National University of Singapore
Faculty of Engineering

ME5402/EE5016R Advanced Robotics

Sample Solution to Exercise 2

There can be more than one correct answer/solution to some of the questions. Email me at mpecck@nus.edu.sg if you have found any errors in the solution. – CK Chui

1. Figure 1 shows the schematic diagram of the Intelledex Robot Model 605T. This robot is a six-axis manipulator consisting of all rotational joints with axes 1, 2 and 3 always co-intersecting at a common joint. (Axis 6 intersects at the same co-intersection point only at the configuration shown in Fig. 1.)

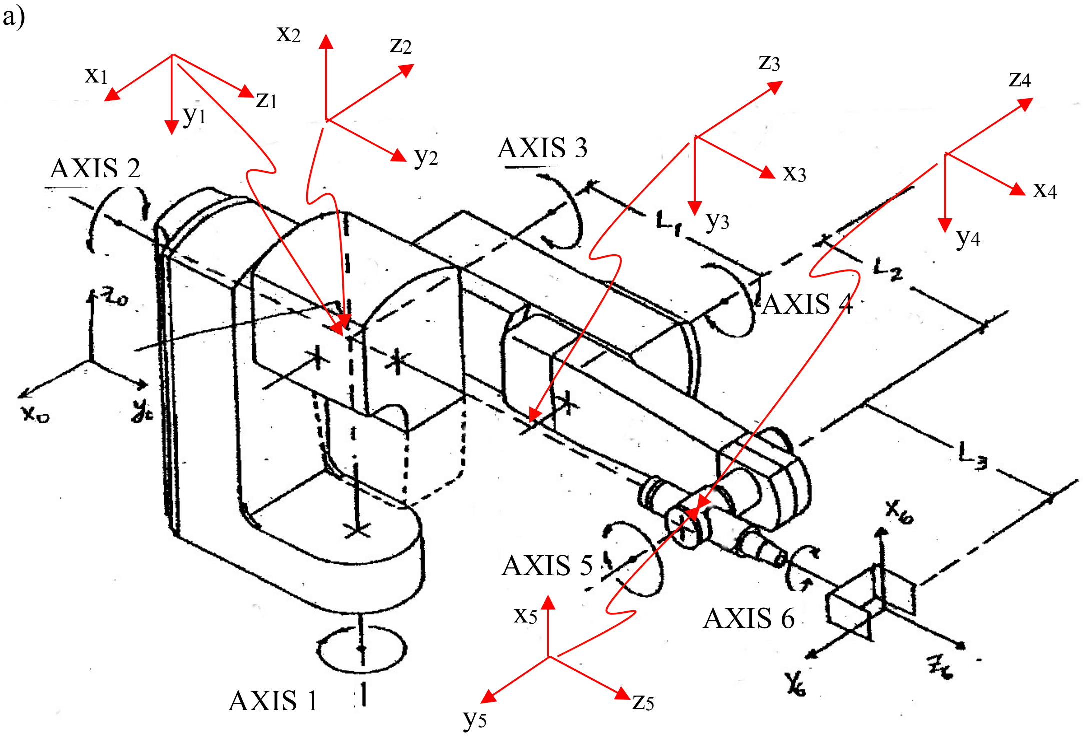
a. Assign coordinate frames to each link according to the Denavit-Hartenberg convention and the following rules:
   - The base frame (frame 0) should be as indicated in the figure. Its origin should coincide with the co-intersection point of axes 1, 2 and 3.
   - The end-effector frame and the z-axes of the rest of the frames should be as indicated in the figure.
   - To the maximum extent possible, make a_i and d_i be equal to zero.
   - The values of the six joint coordinates ([θ_1 θ_2 θ_3 θ_4 θ_5 θ_6]) for the robot at the configuration shown in Fig. 1 are [0 -90^o 90^o 0 -90^o 0].

b. Identify the kinematic parameters of the robot by filling in the table in Table 1.

c. If at the configuration shown in Figure 1, axis 2 has a joint motion range of ±115^o, determine the joint motion range in terms of θ_2 (joint variable for 2nd joint, assigned according to the Denavit-Hartenberg convention, item a above.).

