Case: Arbitrary but known n^* $(n^* > 1)$

- Propositions 1 and 2 are independent of n^*
  Thus, they are still true here.

- However, $\dfrac{k_m}{R_m(s)}$ not SPR
  Stability analysis does not apply!

In fact stability analysis turns out to be very difficult for this case. The problem was unresolved for about 5 years until 1980.

In 1980, it was independently solved in

[1] Narendra, Lin, Valavani, “Stable Adaptive Control,Part-II: Proof of Stability,” IEEE Trans. AC-25, pp440-448, Jun. 1980.

[2] Morse, “Globally Stable Parameter Adaptive Control System,” IEEE-AC-25, pp433-439, Jun. 1980.

[3] Goodwin, Ramadge and Caines, “Discrete-time Multivariable Adaptive Control,” IEEE AC-25, pp449-456, Jun. 1980.

Adaptive Controller for $n^* > 1$

Plant

$$R_p y = k_p Z_p u$$

Control Law:

$$y^{f_1} = \frac{1}{T} y, \qquad u^{f_1} = \frac{1}{T} u$$

$$\omega \stackrel{\Delta}{=} \left[p^{n-1} y^{f_1}, p^{n-2} y^{f_1}, \cdots, y^{f_1}, p^{n-1} u^{f_1}, p^{n-2} u^{f_1}, \cdots, u^{f_1} \right]^T$$

$$\overline{\omega} \stackrel{\Delta}{=} \left[\omega^T \quad r \right]$$

$$u(t) = \theta(t)^T \omega(t) + k(t) r(t) = \overline{\theta}^T \overline{\omega}$$

Control Law is the same as $n^* = 1$!! (Why?)

Adaptive Law

$$W_m(p) \stackrel{\Delta}{=} \frac{k_m}{R_m(p)}$$ The reference model

[$n^* > 1$ Not SPR]

$$\xi(t) = W_m \omega, \qquad \overline{\xi}(t) = W_m \overline{\omega}$$

$$e_1(t) = y(t) - y_m(t)$$

$$e_2(t) = \overline{\theta}^T \overline{\xi} - W_m \left\{ \overline{\theta}^T \overline{\omega} \right\}$$ Auxiliary error

$$\varepsilon(t) = e_1(t) + \theta_{e_2}^T(t) e_2(t)$$ Augmented error

Γ s.p.d.

$\gamma_2 > 0$

$$\dot{\overline{\theta}}(t) = -\text{sgn}(k_p) \Gamma \frac{\overline{\xi}(t) \varepsilon(t)}{\lambda\, r_1^2(t)}$$

$$\dot{\theta}_{e_2}(t) = -\gamma_2 \frac{e_2(t) \varepsilon(t)}{\lambda\, r_1^2(t)}$$

$$r_1(t) = \sqrt{1 + \left\| \overline{\xi}(t) \right\|^2}$$

The adaptive controller is rather complicated.

Why does Adaptive Law have to be different?

[EE6104 CA1 mini-project will investigate this ...]

* Proof of the boundedness of $\|\theta(t)\|$ is relatively straightforward.

* It is difficult to prove the boundedness of $\|\omega(t)\|$.

Proof of stability: (a) Boundedness of $\|\bar{\theta}(t)\|$

From earlier result, there exist θ^* and k^* such that for

This, same as for $n^* = 1$

$$\phi(t) \stackrel{\Delta}{=} \theta(t) - \theta^*$$

$$\phi_r(t) \stackrel{\Delta}{=} k(t) - k^*$$

$$e_1(t) = \frac{k_p}{k_m} W_m(p)\{\phi^T \omega + \phi_r r\}$$

$$e_1(t) = \frac{k_p}{k_m} W_m(p)\{\bar{\phi}^T \bar{\omega}\}$$

p.55 , $k_p k^* = k_m$

$$\boxed{e_1 = \frac{1}{k^*} W_m \bar{\phi}^T \bar{\omega} = \frac{k_p}{k_m} W_m \bar{\phi}^T \bar{\omega}}$$

