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Adaptive Systems

Error Dynamical Models

Error Dynamical Model I

$$\dot{e} = A_m e + k_p b \phi^T w$$

Ia

$$\left. \begin{array}{l} k_p \in \mathbb{R}^1 \\ \text{with known sign} \end{array} \right\} \begin{array}{l} e \in \mathbb{R}^n; \\ \phi, w \in \mathbb{R}^{n_1} \end{array}$$

$$\dot{\phi} = -\text{sgn}(k_p) T^T e P b w$$

Ib

where $T \in \mathbb{R}^{n_1 \times n_1} > 0$

sym p-d.

$A_m \in \mathbb{R}^{n \times n}$ is a stability matrix

and P is the sym p-d. soln

$$\text{to } A_m^T P + P A_m = -Q$$

where Q is any suitably chosen sym p-d. matrix.

for the system $(Ia) \Delta (Ib)$, we have the result that:

• $\|e\|, \|\phi\|$ are bounded for all $t \geq t_0$

- If $\|w\|_2$ is bounded, we also have

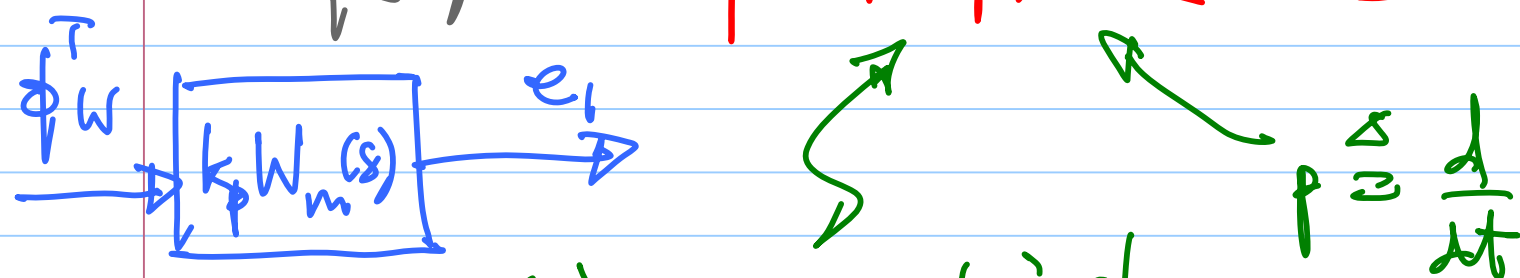
$$\lim_{t \rightarrow \infty} \|e\| = 0$$

△△

Error Dynamical Model II

IIa

$$e_1(t) = k_p W_m(p) [\phi^T w]$$



$W_m(s)$ is a strictly positive-real transfer function in s .

$$e_1(t) \in \mathbb{R}^1; \quad k_p \in \mathbb{R}^1 \quad \begin{matrix} \text{known} \\ \text{sgn}(k_p) \end{matrix}$$

$$\phi, w \in \mathbb{R}^n$$

$$\dot{\phi} = -\text{sgn}(b_p) \Gamma^T e_1(t) w(t)$$

$$\text{--- } \Pi b$$

For the system Πa and Πb ,
we have the result that

$\|\phi\|$ is bounded for all $t \geq t_0$,

and if $\|w\|$ is bounded, then

we also have:

$$\lim_{t \rightarrow \infty} |e_1(t)| = 0$$

$$t \rightarrow \infty$$

$$\Delta \Delta$$

Remark II :

$$a_m < 0$$

the system :

$$\dot{e}_1 = a_m e_1 + k_p \{ \phi y_p + \phi_r r \}$$

$$\begin{bmatrix} \dot{\phi} \\ \dot{\phi}_r \end{bmatrix} = -\operatorname{sgn}(k_p) \begin{bmatrix} \gamma_1 & 0 \\ 0 & \gamma_2 \end{bmatrix} \begin{bmatrix} y_p \\ r \end{bmatrix} e_1$$

is a special case of
the above!

△△

Error Dynamical Model 0

$$e_1(t) = k_p \phi^T w$$

0a

$$e_1 \in \mathbb{R}^1; \quad \phi, w \in \mathbb{R}^{n_1}$$

$$k_p \in \mathbb{R}^1; \quad \text{sgn}(k_p) \text{ known}$$

$$\dot{\phi} = -\text{sgn}(k_p) \Gamma^T w e_1$$

Γ symmetric p.d.

0b

For the system 0a and 0b, we have the result that

$\Delta\Delta$

$\|\phi\|$ is bounded for all $t > t_0$



Remark 0:

If, in addition, the signal $w(t)$ is "persistently exciting", then we also have:

$$\lim_{t \rightarrow \infty} \|\phi(t)\| = 0$$

But the condition of $w(t)$ being "persistently exciting" is a rather stringent & difficult one.



How?

A signal $w(t) \in \mathbb{R}^n$ is p.e. iff there exists $\delta, \epsilon > 0$ s.t.

$$\int_t^{t+\delta} w(\tau)^T w(\tau) d\tau \geq \epsilon$$

Stability Analysis of

Error Dynamical Model

for all $\|W\| = 1$
with $W \in \mathbb{R}^n$;
and all $t > t_0$.

Consider the quadratic form

$$V(t) = \|\phi\|^2 = \phi^T \phi \quad \text{for } P = I$$

then

$$\dot{V}(t) = 2 \phi^T \dot{\phi}$$

$$= 2 \phi^T \left\{ -\text{sgn}(k_p) W e_1 \right\}$$

$$= -2 \text{sgn}(k_p) \frac{1}{k_p} e_1 e_1$$

$$= -2 \text{sgn}(k_p) \frac{\text{sgn}(k_p)}{|k_p|} e_1^2$$

≤ 0

$\Delta \Delta$

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Parts of stability analysis
for $u^* > 1$ case