ORIGINAL

NATIONAL UNIVERSITY OF SINGAPORE

EXAMINATION FOR

(Semester II: 2021/2022)

EE6104 – ADAPTIVE CONTROL SYSTEMS

April/May 2022 – Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES:

1. This question paper contains **FOUR (4)** questions and comprises **SIXTEEN (16)** printed pages.
2. Answer all **FOUR (4)** questions.
3. This is a OPEN BOOK examination.
4. Note carefully that the questions **do not** carry equal marks.
5. Relevant data are provided at the end of this examination paper.
6. Total Marks: 100

Q.1 The need often arises to consider an approach of adaptive control for the system

$$\frac{Y(s)}{U(s)} = \frac{b_0}{s^2 + a_1 s + a_2}$$

when only the signals $y(t)$ and $u(t)$ are the only measurable signals.

a) In this type of situation, one applicable approach would be to consider the related polynomials

$$\begin{aligned} R_m(s) &= s^2 + a_{1m}s + a_{2m} \\ T(s) &= s^2 + t_1 s + t_2 \\ R_p(s) &= s^2 + a_1 s + a_2 \end{aligned}$$

For this specific situation, calculate exactly the coefficients of the resulting polynomials $E(s)$ and $F(s)$ in the polynomial identity

$$R_m(s)T(s) = R_p(s)E(s) + F(s)$$

Using this specific approach above (which is also the approach developed in our lectures), develop in full detail the specific structure of a **control law** which would allow the formulation of an adaptive controller which has properties of uniform boundedness of all the signals in the overall system, and where the appropriate output tracking error

$$e_1(t) = y(t) - y_m(t)$$

would asymptotically converge to zero. (**Important Note:** Here, it is only required to fully develop the the specific structure of the appropriate control law. It is not required to develop the associated stability analysis and convergence.)

(10 marks)

b) For the type of adaptive controller considered in Question 1, show in detail (with full supporting analyses) that the signals generated, when taken together, constitute the state-variables of a non-minimal realization of the system

$$\frac{Y(s)}{U(s)} = \frac{b_0}{s^2 + a_1 s + a_2}$$

Additionally, since the appropriate controller structure developed in Question 1 above utilizes these signals, it is thus actually a situation of state-feedback with this non-minimal state realization. Under these circumstances, thus show (with full supporting analyses) the location of all the $2n = 4$ closed-loop poles for the "perfect" case when the "perfect" control-gains are applied to the controller structure.

(10 marks)

Q2 In a particular hardware set-up, it is desired to use the d.c. motor system (shown in Figure 1) to be the basis of various control system design experiments.

The d.c. motor system has the nominal dynamic model as shown in Figure 2, with the transfer function:

$$\frac{\Theta(s)}{U(s)} = \frac{K}{s(1+s\tau)}$$

where $\Theta(s)$ is the Laplace transform of the angular position signal $\theta(t)$ and $U(s)$ is the Laplace transform of the motor drive input voltage $u(t)$. Calibration tests on the d.c. motor system, using the LabView real-time system connections of Figure 3, has yielded the data listed in Tables 1 and 2.

However, for Table 2, it is also known that the steady-state relationship between the motor drive input voltage $u(t)$ and the tachogenerator output voltage (while constant for each operation) can change in different day-to-day operations, and thus <u>cannot</u> be regarded as being known accurately. Further, simple step-response tests (which <u>cannot</u> be used as accurate calibration data) on the angular velocity has also indicated that

$$\tau \approx 190 \quad \text{milliseconds}$$

for the d.c. motor system, and that a positive-valued drive input voltage $u(t)$ results in a positive-valued angular velocity $\dot{\theta}(t)$.

For this system above, consider the <u>**specific**</u> situation where all the measurements of the input $u(t)$, the angular position output $\theta(t)$ and also the angular velocity output $\dot{\theta}(t)$ are available. Write thus a suitable state-variable description of the d.c. motor system, where the state-variables are chosen as:

$$\begin{aligned} x_1(t) &= \theta(t) \\ x_2(t) &= \dot{\theta}(t) \end{aligned}$$

Your state-variable description can include unknown constants for system coefficients which are not accurately known based on the conditions described above.

In the control system then to be used for the overall positioning mechanism, a simple way would be to use an adaptive state-feedback controller of the configuration shown in Figures 4, 5 and 6. Here, the reference input $r(t)$ is an angular position reference/command signal where step changes are made in its value, to various different constant values, at intervals of 45 seconds or more.