Also see Narendra, p.210

$\bar{\theta}^* \equiv$ constant

Note next that

$$e_2(t) = \bar{\theta}^T \underbrace{W_m \bar{\omega}}_{\bar{\xi}} - W_m \bar{\theta}^T \bar{\omega}$$

$$= \bar{\theta}^T W_m \bar{\omega} - W_m \bar{\theta}^T \bar{\omega} \quad - \bar{\theta}^{*T} W_m \bar{\omega} + W_m \bar{\theta}^{*T} \bar{\omega} \quad (= 0)$$

$$= \bar{\phi}^T W_m \bar{\omega} - W_m \bar{\phi}^T \bar{\omega}$$

$$\boxed{e_2(t) = \bar{\phi}^T \bar{\xi} - W_m \bar{\phi}^T \bar{\omega}}$$

auxiliary error signal

and clearly

$$\frac{k_p}{k_m} e_2 = \frac{k_p}{k_m} \bar{\phi}^T \bar{\xi} - \frac{k_p}{k_m} W_m \bar{\phi}^T \bar{\omega} = \frac{k_p}{k_m} \bar{\phi}^T \bar{\xi} - e_1$$

intermediate result ...

Thus

$e_1(t)$ from above ...

$$\varepsilon = e_1 + \theta_{e_2} e_2 = \underbrace{\frac{k_p}{k_m}\bar{\phi}^T \bar{\xi} - \frac{k_p}{k_m} e_2}_{e_1(t)} + \theta_{e_2} e_2$$

$$\boxed{\varepsilon(t) = \frac{k_p}{k_m}\bar{\phi}^T \bar{\xi} + \phi_{e_2} e_2} \qquad \boxed{(1)}$$

Augmented error signal

where

$$\phi_{e_2} \stackrel{\Delta}{=} \theta_{e_2}(t) - \frac{k_p}{k_m}$$

For this $n^* > 1$ case, we can only work with the error equation (1) first.

Consider

$$V\left(\bar{\phi}, \phi_{e_2}\right) = \frac{|k_p|}{k_m}\bar{\phi}^T \bar{\phi} + \phi_{e_2}^2$$

Different from previously, we only have a Lyapunov function candidate in ϕ, ϕ_{e_2}. Therefore, condition about the boundedness of ω is not possible from this V. It has to be shown separately. (Why?)

$\dot{\theta}$ $\dot{\theta}_{e_2}$

$$\dot{V} = 2\frac{|k_p|}{k_m}\bar{\phi}^T\dot{\bar{\phi}} + 2\phi_{e_2}\dot{\phi}_{e_2}$$

$$= -2\,\mathrm{sgn}(k_p)\frac{|k_p|}{k_m}\frac{\bar{\phi}^T\bar{\xi}\varepsilon}{r_1^2} - 2\frac{\phi_{e_2}e_2\varepsilon}{r_1^2}$$

$$= -\frac{2}{r_1^2}\left\{\frac{k_p}{k_m}\bar{\phi}^T\bar{\xi} + \phi_{e_2}e_2\right\}\varepsilon$$

$$= -2\frac{\varepsilon^2}{r_1^2} \le 0$$

C1: Thus, $\|\bar{\theta}(t)\|$ and $|\theta_{e_2}(t)|$ are bounded (for ϕ, ϕ_{e_2} are bounded plus for θ^*, k^* are bounded) .

C2: In addition, $\int_0^\infty \frac{\varepsilon^2(\tau)}{r_1^2(\tau)}d\tau \le c_1$

Notationally, $\frac{\varepsilon(t)}{r_1(t)} \in L^2[0,\infty)$ ※

Recall, from p 59, $r_1(t) = \sqrt{1 + \xi(t)^2}$

(C3) Since $\frac{\|\bar{\xi}(t)\|}{r_1(t)} \le 1$, this also means that

$\dot{\bar{\phi}}(t) \in L^2[0,\infty)$

$$\boxed{\begin{aligned}\dot{\bar{\phi}} = \dot{\bar{\theta}} - \dot{\bar{\theta}}^* &= -\mathrm{sgn}(k_p)\frac{\bar{\xi}\varepsilon}{r_1^2(t)} \\ &= -\mathrm{sgn}(k_p)\frac{\bar{\xi}}{r_1(t)}\,\frac{\varepsilon}{r_1(t)}\end{aligned}}$$

Roughly speaking, if $\beta(t) \in L^2[0,\infty)$, approximately $\lim_{t\to\infty}\beta(t) = 0$

* $PC_{[0,\infty)}$ = The set of all real piecewise continuous defined on $[0,\infty)$ which have bounded discontinuities. -----Narendra, p.476

* **Definition 1:** Narendra, p.477
Let $x_1 : \Re \to \Re$, and $x_2 : \Re \to \Re \in PC_{[0,\infty)}$. $\Re$ is the set of real numbers. Then

$$x_1(t) = O[x_2(t)]$$

is defined by

$$x_1(t) \le c_1 x_2(t) + c_2$$

where c_1, c_2 are positive real numbers.

