Note Title 10/17/2012

7

Supplement.

Discrete-time Adaptive Control

Note that, for Adaptive Control,
we consider the plant:

$$A(q^{-1})\, y(t) = q^{-d} B(q^{-1})\, u(t)$$

$$t = 0, 1, 2, 3, \dots$$

$$A(q^{-1}) = 1 + a_1 q^{-1} + \dots + a_n q^{-n}$$

$$B(q^{-1}) = b_0 + b_1 q^{-1} + \dots + b_m q^{-m}$$

—— (7.01)

Note that the control strategy that will be used is the "d-step ahead controller" (also called "minimum variance controller", or "minimum prediction error controller").

Thus, see also the earlier part where the reasons/basis for the control strategy were explained/discussed.

Using these ideas, thus, consider the so-called "prediction identity":

$$1 = A(q^{-1})E(q^{-1}) + q^{-d}F(q^{-1})$$

with

$$E(q^{-1}) = 1 + e_1 q^{-1} + \dots + e_{d-1} q^{-(d-1)}$$

$$F(q^{-1}) = f_0 + f_1 q^{-1} + \dots + f_{n-1} q^{-(n-1)} \qquad \text{--- } (7.02)$$

Check this !!

Using (7.01) and (7.02), we can see :

$$A(q^{-1})\, y(t) = q^{-d} B(q^{-1})\, u(t)$$

$$\underbrace{E(q^{-1})\, A(q^{-1})}_{\text{this is } 1 - q^{-d} F(q^{-1})}\, y(t) = q^{-d}\, \underbrace{E(q^{-1})\, B(q^{-1})}_{\text{call this } G(q^{-1})}\, u(t) \qquad \text{--- } (7.11)$$

and $G(q^{-1}) = g_0 + g_1 q^{-1} + \dots + g_{m+d-1} q^{-(m+d-1)}$

Thus, from (7.11), we have:

$$\left\{1 - q^{-d} F(q^{-1})\right\} y(t) = q^{-d} G(q^{-1}) u(t)$$

or

$$y(t) = q^{-d} F(q^{-1}) y(t) + q^{-d} G(q^{-1}) u(t)$$

$$= F(q^{-1}) y(t-d) + G(q^{-1}) u(t-d)$$

$$\text{——— (7.21)}$$

Stepping this d-steps ahead, we have

$$y(t+d) = F(q^{-1}) y(t) + G(q^{-1}) u(t)$$

$$\text{——— (7.22)}$$

The expressions (7.21) and (7.22) are key expressions from which to develop a suitable discrete-time adaptive controller ...

Expression (7.21) is the basis for the "estimation" or "<u>adaptive law</u>".

Expression (7.22) is the basis for the "<u>control law</u>".

Thus, for a suitable adaptive controller, note that (7.21) gives

$$y(t) = F(q^{-1})\, y(t-d) + G(q^{-1})\, u(t-d)$$

$$= \underbrace{\begin{bmatrix} f_0 & f_1 & \cdots & f_{n-1} & g_0 & g_1 & \cdots & g_{m+d-1} \end{bmatrix}}_{\text{Call this } \theta^*} \underbrace{\begin{bmatrix} y(t-d) \\ y(t-d-1) \\ \vdots \\ u(t-d) \\ u(t-d-1) \\ \vdots \end{bmatrix}}_{\text{Call this } w(t-d)}$$

or :

$$y(t) = \theta^{*T} w(t-d) \qquad (7\text{-}25)$$

Based on this, consider the "Adaptive Law" :

$$\hat{y}(t) = \hat{\theta}(t)^T w(t-d) \qquad \text{— } (7.31a)$$

$$e_1(t) = \hat{y}(t) - y(t) \qquad \text{— } (7.31b)$$

$$\Delta\hat{\theta}(t) \triangleq \hat{\theta}(t+1) - \hat{\theta}(t)$$

$$= \frac{-\gamma\, w(t-d)\, e_1(t)}{1 + \| w(t-d) \|^2}$$

— (7.31c)

Note that, clearly, $\hat{\theta}(t)$ will have the form:

$$\hat{\theta}(t) = \left[\hat{f}_0(t) \;\; \hat{f}_1(t) \ldots \hat{f}_{n-1}(t) \;\; \hat{g}_0(t) \ldots \hat{g}_{m+d-1}(t) \right]^T \quad \text{--- (7.31d)}$$

With each $\hat{\theta}(t)$ updated according to the "Adaptive Law" (7.31a)–(7.31d), we can have the "Control Law":

$$r(t) = \hat{\theta}(t)^T w(t)$$

$$= \left[\hat{f}_0(t) \;\; \hat{f}_1(t) \ldots \hat{g}_0(t) \;\; \hat{g}_1(t) \ldots \right] \begin{bmatrix} y(t) \\ y(t-1) \\ \vdots \\ \boxed{u(t)} \\ u(t-1) \\ \vdots \end{bmatrix}$$

Based on def^n of $w(t-d)$ in (7.25), this is clearly $w(t)$!!!

Call above Equation (7.41)

In the above, note that everything is available to compute $u(t)$!!

Thus, this is the Control Law.

Check carefully, for yourself, that

if $\hat{\theta}(t) \equiv \theta^*$ in (7.41)

above, that this results in the

exact "d-step ahead controller" !!

For the "Adaptive Controller" above, for $0<\gamma<2$

we have shown in class that the "Adaptive Law" (7.31a) – (7.31d)

gives:

(i) $\|\hat{\theta}(t)\|$ is bounded for all t; and

(ii) $\|\hat{\theta}(t+1)\|^2 - \|\hat{\theta}(t)\|^2 \leq 0$

— (7.51)

and additional other properties ....

Further, using the "Key Technical Lemma", it is further possible to show (we skipped this part; and thus ~~not~~ considered for exams) that we additionally have:

(a) $\{u(t)\}$ and $\{y(t)\}$ are bounded for all t; and

(b) $\lim_{t \to \infty} \{ y(t+d) - r(t) \} = 0$ #

— (7.52)

Thus, the Discrete-time d-step ahead Adaptive Controller may be summarised as:

<u>Plant</u>:

$$A(q^{-1})\,y(t) = q^{-d}B(q^{-1})\,u(t)$$

<u>Adaptive Law</u>

$$\hat{y}(t) = \hat{\theta}(t)^T w(t-d)$$

$$e_1(t) = \hat{y}(t) - y(t)$$

$$\Delta\hat{\theta}(t) \triangleq \hat{\theta}(t+1) - \hat{\theta}(t)$$

$$= \frac{-\gamma\, w(t-d)\, e_1(t)}{1 + \|w(t-d)\|^2}$$

with $0 < \gamma < 2$

<u>Control Law</u>

$$r(t) = \hat{\theta}(t)^T \underbrace{\begin{bmatrix} y(t) \\ y(t-1) \\ \vdots \\ u(t) \\ u(t-1) \\ \vdots \end{bmatrix}}_{w(t)}$$

The Discrete-time d-step ahead Adaptive Controller described above results in:

$$(\mathrm{I}) \ \{\|\hat{\theta}(t)\|\}, \ \{u(t)\}, \ \{y(t)\}$$

bounded for all t; and

$$(\mathrm{II}) \ \lim_{t \to \infty} \{y(t+d) - r(t)\} = 0$$

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The above summarises the key points in the class lecture. For your enjoyment!!! :) :)