However, arising from additional requirements, it is required <u>**instead**</u> to include "integral control action" in the adaptive control system. A simple direct way to do this is to additionally generate the "augmented" signal

$$\dot{x}_I(t) = \dot{x}_3(t) = r(t) - x_1(t)$$

For this situation, in this case, show that it will always be possible to develop an adaptive state-feedback controller which includes this "integral control action" in the adaptive control system; preserving the properties of uniform boundedness of all signals in the system and convergence of the <u>**state-tracking errors**</u> to zero. Carefully include all relevant equations and detailed descriptions.

(15 marks)

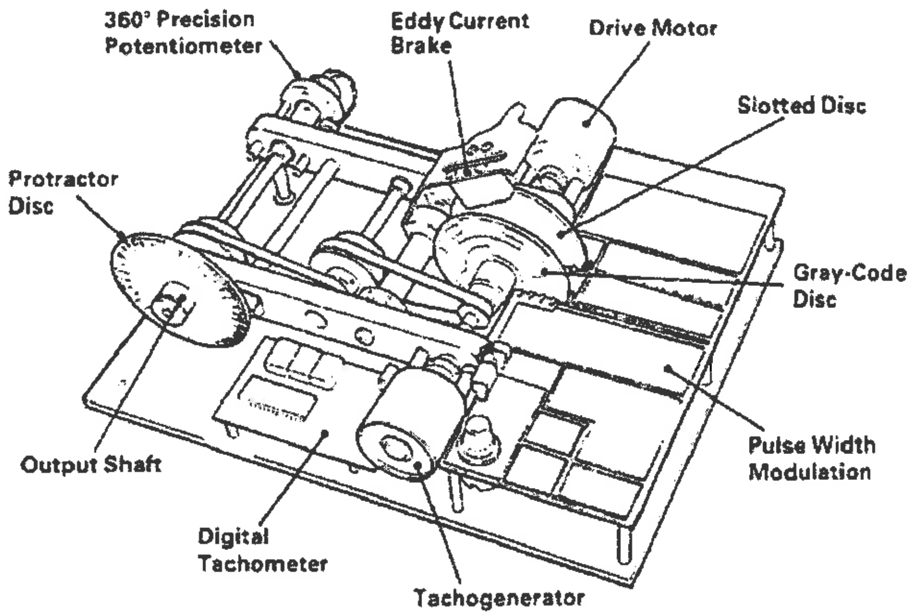


Figure 1

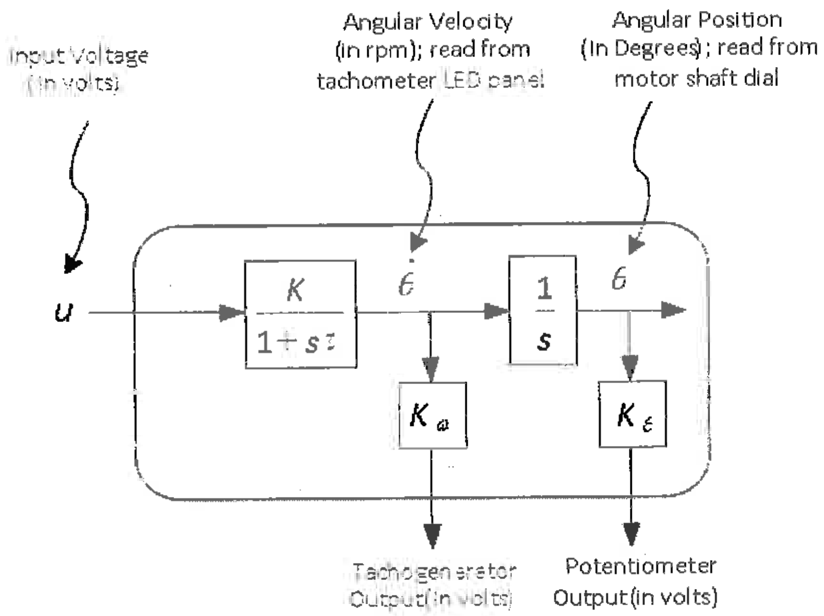


Figure 2

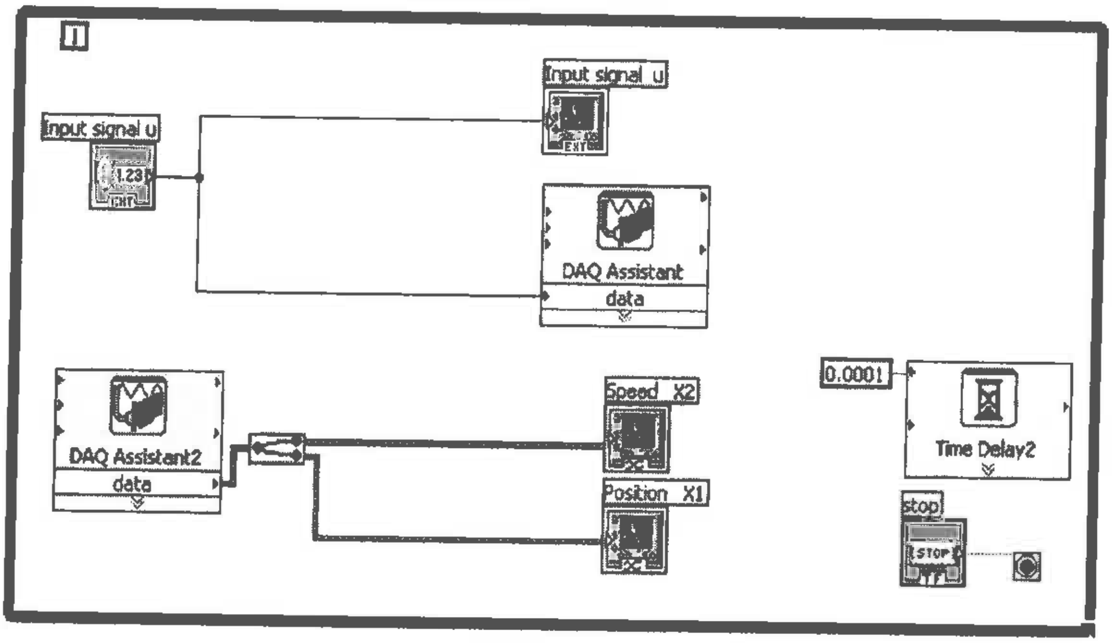

Input signal u
1.23
Input signal u
DAQ Assistant
data
DAQ Assistant2
data
Speed X2
Position X1
0.0001
Time Delay2
stop
STOP

Figure 3

Calibration Results for Part 1

Potentiometer Output (in volts)	Angular Position (in degrees)
-5	-180
-4	-144
-3	-108
-2	-72
-1	-36
0	0
1	36
2	72
3	108
4	144
5	180

Table 1 shows the results for the calibration of the potientiometer

Table 1

Calibration Results for Part 1

Input Voltage (volts)	Tachogenerator Output (volts)	Angular Velocity (rpm)	Angular Velocity (rad/sec)
-5	-4.03	-301	-31.52
-4	-3.17	-237	-24.82
-3	-2.3	-172	-18.01
-2	-1.45	-108	-11.31
-1	-0.6	-45	-4.71
0	0	0	0
1	0.62	48	5.03
2	1.48	111	11.62
3	2.33	175	18.33
4	3.2	239	25.03
5	4.06	303	31.73

