Note Title 8/30/2011

3b

Review of key ideas from last class

Adaptive control of continuous-time system with only input & output measurable

Thus, we consider the system:

$$R_p(p)\, y(t) = k_p Z_p(p)\, u(t)$$

$$p \triangleq \frac{d}{dt}$$

$$R_p(p) = p^n + a_1 p^{n-1} + \dots + a_n \qquad (1.1)$$

$$Z_p(p) = p^m + b_1 p^{m-1} + \dots + b_m$$

i.e. R_p is monic, of deg n

Z_p is monic, of deg m

and "relative degree" $n^* \triangleq n-m$ — (1.2)

Next, consider the polynomial "Diophantine"/"Bezout"/division (!!!) Identity:

$$\underbrace{T(p)}_{\substack{\text{monic} \\ \text{deg } n}} \underbrace{R_m(p)}_{\substack{\text{monic} \\ \text{deg } n^*}} = R_p(p) \underbrace{E(p)}_{\substack{\text{monic} \\ \text{deg } n^*}} + \underbrace{F(p)}_{\text{deg}(n-1)} \quad (1.5)$$

Noting that we are working with LTI differential operators here, we can write (from Eqn (1.1)):

$$E(p)\, R_p(p)\, y = k_p E(p)\, Z_p(p)\, u$$

i.e. from (1.5), we have:

$$\left\{ T R_m - F \right\} y(t) = k_p E Z_p\, u(t)$$

i.e.

$$T R_m\, y(t) = F\, y(t) + k_p E Z_p\, u(t) \qquad \text{--- (2.1)}$$

monic, deg n

and writing $G(p) = E(p)\, Z_p(p)$

monic, deg n^* monic, deg m

Then, if we choose $T(p)$ to be a Hurwitz (stable) polynomial, we can further write (2.1) as:

$$R_m(p)\,y(t) = \frac{F(p)}{T(p)}\,y(t) + k_p \frac{\overline{G}(p)}{T(p)}\,u(t)$$

$$= k_p \left\{ \frac{\overline{F}(p)}{T(p)}\,y(t) + \frac{\overline{G}(p)}{T(p)}\,u(t) \right\} \quad \text{--- } (2.11)$$

Noting that $\overline{G}(p)$ is monic, and of degree n, we can further write:

$$\frac{\overline{G}(p)}{T(p)} = 1 + \frac{G_1(p)}{T(p)}$$

and have:

$$G_1(p) = g_1 p^{n-1} + g_2 p^{n-2} + \dots + g_n$$

and $\bar{F}(p) = f_1 p^{n-1} + f_2 p^{n-2} + \dots + f_n$

Next, write (2.11) as:

$$R_m(p)\, y(t) = k_p \left\{ \underline{\frac{\bar{F}}{T} y + \frac{G_1}{T} u + u} \right\} \quad \text{— (2.12)}$$

and consider that if we can __choose__:

$$\left\{ \underline{\qquad} \right\} = k^* r(t) \quad \text{— (2.13)}$$

with $\underline{k_p k^* \triangleq k_m}$

Then, if this is possible, it results in (2.12) becoming

$$R_m(p)\, y(t) = k_m\, r(t) \qquad (2.14)$$

Note then, that this requires the "perfect" Control Law (from (2.13)) of =

$$u(t) = -\frac{F(p)}{T(p)}\, y(t) - \frac{G_1(p)}{T(p)}\, u(t) + k^* r(t) \qquad (2.15)$$

"perfect"

So, is the ^ Control Law (2.15) realizable ?

For this, as in class, consider what it would look like in the n=2 case.

