Note Title 9/5/2011

Stability Analysis for $k^* = 1$

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$$R_p(p)\, y(t) = k_p Z_p(p)\, u(t)$$

From Proposition 1, the plant can be re-written as:

$$\dot{w} = \bar{\bar{A}}_p w + \bar{\bar{b}}_p u$$

$$y = \bar{\bar{C}}_p^T w$$

Our control law is

$$u(t) = \theta^T(t)\, w(t) + k(t)\, r(t)$$

$$= \left\{ \theta^{*T} + \phi^T(t) \right\} w(t) + \left\{ k^* + \phi_k(t) \right\} r(t)$$

Thus, with this control law, we now have

$$\dot{w} = \bar{\bar{\bar{A}}}_p w + \bar{\bar{\bar{b}}}_p u(t)$$

$$y = \bar{\bar{\bar{c}}}_p^T w$$

i.e.

$$\dot{w} = \bar{\bar{\bar{A}}}_p w + \bar{\bar{\bar{b}}}_p \left\{ \left[\theta^{*T} + \phi^T(t) \right] w(t) + \left[k^* + \phi_k(t) \right] r(t) \right\}$$

$$= \underbrace{\left\{ \bar{\bar{\bar{A}}}_p + b_p \theta^{*T} \right\}}_{\bar{\bar{\bar{A}}}_m} w(t) + \underbrace{k^* \bar{\bar{\bar{b}}}_p}_{\bar{\bar{\bar{b}}}_m} r(t) + \bar{\bar{\bar{b}}}_p \phi^T(t) w(t) + \bar{\bar{\bar{b}}}_p \phi_k(t) r(t)$$

$$y = \underbrace{\bar{\bar{\bar{c}}}_p^T}_{\bar{\bar{\bar{c}}}_m} w \qquad \text{——} (3.1)$$

where, by Propositions 1 & 2, it is implicitly possible to write

$$R_m(p)\, y_m(t) = k_m\, r(t)$$

as the $2n$-order non-minimal realization

$$\dot{w}_m = \bar{\bar{\bar{A}}}_m w_m + \bar{\bar{\bar{b}}}_m r(t)$$

$$y_m = \bar{\bar{\bar{c}}}_m^T w_m \qquad \text{— (3.2)}$$

By Propositions 1 & 2, this w_m exists, but we do not know its values!!!

equivalent state-vector of the Reference Model in non-minimal realisation!!

Then, for

$$e(t) \triangleq w(t) - w_m(t)$$

$$e_1(t) \triangleq y(t) - y_m(t)$$

From (3.1) and (3.2), we have

$$\dot{e} = \bar{A}_m e + \frac{1}{k^*} \bar{b}_m \underbrace{\begin{bmatrix} \phi^T & \phi_k \end{bmatrix}}_{\bar{\phi}^T} \underbrace{\begin{bmatrix} w \\ r \end{bmatrix}}_{\bar{w}}$$

$$e_1 = \bar{c}_m^T e$$

i.e. we have the "error dynamical system"

$$\dot{e} = \bar{A}_m e + \frac{1}{k^*} \bar{b}_m \phi^T \bar{w}$$

$$e_1 = \bar{c}_m^T e$$

where

$$\bar{\bar{c}}_m^T \{ sI - \bar{\bar{A}}_m \}^{-1} \bar{\bar{b}}_m = \frac{k_m}{R_m(s)}$$

is strictly positive-real. [for the case with $n^* = 1$]

Additionally, recall that the "adaptive law" is

$$\dot{\bar{\theta}}(t) = \dot{\bar{\Phi}}(t) = -\operatorname{sgn}(k_p) \Gamma \bar{\omega} e_1$$

Thus, now, consider the quadratic form

$$V(e, \bar{\Phi}) = e^T P e + \bar{\Phi} \Gamma^{-1} \bar{\Phi}$$

where, from the Kalman-Yakubovich Lemma, we have

$$\bar{\bar{A}}_m^T P + P\bar{\bar{A}}_m = -qq^T - \varepsilon L = -Q$$

Thus,

$$\dot{V} = 2e^T P \dot{e} + 2\tilde{\phi}\Gamma^{-1}\dot{\tilde{\phi}}$$

$$= 2e^T P \left\{ \bar{\bar{A}}_m e + \frac{1}{k^*} \bar{\bar{b}} \tilde{\phi}^T \bar{w} \right\}$$

$$+ 2\tilde{\phi}\Gamma^{-1}\dot{\tilde{\phi}}$$

$$= -e^T Q e + 2e^T \frac{1}{k^*} P b_m \tilde{\phi}^T \bar{w}$$

$$+ 2\tilde{\phi}\Gamma^{-1}\dot{\tilde{\phi}}$$

[Note also that $k_m = k_p k^*$]

Recall that the Kalman-Yacubovich Lemma states that for the spr transfer function

$$\frac{1}{|k^*|}\frac{k_m}{R_m(s)}$$

with state-realisation $\left\{\bar{\bar{A}}_m, \frac{1}{|k^*|}\bar{\bar{b}}_m, \bar{\bar{c}}_m\right\}$

- $\bar{\bar{A}}_m^T P + P\bar{\bar{A}}_m = -qq^T - \varepsilon L = -Q$ — (4.1a)
- $P\frac{1}{|k^*|}\bar{\bar{b}}_m = \bar{\bar{c}}_m$ — (4.1b)

$k_m > 0$

Noting that $k_m = k_p k^*$;

i.e. $\operatorname{sgn}(k_p) = \operatorname{sgn}(k^*)$

i.e.

$$\dot V = -e^T Q e + 2e^T \frac{sgn(k^*)}{|k^*|} P b_m \bar\phi^T \bar w + 2\bar\phi^T \Gamma^{-1}\left\{-sgn(k_p)\Gamma \bar w e_1\right\}$$

$\equiv c_m$

i.e.

$$\dot V = -e^T Q e$$

$$\cancel{+\,2\, e_1(t)\, sgn(k^*)\, \bar\phi^T \bar w}$$

$$\cancel{+\,2\,\bar\phi^T \bar w e_1 \left\{-sgn(k_p)\right\}}$$

⋮

leading to $\|\tilde{w}\|$ and $\|\tilde{\theta}\|$

bounded for all $t \geq 0$,

and $\lim_{t \to \infty} e_1(t) = 0$

$\Delta \Delta \Delta$