Note Title 8/12/2013

EE 5104 / 6104

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CA – 70% of module grade — 3 assignments (of which 1 is a hardware mini-project)

Exam – 30% of module grade

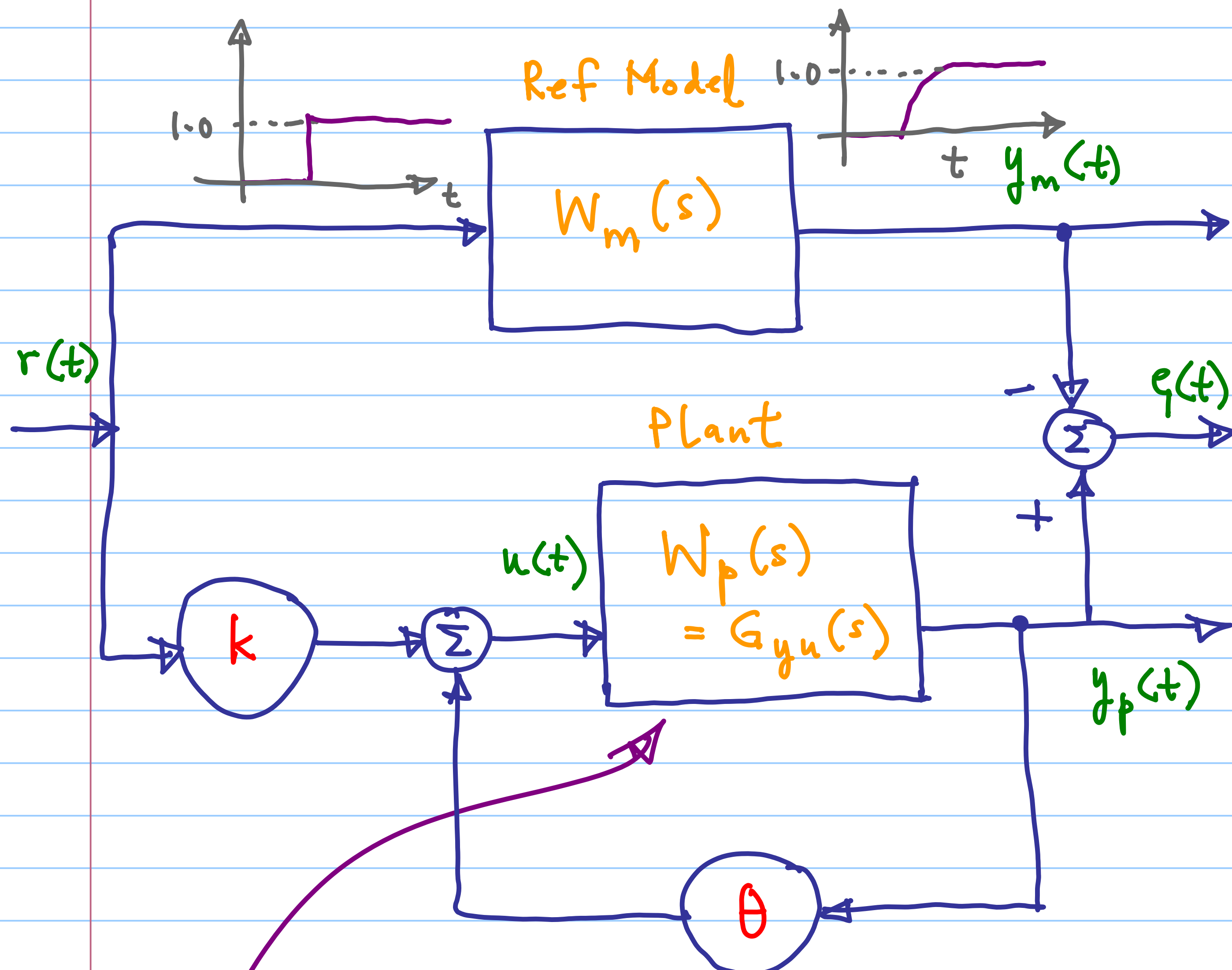
Year 1 graduate students – ?

Year 2 & above, grad students – generally ok

Exchange students – ?

[Appropriate background in Linear Systems & State-Variables; and Electrical Engineering needed.]

Motivating the subject matter...



$$\dot{y}_p = a_p y_p + k_p u$$

consider, as a start,
this simple case...