Table 2 shows the results for the calibration of the tachogenerator

Table 2

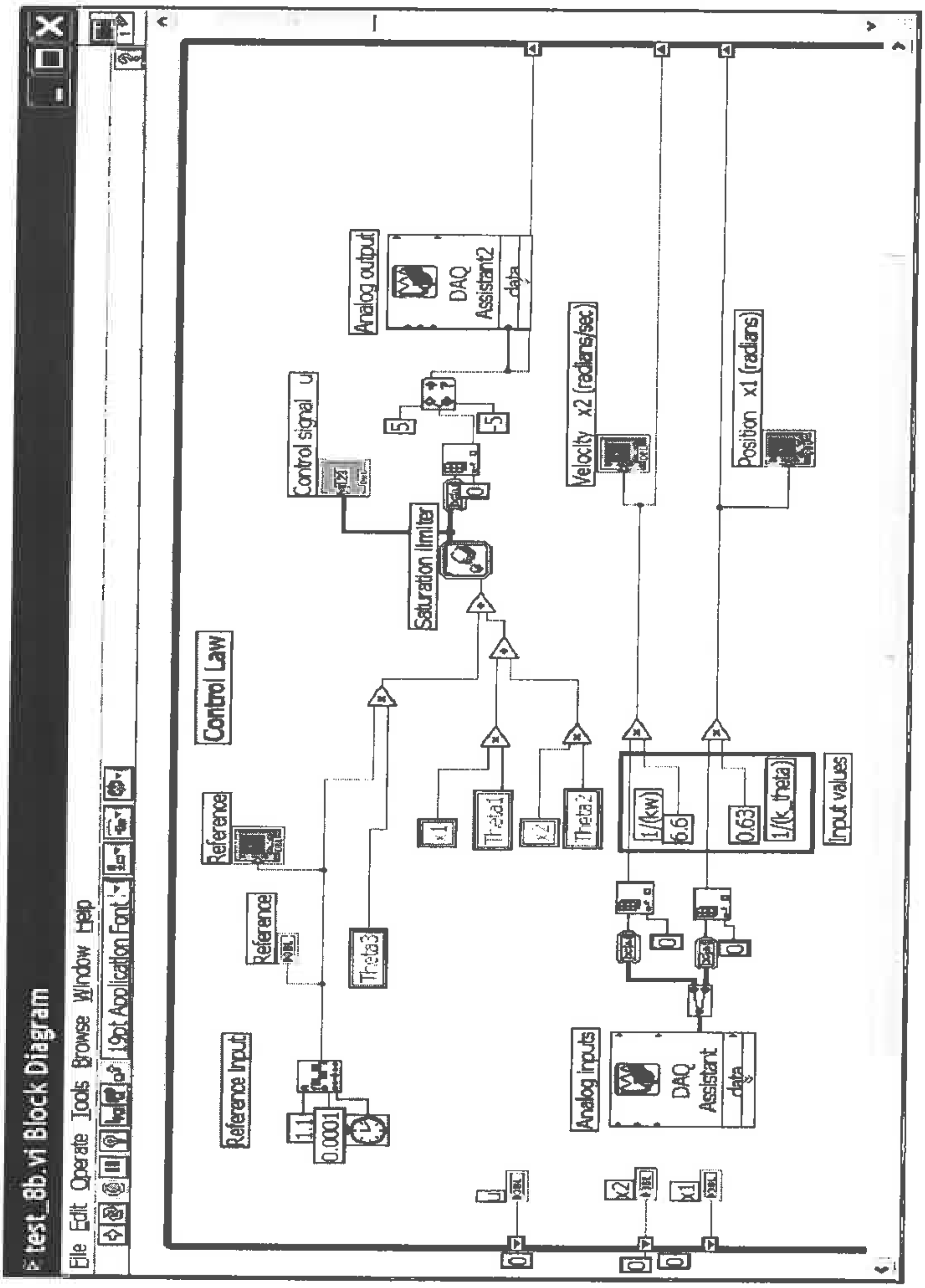
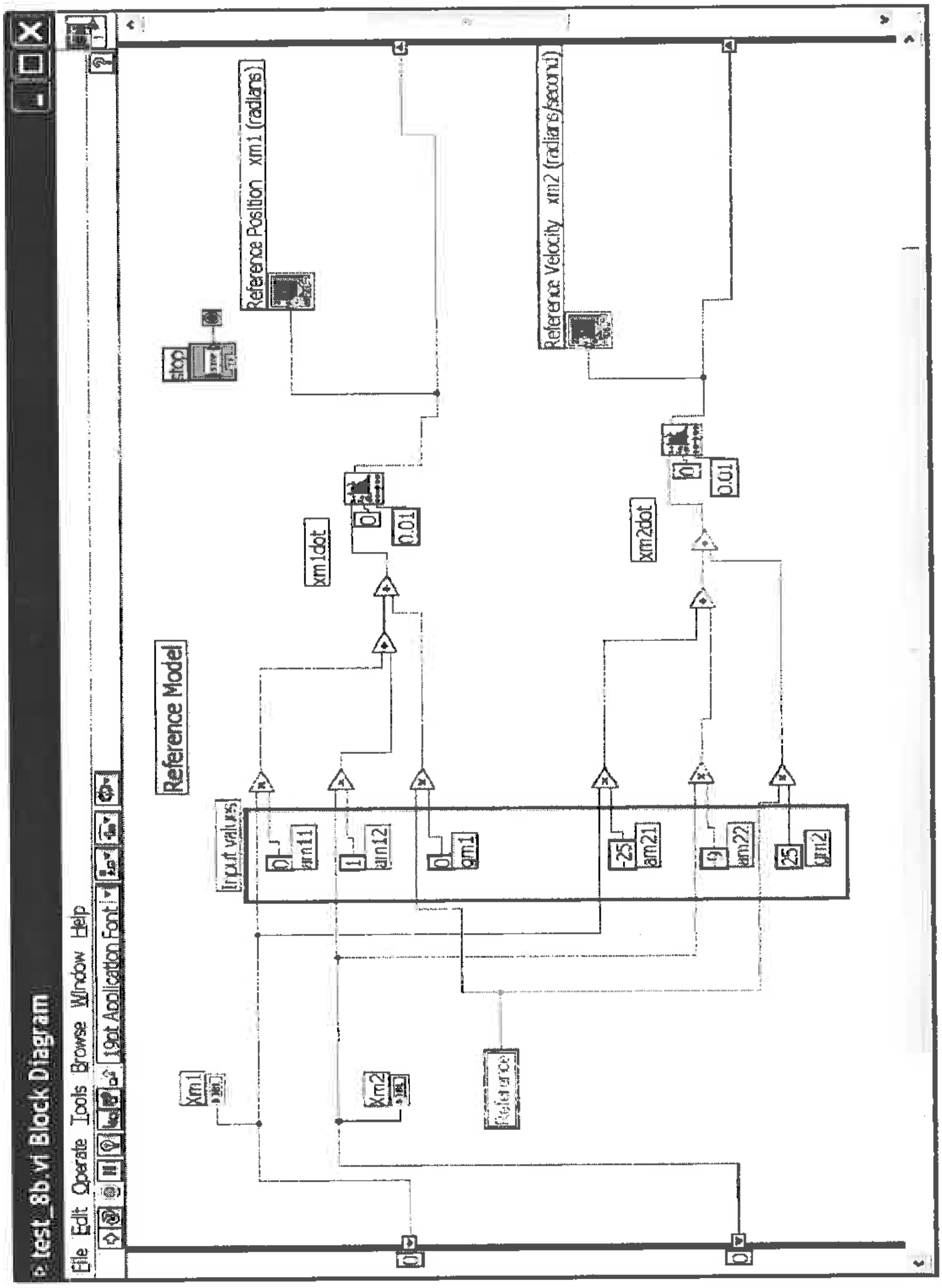