Thus, for n=2, we can define the signal $y^{f_1}(t)$ [now using the notation & symbols in the Lecture Notes :) :) !!] with :

$$y^{f_1}(t) \triangleq \frac{1}{T(p)} y(t) \quad \text{—— (2.31)}$$

$$T(p) = p^2 + t_1 p + t_2$$

Set up (2.31) as a state-variable system. We thus have:

$$T(p)\, y^{f_1}(t) = y(t)$$

$$\{p^2 + t_1 p + t_2\}\, y^{f_1}(t) = y(t) \qquad \text{— (2.32)}$$

$$\dot{y}^{f_1} = p\, y^{f_1} \qquad \text{— (2.33a)}$$

$$\{\dot{p y^{f_1}}\} = p^2 y^{f_1} = -t_2 y^{f_1} - t_1 p y^{f_1} + y$$

$$\text{— (2.33b)}$$

Note that (2.33a) and (2.33b) is a straightforward set of state-variable equations with =

$$\begin{bmatrix} \dot{y}^{f_1} \\ p\,y^{f_1} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -t_2 & -t_1 \end{bmatrix} \begin{bmatrix} y^{f_1} \\ p\,y^{f_1} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} y(t)$$

— (2.34)

And clearly, similarly for

$$u^{f_1}(t) \triangleq \frac{1}{T(p)} u(t)$$

and for this $\underline{\underline{n=2}}$ case, We can also generate the straightforward set of state-variable equations with =

$$\begin{bmatrix} \dot{u}^{f_1} \\ pu^{f_1} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -t_2 & -t_1 \end{bmatrix} \begin{bmatrix} u^{f_1} \\ pu^{f_1} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \qquad (2.35)$$

Clearly, the ideas generalise for all n !!

Further note that after setting up these state-variable systems, we can write (for n=2):

$$\frac{F(p)}{T(p)} y(t) = \{f_1 p + f_2\} \frac{1}{T(p)} y(t)$$

$$= \{f_1 p + f_2\} y^{f_1}(t)$$

$$= f_1 p y^{f_1}(t) + f_2 y^{f_1}(t)$$

— (2.41a)

and clearly,

$$\frac{G_1(p)}{T(p)} u(t) = g_1 p u^{f_1}(t) + g_2 u^{f_1}(t)$$

— (2.41b)

