EE6104 ADAPTIVE CONTROL SYSTEMS (ADVANCED)

Liu Weihao

A0232935A

Continuous Assessment 1

Department of Electrical and Computer Engineering

National University of Singapore

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Contents

1 Introduction

It is desired to design a control system for a continuous-time system (the plant) which is known to have the following structure:

$$\frac{Y(s)}{U(s)} = \frac{b_0 s + b_1}{(s^2 + a_1 s + a_2)} \tag{1}$$

Where $Y(s)$ and $U(s)$ are the Laplace transforms of the time domain signals $y(t)$ (the plant output) and $u(t)$ (the input to the plant). In the physical setup, only $y(t)$ and $u(t)$ are measurable. The exact values of the transfer function coefficients are not known; it is only known that b_0 is a negative number, and that the plant has no zeros in the right-half of the s-plane.

(**Note:** In the simulation experiments required later, the plant is to be simulated as

$$\frac{Y(s)}{U(s)} = \frac{-0.5s - 1}{(s^2 + 0.22s + 6.1)} \tag{2}$$

This information is to be used for simulation of the plant only; the design of the controller may not explicitly use this knowledge.)

2 Part-I

Write down the continuous-time algorithm for an adaptive controller for the given plant which meets the following specifications:

1. the asymptotic closed loop attained should be reasonably fast and have no steady-state offset for step changes in setpoint command signals;
2. the design should be one that ensures boundedness of y(t) and u(t) in the adaptation process.

The model of the plant can be expressed as eqn. 3

$$R_p(p)y(t) = k_p Z_p(p)u(t) \tag{3}$$

The model of the idle model can me expressed as eqn. 4

$$R_m(p)y(t) = k_m r(t) \tag{4}$$

The designed control law can be expressed as the following equations:

$$w_y = \frac{1}{T(p)}y(t), w_u = \frac{1}{T(p)}u(t) \tag{5}$$

$$\begin{aligned} u(t) &= \bar{\theta}^T \bar{w}(t) = \theta_y^T w_y(t) + \theta_u^T w_u(t) + kr(t) \\ &= \begin{bmatrix} -f_1(t) & -f_2(t) & -g_1(t) & -g_2(t) & k(t) \end{bmatrix} \begin{bmatrix} pw_y(t) \\ w_y(t) \\ pw_u(t) \\ w_u(t) \\ r(t) \end{bmatrix} \end{aligned} \tag{6}$$

The adaptive law can be expressed as eqn. 7

$$\begin{aligned} e(t) &\equiv y(t) - y_m(t) \\ \bar{\theta} &= -sgn(k_p)\bar{\Gamma}\bar{w}e(t) \end{aligned} \tag{7}$$

3 Part-II

Using any programming language, run simulations to show the performance of your adaptive controller when the reference signal $r(t)$ is a square wave of an appropriately chosen period. Show plots of the output of the plant $y(t)$ and the output of the reference model $y_m(t)$. Show also some representative plots of the adapted controller gains, comparing these with the exact controller gains. (Note that as this is a simulation project with a simulated plant, the necessary exact controller gains can be calculated for the comparison.)

