JACOBIANS

Jacobian matrix is a multidimensional form of derivative. For example,

$$y_1 = f_1(x_1, x_2, x_3, x_4, x_5, x_6)$$
$$y_2 = f_2(x_1, x_2, x_3, x_4, x_5, x_6)$$
$$\vdots$$
$$y_6 = f_6(x_1, x_2, x_3, x_4, x_5, x_6)$$

vector notation:

$$Y = F(X)$$

Differentials of y_i as a function of differentials of x_j,

$$\delta y_1 = \frac{\partial f_1}{\partial x_1}\delta x_1 + \frac{\partial f_1}{\partial x_2}\delta x_2 + \dots + \frac{\partial f_1}{\partial x_6}\delta x_6$$
$$\delta y_2 = \frac{\partial f_2}{\partial x_1}\delta x_1 + \frac{\partial f_2}{\partial x_2}\delta x_2 + \dots + \frac{\partial f_2}{\partial x_6}\delta x_6$$
$$\vdots$$
$$\delta y_6 = \frac{\partial f_6}{\partial x_1}\delta x_1 + \frac{\partial f_6}{\partial x_2}\delta x_2 + \dots + \frac{\partial f_6}{\partial x_6}\delta x_6 .$$

vector notation:

$$\delta Y = \underline{\frac{\partial F}{\partial X}}\,\delta X$$

6×6 matrix of partial derivatives == Jacobian J.

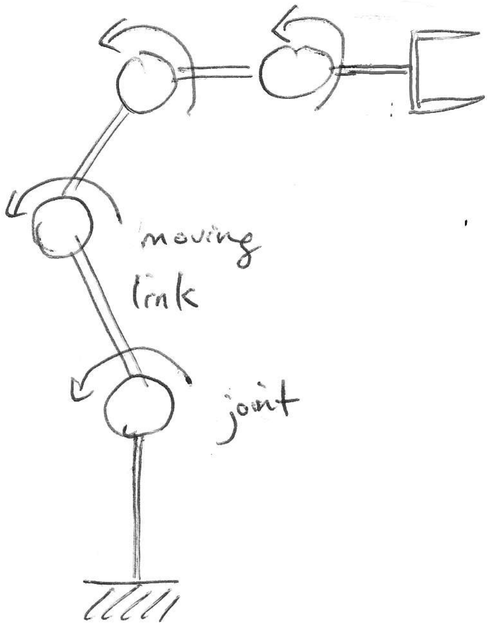
$$\delta Y = J(X)\,\delta X.$$

Dividing both sides by differential time element,

$$\dot{Y} = \underline{J(X)}\,\dot{X}$$

time-varying linear transformation
== maps velocity in X to those in Y.
In robotics == relates joint velocities to Cartesian velocities

Robot manipulator



n moving links

n 1-DOF joints

link : a rigid body (6 parameters — 3 positions and 3 orientations)

