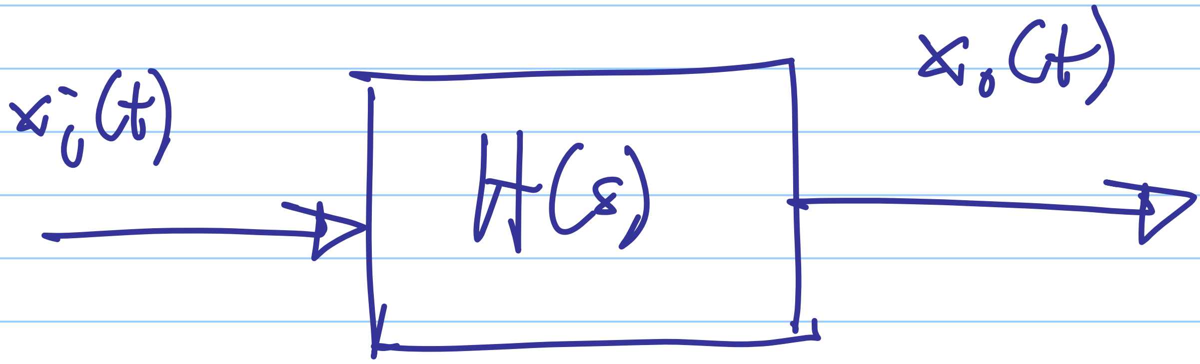
Note Title 8/30/2011

3



If $H(s)$ is strictly positive-real, then, there exist a $t_1 > t_0$ s.t.

$$\int_{t_0}^{t} x_i(\tau)\, x_o(\tau)\, d\tau > c_1$$

for all $t \geq t_1$.

Continuous-time adaptive control of Linear Systems with only input and output measurable

$$R_p(p)\, y(t) = k_p\, Z_p(p)\, u(t)$$

$$p \triangleq \frac{d}{dt}$$

$$R_p(p) = p^n + a_1 p^{n-1} + \dots + a_n$$

$$Z_p(p) = p^m + b_1 p^{m-1} + \dots + b_m$$

Relative degree $n^* = n - m$

divisor quotient remainder

$$T(p)\,R_m(p) = R_p(p)\,E(p) + F(p)$$

degree n monic

degree n^* monic

degree n^* monic

degree $(n-1)$

Now, note that our "plant" is given by:

$$R_p\, y = k_p\, Z_p\, u$$

Next consider:

$$E\, R_p\, y = k_p\, E\, Z_p\, u$$

Then, we have

$$\left\{ T R_m - F \right\} y = k_p \underbrace{E Z_p}_{\bar{G}} u$$

$$R_m y = \frac{F}{T} y + k_p \frac{\bar{G}}{T} u \quad \because T(p) \text{ is Hurwitz}$$

$$= k_p \left\{ \frac{\bar{F}}{T} y + \frac{\bar{G}}{T} u \right\} \qquad \text{—} \quad (1.11)$$

Note that $\bar{G} = \underbrace{E}_{n^{*}=n-m} \underbrace{Z_p}_{m}$

$\therefore$, we can write further $\dfrac{\bar{G}}{T}$ as

$$\frac{\bar{G}}{T} = 1 + \frac{\underbrace{G_1}_{n-1}}{T}$$

Now, write these as:

$$G_1(p) = g_1 p^{n-1} + g_2 p^{n-2} + \dots + g_n$$

$$\bar{F}(p) = f_1 p^{n-1} + f_2 p^{n-2} + \dots + f_n$$

From (1-11), we now have

$$R_m y = k_p \underbrace{\left\{ \frac{\bar{F}}{T} y + \frac{G_1}{T} u + u \right\}}_{= k^* r(t)} \quad \text{--- (1-20)}$$

i.e., achieving the closed-loop

$$R_m(p) y(t) = k_p k^* r(t)$$

with $R_m(p)$ Hurwitz

degree $= n^*$

requires

i.e., a "perfect" control law

given by =

$$\boxed{u(t) = -\frac{\bar{F}}{\bar{T}}\, y - \frac{G_1}{\bar{T}}\, u + k^{*} r(t)} \qquad \text{--- (1.2.1)}$$

<u>**Example:**</u>

Consider the specific case with $(n=2)$:

$$F(p) = f_1 p + f_2$$

$$G_1(p) = g_1 p + g_2$$

i.e. with some chosen

$$T(p) = p^2 + t_1 p + t_2 \quad \text{Hurwitz}$$

Write first the auxiliary system

$$w_y(t) = \frac{1}{T(p)} y(t)$$

i.e. set up a state-variable system

$$(p^2 + t_1 p + t_2)\, w_y(t) = y(t)$$

$$x_{y_1} = w_y$$

$$x_{y_2} = p\, w_y = \dot{x}_{y_1}$$

Then

$$\dot{x}_{y_1} = x_{y_2}$$

$$\dot{x}_{y_2} = p^2 w_y$$

$$= -t_2 x_{y_1} - t_1 x_{y_2} + y$$

Similarly for

$$w_u(t) = \frac{1}{T(p)} u(t)$$

$$\vdots$$

$$x_{u_1} \triangleq w_u(t) \qquad \dot{x}_{u_1} = x_{u_2}$$

$$x_{u_2} \triangleq p\, w_u(t) \qquad \dot{x}_{u_2} = -t_2 x_{u_1} - t_1 x_{u_2} + u$$

