EE5103R/ME5403 Computer Control Systems: Homework #3 Solution

Semester 1 Y2021/2022

Q1

The controller is required to be in form of

$$R(q)u(k) = T(q)u_c(k) - S(q)y(k), \tag{0.1}$$

whose z transform is

$$U(z) = \frac{T(z)}{R(z)}U_c(z) - \frac{S(z)}{R(z)}Y(z). \tag{0.2}$$

Therefore, the block diagram for this control system is

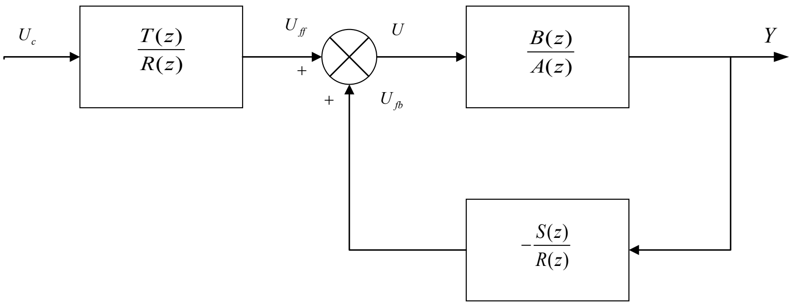

Figure 1 Block diagram of the closed-loop system.

And we have

$$\frac{B(z)}{A(z)} = H(z) = \frac{z+0.8}{z^2 - 1.5z + 0.5} \tag{0.3}$$

By designing the above two degree-of-freedom controller, we aim to obtain a closed-loop system with the following characteristic polynomial

$$A_m(z) = z^2 - 1.8z + 0.9 \tag{0.4}$$

To match the specified closed-loop denominator (poles), we mainly need to manipulate the feedback term $S(z)/R(z)$ since the closed-loop transfer function caused from the control law (1.2) is computed to be

$$\frac{Y(z)}{U_c(z)} = \frac{B(z)T(z)}{A(z)R(z)+B(z)S(z)} \triangleq \frac{B(z)T(z)}{A_{cl}(z)}. \tag{0.5}$$

Now the question is: what is the closed-loop polynomial $A_{cl}(z)$ in (1.5) ? Since we are required to achieve $A_m(z)$ as the closed-loop denominator at the end, $A_{cl}(z)$ must originally be in some form of $A_{cl}(z) = A_o(z)A_m(z)$ and then reduced to $A_m(z)$ by cancelling $A_o(z)$. Now the remaining task is to determine $A_o(z)$. Generally, $A_o(z)$ may stem simply from polynomial order matching (when $A_{cl}(z)$ has a higher order than the required $A_m(z)$) or from additional requirements like zero cancellation, disturbance rejection etc. This is a basic principle for model matching of lecture 5, which applies to all the problems in this assignment.

a)

If the process zero is cancelled, that is, there is no $B(z)$ in the final form of (1.5), then it can be inferred that the closed-loop denominator before cancellation is

$$A_{cl}(z) = B(z)A_m(z) = (z+0.8)A_m(z) = (z+0.8)(z^2-1.8z+0.9) \tag{0.6}$$

Obviously, we have $\deg(R(z)) = \deg(A_{cl}(z)) - \deg(A(z)) = 1$. Therefore, the general form of $R(z) = z + r_1$. Additionally, since there is a factor $(z+0.8)$ in $A_{cl}(z)$, the $R(z)$ must be in this form $R(z) = (z+0.8)$. Further, suppose $S(z) = s_o z + s_1$ due to $\deg(S(z)) \le \deg(R(z))$, then we have

$$\begin{aligned} A_{cl}(z) &= A(z)R(z) + B(z)S(z) \\ &= (z^2 - 1.5z + 0.5)(z+0.8) + (z+0.8)(s_o z + s_1) \\ &= (z+0.8)[z^2 + (s_0 - 1.5)z + 0.5 + s_1]. \end{aligned} \tag{0.7}$$