1-DOF joint = 5 constraints

n moving links : $6n$ parameters

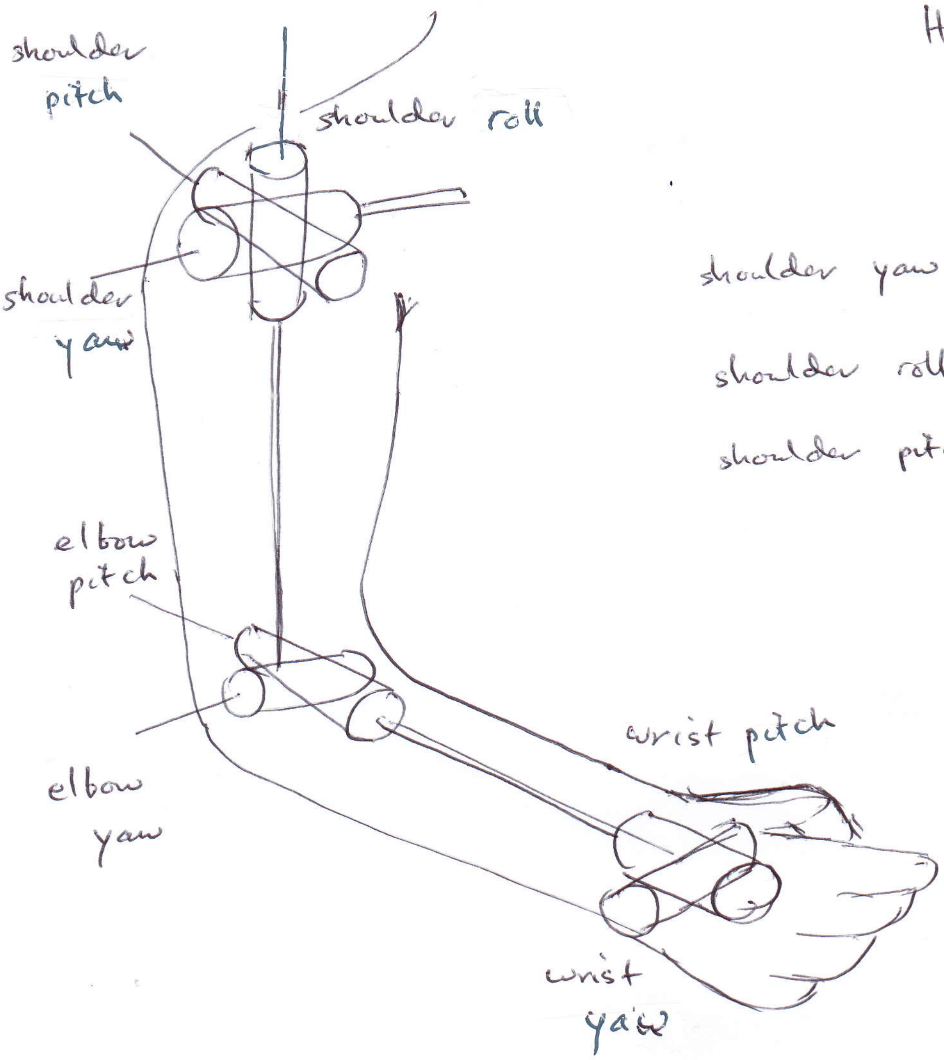
n 1-DOF joints : $5n$ constraints

D.O.F of the system : $6n - 5n = n$ #

Industrial Robot - Every mechanical point on a robot, except in the gripper or tool, at which some form of drive induces motion in a robot part is called a degree of freedom.