The Simulink model is shown as Fig. 1

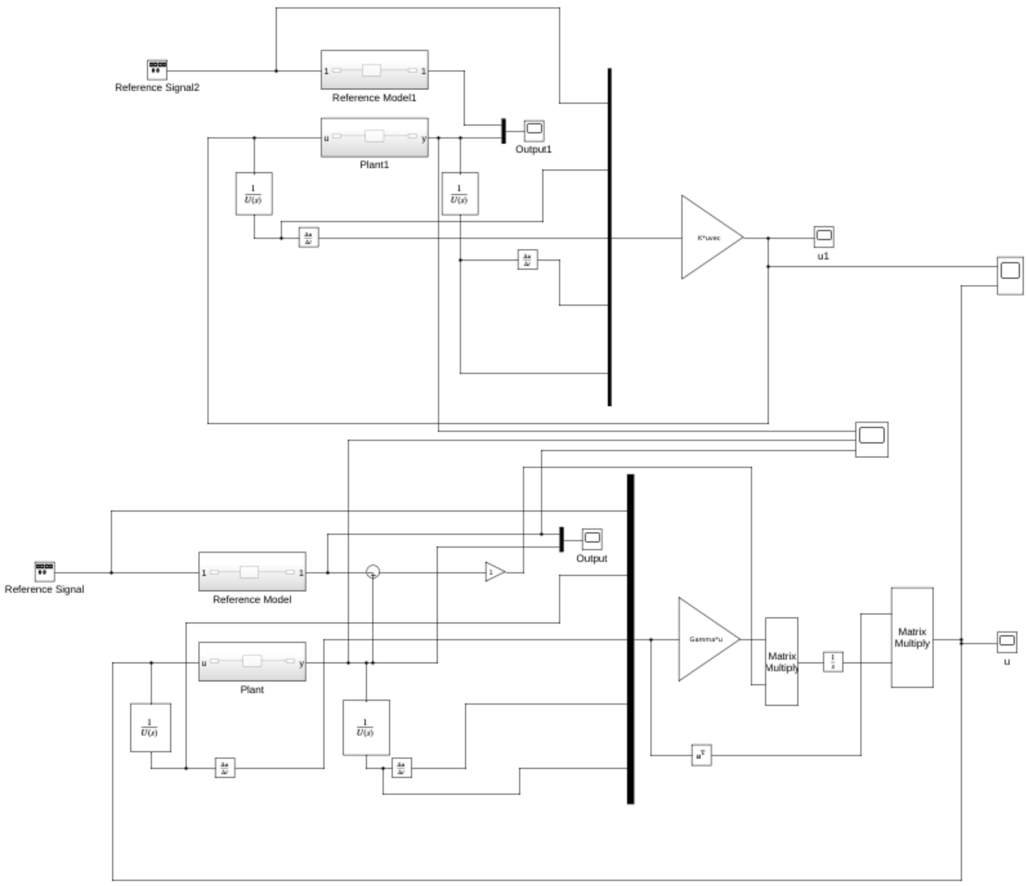
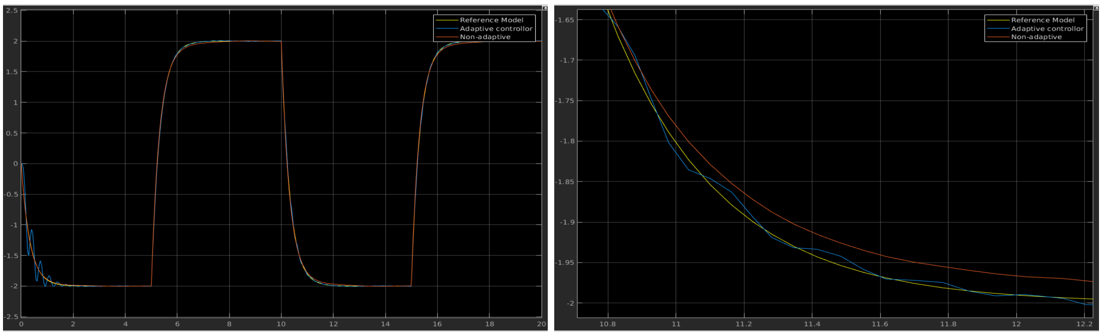


Figure 1: Simulink model.

The output of idle model, non-adaptive controller and adaptive controller is shown in Fig. 2a, the zoom figure is Fig. 2b. The controller gain is shown in Fig. 3



(a) Overall output. (b) Zoom in the figure.

Figure 2: Square wave signal.