From (1.6) and (1.7), it can be inferred that

$$z^2 + (s_0 - 1.5)z + 0.5 + s_1 = z^2 - 1.8z + 0.9 \tag{0.8}$$

And it can be solved as $s_0 = -0.3$, $s_1 = 0.4$. Hence

$$\frac{S(z)}{R(z)} = \frac{-0.3z + 0.4}{z+0.8} \tag{0.9}$$

After we get $R(z)$ and $S(z)$, it is time now to choose $T(z)$. The requirement is that the steady state gain from $U_c(z)$ to $Y(z)$ is one. For the closed-loop transfer function (1.5), the steady state gain can be computed by $Y(1)/U_c(1)$. We can assume the simplest form for $T(z) = t_0$. Then, we have

$$\frac{Y(1)}{U_c(1)} = \left.\frac{B(z)T(z)}{A_{cl}(z)}\right|_{z=1} = \frac{t_0}{1-1.8+0.9} = 1, \tag{0.10}$$

which further gives

$$t_0 = 0.1. \tag{0.11}$$

Thus, $$T(z) = 0.1. \tag{0.12}$$

The whole controller is designed as follows:

$$\boxed{\begin{aligned} &R(z) = z + 0.8 \\ &S(z) = -0.3z + 0.4 \\ &T(z) = 0.1 \\ &U(z) = \frac{T(z)}{R(z)} U_c(z) - \frac{S(z)}{R(z)} Y(z) \end{aligned}} \tag{0.13}$$

In this case, from (1.5), the closed-loop transfer function is

$$G(z) = \frac{B_m(z)}{A_m(z)} = \frac{Y(z)}{U_c(z)} = \frac{0.1}{z^2 - 1.8z + 0.9}. \tag{0.14}$$

The controller is:

$$u(k) = 0.2188u(k-1) + 0.0556u_c(k) + 0.0812y(k) - 0.1368y(k-1)$$

b)

If the process zero is not cancelled, based on the above analysis, the order of the closed-loop characteristic polynomial is $\deg(A_{cl}(z)) = \deg(A(z)R(z) + B(z)S(z)) > \deg(A(z)) = 2$ since the order of $R(z)$ is at least one. Then, because the desired one is just $\deg(A_m(z)) = 2$, we have to include another auxiliary factor $A_o(z)$ such that the closed-loop characteristic polynomial can be matched as

$$A_{cl}(z) = A_o(z)A_m(z) = zA_m(z) = z(z^2 - 1.8z + 0.9) \tag{0.15}$$

Here we choose $A_o(z) = z$ because we want the lowest order one whose poles are all at the origin.

Now it can be inferred that $\deg(R(z)) = \deg(A_{cl}(z)) - \deg(A(z)) = 1$. Similarly, we shall assume $R(z) = r_0 z + r_1$, $S(z) = s_o z + s_1$. Then, we notice that the leading coefficient of both $A_{cl}(z)$ and $A(z)$ is 1, which indicates that $r_0 = 1$ since there exists $A(z)R(z) + B(z)S(z) = A_{cl}(z)$. Now we have $R(z) = z + r_1$.

Then, note that the additional $A_o(z)$ in (1.15) must be cancelled in the final form because the

$A_m(z)$. This is the responsibility of $T(z)$, which is designed to be $T(z) = t_0 z$ since $T(z)$ should include the factor $A_o(z) = z$.

Finally, the closed-loop transfer function is

$$\begin{aligned} G(z) = \frac{Y(z)}{U_c(z)} &= \frac{B(z)T(\mathrm{z})}{A(z)R(z)+B(z)S(z)} = \frac{t_0 z(z+0.8)}{(z^2-1.5z+0.5)(z+r_1)+(z+0.8)(s_o z+s_1)} \\ &= \frac{t_0 z(z+0.8)}{z^3+(r_1+s_0-1.5)z^2+(0.5-1.5r_1+\ s_1+0.8s_0)z+0.5r_1+0.8s_1} \end{aligned} \tag{0.16}$$

According to (1.15) and (1.16), we have

$$z^3+(r_1+s_0-1.5)z^2+(0.5-1.5r_1+\ s_1+0.8s_0)z+0.5r_1+0.8s_1 = z(z^2-1.8z+0.9) \tag{0.17}$$

Thus, the following equations are derived

$$\begin{cases} r_1+s_0-1.5=-1.8 \\ 0.5-1.5\mathrm{r}_1+\mathrm{s}_1+0.8\mathrm{s}_0=0.9 \\ 0.5r_1+0.8s_1=0 \end{cases} \tag{0.18}$$

whose solutions is

$$\begin{cases} r_1=-0.2188 \\ s_0=-0.0812 \\ s_1=0.1368 \end{cases} \tag{0.19}$$

