Note Title 8/2/2010

EE 5104 / 6104

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CA - 70% of module grade — 3 assignments (of which 1 is a hardware mini-project)

Exam — 30% of module grade

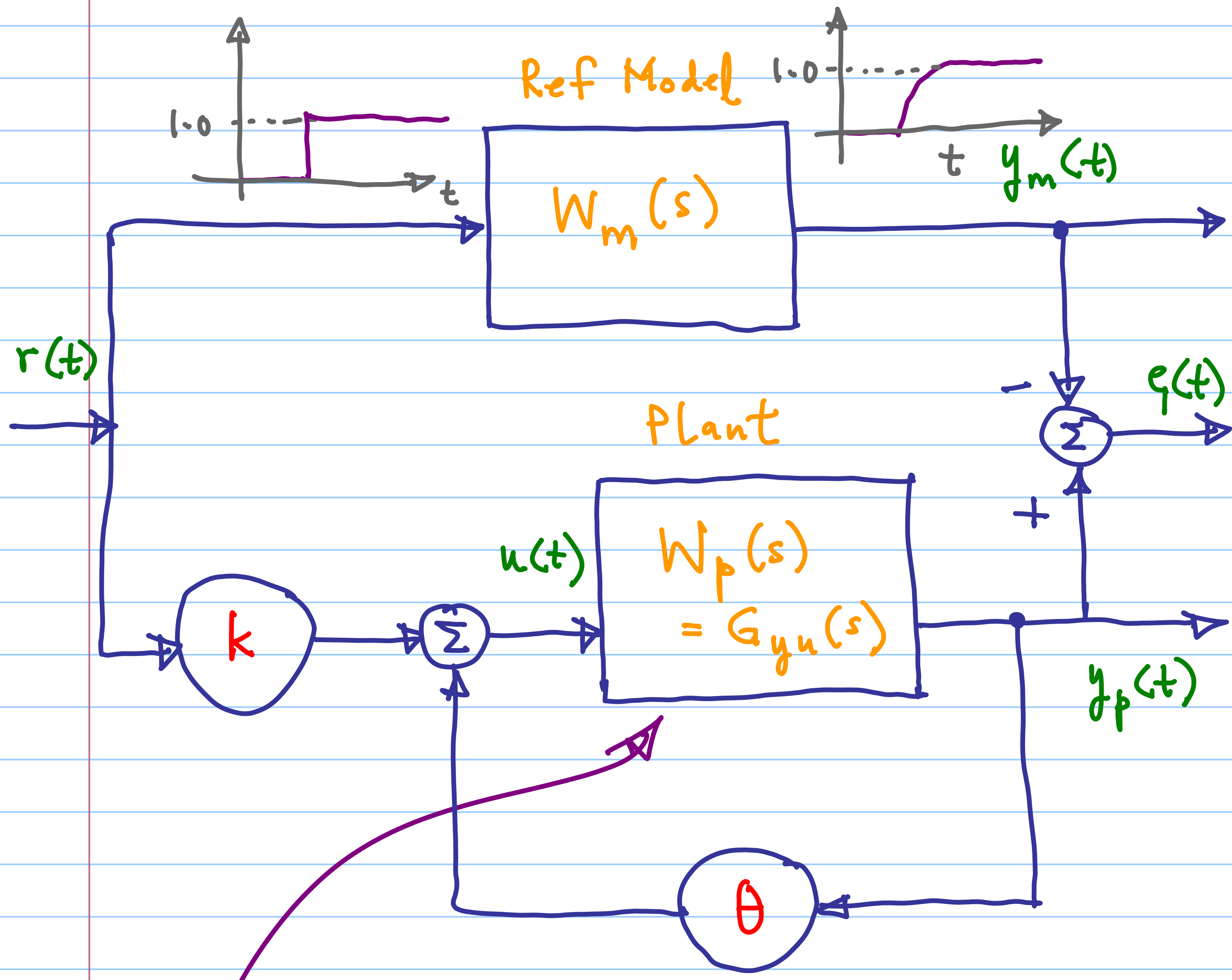
Year 1 graduate students — ?

Year 2 & above, grad students — generally ok

Exchange students — ?

[Appropriate background in Linear Systems & State-Variables; and Electrical Engineering needed.]

Motivating the subject matter...



$$\dot{y}_p = a_p y_p + k_p u$$

consider as a start, this simple case...

