National University of Singapore

Department of Mechanical Engineering

ME5401/EE5101 Linear System 2021/2022

Tutorial 1

Note that Questions 8-10 are optional.

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1. Find the inverse of the following matrices, if they exist.

(a) A= $\begin{bmatrix} 2 & 5 \\ 10 & -1 \end{bmatrix}$ (b) A= $\begin{bmatrix} 3 & 0 & -1 \\ -2 & 1 & 2 \\ 0 & 1 & -1 \end{bmatrix}$

(c) A= $\begin{bmatrix} 1 & 3 & 4 \\ -1 & 1 & 0 \\ -1 & 0 & -1 \end{bmatrix}$

2. Let $A \in R^{n\times n}$. Consider the definition

$$e^{At} = I + At + \frac{1}{2!}A^2t^2 + \frac{1}{3!}A^3t^3 + ...$$

Show that A and e^{At} commute, i.e., $Ae^{At} = e^{At}A$.

Using Laplace transform or otherwise, show that

$$(sI - A)^{-1} = \frac{I}{s} + \frac{A}{s^2} + \frac{A^2}{s^3} + \frac{A^3}{s^4} + ...$$

3. Consider the a system where the input is $u(t)$ and the output $y(t) = \frac{d}{dt}(tu(t))$.

Show that the system is linear.

4. Find the eigenvector and eigenvalues of the following matrix.

$$\begin{bmatrix} -4 & 1 & 0 \\ 0 & -3 & 1 \\ 0 & 0 & -2 \end{bmatrix}$$

5. Given A= $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$. Verify the Caley-Hamilton Principle.

6. Determine the rank and nullity of the following matrix:

$$\begin{bmatrix} 4 & 2 & 1 & 3 \\ 6 & 3 & 1 & 4 \\ 2 & 1 & 0 & 1 \end{bmatrix}$$

Find a basis for its range space and its null space.

7. Let $A \in R^{n\times n}$ be a non-singular matrix. Show that if λ_i is an eigenvalue of A, then $\dfrac{1}{\lambda_i}$ is an eigenvalue of A^{-1}.

8. Let $A \in R^{n\times n}$ be a matrix with *n* distinct eigenvalues. Prove that the set of *n* eigenvectors are linearly independent.

9. Let $A \in R^{n\times r}$ and $B \in R^{r\times n}$ be arbitrary matrices so that *AB* and *BA* are $n \times n$ and $r \times r$ matrices respectively. Assume that $n \geq r$ and prove that
   (a) The scalar λ_i is a nonzero eigenvalue of *AB* if and only if it is a non-zero eigenvalue of *BA*.
   (b) If *v* is an eigenvector of *AB* associated with a non-zero eigenvalue, then $\zeta = Bv$ is an eigenvector of *BA*.
   (c) *AB* has at least *n-r* zero eigenvalues.

10. Let $A \in R^{n\times n}$ be a symmetric matrix. Show that all its eigenvalues are real numbers.