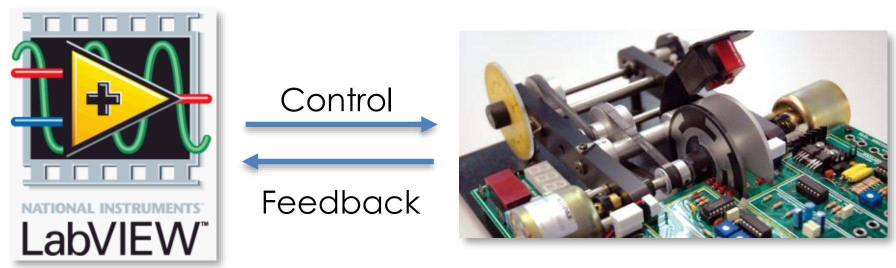
EE5104/6014 ADVANCED/ADAPTIVE CONTROL SYSTEMS

Briefing Notes for CA3 (Mini Project):
Adaptive Control of the Angular Position/Velocity of the DC Motor
Using State Feedback

CONTENTS

OBJECTIVE

- Design an adaptive controller for the DC motor in order to control its angular position (EE5104/EE6104) and angular velocity (EE6104 only).
- Program the controller in LabVIEW and test the controller on the L.J. Electronics DC motor apparatus.



- Both the angular positon and the angular velocity of the motor are measurable in real-time.

EXTRA

INTRODUCTION TO LABVIEW AND THE DC MOTOR APPARATUS

- LABVIEW

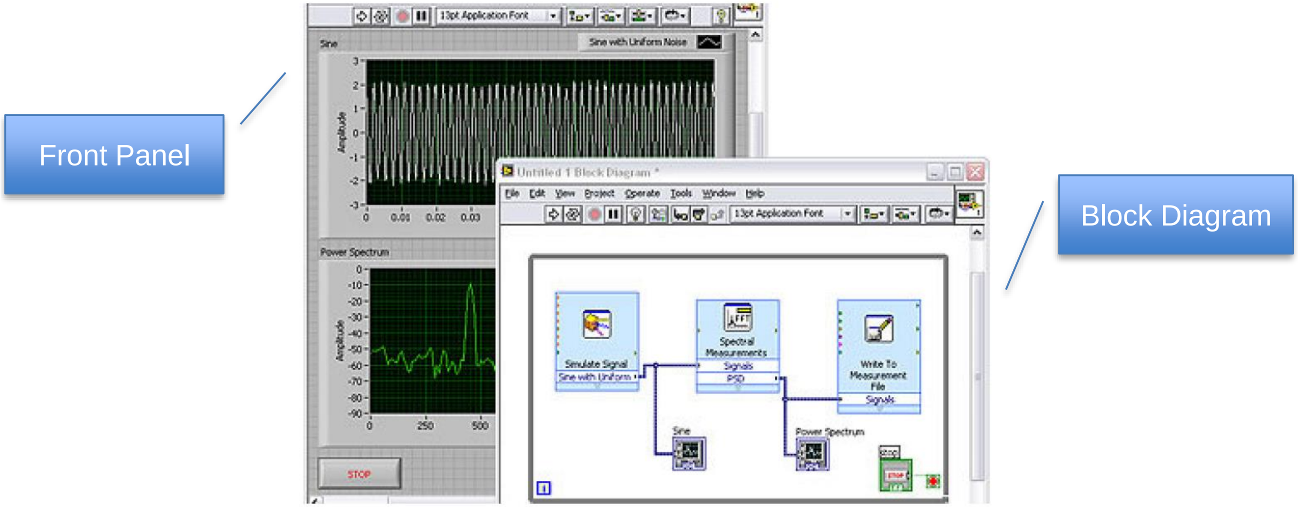
- LabVIEW (Laboratory Virtual Instrument Engineering Workbench) is systems engineering software for applications that require test, measurement and control with rapid access to hardware and data insights.*

- Keywords:
  - Visual Programming Language (Graphical Language, “G-Code”)
  - Integrated Data acquisition, Instrument Control and Industrial Automation.

* National Instruments, http://www.ni.com/

INTRODUCTION TO LABVIEW AND THE DC MOTOR APPARATUS
- LABVIEW

- A basic LabVIEW program is composed of 2 parts: the front panel and the block diagram.
- The front panel is the user interface of a LabVIEW Program. After the front panel window is designed, graphical representations of functions to control the front panel objects need to be added. The block diagram window contains this graphical source code.