test_8b.vi Block Diagram
File Edit Operate Tools Browse Window Help
19pt Application Font
Reference Input
Reference
Reference
1.1
0.0001
Theta3
Control Law
x1
Theta1
x2
Theta2
Saturation limiter
Control signal u
5
-5
Analog output
DAQ Assistant2
data
Velocity x2 (radians/sec)
Position x1 (radians)
1/(kw)
6.6
0.63
1/(k_theta)
Input values
Analog inputs
DAQ Assistant
data
u
x2
x1
0

Figure 4


test_8b.vi Block Diagram
File Edit Operate Tools Browse Window Help
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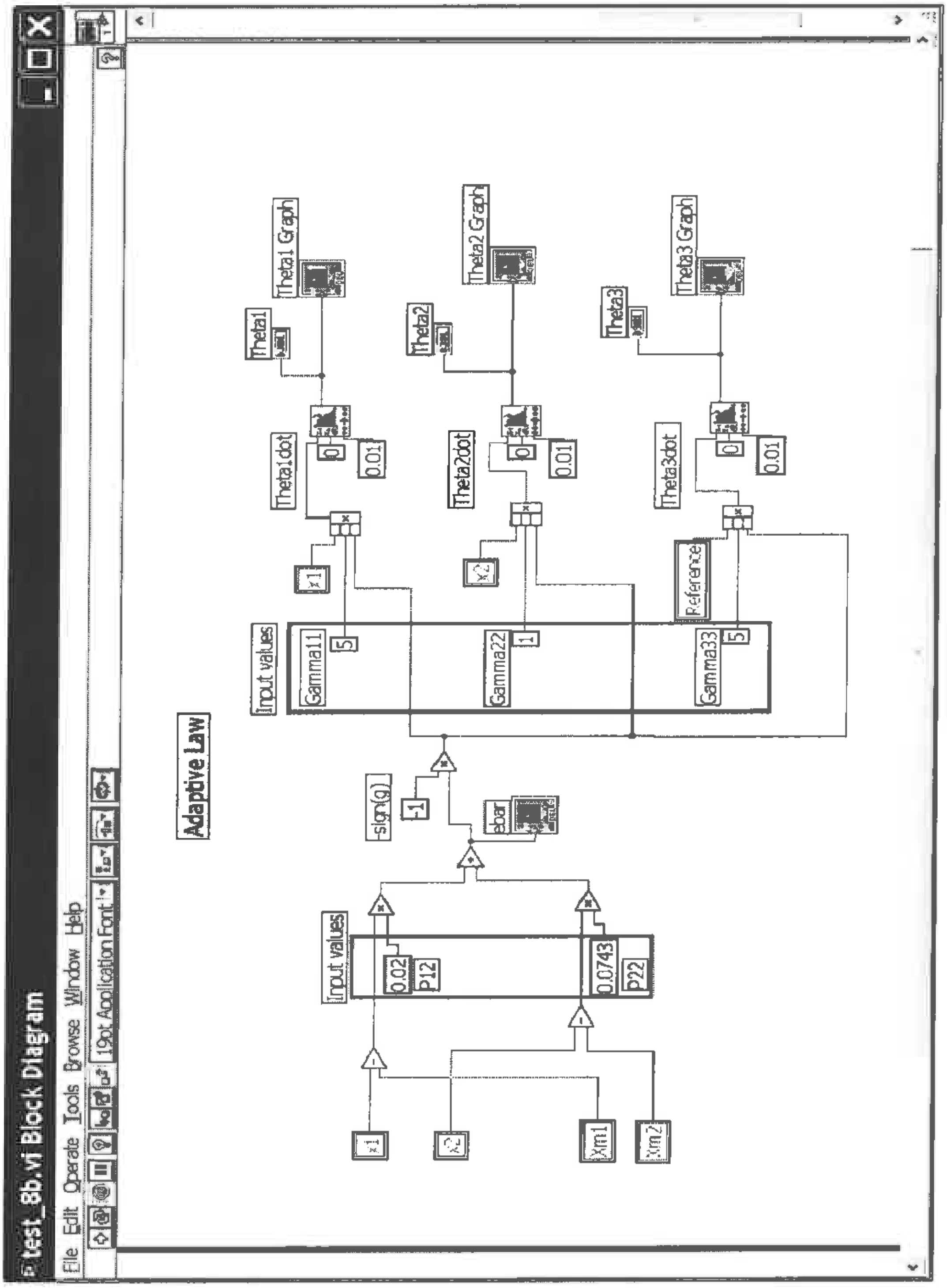
Reference Model
stop
Reference Position xm1 (radians)
Reference Velocity xm2 (radians/second)
xm1dot
xm2dot
0.01
0.01
Input values
0
am11
1
am12
0
gm1
-25
am21
-9
am22
25
gm2
Xm1
Xm2
Reference