And then from (1.16) and (1.17) we get

$$G(z) = \frac{t_0(z+0.8)}{z^2-1.8z+0.9} \tag{0.20}$$

Besides, it is required that $G(1)=1$ for a unit steady-state gain:

$$G(1) = \frac{1.8t_0}{0.1} = 1 \Rightarrow t_0 = \frac{1}{18} \approx 0.0556 \tag{0.21}$$

After all, the controller is designed as follows

$$\boxed{\begin{aligned} &R(z) = z - 0.2188 \\ &S(z) = -0.0812z + 0.1368 \\ &T(z) = \frac{1}{18}z \\ &U(z) = \frac{T(z)}{R(z)}U_c(z) - \frac{S(z)}{R(\mathrm{z})}Y(z) \end{aligned}} \tag{0.22}$$

And the closed-loop transfer function in this case is

$$G(z) = \frac{Y(z)}{U_c(z)} = \frac{z + 0.8}{18(z^2 - 1.8z + 0.9)} \tag{0.23}$$

The controller is:

$$u(k) = 0.2188u(k-1) + 0.0556u_c(k) + 0.0812y(k) - 0.1368y(k-1)$$

c)

After we finished the design, a Simulink model can be easily built according to Figure 1, which is exhibited in Figure 2. Run the simulation with a sampling period of 1ms.

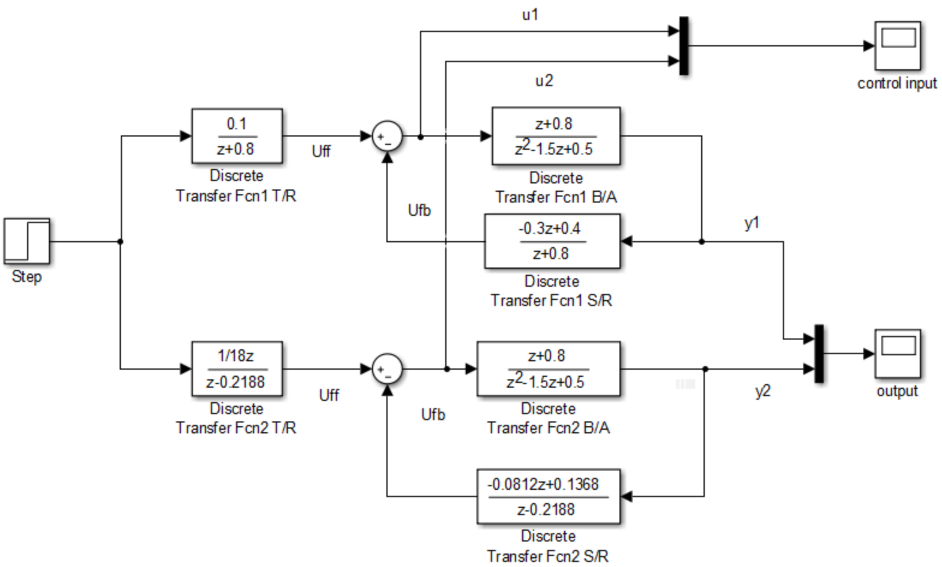

Figure 2 Simulink model.

The closed-loop output and control input profiles are shown in Figure 3 and Figure 4 respectively.