EXTRA

INTRODUCTION TO LABVIEW AND THE DC MOTOR APPARATUS

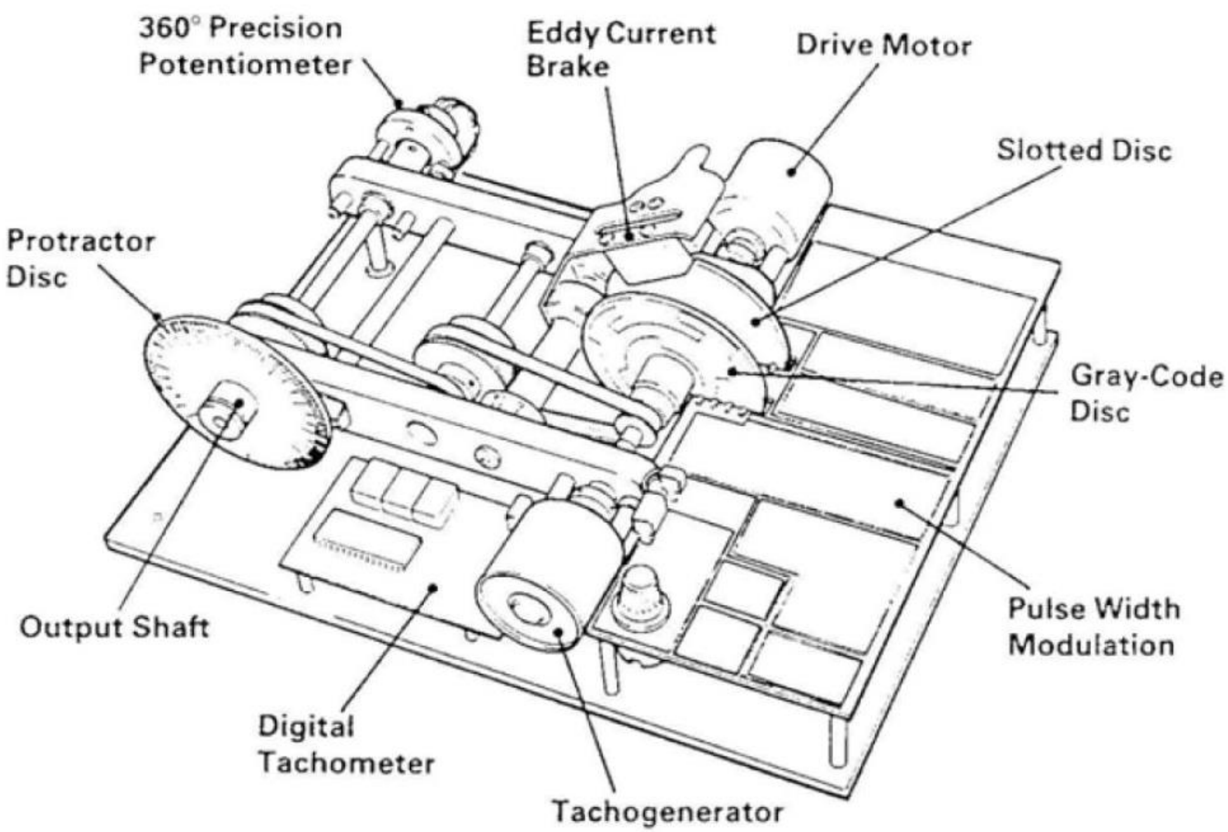
- DC MOTOR APPARATUS

- The L.J. Electronics 207-15 DC motor control trainer has been designed to allow the user to perform numerous control experiments using either an analog or a digital controller.*