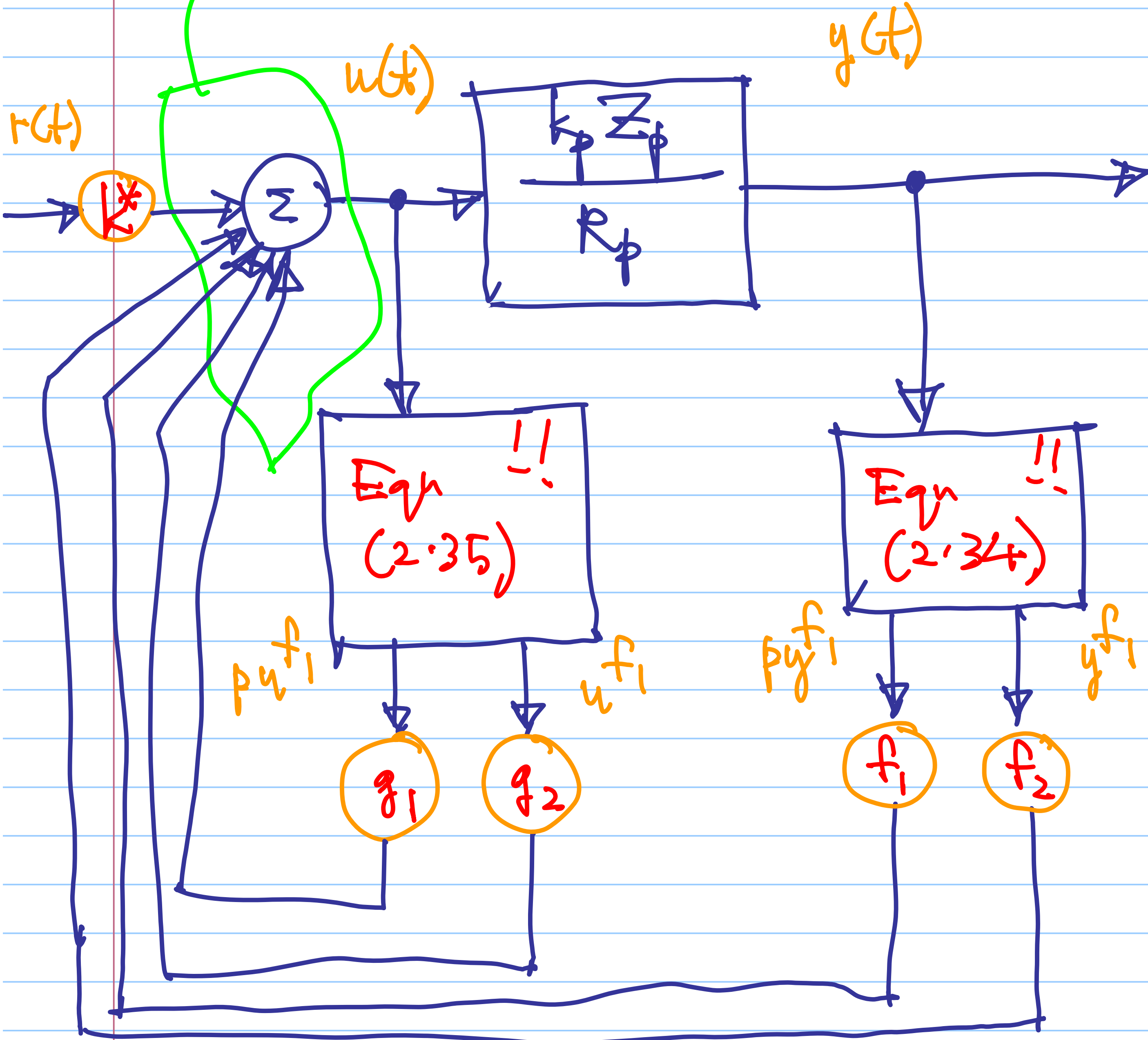
and the necessary "perfect" Control Law for $n=2$, is realized as:

$$u(t) = -\frac{F(p)}{T(p)}\,y(t) - \frac{G_1(p)}{T(p)}\,u(t) + k^* r(t)$$

$$= -\left\{ f_1\, p\, y^{f_1} + f_2\, y^{f_1} \right\} - \left\{ g_1\, p\, u^{f_1} + g_2\, u^{f_1} \right\} + k^* r \qquad (2.42)$$

and as a realizable patching diagram, It can be set up as:

this implements (2.15) / (2.42) !!!!



ΔΔΔ

Thus, clearly the "perfect" Control Law (2.15) is realizable as:

$$T(p)\, y^{f_1}(t) = y(t)$$

$$\{p^n + t_1 p^{n-1} + \dots t_n\}\, y^{f_1} = y(t)$$

$$T(p)\, u^{f_1}(t) = u(t)$$

$$\{p^n + t_1 p^{n-1} + \dots + t_n\}\, u^{f_1} = u(t)$$

and
$$u(t) = -\frac{F}{T} y - \frac{G_1}{T} u + k^* r$$

$$= \underbrace{\left[-f_1 \; -f_2 \cdots -f_n \; -g_1 \; -g_2 \cdots -g_n\right]}_{\theta^* = \begin{bmatrix} \theta^*_y \\ \theta^*_u \end{bmatrix}} \underbrace{\begin{bmatrix} p^{n-1} y^{f_1} \\ \vdots \\ y^{f_1} \\ p^{n-1} u^{f_1} \\ \vdots \\ u^{f_1} \end{bmatrix}}_{w(t) = \begin{bmatrix} w_y(t) \\ w_u(t) \end{bmatrix}} + k^* r \quad \text{--- (2.51)}$$

i.e. this "perfect" Control Law

can be written as:

$$u(t) = \theta^{*T} w(t) + k^* r(t)$$

$$= \left[\theta^{*T} \quad k^*\right] \begin{bmatrix} w(t) \\ r(t) \end{bmatrix} = \bar{\theta}^{*T} \bar{w}(t) \qquad -(2.52)$$