Figure 5


test_8b.vi Block Diagram
File Edit Operate Tools Browse Window Help
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Adaptive Law
Input values
0.02
P12
0.0743
P22
x1
x2
Xm1
Xm2
ebar
-sign(g)
-1
Input values
Gamma11
5
Gamma22
1
Gamma33
5
Reference
x1
x2
Theta1dot
Theta2dot
Theta3dot
0
0.01
Theta1
Theta2
Theta3
Theta1 Graph
Theta2 Graph
Theta3 Graph

Figure 6

Q.3 Consider the following constant-velocity state-space model that can be used in tracking problem

$$\begin{aligned} x(k+1) &= Ax(k) \\ y(k) &= Cx(k) + e(k) \end{aligned} \tag{1}$$

where

$$\begin{aligned} x(k) &= \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} \\ A &= \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \\ C &= \begin{bmatrix} 1 & 0 \end{bmatrix} \end{aligned}$$

The position and velocity of the target are given by $x_1(k)$ and $x_2(k)$ respectively. The sampling interval is 1 second and $e(k)$ is a zero-mean independent Gaussian random variable. The velocity is constant for all $k = 1, 2, 3, \ldots, N$ as Equation (1) gives $x_2(k+1) = x_2(k)$. Equation (1) also gives $x_1(k+1) = x_1(k) + x_2(k)$ i.e. the next position is given by the current position plus the product, current velocity $\times$ 1 (time interval).

For this question, although numerical values of A and C are given, you can leave your answers in term of A and C. There is no need to substitute them with their numerical values.

a) By iterating from the initial condition $x(1)$, express $y(N)$ in terms of A, C, $x(1)$ and $e(N)$.

(10 marks)

b) Consider the least-squares cost function

$$J = \frac{1}{2}\sum_{k=1}^{N} \lambda^{N-k} e(k)^2$$

where λ is the forgetting factor. By differentiating J with respect to $x(1)$, obtain the least-squares estimate of the initial condition, $\hat{x}(1)$, with $\lambda = 1$.

(10 marks)

c) By defining the least-squares estimate of $\hat{x}(1)$ at the N^{th} recursion as $\hat{\theta}(N)$ and expressing the covariance matrix $P(N)$ and vector $\phi(N)$ in terms of A and C, write down the recursive least-squares algorithm for $\hat{\theta}(N)$ with $\lambda \leq 1$.

(15 marks)

d) Find $\hat{x}(N)$ in terms of $\hat{x}(1)$.

(5 marks)

Q.4 The input, $u(t)$, and output, $y(t)$, of the plant

$$G_p(s) = \frac{Y(s)}{U(s)} = \frac{K}{(sT+1)^3}$$

connected to a relay in a negative feedback loop are shown in Figures Q4.

a) Find the ultimate gain K_u and ultimate period T_u.

(10 marks)

b) Find K and T.

(10 marks)

c) Sketch the input, $u(t)$, and output, $y(t)$, if the amplitude of the relay and the gain, K, of the plant are halved.