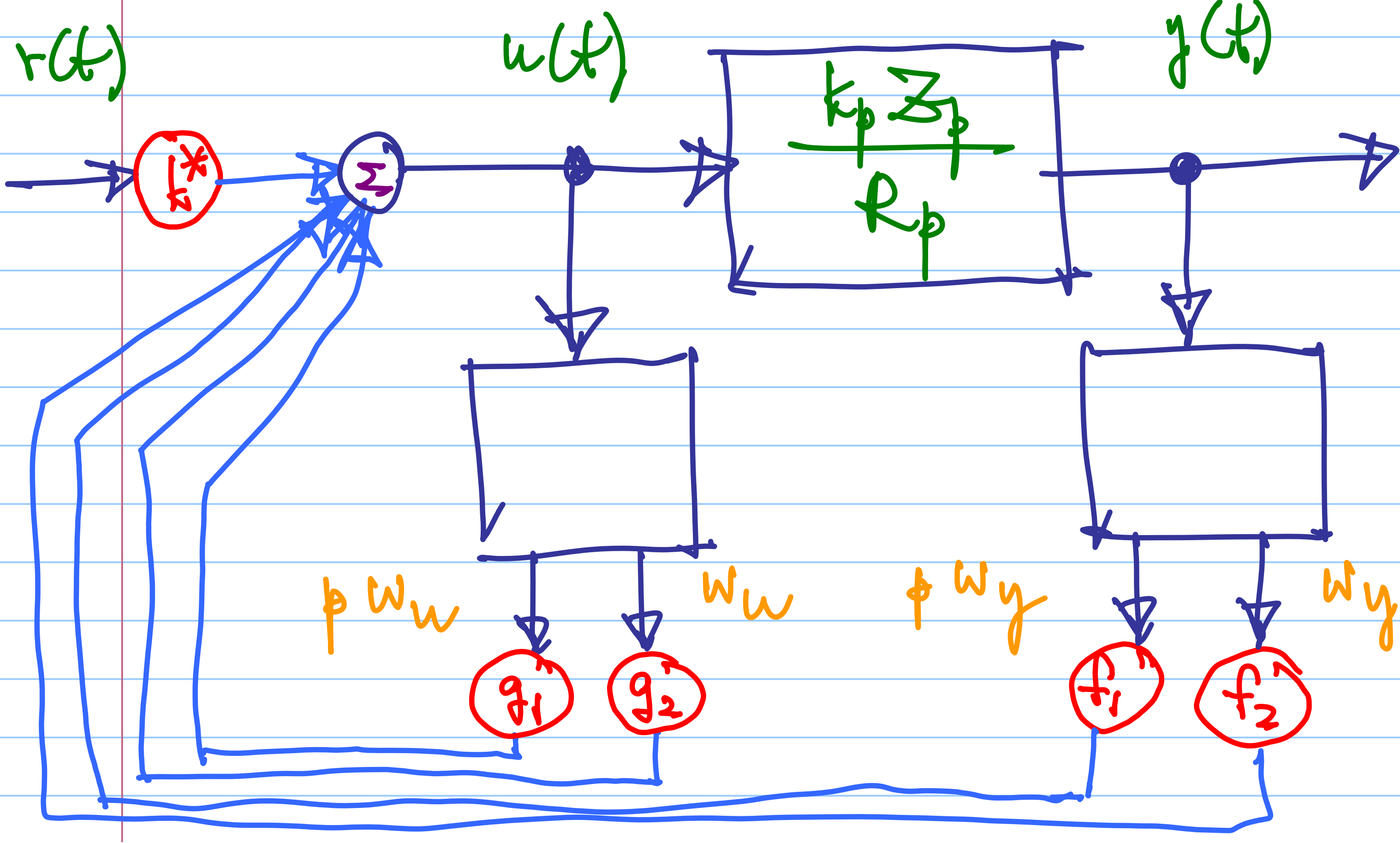
Then, further

$$\frac{\bar{F}}{T} y = (f_1 p + f_2)\; w_y(t)$$

$$= f_1\; p\, w_y(t) + f_2\; w_y(t)$$

$$\frac{G_1}{T} u = (g_1 p + g_2)\; w_u(t)$$

$$= g_1\; p\, w_u(t) + g_2\; w_u(t)$$



i.e., here, the required "perfect" control law is:

$$u(t) = \underbrace{\begin{bmatrix} -f_1 & -f_2 & -g_1 & -g_2 & k^* \end{bmatrix}}_{\bar{\theta}^*} \underbrace{\begin{bmatrix} p\,w_y \\ w_y \\ p\,w_u \\ w_u \\ r \end{bmatrix}}_{\bar{w}}$$

Thus, from (1-20) & (1-21) above, we have:

$$R_m y = k_p \left\{ \frac{\bar{F}}{T} y + \frac{G_1}{T} u + u \right\}$$

$$= k_p \left\{ \left[\underbrace{\frac{\bar{F}}{T} y + \frac{G_1}{T} u - k^* r} \right] + k^* r + u \right\}$$

$$= -\bar{\theta}^{*T} \bar{w}(t)$$

from earlier

And, to develop the adaptive control, we will choose:

$$u(t) = \bar{\theta}(t)^T \bar{w}(t)$$

i.e. this new phrase

$$R_m y = k_p\left\{ -\bar{\theta}^{*T}\bar{w} + k^* r + \bar{\theta}(t)^T \bar{w} \right\}$$

$$= k_p\left\{ \bar{\Phi}^T \bar{w} + k^* r \right\} \quad \text{(1.41)}$$

for $\bar{\Phi} \triangleq \bar{\theta}(t) - \bar{\theta}^*$

And if the reference model is chosen:

$$R_m y_m = k_m r \quad \text{(1.42)}$$

where $k_p k^* \triangleq k_m$

This leads to the "error dynamical system"

$$e_1(t) \triangleq y(t) - y_m(t)$$

$$\boxed{R_m(p)\, e_1(t) = k_p\, \bar{\phi}^T(t)\, \bar{\omega}(t)}$$