$\bar{\theta}^{*T}$ $\quad$ $\bar{w}(t)$

And since we do not know $\bar{\theta}^*$,

we will develop an adaptive system

using Control Law:

$$u(t) = \left[\theta_y(t) \quad \theta_u(t) \quad k(t)\right] \begin{bmatrix} w_y(t) \\ w_u(t) \\ r(t) \end{bmatrix} = \bar{\theta}^T(t)\, \bar{w}(t)$$

How, next, do we generate the

Adaptive Law ?

For this, we need to note :

Proposition 1 : The set of $2n$

signals :

$$w(t) = \begin{bmatrix} w_y(t) \\ \\ w_u(t) \end{bmatrix} = \begin{bmatrix} p^{n-1} y^{f_1} \\ \vdots \\ y^{f_1} \\ p^{n-1} u^{f_1} \\ \vdots \\ u^{f_1} \end{bmatrix}$$

is actually the state-vector of a $(2n)$-order non-minimal realization of the plant (1.1). △△

<u>Proposition 2:</u> For the plant (1.1), and the "perfect" control law (2.52), the gain vector $\bar{\theta}^*$ exists, and for (2.52) results in a closed-loop with $2n$ poles which are the roots of $T(p)R_m(p)Z_p(p)$; and the closed-loop input-output relationship, with (2.52), reduces to:

$$R_m(p)\, y(t) = k_p k^* r(t)$$

$$= k_m r(t)$$

$\Delta\Delta\Delta$

Then, using Proposition 1; Proposition 2; and for the Case of $n^* = 1$; we can note that the chosen Reference Model will be:

$$R_m(p)\, y_m(t) = k_m\, r(t)$$

$\deg n^* = 1$

$R_m(p)$ to be Hurwitz

i.e. $R_m(p) = p + a_m$

i.e. $a_m > 0$

And it is straightforward to check that $H(s) = \dfrac{k_m}{s + a_m}$ is strictly positive-real

Then, with this, it will be possible to use the Adaptive Law:

$$\dot{\bar{\theta}}(t) = -\operatorname{sgn}(k_p)\,\Gamma\,\bar{w}(t)\,e_1(t)$$

where $e_1(t) \triangleq y(t) - y_m(t)$

Γ = any symm p.d. matrix

Using all of the above, we can show that for this adaptive system,

- $\|\bar{\theta}(t)\|$ and $\|\bar{w}(t)\|$ are bounded for all $t \geq 0$

- $\lim_{t\to\infty} e_1(t) = 0$ $\Delta\Delta\Delta$!!!

We will develop the stability proof in next class .....

Summary: [$n^* = 1$ Case]

Plant: $R_p(p)\, y(t) = k_p Z_p(p)\, u(t)$

Ref Model: $R_m(p)\, y_m(t) = k_m r(t)$

$R_m(p) = p + a_m \; ; \; a_m > 0$

Control Law: $u(t) = \bar{\theta}(t)^T \bar{w}(t)$

Adaptive Law: $\dot{\bar{\theta}}(t) = -\mathrm{sgn}(k_p)\, \Gamma\, \bar{w}(t)\, e_1(t)$

Result: Above system leads to

- $\|\bar{\theta}(t)\|$ & $\|\bar{w}(t)\|$ bounded for all $t \geq 0$
- $\lim_{t \to \infty} e_1(t) = 0$

$\Delta\Delta\Delta$

These notes essentially review what we developed in last class.

I just re-wrote everything to put all concisely together.

Took some hours to write!! :)

Hope this helps further ....