Consider the simple "plant":

$$\dot{y}_p = a_p y_p + k_p u$$

We are interested to obtain the closed-loop specified by:

$a_m < 0$

$$\dot{y}_m = a_m y_m + k_m r(t)$$

Note that for this simple system, we can use:

$$u(t) = \theta y_p(t) + k r(t) \longrightarrow (3)$$

With this, we clearly have:

$$\dot{y}_p(t) = a_p\, y_p(t) + k_p\, u(t)$$

$$= a_p\, y_p(t) + k_p \left\{ \theta\, y_p(t) + k\, r(t) \right\}$$

$$= \underbrace{\left[a_p + k_p \theta \right]}_{= a_m} y_p(t) + \underbrace{k_p k}_{k_m}\, r(t)$$

And here, clearly, for

θ^* defined by

$$a_p + k_p \theta^* \triangleq a_m$$

and for k^* defined by

$$k_p k^* \triangleq k_m$$

and if we have
$\theta \equiv \theta^*$
$k \equiv k^*$

we have:

$$\dot{y}_p = \left\{ a_p + k_p \theta^* \right\} y_p + k_p k^* r(t)$$

$$= a_m y_p(t) + k_m r(t)$$

and for the tracking error

$$e_1(t) \triangleq y_p(t) - y_m(t)$$

we have

$$\dot{e}_1(t) = \dot{y}_p(t) - \dot{y}_m(t)$$

$$= \left\{ a_m y_p(t) + k_m r(t) \right\}$$

$$- \left\{ a_m y_m(t) + k_m r(t) \right\}$$

$$= a_m e_1(t) \quad ; \quad \text{with } a_m < 0$$

But we do <u>not</u> know a_p & k_p, and thus, cannot exactly calculate θ^* and k^*

Thus, consider the following control law:

$$u(t) = \theta(t)\, y_p(t) + k(t)\, r(t)$$

i.e. time-varying "control gains".

How to specify these time-varying "control gains"?

Answer: Use the following adaptive law:

$$\dot{\theta}(t) = -\operatorname{sgn}(k_p)\, \gamma_1\, y_p(t)\, e_1(t)$$

$$\dot{k}(t) = -\operatorname{sgn}(k_p)\, \gamma_2\, r(t)\, e_1(t)$$

$\gamma_1, \gamma_2 > 0$; $e_1(t) = y_p(t) - y_m(t)$

How does this work?

For a start, we should be interested in the following error signals:

$$e_1(t) \triangleq y_p(t) - y_m(t)$$

$$\phi(t) \triangleq \theta(t) - \theta^*$$

$$\psi(t) \triangleq k(t) - k^*$$

and thus also in the "energy" function =

$$V\left(e_1(t), \phi(t), \psi(t)\right)$$

$$= \frac{1}{2}\left\{ e_1^2(t) + \frac{|k_p|}{\gamma_1}\phi^2(t) + \frac{|k_p|}{\gamma_2}\psi^2(t) \right\}$$

Consider then, for this system as described above, how this "energy" function evolves with time ...

i.e. consider

$$\frac{d}{dt} V(e_1, \phi, \psi)$$

$$= e_1 \dot{e}_1 + \frac{|k_p|}{\gamma_1} \phi \dot{\phi} + \frac{|k_p|}{\gamma_2} \psi \dot{\psi} \qquad \text{--- (21)}$$

Now, note that:

$$\dot{e}_1(t) = \dot{y}_p(t) - \dot{y}_m(t)$$

$$= \left\{ a_p y_p(t) + k_p \left[\phi(t) + \theta^* \right] y_p(t) + k_p \left[\psi(t) + k^* \right] r(t) \right\}$$

$$- \left\{ a_m y_m(t) + k_m r(t) \right\}$$

$\vdots$

$$= a_m e_1(t) + k_p \left\{ \phi(t) y_p(t) + \psi(t) r(t) \right\}$$

Further, note that:

$$\dot{\phi}(t) = \dot{\left\{ \theta - \theta^* \right\}} = \dot{\theta} = -\text{sgn}(k_p)\, \gamma_1 e_1 y_p$$

$$\dot{\psi}(t) = \quad \ldots \quad = -\text{sgn}(k_p)\, \gamma_2 e_1 r$$

Thus, (21) becomes:

$$\dot{V}(e, \phi, \psi)$$

$$= e_1 \left\{ a_m e_1 + \cancel{k_p \phi y_p} + \cancel{k_p \psi r} \right\}$$

$$+ \frac{|k_p|}{\gamma_1} \phi \left\{ \cancel{-\operatorname{sgn}(k_p)\, \gamma_1 y_p e_1} \right\}$$

$$+ \frac{|k_p|}{\gamma_2} \psi \left\{ \cancel{-\operatorname{sgn}(k_p)\, \gamma_2 r e_1} \right\}$$

yields:

$$\dot{V}(e, \phi, \psi) = a_m e_1(t)^2 \leq 0$$

$$a_m < 0$$

For such classes of NLTV systems, note now that:

(a) $$V(e_1, \phi, \psi) = \frac{1}{2}\left\{ e_1^2 + \frac{\|k_p\|}{\gamma_1}\phi^2 + \frac{\|k_p\|}{\gamma_2}\psi^2 \right\}$$ — (51)

we have shown that the "Control Law" and the "Adaptive Law" yields — (52)

$$\dot{V}(e_1, \phi, \psi) = a_m e_1^2 \leq 0$$

since the closed-loop specification means $a_m < 0$

(b) Eqns (51) and (52)