Solution:

a)



b)

Table 1:

Link number	θ_i	d_i	α_i	a_i
1	θ_1	0	-90°	0
2	θ_2	0	90°	0
3	θ_3	0	0	L_1
4	θ_4	0	0	L_2
5	θ_5	0	-90°	0
6	θ_6	L_3	0	0

$$
{}^0_1\mathbf{A} = \begin{bmatrix} c_1 & 0 & -s_1 & 0 \\ s_1 & 0 & c_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
{}^1_2\mathbf{A} = \begin{bmatrix} c_2 & 0 & s_2 & 0 \\ s_2 & 0 & -c_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
{}^2_3\mathbf{A} = \begin{bmatrix} c_3 & -s_3 & 0 & L_1c_3 \\ s_3 & c_3 & 0 & L_1s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^3_4\mathbf{A} = \begin{bmatrix} c_4 & -s_4 & 0 & L_2c_4 \\ s_4 & c_4 & 0 & L_2s_4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
{}^4_5\mathbf{A} = \begin{bmatrix} c_5 & 0 & -s_5 & 0 \\ s_5 & 0 & c_5 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
{}^5_6\mathbf{A} = \begin{bmatrix} c_6 & -s_6 & 0 & 0 \\ s_6 & c_6 & 0 & 0 \\ 0 & 0 & 1 & L_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

c) $-90^\circ-115^\circ \le \theta_2 \le -90^\circ+115^\circ$

2. Figure 2 shows a 3-joint robot with one translational joint. It is a cylindrical robot whose first two joints are analogous to polar coordinates when viewed from above. The last joint provides "roll" for the hand.

a. Assign a coordinate frame to each link according to the Denavit-Hartenberg convention.
b. Identify and tabulate the Denavit-Hartenberg parameters.
c. Compute ${}^{0}_{3}T$.
d. Describe the three degrees-of-freedom of the robot in Cartesian space. Sketch the reachable workspace of the robot.
e. Derive the complete inverse kinematic equations for the robot.

Solution:

a)

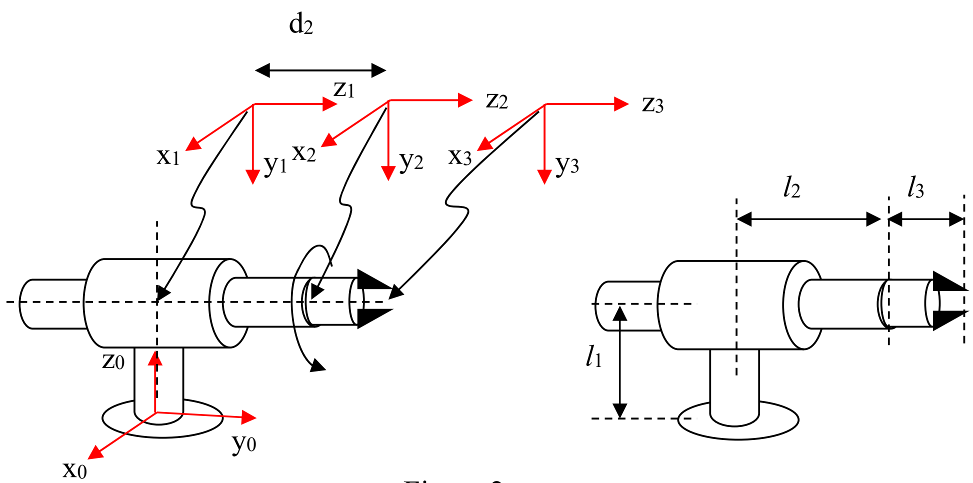


Figure 2

b)

Link number	θ_i	d_i	a_i	α_i
1	θ_1	l_1	0	-90°
2	0	d_2	0	0
3	θ_3	l_3	0	0

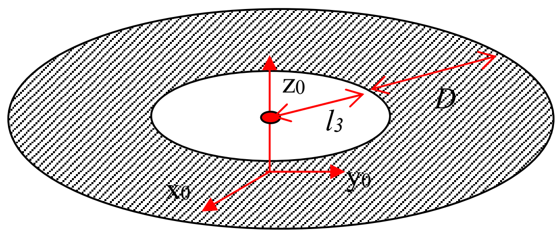
c)

$$
{}^{0}_{1}\mathbf{A} = \begin{bmatrix} c_1 & 0 & -s_1 & 0 \\ s_1 & 0 & c_1 & 0 \\ 0 & -1 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\quad
{}^{1}_{2}\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\quad
{}^{2}_{3}\mathbf{A} = \begin{bmatrix} c_3 & -s_3 & 0 & 0 \\ s_3 & c_3 & 0 & 0 \\ 0 & 0 & 1 & l_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{0}_{3}\mathbf{T} = {}^{0}_{1}\mathbf{A}\,{}^{1}_{2}\mathbf{A}\,{}^{2}_{3}\mathbf{A} = \begin{bmatrix} c_1c_3 & -s_3c_1 & -s_1 & -(l_3+d_2)s_1 \\ s_1c_3 & -s_3s_1 & c_1 & (l_3+d_2)c_1 \\ -s_3 & -c_3 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

d)