* **Definition 2:**
Let $x_1 : \Re \to \Re$, and $x_2 : \Re \to \Re \in PC_{[0,\infty)}$. $\Re$ is the set of real numbers. Then

$$x_1(t) = o[x_2(t)]$$

is defined by

$$x_1(t) = \beta_1(t) x_2(t)$$

where $\beta_1(t) \to 0$ as $t \to \infty$.

* **Definition 3:**
Let $x_1 : \Re \to \Re$, and $x_2 : \Re \to \Re \in PC_{[0,\infty)}$. $\Re$ is the set of real numbers. Then

$$x_1(t) \sim x_2(t)$$

is defined by

$$x_1(t) = O[x_2(t)], \qquad x_2(t) = O[x_1(t)]$$

For your reading pleasure only. Not for exams!

Proof of stability: (b) Boundedness of $\|\omega(t)\|$

Recall

$$\dot{\omega} = A_m\omega + b_p\left(\phi^T\omega + kr\right)$$

We already know $\|\phi\|$, $|k|$ are bounded. Thus,

$$\|\dot{\omega}\| \le c_3\|\omega\| + c_4 \tag{7}$$

Note that

$$\xi(t) = W_m(p)\omega(t) \tag{8}$$

$W_m(p)$ is a stable, proper operator, we have

$$\|\xi(t)\| = \mathrm{O}[\sup_{\tau\le t}\|\omega(\tau)\|]$$ ----------------Narendra p.215

In conjunction with (7) above, the reverse is also true

$$\|\omega(t)\| = \mathrm{O}[\sup_{\tau\le t}\|\xi(\tau)\|]$$

Thus

$$\sup_{\tau\le t}\|\xi(\tau)\| \sim \sup_{\tau\le t}\|\omega(\tau)\|$$

i.e. ξ and ω must "grow" at the same rate.

. . .

It can be shown that

$$TR_m Z_p y^{f_1} = k_m Z_p r + k_p Z_p \left\{ \bar{\phi}^T \bar{\omega} \right\} \tag{8+}$$

[**HOME WORK!**]

i.e.,

$$y^{f_1} = \frac{1}{T}\frac{k_m}{R_m} r + \frac{1}{T}\frac{k_p}{k_m}\frac{k_m}{R_m}\left\{ \bar{\phi}^T \bar{\omega} \right\}$$

$$= \frac{1}{T} W_m r + \frac{1}{T}\frac{k_p}{k_m} W_m \left\{ \bar{\phi}^T \bar{\omega} \right\}$$

and $$u^{f_1} = \frac{R_p}{k_p Z_p} y^{f_1}$$

Thus,

$$y^{f_1} = \frac{1}{T} W_m r + \frac{1}{T}\frac{k_p}{k_m}\left\{ \bar{\phi}^T \bar{\xi} - e_2 \right\}$$

$$= \frac{1}{T} W_m r + \frac{1}{T}\left\{ \varepsilon - \theta_{e_2} e_2 \right\} \tag{8a}$$

$$u^{f_1} = \frac{R_p}{k_p Z_p T} W_m r + \frac{R_p}{k_p Z_p T}\left\{ \varepsilon - \theta_{e_2} e_2 \right\} \tag{8b}$$

Note that θ_{e_2} is bounded, and because $\dot{\bar{\phi}} \in L^2[0,\infty)$

$$e_2(t) = o\left[\sup_{\tau \le t} \|\omega(\tau)\| \right]$$

But y^{f_1} and u^{f_1} form the basis for ω(t). In addition, Z_p and T are stable. Thus, (8a) and (8b) imply

$$\sup_{\tau \le t} \|\omega(\tau)\| = \mathrm{O}[\sup_{\tau \le t} \|\varepsilon(\tau)\|] \tag{9}$$

However, condition (C2) of the adaptive law implies

$$\frac{\varepsilon(t)}{r_1(t)} = \beta_1(t), \qquad \beta_1(t) \in L^2[0,\infty)$$

Thus $$|\varepsilon(t)| = \beta_1(t)\sqrt{1+\|\xi^2\|}$$
$$= o\left[\sup_{\tau\le t}\|\xi(\tau)\|\right] = o\left[\sup_{\tau\le t}\|\omega(\tau)\|\right] \qquad (10)$$

Equations (9) and (10) are impossible if $\|\omega(t)\|$ is unbounded.

