatrix} 0.866 & -0.5 & 0 & 10 \\ 0.5 & 0.866 & 0 & 0 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^{B}_{C}\mathbf{T} = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and are moving with following generalized velocity vectors (the first three components represent the translational velocity while the last three components represent the angular velocity):

$${}^{A}V_{B} = \begin{bmatrix} 0 \\ 2 \\ -3 \\ 1.414 \\ 1.414 \\ 0 \end{bmatrix}; \quad {}^{B}V_{C} = \begin{bmatrix} 1 \\ 4 \\ -5 \\ 0.5 \\ 1 \\ 2 \end{bmatrix}$$

Find the generalized velocity ${}^{A}V_{C}$ of Frame C as seen from Frame A.

Ans: $${}^{A}V_{C} = \begin{bmatrix} 5.936 \\ -1.106 \\ -1.826 \\ 1.347 \\ 2.53 \\ 2 \end{bmatrix}$$

5. A single-link robot with a rotary joint is motionless at $\theta = -5^{o}$. It is desired to move the joint in a smooth manner to $\theta = 80^{o}$ in 4 seconds. Find the coefficients of a cubic which accomplishes this motion and brings the arm to rest at the goal. Plot the position, velocity, and acceleration of the joint as a function of time.

6. A single-link robot with a rotary joint is motionless at $\theta = -5^{o}$. It is desired to move the joint in a smooth manner to $\theta = 80^{o}$ in 4 seconds and stop smoothly. Compute the corresponding parameters of a linear trajectory with parabolic blends. Plot the position, velocity, and acceleration of the joint as a function of time.