Note Title 9/21/2011

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Estimator

$$R_p(p)\, y(t) = Z_p(p)\, u(t)$$

$$R_p(p) = p^n + a_1 p^{n-1} + \dots + a_n$$

$$Z_p(p) = b_0 p^m + b_1 p^{m-1} + \dots + b_m$$

Typical development is to work out a suitable "Linear-in-the-parameters" structure.

One way, here, is as in the Lecture Notes approach ...

Thus, consider for example, $n=2$

$$R_p(p) = p^2 + a_1 p + a_2$$

$$Z_p(p) = b_0 p + b_1$$

Then, choose a suitable

$$T_2(p) = p^3 + t_1 p^2 + t_2 p + t_3$$

Hurwitz !!
(order $= n+1$)

Define

$$y^{f_2}(t) = \frac{t_3}{T_2(p)} y(t) \triangleq w_{1_y}(t) \text{ say}$$

Then, generate the signals

$$\dot{w}_{1_y}(t) = w_{2_y}(t) \quad = p\, y^{f_2}$$

$$\dot{w}_{2_y}(t) = w_{3_y}(t) \quad = p^2 y^{f_2}$$

and $$\dot{w}_{3_y}(t) = p^3 y^{f_2}(t)$$

$$= -t_3 w_{1_y} - t_2 w_{2_y} - t_1 w_{3_y} + t_3 y$$

And generate a similar set for

$$w^{f_2}(t) = \frac{t_3}{T_2(p)} u(t) \quad \triangleq w_{1_u}(t)$$

Then, for the system

$$(p^2 + a_1 p + a_2)\, y(t) = (b_0 p + b_1)\, u(t)$$

Filter both sides with : $\dfrac{t_3}{T_2(p)}$

We can write as

$$(p^2 + a_1 p + a_2)\, y^{f_2}(t) = (b_0 p + b_1)\, u^{f_2}$$

OR

$$W_{3_y}(t) + a_1 W_{2_y}(t) + a_2 W_{1_y}(t)$$

$$= b_0 W_{2_u}(t) + b_1 W_{1_u}(t)$$

OR

$$W_{3_y}(t) = \begin{bmatrix} -a_1 & -a_2 & b_0 & b_1 \end{bmatrix} \begin{bmatrix} W_{2_y}(t) \\ W_{1_y}(t) \\ W_{2_u}(t) \\ W_{1_u}(t) \end{bmatrix}$$

parameters to be estimated

available generated signals

$$= \theta^{*^T} w(t)$$

a "Linear-in-the-Parameters" form !!!

This is, of course, not the only way to do this.

Think: Can you do something equivalent (not identical) using a filter $\frac{t_2}{T_1(p)}$, with

$$T_1(p) = p^2 + t_1 p + t_2$$

Hurwitz !!

i.e. using only a Hurwitz filter of order = n ?

△△

Once we have a "Linear-in-the-parameter" structure:

$$z(t) = \theta^{*T} w(t) \qquad (6.11)$$

available signal/measurement

parameters to be estimated

available signals

It is quite straightforward to note that we can use an estimation structure of the form:

$$\hat{z}_1(t) = \hat{\theta}^T(t)\, w(t) \qquad \text{— (6.12a)}$$

$$\dot{\hat{\theta}}(t) = -\Gamma\, w(t)\, e_1(t) \qquad \text{— (6.12b)}$$

where $\quad e_1(t) = \hat{z}(t) - z(t)$

$$\text{— (6.12c)}$$

Should be able to see now that for the error

$$\tilde{\theta}(t) \triangleq \hat{\theta}(t) - \theta^*$$

For the above (6.11) and (6.12), we will have

$\phi^T \Gamma^{-1} \phi$

$$\frac{d}{dt}\left\{ \tilde{\theta}^T(t)\, \Gamma^{-1}\, \tilde{\theta}(t) \right\} \leq 0$$

i.e. estimation errors which
are guaranteed "non-increasing".

$\triangle\triangle\triangle$

Then, with the estimates of
$\hat{\theta} = \{ \hat{a}_1, \hat{a}_2, \hat{b}_0, \hat{b}_1 \}$, use
any preferred control law
computation from EE 5101 ....

No complete rigorous stability
proof; but works well in
many situations ....