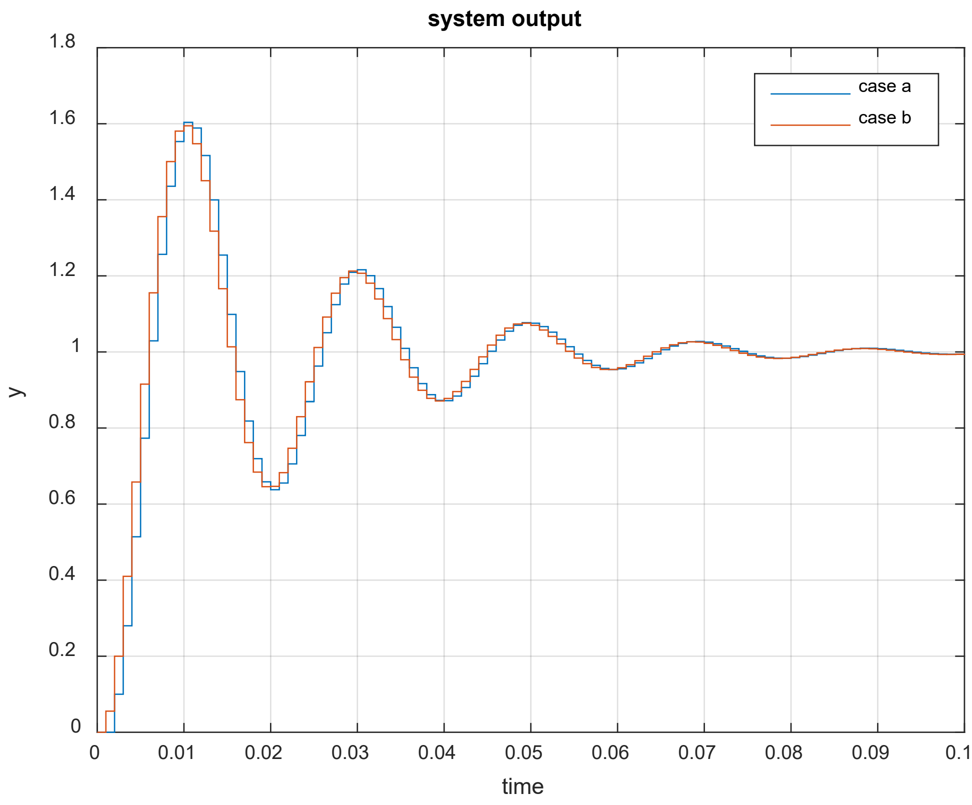

Figure 3 System output for the two cases.

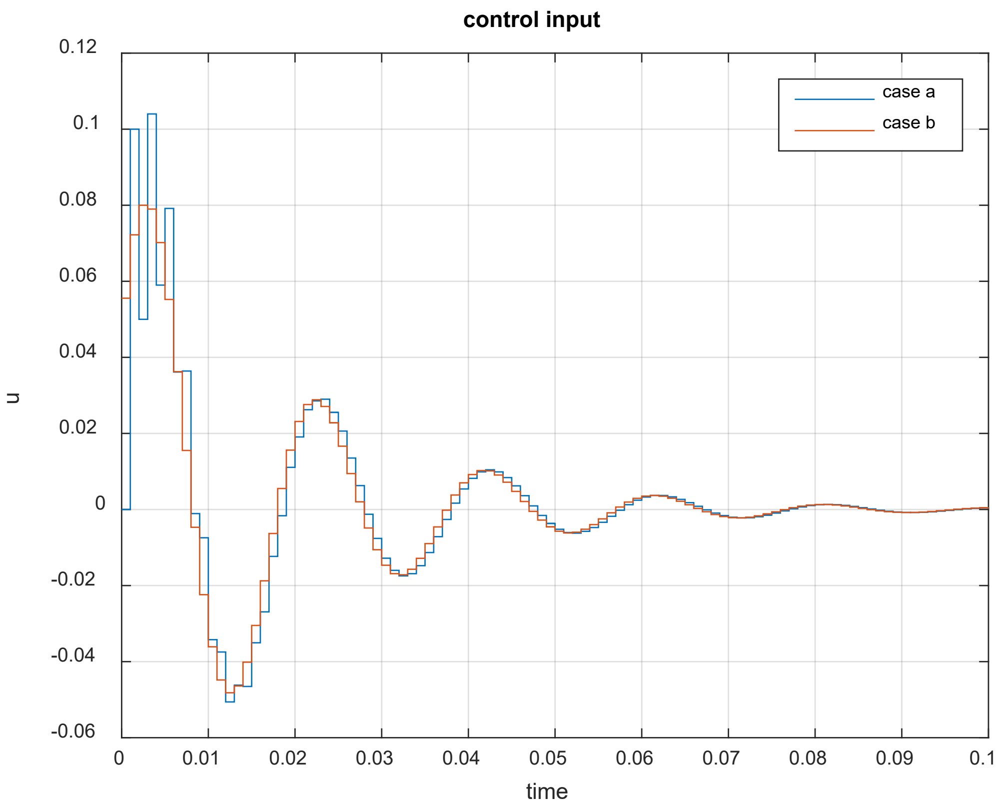

Figure 4 Control input signals for the two cases.