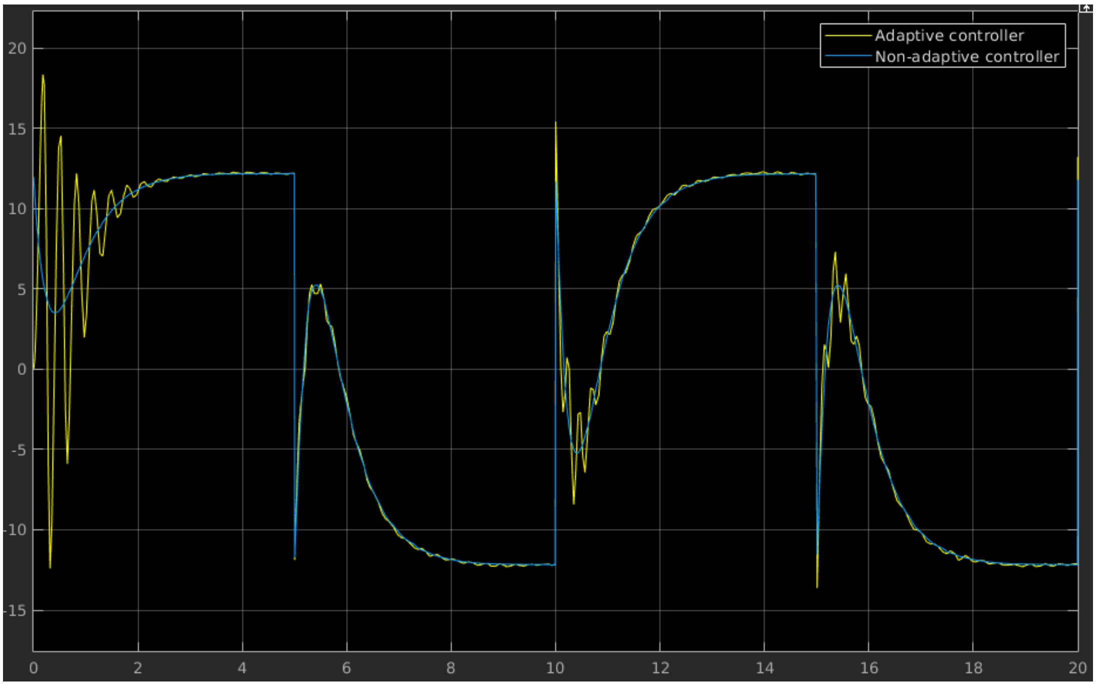


Figure 3: Controller gain.

4 Part-III

Investigate the effects of different choices of the observer polynomial (denoted as $T(p)$ in the class notes).

In class notes, the $T(p)$ can be expressed as eqn. 8

$$T(p) = p^2 + 2\eta\omega_n p + \omega^2 \tag{8}$$

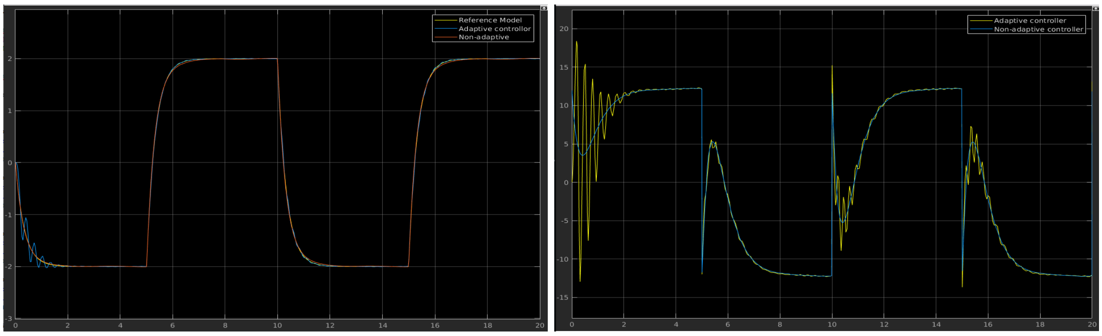
In this part, our parameters η and ω is selected from Table 1 The results are shown as Figs. 4 - 9

Conclusion:

When the parameter η and ω are acceptable, the output are slightly infected. However, the controller gain oscillate heavily.