The first two joints can be viewed as the polar coordinates for the wrist. The third joint (wrist) does not cause any change to the end-point position. The reachable workspace is shown below (assume $0 \le d_2 \le D$ and θ_1 is unlimited):



e)

$$
{}^{0}_{3}\mathbf{T} = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c_1c_3 & -s_3c_1 & -s_1 & -(l_3+d_2)s_1 \\ s_1c_3 & -s_3s_1 & c_1 & (l_3+d_2)c_1 \\ -s_3 & -c_3 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

=>

$a_x = -s_1$

$a_y = c_1$ => $\theta_1 = \text{Atan2}(s_1, c_1) = \text{Atan2}(-a_x, a_y)$

$p_x = -(l_3 + d_2)s_1$ => $d_2 = -l_3 - p_x/s_1$

$n_z = -s_3$

$o_z = -c_3$ => $\theta_3 = \text{Atan2}(s_3, c_3) = \text{Atan2}(-n_z, -o_z)$

3. Coordinate frame N is attached to an end-effector as shown in Figure 3. It is desired to design an N-joint robot that can provide the following position and orientation of the end-effector:

$$ {}^{0}_{N}T = \begin{bmatrix} n_x & o_x & 0 & p_x \\ n_y & o_y & 0 & p_y \\ 0 & 0 & -1 & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

where n_x, n_y, o_x, o_y, p_x, p_y, and p_z are functions of the robot joint coordinates.

a. What is the minimum number of degrees-of-freedom required of the robot? (That is, what is the minimum number of joints?)

b. Suggest a robot structure/configuration that can satisfy the task ${}^{0}_{N}T$. That is, identify the number and type of joints, draw the base frame 0 and provide a schematic diagram of the robot including the end-effector and its frame N.

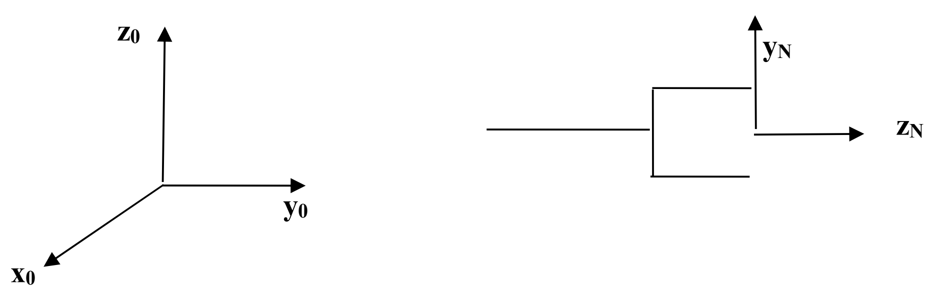


Figure 3

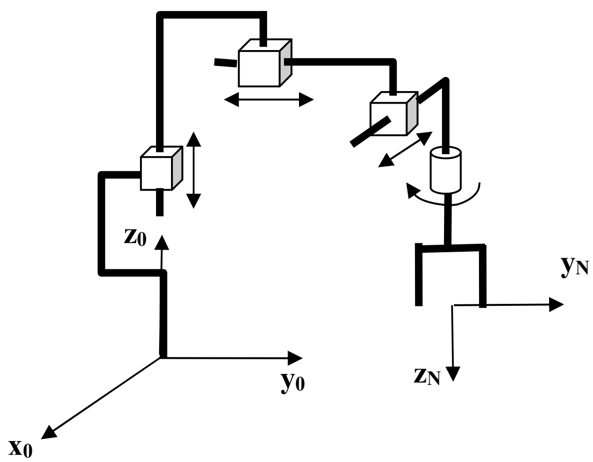
Solution:

a) 7 parameters with three constraint equations:

$n_x^2+n_y^2 = 1$ (unit vector for **n**)
$o_x^2+o_y^2 = 1$ (unit vector for **o**)
$n_x o_x + n_y o_y = 0$ (orthonormal between **n** and **o**)

Hence, 7-3 = 4 dof are required.

b) Example: 3 prismatic joints and 1 revolute joint



4. A three-degree-of-freedom RPR robot is as shown in Figure 4. The joint variables are (θ_1, d_2, θ_3) and $l_3 = 1\ m$.

   a. Assign the remaining coordinate frames based on Denavit-Hartenberg notation and fill out the link parameters table.
   b. Obtain the ${}^0_3T$ matrix that describes the position and orientation of Frame {3} relative to Frame {0}.
   c. Given the desired position vector of the tip of the arm, ${}^0p = [p_x, p_y, 0]^T$ and the desired x_3 axis direction expressed in terms of angle ϕ, which is measured anti-clockwise from x_0, find the expressions of the joint variables in terms of p_x, p_y and ϕ. Assume $d_2 > 0$.

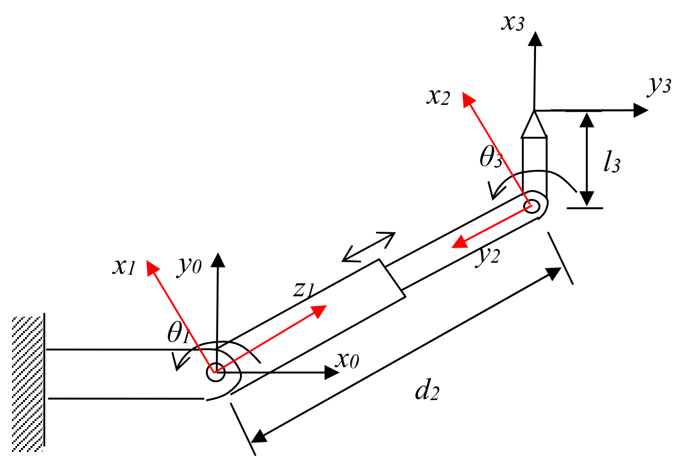


Figure 4

Solution:

a)

For the frames chosen for links 1 and 2 as shown above:

Link number	θ_i	d_i	a_i	α_i
1	θ_1	0	0	90°
2	0	d_2	0	-90°
3	θ_3	0	l_3	180°

(Note: The solutions depend on the frame orientations chosen for links 1 and 2.)

b)

$$ {}^0_1\mathbf{A} = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^1_2\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^2_3\mathbf{A} = \begin{bmatrix} c_3 & s_3 & 0 & c_3 \\ s_3 & -c_3 & 0 & s_3 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^0_3T = {}^0_1A\,{}^1_2A\,{}^2_3A = \begin{bmatrix} c_{13} & s_{13} & 0 & c_{13} + d_2 s_1 \\ s_{13} & -c_{13} & 0 & s_{13} - d_2 c_1 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

c)

$$ \begin{bmatrix} p_x \\ p_y \\ \phi \end{bmatrix} = \begin{bmatrix} c_{13} + d_2 s_1 \\ s_{13} - d_2 c_1 \\ \theta_1 + \theta_3 \end{bmatrix} $$

That is,

$p_x = c_{13} + d_2 s_1$ (1)

$p_y = s_{13} - d_2 c_1$ (2)

Rearranging equations (1) and (2),

$p_x - c_{13} = d_2 s_1$ (3)

$p_y - s_{13} = -d_2 c_1$ (4)

Squaring equations (3) and (4),

$p_x^2 - 2p_x c_{13} + c_{13}^2 = d_2^2 s_1^2$ (5)

$p_y^2 - 2p_y s_{13} + s_{13}^2 = d_2^2 c_1^2$ (6)

(5)+(6),

$p_x^2 + p_y^2 - 2(p_x c_{13} + p_y s_{13}) + 1 = d_2^2$

Therefore,

$$d_2 = \sqrt{p_x^2 + p_y^2 - 2(p_x c_\phi + p_y s_\phi) + 1} \qquad \text{(given } d_2 > 0\text{)}$$

From equations (1) and (2),

$$s_1 = \frac{p_x - c_\phi}{d_2}, c_1 = \frac{s_\phi - p_y}{d_2}$$

$$\theta_1 = A\tan 2(s_1, c_1)$$

Finally, $\theta_3 = \phi - \theta_1$