Thus, $\|\omega(t)\|$ is in fact bounded.

Then, we have

$$e_2(t) = o\left[\sup_{\tau\le t}\|\omega(\tau)\|\right] \Rightarrow \lim_{t\to\infty} e_2(t) = 0$$

Equation (10) implies

$$\lim_{t\to\infty} \varepsilon(t) = 0$$

Bounded $\theta_{e_2}(t)$ then implies

$$\lim_{t\to\infty} e_1(t) = \lim_{t\to\infty} \{\varepsilon(t) - \theta_{e_2}(t) e_2(t)\} = 0.$$

(End of "Not for Exams" part)

Result: [Based on overall system in p59]

For the adaptive controller applies to the plant, if

(a1) the relative degree n^* is known;

(a2) the order of plant n is known;

(a3) Z_p is a stable polynomial; and

(a4) $\operatorname{sgn}(k_p)$ is known

then, y(t), u(t), $\bar{\theta}(t)$, $\theta_{e_2}(t)$ are bounded $\forall t \geq 0$,

and $$\lim_{t \to \infty} \left(y(t) - y_m(t) \right) = 0$$

△△△

[EE6104 CA1 mini-project,
you are all set to go...]

Continuous-time adaptive control using only input-output measurements non-rigorous approach

- Thus far, we have considered mathematically rigorous formulation:

  -- boundedness of all signals proved

$$\theta(t), y(t), u(t)$$

  -- convergence of tracking error, i.e. $\lim_{t\to\infty}\left(y(t) - y_m(t)\right) = 0$

  However, this is usually very difficult

- Alternatively, disregard boundedness analysis

  -- simply combine a "good" estimator with a particular control law

  -- then incorporate additional checks to approximately ensure everything work.

Estimator

Plant

$$R_p(p)y(t) = Z_p(p)u(t)$$
$$R_p(p) = p^n + a_1 p^{n-1} + \cdots + a_n$$
$$Z_p(p) = b_0 p^m + b_1 p^{m-1} + \cdots + b_m$$

k_p is absorbed in Z_p

Re-write in a form suitable for parameter estimation (Recall LIP form!)

Define signals

$$y^{f_2}(t) = \frac{t_{n+1}}{T_2(p)} y(t)$$
$$u^{f_2}(t) = \frac{t_{n+1}}{T_2(p)} u(t)$$

where

$$T_2(p) = p^{n+1} + t_1 p^n + t_2 p^{n-1} + \cdots + t_{n+1}$$

is a stable polynomial.

$\frac{t_{n+1}}{T_2(p)} \rightsquigarrow R_p(p)\, y(t) = Z_p(p)\, u(t)$

Then

$$R_p(p) y^{f_2}(t) = Z_p(p) u^{f_2}(t)$$

and it can be re-written as, with new signal $z(t) \in \mathbb{R}^1$,

$$Z(t) = p^n y^{f2}(t) = \theta^{*T} \varphi(t)$$

where

$$\theta^* = \left[-a_1 \cdots -a_n \quad b_0 \cdots b_m\right]^T$$

$$\varphi(t) = \left[p^{n-1} y^{f_2} \cdots y^{f_2} \quad p^m u^{f_2} \cdots u^{f_2}\right]^T$$

This is in the LIP form!

The vector $\varphi(t)$ contains realizable signals.

Construct a suitable estimator:

$$\hat{Z}(t) = \hat{\theta}(t)^T \varphi(t)$$

$$\dot{\hat{\theta}}(t) = -\Gamma \varphi(t) e_1(t), \qquad \Gamma = \Gamma^T > 0$$

$$e_1(t) = \hat{Z}(t) - Z(t)$$

Properties of the estimator

Consider the quadratic form

$$V = \tilde{\theta}(t)^T \Gamma^{-1} \tilde{\theta}(t)$$

where

$$\tilde{\theta}(t) \stackrel{\Delta}{=} \hat{\theta}(t) - \theta^*$$

then

$$e_1(t) = \hat{Z}(t) - Z(t) = \hat{\theta}^T \varphi - \theta^{*T} \varphi$$
$$= \tilde{\theta}(t)^T \varphi(t)$$

and

$$\frac{d}{dt} V = 2\tilde{\theta}^T \Gamma^{-1} \dot{\tilde{\theta}}$$
$$= 2\tilde{\theta}^T \Gamma^{-1} \left(-\Gamma \varphi e_1\right)$$
$$= -2\left[\tilde{\theta}^T \varphi\right]^2 \leq 0$$
$$(\quad = -2e_1^{\,2} \leq 0)$$