$\Rightarrow |e_1(t)|, |\phi(t)|, |\psi(t)|$

are bounded for all $t \geq 0$

(c) Next, note that

$$\dot{V}(t) = a_m e_1^2(t) = -\alpha e_1^2(t)$$

Write
$a_m = -\alpha$
$\alpha > 0$

$$\int_{t=t_0}^{t} \dot{V}(\tau)\, d\tau = -\int_{t=t_0}^{t} \alpha e_1^2(\tau)\, d\tau$$

$$V(t) - V(t_0) = -\alpha \int_{t_0}^{t} e_1^2(\tau)\, d\tau$$

$\Rightarrow$

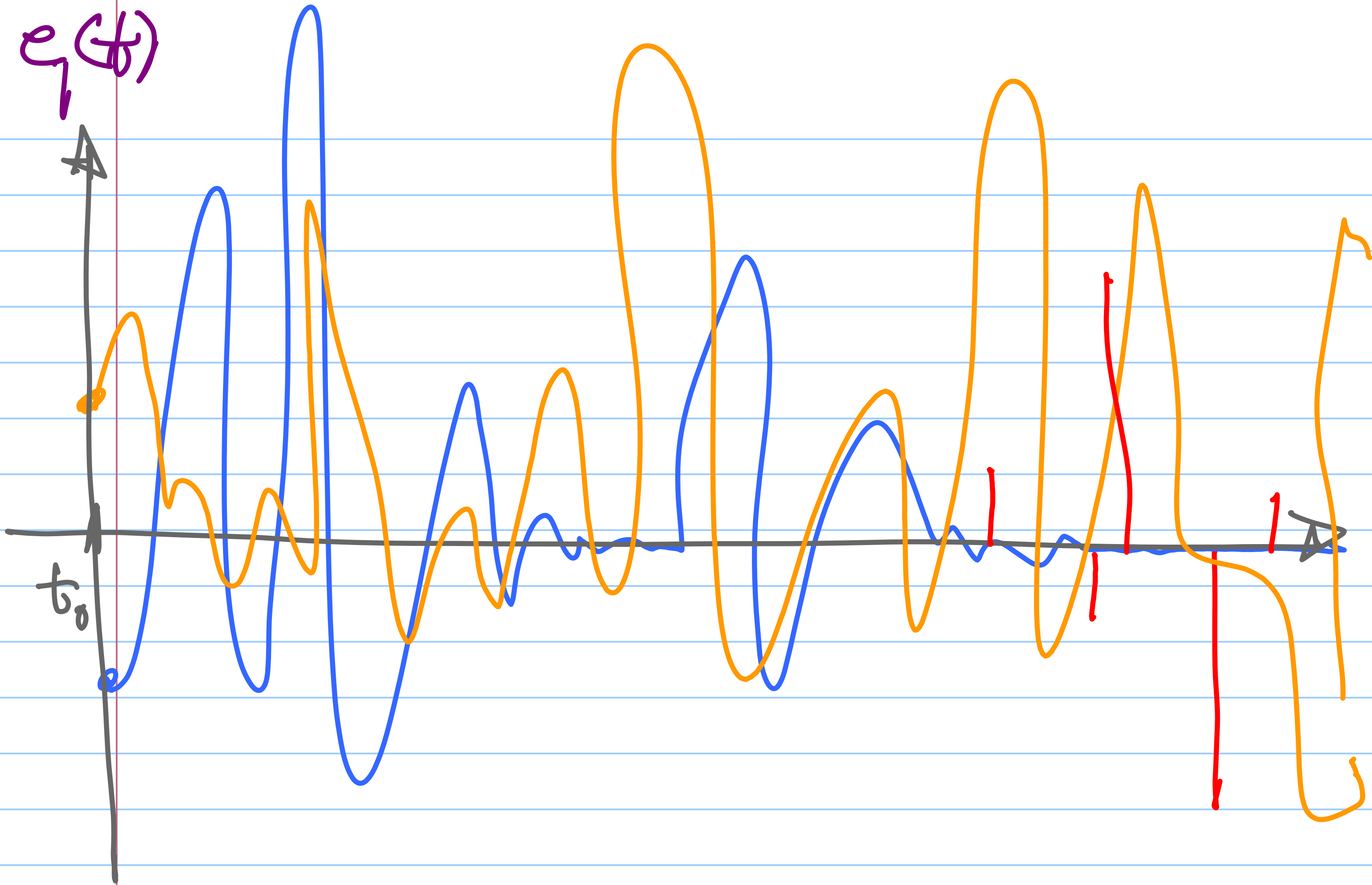
$$\alpha \int_{t_0}^{t} e_1^2(\tau)\, d\tau = V(t_0) - V(t) \leq V(t_0)$$

$\Rightarrow$

$$\int_{t_0}^{t} e_1^2(\tau)\, d\tau \leq \frac{1}{\alpha} V(t_0) = c_1$$

true for all t !!!

— (53)



Next, note further, that

$$\dot{e}_1(t) = a_m e_1(t) + k_p \phi(t) y_p(t) + k_p \psi(t) r(t)$$

and since we already know

$\{e, \phi, \psi\}$ are bounded $\forall t$

we also have

$\dot{e}_1(t)$ is bounded for all $t \geq 0$

— (54)

(53) and (54) together imply :

$$\lim_{t \to \infty} e_1(t) = 0 \qquad (55)$$