Discrete-time Systems

$j = 0, 1, 2, 3 \ldots$

Consider the plant

$$A(q^{-1})\, y(j) = q^{-d} B(q^{-1})\, u(j) + e(j) \qquad \text{(2.1)}$$

with

$$A(q^{-1}) = 1 + a_1 q^{-1} + \ldots + a_n q^{-n}$$

$$B(q^{-1}) = b_0 + b_1 q^{-1} + \ldots + b_m q^{-m}$$

$\{e(j)\}$ uncorrelated noise sequence with variance σ^2

Consider the "Prediction Identity"

$$1 = A(q^{-1})E(q^{-1}) + q^{-d}F(q^{-1})$$

$$\deg(E) = d-1$$
$$\deg(F) = n-1$$

$\Delta\Delta$

Note that this follows from noting that multiplying both sides by $q^n q^{d-1}$ gives:

$$q^n q^{d-1} = A^*(q)E^*(q) + F^*(q) \qquad \text{—— (2.5)}$$

where

$$A^*(q) = q^n + a_1 q^{n-1} + \dots + a_n$$

$$E^{*}(q) = e_0 q^{d-1} + e_1 q^{d-2} + \dots + e_{d-1}$$

$$F^{*}(q) = f_0 q^{n-1} + f_1 q^{n-2} + \dots + f_{n-1}$$

and (2.5) immediately follows as being true (as before!) from simple polynomial division. !!!

$\triangle\triangle$

Then, for the plant (2.1), we can write:

$$A(q^{-1})\, y(j) = q^{-d} B(q^{-1})\, u(j) + e(j)$$

$$E(q^{-1}) A(q^{-1})\, y(j) = q^{-d} E B\, u(j) + E\, e(j)$$

$$\left\{ 1 - q^{-d} F \right\} y(j) = q^{-d} E B\, u(j) + E\, e(j)$$

i.e.

$$y(j) = q^{-d} F(q^{-1})\, y(j) + q^{-d} E(q^{-1}) B(q^{-1})\, u(j) + E(q^{-1})\, e(j)$$

OR equivalently:

$G(q^{-1})$ deg = $m+d-1$

$-r(j)$

$$y(j+d) = \underline{F(q^{-1})\, y(j) + \overbrace{E(q^{-1}) B(q^{-1})}^{G(q^{-1})}\, u(j)} + E(q^{-1})\, e(j+d)$$

$-r(j)$

$= 0$

$$F(q^{-1}) = f_0 + f_1 q^{-1} + \dots + f_{n-1} q^{-(n-1)}$$

$$G(q^{-1}) = g_0 + g_1 q^{-1} + \dots + g_{m+d-1} q^{-(m+d-1)}$$

$$E(q^{-1}) = e_0 + e_1 q^{-1} + \dots + e_{d-1} q^{-(d-1)}$$

i.e. $E(q^{-1})\, e(j+d) = e_0\, e(j+d) + e_1\, e(j+d-1) + \dots + e_{d-1}\, e(j+1)$

Based on the above, and because in a causal system, it is not possible for $u(j)$ to "anticipate" $e(j+l)$ for $l \geq 1$, in the above, $E\{[y(j+d)]^2\}$ the "Minimum Variance Control Law" must be to set:

$$F(q^{-1})\,y(j) + \underbrace{E(q^{-1})B(q^{-1})}_{G(q^{-1})}u(j) = 0$$

$$\left\{f_0 + f_1 q^{-1} + \ldots f_{n-1} q^{-(n-1)}\right\} y(j)$$

$$+ \left\{g_0 + g_1 q^{-1} + \ldots + g_{m+d-1} q^{-(m+d-1)}\right\} u(j)$$

$$= 0$$

$\Delta\Delta$

Extending from the above, it is straightforward to see that "Minimum Variance for the Set-point tracking" as given by

$$E\left\{\left[y(j+d) - r(j)\right]^2\right\}$$

where $\{r(j)\}$ is the set-point sequence is thus given by

$$\left\{f_0 + f_1 q^{-1} + \dots f_{n-1} q^{-(n-1)}\right\} y(j) + \left\{g_0 + g_1 q^{-1} + \dots + g_{m+d-1} q^{-(m+d-1)}\right\} u(j) = r(j)$$

$\Delta\Delta$