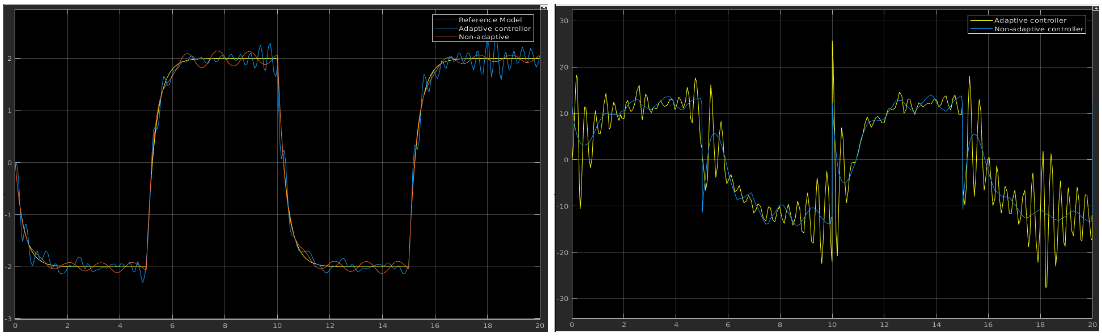
Table 1: Simulation Parameters

Parameter	Value
η	$\{0.01, 0.1\}$
ω	$\{1, 5, 10\}$



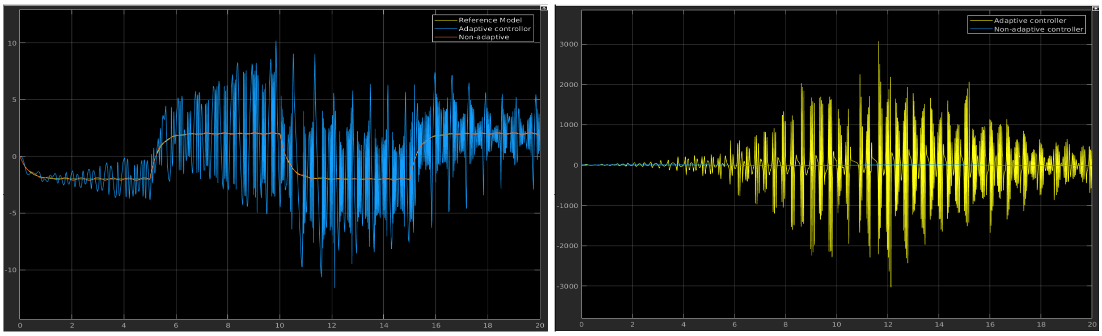
(a) Output (b) Controller gain.

Figure 4: η=0.01,ω=1.



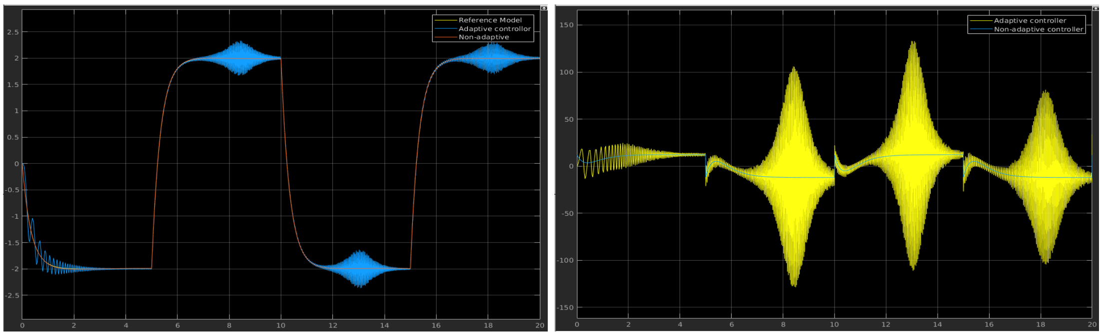
(a) Output (b) Controller gain.

Figure 5: η=0.01,ω=5.



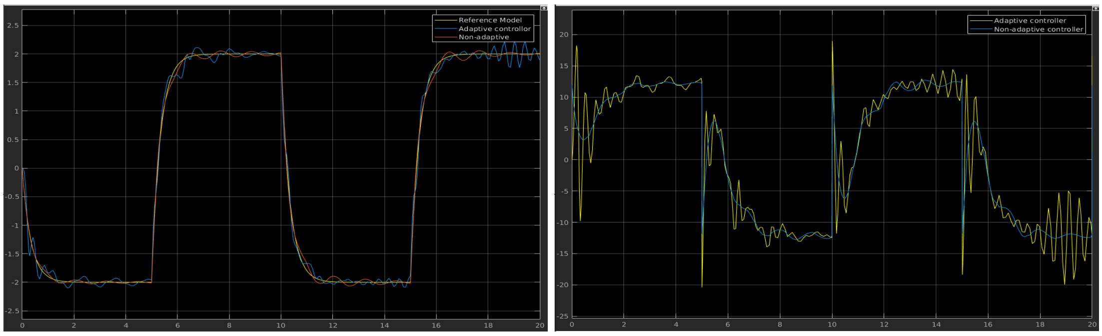
(a) Output (b) Controller gain.

Figure 6: η=0.01,ω=10.