(5 marks)

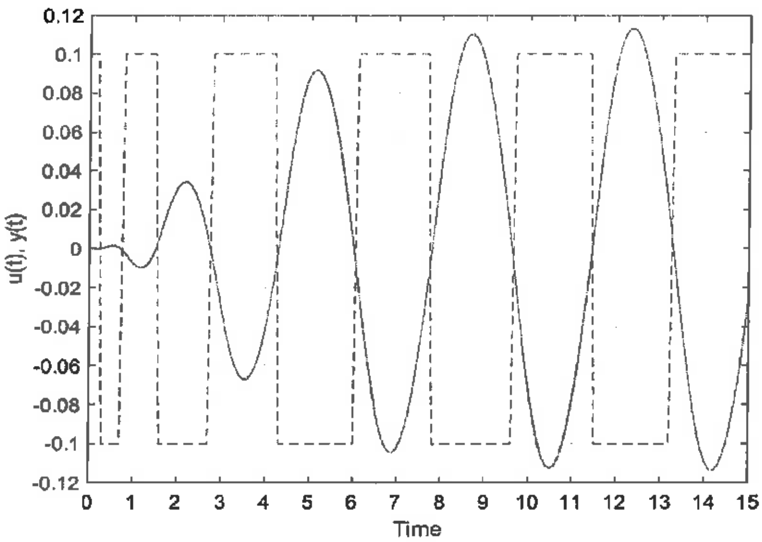

Figure Q4: The plant input, $u(t)$, and output, $y(t)$, are given by the dashed-line and solid-line respectively.

– End of Questions –

DATA SHEET:

0. Prototype Response Tables

	k	Pole Locations for $\omega_0 = 1\ rad/s$[a]
ITAE	1	$s+1$
	2	$s+0.7071 \pm 0.7071j$[b]
	3	$(s+0.7081)(s+0.5210 \pm 1.068j)$
	4	$(s+0.4240 \pm 1.2630j)(s+0.6260 \pm 0.4141j)$
	5	$(s+0.8955)(s+0.3764 \pm 1.2920j)(s+0.5758 \pm 0.5339j)$
Bessel	1	$s+1$
	2	$s+0.8660 \pm 0.5000j$[b]
	3	$(s+0.9420)(s+0.7455 \pm 0.7112j)$
	4	$(s+0.6573 \pm 0.8302j)(s+0.9047 \pm 0.2711j)$
	5	$(s+0.9264)(s+0.5906 \pm 0.9072j)(s+0.8516 \pm 0.4427j)$

[a] Pole locations for other values of ω_0 can be obtained by substituting s/ω_0 for s.
[b] The factors $(s+a+bj)(s+a-bj)$ are written as $(s+a\pm bj)$ to conserve space.

1. The Lyapunov Equation states that given any $n \times n$ stability matrix A_m, for every symmetric positive definite matrix Q, there exists a unique symmetric positive definite matrix P that is the solution to the equation

$$A_m^\top P + PA_m = -Q.$$

In addition, the error system dynamics (with $\mathbf{e} \in \mathbf{R}^n$ and Γ an $n \times n$ symmetric positive-definite matrix) given by

$$\begin{aligned} \dot{\mathbf{e}}(t) &= A_m \mathbf{e}(t) + g\mathbf{b}\phi(t)^\top \mathbf{x}(t) \\ \dot{\phi}(t) &= -\mathrm{sgn}(g)\Gamma \mathbf{e}(t)^\top P\mathbf{b}\,\mathbf{x}(t) \end{aligned}$$

has the properties that $\|\mathbf{e}(t)\|$ and $\|\phi(t)\|$ are bounded, and if it should also be known that $\|\mathbf{x}(t)\|$ is bounded, then additionally

$$\lim_{t\to\infty} \mathbf{e}(t) = 0$$

2. For the triple

$$\begin{aligned} A_m &= \begin{bmatrix} 0 & 1 & 0 \\ a_1 & a_2 & a_3 \\ 1 & 0 & 0 \end{bmatrix} \\ b_m &= \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \\ c_m &= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \end{aligned}$$

the equivalent transfer function is

$$c_m^\top[sI - A_m]^{-1}b_m = \frac{-a_3}{s^3 - a_2s^2 - a_1s - a_3}$$

3. For

$$A_m = \begin{bmatrix} 0 & 1 & 0 \\ -21 & -12 & -10 \\ 1 & 0 & 0 \end{bmatrix}$$

and

$$Q = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

the solution to

$$A_m^\top P + PA_m = -Q.$$

is

$$P = \begin{bmatrix} 9.30 & 0.38 & 5.40 \\ 0.38 & 0.24 & 0.25 \\ 5.40 & 0.25 & 9.01 \end{bmatrix}$$

and the eigenvalues of P are $\lambda = 14.57, 3.76, 0.22$.

4. For

$$A_m = \begin{bmatrix} 0 & 1 & 0 \\ -11 & -7 & -5 \\ 1 & 0 & 0 \end{bmatrix}$$

and

$$Q = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

the solution to

$$A_m^\top P + PA_m = -Q.$$

is

$$P = \begin{bmatrix} 9.92 & 0.76 & 5.83 \\ 0.76 & 0.47 & 0.50 \\ 5.83 & 0.50 & 9.28 \end{bmatrix}$$

and the eigenvalues of P are $\lambda = 15.49, 3.77, 0.40$.