As can be seen, the outputs in the two cases are quite similar. However, in Figure 4 it is noted that the control input signal for case 1 (with zero cancellation) is more seriously oscillating before arriving at the steady state. Besides, the control input for case 1 has a slightly larger magnitude. What is the reason for such behavior of the control input? Since we are studying the control input $u(k)$, we first write down the transfer function from the command signal $u_c(k)$ to the control input $u(k)$ as follows (for both the two cases)

$$\begin{cases} U(z) = \dfrac{T(z)}{R(z)}U_c(z) - \dfrac{S(z)}{R(z)}Y(z) \\ \dfrac{Y(z)}{U(z)} = \dfrac{B(z)}{A(z)} \end{cases} \Rightarrow H_u(z) = \frac{U(z)}{U_c(z)} = \frac{A(z)T(z)}{A(z)R(z)+B(z)S(z)} = \frac{A(z)T(z)}{A_{cl}(z)}. \quad (0.24)$$

By inserting $A_{cl}(z)$ from (1.6) and (1.15) into the above equation for case 1 and case 2 respectively, we get $H_u(z)$ for the two cases:

$$\begin{array}{ll} \text{case } a & H_u(z) = \dfrac{0.1(z^2-1.5z+0.5)}{(z+0.8)(z^2-1.8z+0.9)} \\ \text{case } b & H_u(z) = \dfrac{0.0556z(z^2-1.5z+0.5)}{z(z^2-1.8z+0.9)} = \dfrac{0.0556(z^2-1.5z+0.5)}{z^2-1.8z+0.9} \end{array} \quad (0.25)$$

The poles of the above two transfer functions differ from each other, as follows

$$\begin{array}{ll} \text{case } a \text{ poles} & [-0.8, \quad 0.9+0.3i, \quad 0.9-0.3i] \\ \text{case } b \text{ poles} & [0.9+0.3i, \quad 0.9-0.3i] \end{array} \quad (0.26)$$

The additional pole of case a has a negative real part. We know that a pole with a negative real part in z transfer function would cause oscillation and this is the main reason for the above phenomenon we have observed.

In practice, we would favor a smoother control input with a smaller magnitude, which is easier to implement and more cost-friendly. Hence with the comparison, the second controller (without zero cancellation) is preferred.

Q2

a)

Applying z-transform to the state-space representation with zero initial conditions yields:

$$\begin{cases} zX(z) = \Phi X(z) + \Gamma U(z) + \Phi_{xv}V(z) \\ Y(z) = CX(z) \end{cases}$$

Where $\Phi = \begin{bmatrix} 0.5 & 1 \\ 0.5 & 0.8 \end{bmatrix}, \Gamma = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix}, \Phi_{xv} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \end{bmatrix}$.

Then we can get the transfer function

$$\begin{aligned} Y(z) &= C(zI-\Phi)^{-1}\Gamma U(z) + C(zI-\Phi)^{-1}\Phi_{xv}V(z) \\ &= G_u U(z) + G_v V(z) \end{aligned}$$

By superposition principle, we can firstly assume the disturbance to be zero, so the transfer function from input to output is

$$G_u(z) = \frac{Y(z)}{U(z)} = C(zI - \Phi)^{-1}\Gamma = \frac{0.2z - 0.06}{z^2 - 1.3z - 0.1} \triangleq \frac{B(z)}{A(z)}$$

Secondly, we will assume the input is zero, so the transfer function from the disturbance to the output is

$$G_v(z) = \frac{Y(z)}{V(z)} = C(zI - \Phi)^{-1}\Phi_{xv} = \frac{z - 0.8}{z^2 - 1.3z - 0.1} \triangleq \frac{B_v(z)}{A(z)}$$

Then the disturbance rejection controller can be designed with output feedback as shown in Figure 5.

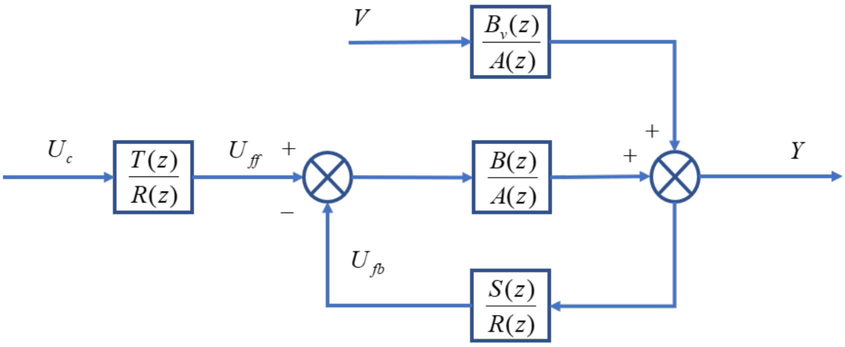

Figure 5 Diagram graph of the control system.