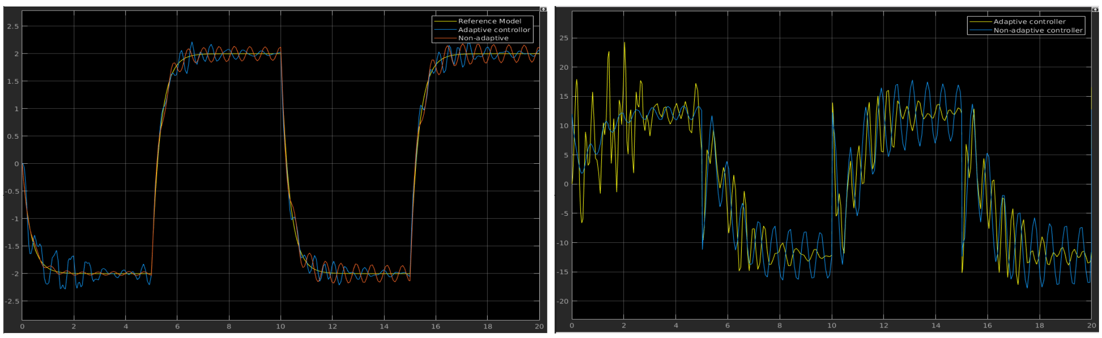
(a) Output (b) Controller gain.

Figure 7: η=0.1,ω=1.



(a) Output (b) Controller gain.

Figure 8: η=0.1,ω=5.



(a) Output (b) Controller gain.

Figure 9: η=0.1,ω=10.

5 Part-IV

For a particular choice of $T(P)$ which you consider best in some sense, investigate the specific case where the reference signal is the single sinusoid:

$$r(t) = 10sin(0.5t) \tag{9}$$

Discuss the simulation results you observe for this specific case, noting especially the output tracking error and the adapted controller gains.

Compare this case of the single sinusoid reference signal with the cases where:

1. The reference signal is a square wave of comparable period and amplitude;
2. The reference signal is a sum of five or more sinusoidal signals of different but comparable periods, with a comparable overall amplitude. Discuss your observations.

The results are shown as Figs. 10 and 11

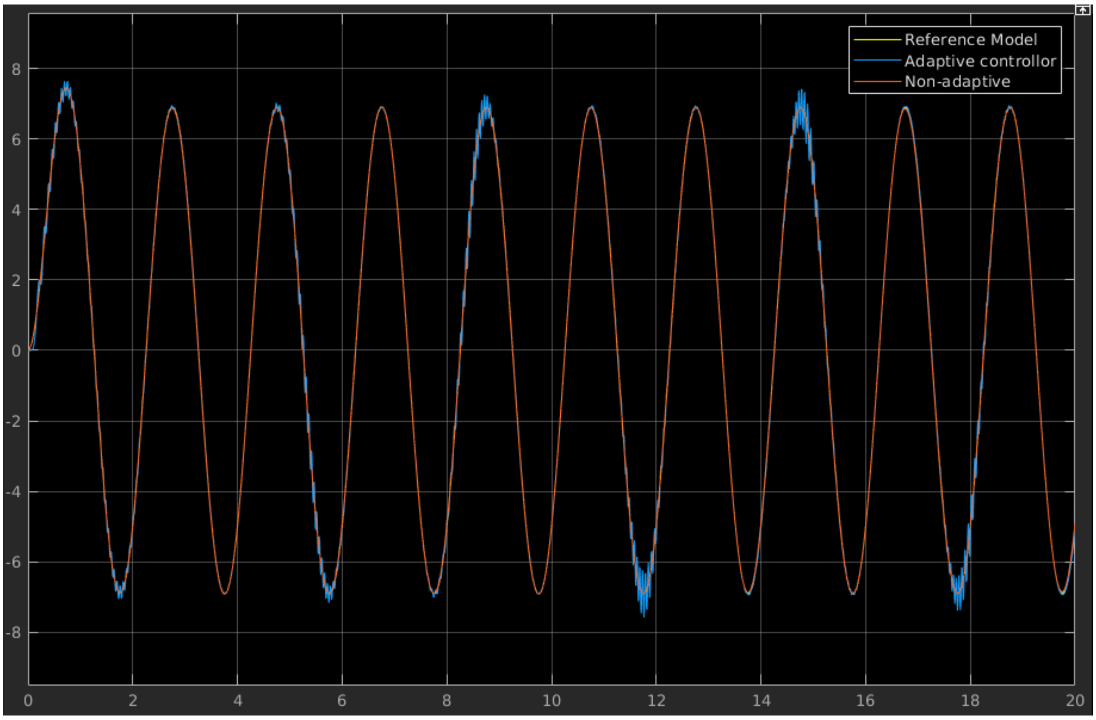


Figure 10: Output Signal.

