Note Title 9/8/2011

(5)

Adaptive Systems

Error Dynamical Models

Error Dynamical Model I

$$\dot{e} = A_m e + k_p b \phi^T w \qquad \text{(Ia)}$$

$k_p \in \mathbb{R}^1$ with known sign; $e \in \mathbb{R}^n$; $\phi, w \in \mathbb{R}^{n_1}$

$$\dot{\phi} = -\operatorname{sgn}(k_p)\, \Gamma e^T P b w \qquad \text{(Ib)}$$

where $\Gamma \in \mathbb{R}^{n_1 \times n_1} > 0$

sym p-d.

$A_m \in \mathbb{R}^{n \times n}$ is a stability matrix

and P is the sym p-d. soln

to $A_m^T P + P A_m = -Q$

where Q is any suitably chosen sym p-d. matrix.

$\Delta\Delta$

For the system (Ia) & (Ib), we have the result that:

- $\|e\|, \|\Phi\|$ are bounded for all $t \geq t_0$

• If $\|w\|$ is bounded, we also have

$$\lim_{t\to\infty} \|e\| = 0$$

$\Delta\Delta$

Error Dynamical Model II

IIa

$$e_1(t) = k_p W_m(p)\left[\phi^T w\right]$$

$$p \triangleq \frac{d}{dt}$$

$$\phi^T w \longrightarrow \boxed{k_p W_m(s)} \xrightarrow{e_1}$$

$W_m(s)$ is a strictly positive-real transfer function in s.

$e_1(t) \in \mathbb{R}^1$; $k_p \in \mathbb{R}^1$ with $\mathrm{sgn}(k_p)$ known

$\phi, w \in \mathbb{R}^{n}$,

$$\dot{\phi} = -\mathrm{sgn}(k_p)\,\Gamma\, e_1(t)\, w(t) \qquad \text{--- } \mathrm{II}b$$

$\Delta\Delta$

For the system $\mathrm{II}a$ and $\mathrm{II}b$,

we have the result that

$\|\phi\|$ is bounded for all $t \geq t_0$,

and if $\|w\|$ is bounded, then

we also have:

$$\lim_{t \to \infty} |e_1(t)| = 0$$

$\Delta\Delta$

Remark II :

$a_m < 0$

The system :

$$\dot{e}_1 = a_m e_1 + k_p \{ \phi y_p + \phi_r r \}$$

$$\begin{bmatrix} \dot{\phi} \\ \dot{\phi}_r \end{bmatrix} = -\operatorname{sgn}(k_p) \begin{bmatrix} \gamma_1 & 0 \\ 0 & \gamma_2 \end{bmatrix} \begin{bmatrix} y_p \\ r \end{bmatrix} e_1$$

is a special case of the above !

$\Delta\Delta$

Error Dynamical Model ①

$$e_1(t) = k_p \phi^T w \quad \text{--- } ①_a$$

$e_1 \in \mathbb{R}^1$; $\phi, w \in \mathbb{R}^{n_1}$

$k_p \in \mathbb{R}^1$; $\mathrm{sgn}(k_p)$ known

$$\dot{\phi} = -\mathrm{sgn}(k_p)\,\Gamma w e_1$$

Γ symmetric p.d. $\text{--- } ①_b$

$\Delta\Delta$

For the system $①_a$ and $①_b$,
we have the result that

$\|\phi\|$ is bounded for all $t > t_0$

Remark ①:

If, in addition, the signal $w(t)$ is "persistently exciting", then we also have:

$$\lim_{t \to \infty} \| \phi(t) \| = 0$$

But the condition of $w(t)$ being "persistently exciting" is a rather stringent & difficult one.

Aside:

A signal $w(t) \in \mathbb{R}^{n_1}$ is p.e. iff there exists $\delta, \epsilon > 0$ s.t.

$$\int_t^{t+\delta} w(\tau)^T w \, d\tau \geq \epsilon$$

Stability Analysis of Error Dynamical Model

for all $\|\mathbf{x}\| = 1$ with $\mathbf{x} \in \mathbb{R}^{n_1}$; and all $t > t_0$.

Consider the quadratic form $\phi^T \Gamma^{-1} \phi$

$$V(t) = \|\phi\|^2 = \phi^T \phi$$

for $\Gamma = I$

Then

$$\dot{V}(t) = 2\phi^T \dot{\phi}$$

$$= 2\phi^T \left\{ -\operatorname{sgn}(k_p)\, w\, e_1 \right\}$$

$$= -2\operatorname{sgn}(k_p) \frac{1}{k_p} e_1 e_1$$

$$= -2\operatorname{sgn}(k_p) \frac{\operatorname{sgn}(k_p)}{|k_p|} e_1^2$$

$$\leq 0$$

$\Delta\Delta$

⋮

Cases of stability analysis

for $n^* > 1$ case