Assuming the $U_c(z) = 0$, we will get the closed-loop transfer function between the disturbance and the output:

$$H_v(z) = \frac{Y(z)}{V(z)} = \frac{B_v(z)R(z)}{A(z)R(z) + B(z)S(z)} \tag{0.27}$$

To eliminate the disturbance effect at steady state, we require the steady state gain of $H_v(z)$ to be zero, that is, $H_v(1) = 0$. This further request $R(z)$ to contain a factor $(z - 1)$. We suppose that $R(z) = (z - 1)(z + r)$ and $S(z) = s_0 z^2 + s_1 z + s_2$ to match the desired control performance. Since the problem did not give the desired closed-loop characteristic polynomial, we can choose the simplest one as $A_m(z) = z^4$. Therefore to match $A_{cl}(z) = A_m(z)$,

$$\begin{aligned} A_{cl}(z) &= A(z)R(z) + B(z)S(z) \\ &= z^4 + (-2.3 + r_1 + 0.2s_0)z^3 + (1.2 - 2.3r_1 - 0.06s_0 + 0.2s_1)z^2 \\ &\quad + (0.1 + 1.2r_1 - 0.06s_1 + 0.2s_2)z + (0.1r_1 - 0.06s_2) \\ &= A_m(z) = z^4 \end{aligned}$$

We will have

$$\begin{cases} -2.3 + r_1 + 0.2s_0 = 0 \\ 1.2 - 2.3r_1 - 0.06s_0 + 0.2s_1 = 0 \\ 0.1 + 1.2r_1 + 0.2s_2 - 0.06s_1 = 0 \\ 0.1r_1 - 0.06s_2 = 0 \end{cases} \Rightarrow \begin{cases} r_1 = -0.2711 \\ s_0 = 12.8554 \\ s_1 = -5.2608 \\ s_2 = -0.4518 \end{cases}$$

Choose $T(z)$ to guarantee the steady state gain to be 1, so choose $T(z) = t_0 z^2$

$$H_{cl}(z)\Big|_{z=1} = \frac{B(z)T(z)}{A_{cl}(z)} = \left.\frac{t_0 z^2 (0.2z - 0.06)}{z^4}\right|_{z=1} = 1 \quad \Rightarrow \quad t_0 = 7.1428 \qquad (0.28)$$

Therefore, the controller is

$$\boxed{\begin{cases} R(z) = (z-1)(z-0.2711) \\ S(z) = 12.8554z^2 - 5.2608z - 0.4518 \\ T(z) = 7.1428z^2 \end{cases}} \qquad (0.29)$$

So, the desired controller is:

$$\begin{aligned} u(k) = {} & 7.1428u_c(k) + 1.2711u(k-1) - 0.2711u(k-2) - 12.8554y(k) \\ & + 5.2608y(k-1) + 0.4518y(k-2), \quad k \geq 2 \end{aligned}$$

Remark:

You can also choose R(z) as R(z)=(z-1)B(z) for both zero cancellation and disturbance rejection , then calculate S(z) and T(z) accordingly.

b)

For this problem, the polynomial approach is simpler because we just need to choose appropriate R,S and T to make the closed loop system stable and to eliminate constant disturbance. There is no need to build the observer.

Q3

a)

The structure of the closed-loop system is the same with the one in Figure 1. The obtained closed-loop transfer function is

$$\frac{Y(z)}{U_c(z)} = \frac{B(z)T(z)}{A(z)R(z)+B(z)S(z)} \triangleq \frac{B(z)T(z)}{A_{cl}(z)}.$$

Comparing the transfer function with the reference model $A_m(z)$, we can see that the factor $B(z) = z - 0.5$ disappears in the reference model, which implies zero cancellation. Therefore, $R(z)$ must be comprised of $B(z)$ and possibly another factor.

Furthermore, here we should pay attention to another requirement on constant disturbance rejection. Assume the disturbance is $v(k)$. Then, from Figure 5, the transfer function from the disturbance v to the output y is resolved to be

$$G_v(z) = \frac{Y(z)}{V(z)} = \frac{B(z)R(z)}{A(z)R(z)+B(z)S(z)}$$

To eliminate the disturbance effect at steady state, we require the steady state gain of $G_v(z)$ to be zero, that is, $G_v(1)=0$. This further request R(z) to contain a factor (z-1). Therefore, R(z) should be the form (z-1)(z-0.5). Similarly, since there exists zero cancellation, the closed-loop polynomial is in the form of $A_{cl}(z)=(z-0.5)A_m(z)A_o(z)$.