Conclusion:

A sine wave is more straightforward to track than a square wave due to the absence of abrupt changes in function values.

The tracking error is assured to converge to zero. However, for an adapted controller gain, the parameter error remains bounded, but there is no guarantee it will converge to zero.

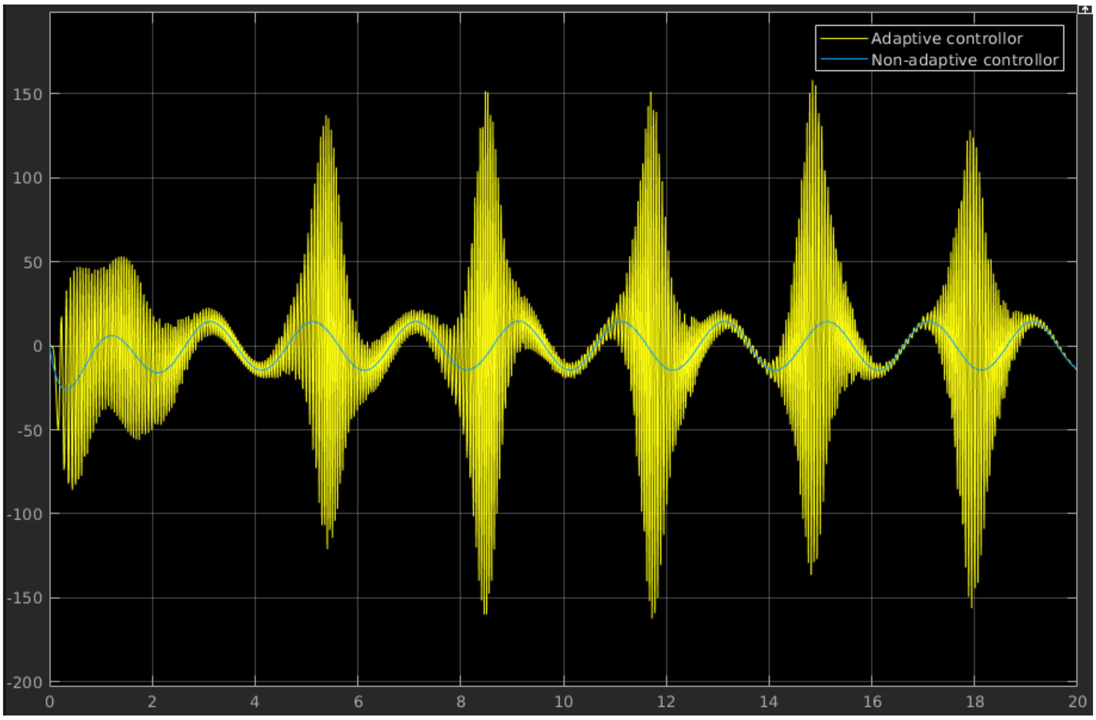


Figure 11: Controller gain.

6 Appendix

```
eta = 0.01;
wn = 1;

ga = 1;
Gamma = 100*diag([1*ga,1*ga,1*ga,1*ga,1*ga]);

a1 = 0.22;
a2 = 6.1;
b0 = -0.5;
b1 = -1;
num = [b0 b1];
dem = [1 2*eta*wn wn^2];
sys = tf(num,dem);
pole(sys)

K_p = b0;
Z_p = [1 b1/b0];
R_p = [1 a1 a2];
T = [1 2*eta*wn wn^2];
a_m = 3;
R_m = [1 a_m];
K_m = a_m;
[E,F] = deconv(conv(T,R_m),R_p);
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```
Fbar = F/K_p;
Gbar = conv(E,Z_p);
G1 = Gbar - T;
K_star = K_m/K_p;
theta_bar_star = [K_star,-G1(3),-G1(2),-Fbar(4),-Fbar(3)];
```