$\therefore$ V is uniformly non-increasing

i.e.

$$\tilde{\theta}(t)^T \Gamma^{-1} \tilde{\theta}(t) \leq \tilde{\theta}(0) \Gamma^{-1} \tilde{\theta}(0) \qquad \text{(i.e., } V(t) \leq V(0))$$

The estimates are “likely to become better”.

Combining estimation and control

- Thus, we have an estimator with some nice properties. Estimator gives estimated $\hat{R}_p$ and $\hat{Z}_p$.

- Use the estimates to construct the control law as if they are the correct values. (Certainty-equivalent strategy)

- We will illustrate using the same control structure as before. However, any control structure can be used.

[E.g. Input-output Pole-placement]

Since we have $\hat{R}_p$ and $\hat{Z}_p$,

(a) Solve for $\hat{E}$ and $\hat{F}$ in

$$T(p)R_m(p) = \hat{R}_p(p)\hat{E}(p) + \hat{F}(p)$$

where T(deg n), R_m (deg n^*) are design polynomials as before.

(b) Generate

$$y^{f_1}(t) = \frac{1}{T(p)} y(t), \qquad u^{f_1}(t) = \frac{1}{T(p)} u(t)$$

(c) For the control law (8), p. 44, use

$$\hat{F}(p) y^{f_1}(t) + \hat{E}(p) \hat{Z}_p(p) u^{f_1}(t) = k_m r(t)$$

[See p.51, $\overline{F} y^{f_1}(t) + \overline{G} u^{f_1}(t) = k^* r(t)$ with k_p explicitly expressed in $R_p y = k_p Z_p u$.

For $R_p y = Z_p u$, note that $\overline{G} = E Z_p$, we have the above]

which is implemented as

$$\hat{F}(p) y^{f_1}(t) + \left\{ \hat{E}(p) \hat{Z}_p(p) - \hat{b}_0 T(p) \right\} u^{f_1}(t) + \hat{b}_0 u(t) = k_m r(t)$$

or

$$u(t) = \frac{1}{\hat{b}_0} \left[-\hat{F} y^{f_1} - \left\{ \hat{E} \hat{Z}_p - \hat{b}_0 T \right\} u^{f_1} + k_m r(t) \right]$$

Everything on R.H.S. is realizable.

[Exercise: Check this out on your own!!]

Thus in this approach

- Choose a suitable estimator .
  Estimates are then obtained for
  $$\hat{R}_p \text{ and } \hat{Z}_p$$

  Depending on your choice of estimators, it is usually possible to know something about the properties of the estimate. In this particular example, the estimator ensure that

  $$\tilde{\theta}(t)^T \Gamma^{-1} \tilde{\theta}(t) \leq \tilde{\theta}(0)^T \Gamma^{-1} \tilde{\theta}(0)$$

- Apply a suitable control law
  $$u(t) = f\left(\hat{\theta}(t), y(t), r(t)\right)$$

Examples:
Almost anything from your EE5101 class!

It is difficult to conclude anything rigorous about this part. However, the control scheme typically works quite well in practice.

* Pole-placement
* Optimal Control
* Model-based control

etc...

Digital Realization

- Possible to implement the previously described adaptive controllers using analog components.

- However, probably better to use micro-processor based implementation

  -- less problems with drifts, reliability
  -- more flexible, can include other supplementary
  tasks like bumpless transfer,
  sequence scheduling of events, anti-reset windup.

- This means that the adaptive controller, designed in continuous-time, has to be coded and realized in discrete-time.

Digital Realization Considerations

Example: Consider the case of Adaptive Control with full state measurable.

From page 29...

Plant:

$$\dot{x}_p = A_p x_p + gbu$$

$x_p \in \Re^n$ measurable, b known

Matching Conditions:

$$A_p + gb\theta_x^{*T} = A_m$$

$$g\theta_r^* = g_m$$

Reference Model:

$$\dot{x}_m = A_m x_m + g_m br$$

Control Law:

$$u(t) = \theta_x(t)^T x_p(t) + \theta_r(t) r(t)$$

Adaptive Law:

$$e = x_p - x_m$$

$A_m^T P + PA_m = -Q$ Choose Q (s.p.d.)