Now what is $A_0(z)$? We know that if we choose $R(z) = (z-1)(z-0.5)$, then the general form of $S(z)$ is $S(z) = s_0z^2 + s_1z + s_2$. Since $A(z)R(z)$ has a degree of 4, we may choose $A_0(z) = z$. Now we have

$$A(z)R(z) + B(z)S(z) = (z^2-1)(z-1)(z-0.5) + (z-0.5)(s_0z^2 + s_1z + s_2)$$
$$A_{cl}(z) = (z-0.5)A_m(z)A_0(z) = (z-0.5)z^3$$

To match corresponding coefficients, we can solve $S(z) = z^2 + z - 1$.

Now the closed-loop transfer function is

$$\frac{Y(z)}{U_c(z)} = \frac{B(z)T(z)}{A(z)R(z)+B(z)S(z)} = \frac{T(z)}{z^3}$$

To eliminate the additional $A_0(z)$ and get $H_m(z)$, it is obvious that $T(z) = z$.

Therefore, the controller is

$$\boxed{\begin{cases} R(z)=(z-1)(z-0.5)=z^2-1.5z+0.5 \\ S(z)=z^2+z-1 \\ T(z)=z \end{cases}}$$

Therefore, the controller is

$$u(k)=1.5u(k-1)-0.5u(k-2)+u_c(k-1)-y(k)-y(k-1)+y(k-2),\ k\ge 2$$

b)

The closed-loop transfer function is

$$G(z)=\frac{Y(z)}{U_c(z)}=\frac{B(z)R(z)}{A(z)R(z)+B(z)S(z)}H_{ff}(z). \tag{0.30}$$

The closed-loop transfer function from the disturbance to the output is still

$$G_v(z)=\frac{Y(z)}{V(z)}=\frac{B_v(z)R(z)}{A(z)R(z)+B(z)S(z)}. \tag{0.31}$$

To reject constant disturbance, we need $R(z)$ involves a $(z-1)$ factor. If $R(z)$ is chosen to be of degree one, then we have $S(z)=s_0z+s_1$. Since the degree of $A(z)R(z)$ is 3, to match the closed-loop polynomials there are 3 equations (the leading coefficient is fixed to be 1) but only 2 unknows s_0 and s_1, which usually has no solutions. Therefore, $R(z)$ cannot be 1st order and $R(z)$ should have a degree of at least 2. Supposing $R(z)=(z-r_1)(z-1)$ and $S(z)=s_0z^2+s_1z+s_2$, we can get

$$A(z)R(z)+B(z)S(z)=(z^2-1)(z-r_1)(z-1)+(z-0.5)(s_0z^2+s_1z+s_2) \tag{0.32}$$

Since $A_m(z)=z^2$ is only second order, we need another term $A_0(z)=z^2$ to match the equation, and get

$$A_{cl}(z)=A_m(z)A_0(z)=z^4 \tag{0.33}$$

Comparing (1.32) and (1.33), we obtain

$$\begin{cases} -1-r_1+s_0=0 \\ -1+r_1-0.5s_0+s_1=0 \\ 1+r_1-0.5s_1+s_2=0 \\ -r_1-0.5s_2=0 \end{cases} \Rightarrow \begin{cases} r_1=0.3333 \\ s_0=1.3333 \\ s_1=1.3333 \\ s_2=-0.6667 \end{cases} \tag{0.34}$$

Therefore, the feedback controller is

$$\boxed{\frac{S(z)}{R(z)} = \frac{1.3333z^2 + 1.3333z - 0.6667}{(z-0.3333)(z-1)}} \tag{0.35}$$

Now we have $R(z)$ and $S(z)$. From (1.30) we further obtain

$$\frac{B(z)R(z)}{A(z)R(z)+B(z)S(z)} H_{ff}(z) = \frac{B(z)R(z)}{A_{cl}(z)} H_{ff}(z) \tag{0.36}$$

To match the closed-loop transfer function (1.36) with the reference model $H_m(z) = B_m(z)/A_m(z)$, we get

$$\frac{B(z)R(z)}{A_{cl}(z)} H_{ff}(z) = \frac{B_m(z)}{A_m(z)} \quad \Rightarrow \quad H_{ff}(z) = \frac{B_m(z)A_{cl}(z)}{A_m(z)B(z)R(z)} = \frac{B_m(z)A_0(z)}{B(z)R(z)} \tag{0.37}$$

