Chapter 3

From Lecture Slides

(3.1) $R_{sys} = 0.99^{10}$

(3.2) CFRs : 0.4, 0.5, 0.6

$\lambda(t) = 0.4 + 0.5 + 0.6 = 1.5$ failures/min

$\Rightarrow$ CFR.

$MTTF = (1/\lambda) = 0.667$ mins

Comparing with individual MTTFs :

$$\begin{cases} MTTF_1 = 1/0.4 = 2.5 \text{ mis} \\ MTTF_2 = 2.0 \text{ mis} \\ MTTF_3 = 1.67 \text{ mins} \end{cases}$$

(3.3) $R_{sys} = 1 - (1-0.8)^4 = 0.9984$

(3.4) $R_1(t) = e^{-0.4t}$

$R_2(t) = e^{-0.5t}$

$R_3(t) = e^{-0.6t}$

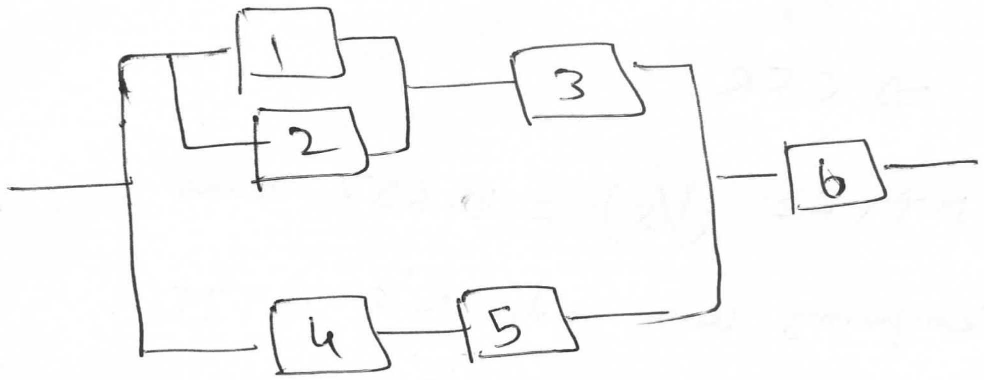
$\Rightarrow R(t) = 1 - (1-e^{-0.4t})(1-e^{-0.5t})(1-e^{-0.6t})$

$MTTF = \int_0^\infty R(t)dt \simeq 4$ mins [Compare (3.2) & this result]

(2) $\lambda(t) = -\frac{dR(t)}{dt} \cdot \frac{1}{R(t)} = \underbrace{f(t)}_{\text{function of } t.}$

$\Rightarrow$ failure rate of the system is clearly not a CFR.

(3.5)



let:

(1,2) : A   (1,2,3) : B   (4,5) : C

(A,B,C) : D

$$\begin{cases} R_A = 1-(1-R_1)(1-R_2) \\ R_B = R_3 \cdot R_A \end{cases}$$

$R_C = R_4 \cdot R_5$

$R_D = 1-(1-R_B)(1-R_C)$

$= \underbrace{1-(1-R_3(1-(1-R_1)(1-R_2)))(1-R_4 \cdot R_5)}$

$\boxed{R = R_6 \cdot R_D}$

From Practice Problems link

① (a) $\int_0^\infty f(t)\,dt = \int_0^{1000} \frac{3t^2}{10^9}\,dt = 1$ &

as $f(t) \geq 0 \ \forall t$, $f(t)$ is a valid PDF.

$$F(t) = \int_0^t f(t)\,dt = \begin{cases} \frac{t^3}{10^9} & ; \ 0 \leq t \leq 1000 \\ 1 & ; \ t > 1000 \end{cases}$$

$$R(t) = 1 - F(t) = \begin{cases} 1 - t^3/10^9 & ; \ 0 \leq t \leq 1000 \\ 1 & ; \ t > 1000 \end{cases}$$

(b) $R(500) = 1 - \frac{500^3}{10^9} = 0.875$

(c) $P(\text{failing in first 100 hrs}) = F(100) = 0.001$

(d) $R(t) = 0.99 = 1 - t^3/10^9$

$\Rightarrow t = 215$ hrs

$R(t) \geq 0.99$ for $0 \leq t \leq 215$. Hence design life-time is 215 hrs.

(e) $\lambda(t) = f(t)/R(t)$

λ is not defined for $t > 1000$ since component fails before 1000 hrs.

4

2 (a) $f(t) = -\frac{dR}{dt} = \frac{2}{t_0}\left(1 - t/t_0\right), 0 \leq t \leq t_0$

$\lambda(t) = f(t)/R(t)$

$$= \frac{\frac{2}{t_0}\left(1 - t/t_0\right)}{\left(1 - t/t_0\right)^2} = \frac{2}{(t_0 - t)}, \quad 0 \leq t \leq t_0$$

Which is a monotonically increasing fuction implying a wear-out phase;

(b) $MTTF = \int_0^{\infty} R(t)\, dt = (t_0/3)$

(c) $R(t) = 0.9 = \left(1 - t/5000\right)^2$

$\Rightarrow t = 257$ hrs. (design life-time)

$R \geq 0.9$ for $0 \leq t \leq 257$ hrs.

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3 MTTF = 1100 hrs

(a) $\lambda = (1/MTTF) = 9.09 \times 10^{-4}\ hr^{-1}$

(b) $R(200) = 0.834 \quad \left[e^{-200/1100}\right]$

(c) $R(t) = 0.95 = e^{-t/1100}$

$\Rightarrow t = 56$ hrs. (design life-time)

4 $\beta = 1.4 \qquad \theta = 550$

(a) $R_{comp}(t) = e^{-(t/550)^{1.4}}$

$$R_{\substack{comp\ in\ the \\ system}}(t) = \begin{cases} R_{comp}(t - t_0) & ; \ t > t_0 \\ 1 & ; \ 0 \le t \le t_0 \end{cases}$$

where $t_0 = 200$ hrs

(b) $R(100) = 1$ since $100 < t_0$.

$$R(300) = R_{comp}(300 - 200) = e^{-\left(\frac{100}{550}\right)^{1.4}}$$
$$= 0.912$$

(c) $MTTF = t_0 + MTTF_{comp}$

$$= t_0 + \theta \cdot \underbrace{\Gamma(1 + 1/\beta)}_{\text{gamma func.}}$$

$$= 200 + 550\ \underbrace{\Gamma(1.714)}_{0.91140}$$

$\Rightarrow$ $\underline{MTTF = 701 \text{ hrs}}$