Example : 7 DOF of Human Arm

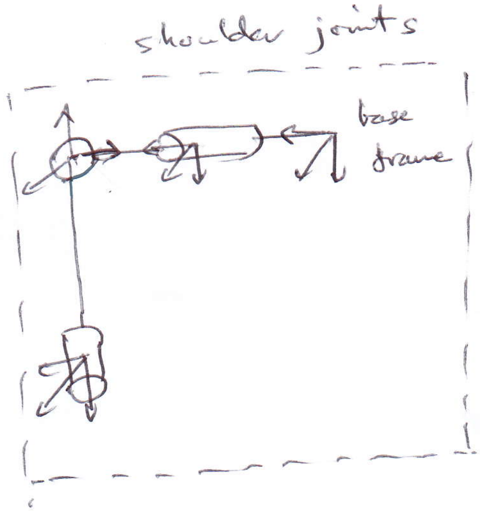
Human hand = 25 DOF

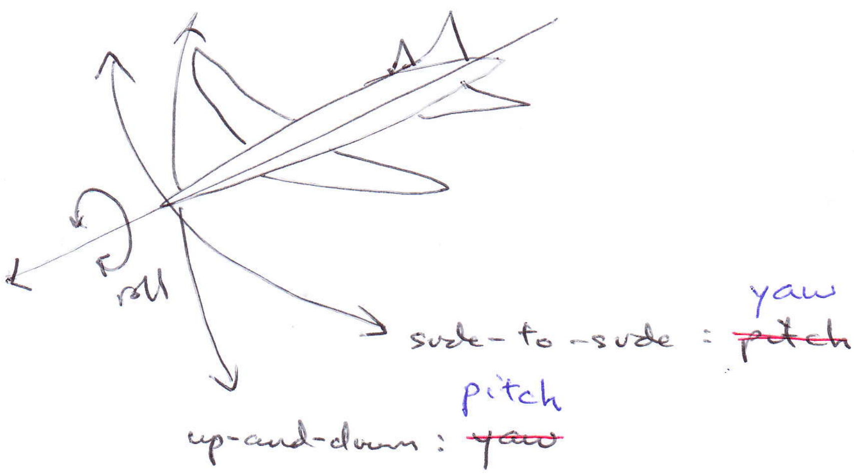


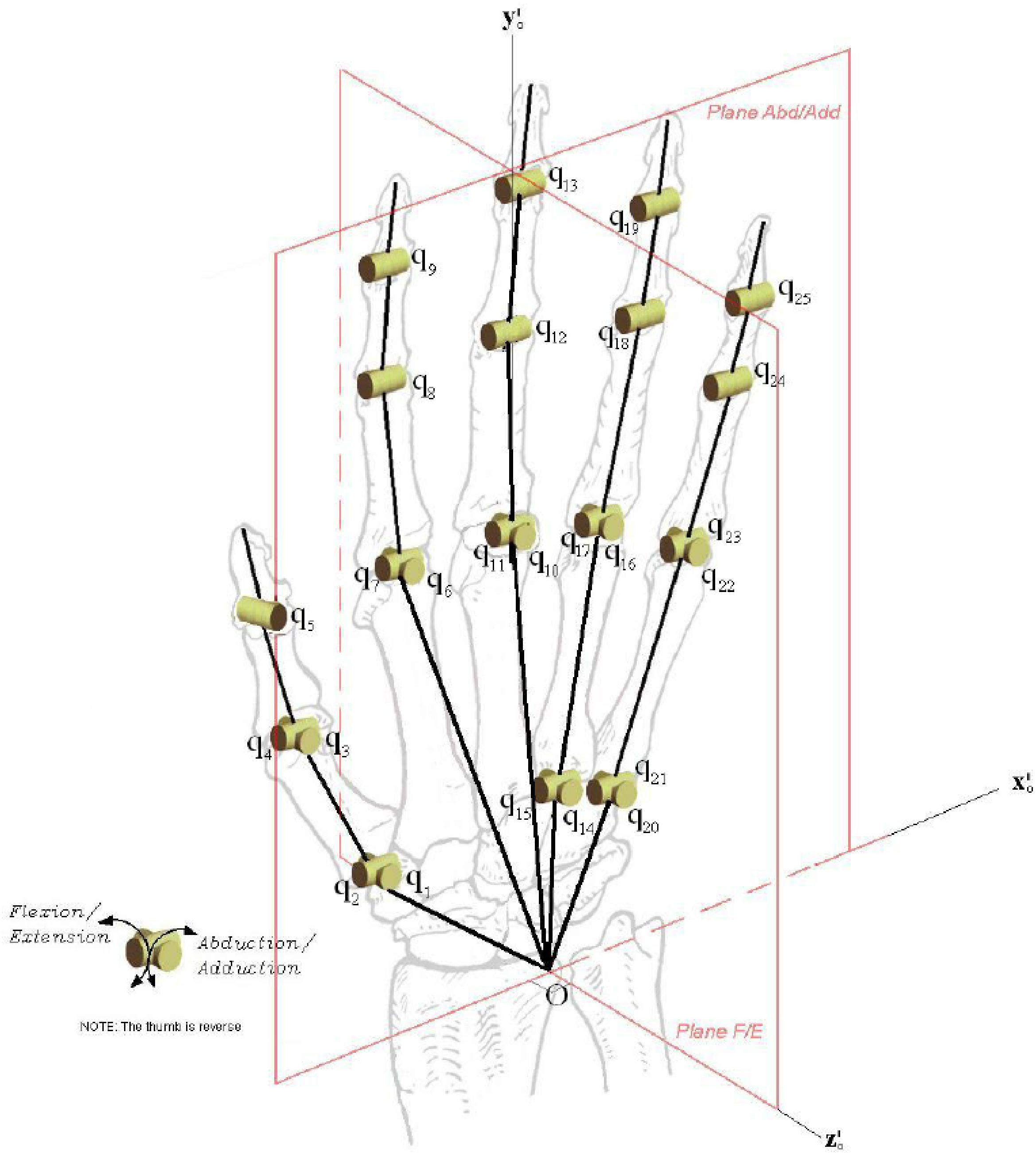
shoulder yaw = internal / external rotation

shoulder roll = abduction / adduction

shoulder pitch = flexion / extension







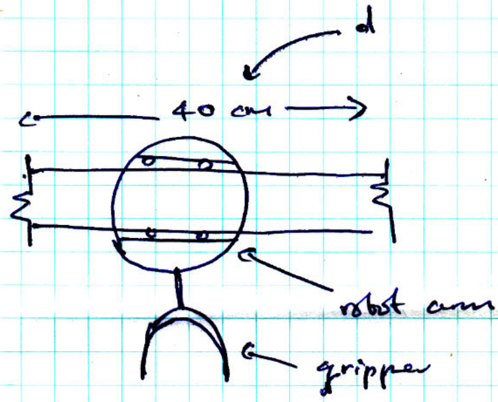
Source: From Internet

C-K Chen
2022/01/09

Resolution, Spatial Resolution, Accuracy and Repeatability

Resolution (or control resolution): smallest increment that the robot controller can control by means of D/A or A/D commands (DACS) for position feedback and control.

Example:



Given: a sliding arm tip can traverse a distance of 40 cm upon receiving $\underbrace{\text{10 bits}}_{n}$ DACS for position feedback and control.

$$\text{Control resolution (CR)} = d/2^n = \frac{40 \text{ cm}}{2^{10}} = \frac{40}{1024} = 0.03906 \text{ cm}$$

Spatial resolution : smallest increment of motion achieved by a robotic manipulator at its tool or end effector. It is the combined resolution including control resolution and motion inaccuracies that originate from mechanical or dynamic interaction of the robot with its environment.

Accuracy = half the control or spatial resolution of a robot.

Repeatability : ability of the robot to return to a taught position, which is influenced by control resolutions and mechanical inaccuracies (gravitational, vibrational, accelerational, deformational).