Consider the simple "plant":

$$\dot{y}_p = a_p y_p + k_p u$$

We are interested to obtain the closed-loop specified by:

$a_m < 0$

$$\dot{y}_m =$$

Note that for this simple system, we can use:

$$u(t) =$$

$$\longrightarrow (3)$$

With this, we clearly have:

$$\dot{y}_p(t) = a_p y_p(t) + k_p u(t)$$

$$= a_p y_p(t) + k_p \{ \quad \}$$

$$= [a_p + k_p \theta] y_p(t) + k_p k \, r(t)$$

And here, clearly, for

θ^* defined by

$$a_p + k_p \theta^* \triangleq a_m$$

and for k^* defined by

$$k_p k^* \triangleq k_m$$

and if we have

$\theta = \theta^*$

$k = k^*$

we have:

$$\dot{y}_p = \{a_p + k_p \theta^*\} y_p + k_p k^* r(t)$$

$$=$$

and for the tracking error

$$e_1(t) \triangleq y_p(t) - y_m(t)$$

we have

$$\dot{e}_1(t) = \dot{y}_p(t) - \dot{y}_m(t)$$

$$= \left\{ a_m y_p(t) + k_m r(t) \right\}$$

$$- \left\{ a_m y_m(t) + k_m r(t) \right\}$$

$$= \qquad\qquad ; \quad \text{with } a_m < 0$$

But we do <u>not</u> know a_p & k_p, and thus, cannot exactly calculate θ^* and k^*

Thus, consider the following control law:

$u(t) =$

i.e. time-varying "control gains".

How to specify these time-varying "control gains"?

Answer: Use the following adaptive law:

$$\dot{\theta}(t) = -\operatorname{sgn}(k_p)\,\gamma_1\, y_p(t)\, e_1(t)$$

$$\dot{k}(t) = -\operatorname{sgn}(k_p)\,\gamma_2\, r(t)\, e_1(t)$$

$\gamma_1, \gamma_2 > 0$; $e_1(t) = y_p(t) - y_m(t)$

How does this work?

For a start, we should be interested in the following error signals:

$$e_1(t) \triangleq y_p(t) - y_m(t)$$

$$\phi(t) \triangleq \theta(t) - \theta^*$$

$$\psi(t) \triangleq k(t) - k^*$$

and thus also in the "energy" function =

$$V\big(e_1(t), \phi(t), \psi(t)\big)$$

$$= \frac{1}{2}\Big\{ \qquad\qquad\qquad\qquad \Big\}$$

Consider then, for this system as described above, how this "energy" function evolves with time ....

i.e. consider

$$\frac{d}{dt} V(e_1, \phi_1, t)$$

$=$

Now, note that: $\qquad$ — (21)

$$\dot{e}_1(t) = \dot{y}_p(t) - \dot{y}_m(t)$$

$$= \left\{ a_p\, y_p(t) + k_p \left[\qquad \right] y_p(t) + k_p \left[\qquad \right] r(t) \right\}$$

$$- \left\{ a_m\, y_m(t) + k_m\, r(t) \right\}$$

$\vdots$

$=$

Further, note that :

$$\dot{\phi}(t) = \dot{\{\theta - \theta^*\}} = \dot{\theta} = -\operatorname{sgn}(k_p)\,\gamma_1 e_1 y_p$$

$$\dot{\psi}(t) = \cdots = -\operatorname{sgn}(k_p)\,\gamma_2 e_1 r$$

Thus, (21) becomes :

$$\dot{V}(e, \phi, \psi)$$

$$= e_1 \left\{ \qquad \right\}$$

$$+ \frac{\|k_p\|}{\gamma_1} \phi \left\{ \qquad \right\}$$

$$+ \frac{\|k_p\|}{\gamma_2} \psi \left\{ \qquad \right\}$$

yields :

$$\dot{V}(e_1, \phi, \psi) =$$

and ≤ 0

For such classes of NLTV systems, note now that:

(a) $$V(e_1, \phi, \psi) = \frac{1}{2}\left\{ e_1^2 + \frac{\|k_p\|}{\gamma_1}\phi^2 + \frac{\|k_p\|}{\gamma_2}\psi^2 \right\}$$

we have shown that the → (51)

"Control Law" and the

"Adaptive Law" yields → (52)

$$\dot{V}(e_1, \phi, \psi) = a_m e_1^2 \leq 0$$

since the closed-loop specification means $a_m < 0$

(b) Eqns (51) and (52)