5. For

$$A_m = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}$$

and

$$Q = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

the solution to

$$A_m^\top P + PA_m = -Q.$$

is

$$P = \begin{bmatrix} 7.50 & 2.50 \\ 2.50 & 2.50 \end{bmatrix}$$

and the eigenvalues of P are $\lambda = 8.54, 1.46$.

6. For

$$A_m = \begin{bmatrix} 0 & 1 \\ -2 & -2 \end{bmatrix}$$

and

$$Q = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

the solution to

$$A_m^\top P + PA_m = -Q.$$

is

$$P = \begin{bmatrix} 6.25 & 1.25 \\ 1.25 & 1.875 \end{bmatrix}$$

and the eigenvalues of P are $\lambda = 6.58, 1.54$.

7. For

$$A_m = \begin{bmatrix} 0 & 1 & 0 \\ -3,600 & -120 & -32,000 \\ 1 & 0 & 0 \end{bmatrix}$$

and

$$Q = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

the solution to

$$A_m^\top P + PA_m = -Q.$$

is

$$P = \begin{bmatrix} 105.1 & 0.2 & 720.2 \\ 0.2 & 0.0225 & 0.0 \\ 720.2 & 0.0 & 6,424.4 \end{bmatrix}$$

and the eigenvalues of P are $\lambda = 6,505.4; 24.1; 0.021$.

8. For

$$A_m = \begin{bmatrix} 0 & 1 \\ -400 & -40 \end{bmatrix}$$

and

$$Q = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

the solution to

$$A_m^\top P + PA_m = -Q.$$

is

$$P = \begin{bmatrix} 25.31 & 0.0063 \\ 0.0063 & 0.0627 \end{bmatrix}$$

and the eigenvalues of P are $\lambda = 25.31, 0.0627$.

6. For

$$A_m = \begin{bmatrix} 0 & 1 \\ -400 & -20 \end{bmatrix}$$

and

$$Q = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

the solution to

$$A_m^\top P + P A_m = -Q.$$

is

$$P = \begin{bmatrix} 50.25 & 0.006 \\ 0.006 & 0.125 \end{bmatrix}$$

and the eigenvalues of P are $\lambda = 50.25, 0.125$.

7. The standard discrete-time gradient estimator is

$$\begin{aligned} \hat{y}(j) &= \hat{\theta}(j)^\top \omega(j) \\ e_1(j) &= \hat{y}(j) - y(j) \\ \hat{\theta}(j+1) &= \hat{\theta}(j) - \frac{\omega(j) e_1(j)}{1 + \|\omega(j)\|^2} \end{aligned}$$

It is applicable to the process

$$y(j) = \theta^{*\top} \omega(j)$$

Laplace Transform Table

Laplace Transform, F(s)	Time Function, f(t)
1	$\delta(t)$ (unit impulse)
$\frac{1}{s}$	$u(t)$ (unit step)
$\frac{1}{s^2}$	t
$\frac{1}{s^n}$	$\frac{t^{n-1}}{(n-1)!}$ (n = positive integer)
$\frac{1}{s+a}$	e^{-at}
$\frac{1}{(s+a)^2}$	te^{-at}
$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$
$\frac{1}{s(s+a)^2}$	$\frac{1}{a^2} - \left(\frac{1}{a^2} + \frac{1}{a}t\right)e^{-at}$
$\frac{1}{s^2(s+a)^2}$	$\left(\frac{2}{a^3} + \frac{1}{a^2}t\right)e^{-at} - \frac{2}{a^3} + \frac{1}{a^2}t$
$\frac{1}{(s+a)^n}$	$\frac{1}{(n-1)!}t^{n-1}e^{-at}$ (n = positive integer)
$\frac{1}{(s+a)(s+b)}$	$\frac{e^{-at}-e^{-bt}}{b-a}$
$\frac{1}{s(s+a)(s+b)}$	$\frac{1}{ab}\left[1 + \frac{1}{a-b}(be^{-at} - ae^{-bt})\right]$
$\frac{\omega}{s^2+\omega^2}$	$\sin\omega t$
$\frac{s}{s^2+\omega^2}$	$\cos\omega t$
$\frac{\omega}{(s+a)^2+\omega^2}$	$e^{-at}\sin\omega t$
$\frac{s+a}{(s+a)^2+\omega^2}$	$e^{-at}\cos\omega t$
$\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\frac{\omega_n}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin\omega_n\sqrt{1-\zeta^2}t$
$\frac{s}{s^2+2\zeta\omega_n s+\omega_n^2}$	$-\frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}t-\phi)$ $\phi = \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}$
$\frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$	$1 - \frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}t+\phi)$ $\phi = \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}$

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