Calculate P (s.p.d.)

$$\begin{bmatrix} \dot{\theta}_x \\ \dot{\theta}_r \end{bmatrix} = -\mathrm{sgn}(g)\Gamma \begin{bmatrix} x_p \\ r \end{bmatrix} e^T Pb$$

Result: All signals $\{x_p, \theta_x, \theta_r\}$ are bounded, and $\lim_{t\to\infty} \|x_p - x_m\| = 0$

Simple first-order plant adaptive control digital realization

- Choose a sampling interval h (factors influencing this will be discussed later)

- Use a suitable approximation for the $p = \frac{d}{dt}$ operator.

  E.g., if we can sample fast enough relative to bandwidth of the overall system, we can use

$$p \simeq \frac{q-1}{h}$$

First-Order Case

$$\dot{y}_m = a_m y_m + k_m r$$

This gives $(t = jh, \quad j \in Z^+)$

Reference Model:

$$y_m(j+1) = (1 + a_m h) y_m(j) + hk_m r(j)$$

$$\frac{d}{dt} y_m = \frac{y_m(j+1) - y_m(j)}{h}$$

Control Law:

$$u(j) = \theta(j) y(j) + k(j) r(j)$$

Adaptive Law:

$$\theta(j+1) = \theta(j) - \operatorname{sgn}(k_p)\gamma_1 h e(j) y(j)$$

$$k(j+1) = k(j) - \operatorname{sgn}(k_p)\gamma_2 h e(j) r(j)$$

$$e(j) = y(j) - y_m(j)$$

$$\theta(0) = \theta_0 \qquad k(0) = k_0$$

Arbitrary starting gains

Time constants to be considered in choice of h

- Consider the more difficult n-state variables case
- The closed-loop time constants are given by the eigenvalues of A_m of the reference model

$$\dot{x}_m = A_m x_m + g_m b r$$

The fastest dynamic here is given approximately by a time constant of

$$\tau = \frac{1}{\max\{|\lambda_i(A_m)|\}}$$

A rule of thumb is

$$h < \frac{1}{20}\tau$$

- In LTI systems, this might already be sufficient. But in adaptive systems, there are dynamics in the overall system which are dynamics of the adaptive law!! The sampling interval must be small enough to handle the fastest adaptation!

Recall that the speed of adaptation is decided partly by the Q matrix. More precisely:

$$A_m^T P + P A_m = -Q$$

$$V = e^T P e + |g| \phi^T \Gamma^{-1} \phi$$

$$\dot{V} = -e^T Q e$$

$$\dot{\bar{V}} + \omega_r \bar{V} = 0$$

i.e. $(s + \omega_r)\bar{V}(s) = 0$

The rate of adaptation might be approximately quantified by

$$\rho = \frac{|\dot{V}|}{V} = \frac{e^T Q e}{e^T P e + |g| \phi^T \Gamma^{-1} \phi}$$

$$\leq \frac{e^T Q e}{e^T P e}$$

$$\leq \frac{\lambda_{\max}(Q)}{\lambda_{\min}(P)} \stackrel{\Delta}{=} \rho_{\max}$$

!! Related to adaptation time-constants

- The choice of h should be able to handle the fastest rate possible

i.e. $$h < \frac{1}{20} \frac{1}{\rho_{\max}} = \frac{1}{20} \frac{\lambda_{\min}(P)}{\lambda_{\max}(Q)}$$

- Thus, if large Q is chosen for fast adaptation, we must have hardware capable of achieving sampling rates chosen above.

Exercise 4

Refer to the controller you designed in Exercise 1. Using the MATLAB language, write the code for a digital realization of your adaptive controller. Discuss the guidelines for the choice of the sampling interval h.

Verify your design using simulation.

Exercise 5

Refer to Simulation 5-1 in pp.197 of Narendra……

Using the methods that we have just considered, design an adaptive controller for the system described by

$$W_p(s) = \frac{s+1}{(s-2)(s-1)} \quad \text{Plant}$$

(Remember that only y and u are measurable)

$$W_m(s) = \frac{1}{s+1}$$

Use simulation to verify your design.