$\Longrightarrow$

are bounded for all $t \geq 0$

(c) Next, note that

$$\dot{V}(t) = a_m e_1^2(t) = -\alpha e_1^2(t)$$

Write
$a_m = -\alpha$
$\alpha > 0$

$$\int_{t=t_0}^{t} \dot{V}(\tau)\, d\tau = -\int_{t=t_0}^{t} \alpha e_1^2(\tau)\, d\tau$$

$$= -\alpha \int_{t_0}^{t} e_1^2(\tau)\, d\tau$$

$\therefore$

$$\alpha \int_{t_0}^{t} e_1^2(\tau)\, d\tau = V(t_0) - V(t)$$

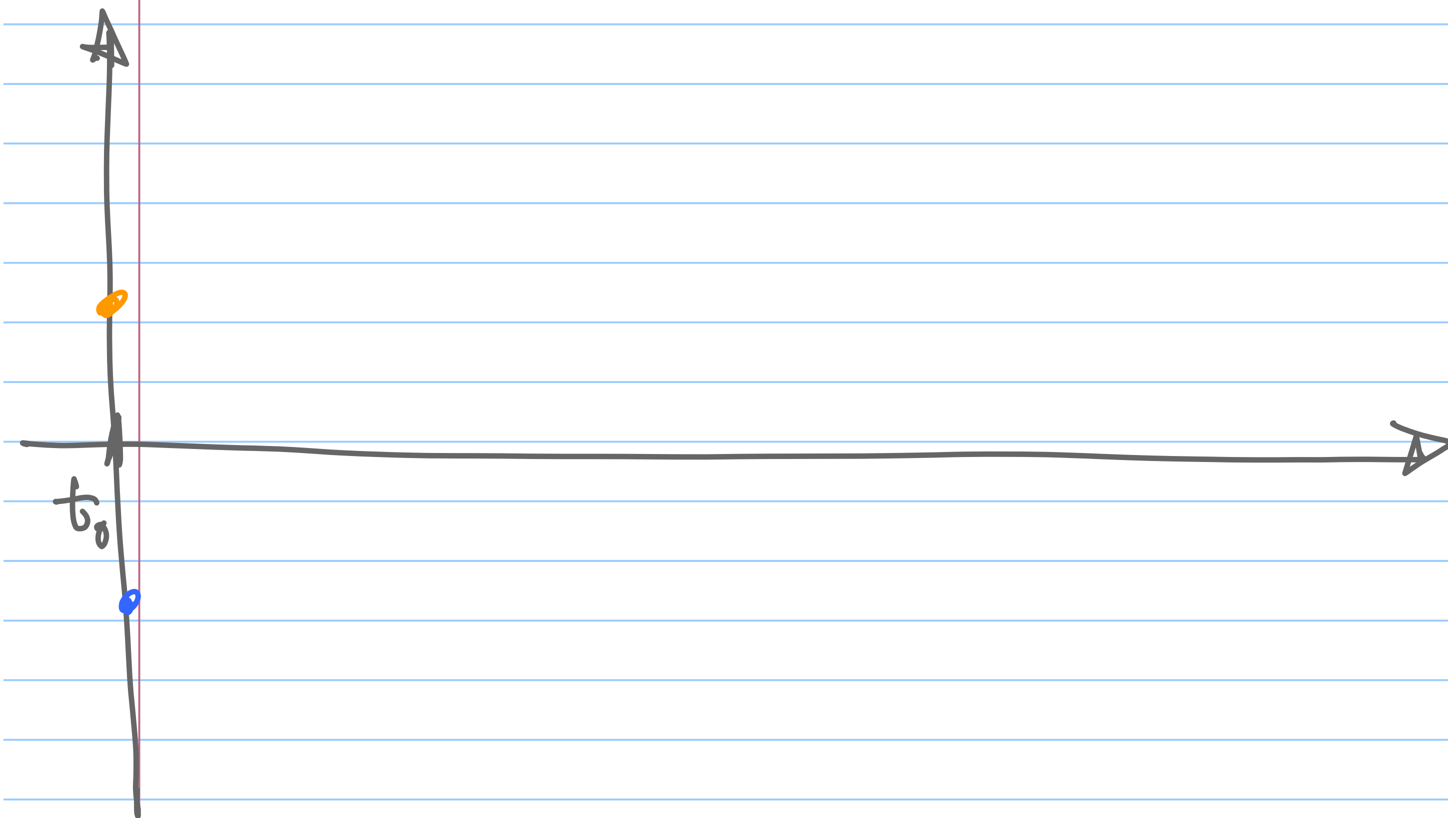
$$\leq V(t_0)$$

$\therefore$

$$\int_{t_0}^{t} e_1^2(\tau)\, d\tau \leq$$

true for all t !!!

— (53)



Next, we further, that

$$\dot{e}_1(t) = a_m e_1(t) + k_p \Phi(t) + k_p \Psi(t)$$

and since we already know

$\{e, \Phi, \Psi\}$ are bounded $\forall t$

we also have

$\dot{e}_1(t)$ is

— (54)

(53) and (54) together imply :

$$\lim_{t \to \infty} e_1(t) = 0 \qquad (55)$$