Inserting the relevant terms into (1.37) gives

$$\boxed{H_{ff}(z) = \frac{z^2}{(z-0.5)(z-0.3333)(z-1)}} \tag{0.38}$$

Therefore, the two-degree of freedom controller has the form of,

$$U(z) = \frac{z^2}{(z-0.5)(z-1)(z-0.3333)} U_c(z) - \frac{1.3333z^2 + 1.3333z - 0.6667}{(z-1)(z-0.3333)} Y(z)$$

Note:

1. For a second thought, you may find that if we keep the $R(z)$ and $S(z)$ in the solution to question a) unchanged and let $H_{ff}(z) = T(z)/R(z)$, we can also meet the requirement. That is to say, the control configuration in a) is exactly a special case of b). However, the $H_{ff}(z)$ structure in b) will give you more freedom since there is no requirement on its denominator form and it is a more general control configuration. For exercise and examination purpose, you should follow different procedures to finish the design in these two cases, although a) can be regarded as a special case of b). In this assignment, if you simply write $H_{ff}(z) = T(z)/R(z)$ by reusing the result obtained in a), you will only get 1 point.

2. In case b), $R(z)$ is only responsible for disturbance rejection since $H_{ff}(z)$ doesn't appear in the transfer function from disturbance to output. All the other work will be done by $H_{ff}(z)$ since it has more freedom than case a).

3. In the above, to make the computation easier, we have chosen a $R(z)$ of a lowest possible degree. Unless explicitly specified, you can of course choose a higher order of $R(z)$. Similarly, we choose a $A_0(z)$ whose poles are all at the origin to facilitate the calculation. You can choose other forms of $A_0(z)$ as long as its poles are stable. Overall, the only

requirement is that the closed-loop transfer function should match the given reference model.

Q4

a)

The given input-output model is

$$y(k+1) = cy(k) + y^2(k-1) + u(k-1) \qquad (1.1)$$

Forward shift by one, we get

$$y(k+2) = cy(k+1) + y^2(k) + u(k) \qquad (1.2)$$

In order to get a predictor, just replacing y(k+1) by (2.1),

$$y(k+2) = c(cy(k) + y^2(k-1) + u(k-1)) + y^2(k) + u(k) \qquad (1.3)$$

In one-step-ahead controller, just make $y(k+2) = r(k+2)$ gives

$$u(k) = r(k+2) - c^2 y(k) - cy^2(k-1) - y^2(k) - cu(k-1) \qquad (1.4)$$

b)

We need to assure that the plant output $y(k)$ and the controller output (the plant input) $u(k)$ are both bounded for perfect tracking.

Since the perfect tracking can be achieved, we will have $y(k) = r(k)$, the plant output is bounded as long as the reference $r(k)$ is bounded. In the case of perfect tracking, we have $y(k+2) = r(k+2)$ and $y(k+1) = r(k+1)$. Based on equation (2.2), the control input is

$$u(k) = r(k+2) - cr(k+1) - r^2(k) \qquad (1.5)$$

For any reference signal $r(k)$, as long as it is bounded, the right hand side of (2.5), i.e., $r(k+2) - cr(k+1) - r^2(k)$, is also bounded. Therefore, as long as r(k) is bounded, the input u(k) is also bounded. So, there is no constraint on parameter c.

Note:

For the discussion on the perfect tracking, we should derive the condition based on the original input-output equation. This is the key step to get the correct result. The reason lies that, when converting the original input-output model, we are very likely to introduce more common poles/ zeros into the model. For example, let's just consider one simple input-output model as

$$y(k+1) = cy(k) + y(k-1) + u(k-1)$$

Then the transfer function is

$$\frac{Y(z)}{U(z)} = \frac{1}{z^2 - cz - 1}$$

which has a stable inverse.

However, in order to get the predictor form, we have

$$y(k+2) = c(cy(k) + y(k-1) + u(k-1)) + y(k) + u(k)$$

Then the transfer function now becomes

$$\frac{z+c}{z^3 - (c^2+1)z - c} = \frac{z+c}{(z^2 - cz - 1)(z+c)}$$

So, the transfer function of the predictor is the original TF plus extra common poles and zeros (z+c). But we cannot use this TF to analyze the stability of the original open-loop system because the extra pole (z+c) is imposed by us.