* LJ CREATE, http://www.ljcreate.com/

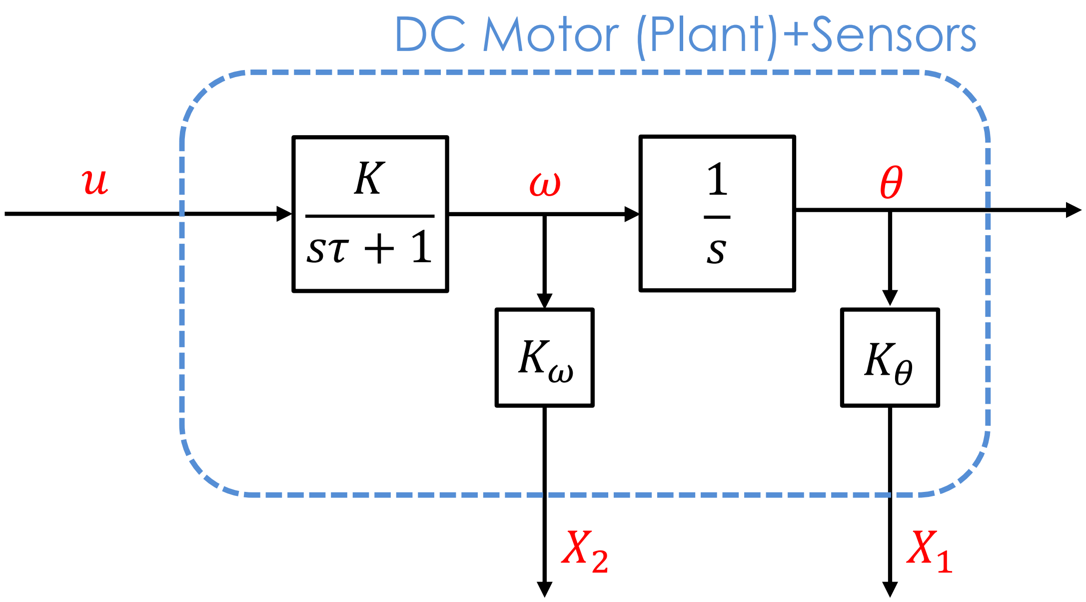
INTRODUCTION TO LABVIEW AND THE DC MOTOR APPARATUS

- DC MOTOR APPARATUS



INTRODUCTION TO LABVIEW AND THE DC MOTOR APPARATUS
- DC MOTOR APPARATUS

- The DC motor can be modeled as follows.



u: Control Signal (in volts); set and read in LabVIEW
ω: Angular Velocity (in rpm); read from tachometer LED panel
θ: Angular Position (in degrees); read from motor shaft dial
X_1: Potentiometer Output (in volts); read in LabVIEW
X_2: Tachogenerator Output (in volts); read in LabVIEW

K, τ: Unknown dynamic system parameters
K_θ, K_ω: Output gains that can be obtained through calibration

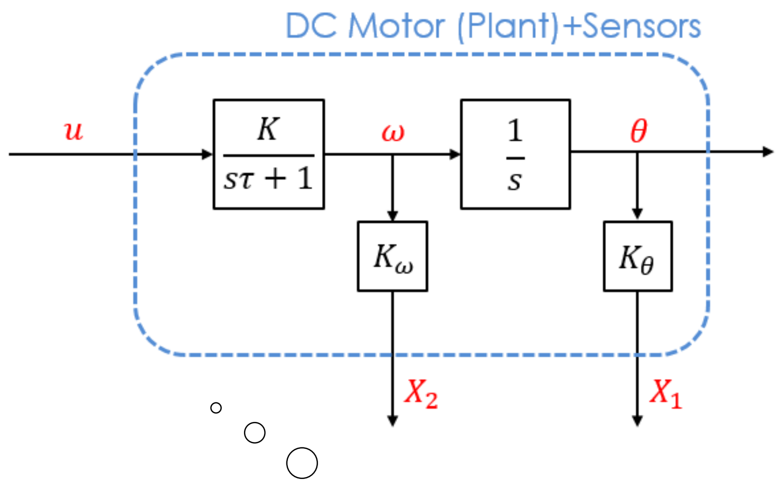
CONTROL SYSTEM DESIGN
- STATE-SPACE REALIZATION OF THE PLANT

- K_θ and K_ω can be obtained through calibration (which will be introduced later).

- With known K_θ and K_ω, the real-time measurement X_1 and X_2 can be transformed into θ and ω. Therefore, we can assume that θ and ω are measurable in real-time.

CONTROL SYSTEM DESIGN
- STATE-SPACE REALIZATION OF THE PLANT

- Define the state vector $x_p = [\theta \quad \omega]^T$, the state space realization of the plant is formulated as follows.



$$\dot{x}_p = A_p x_p + gbu \tag{1}$$

where x_p is measurable and

$$A_p = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{1}{\tau} \end{bmatrix},\ g = \frac{K}{\tau},\ b = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Notice that there are unknown parameters in A_p and g.

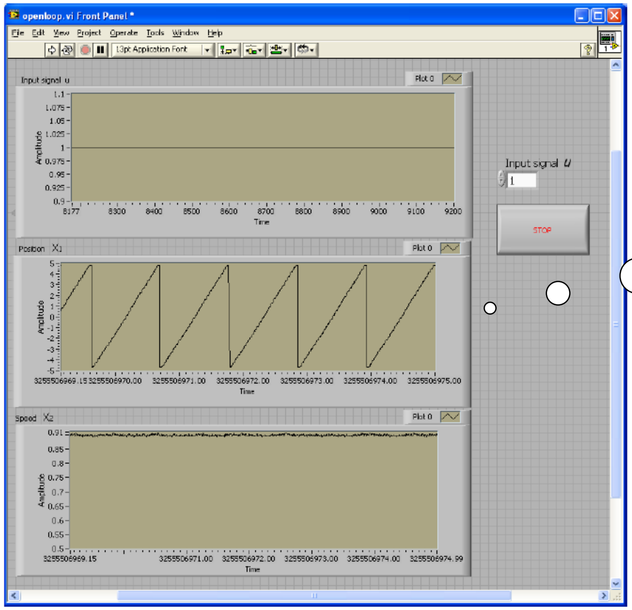
$$\omega(s) = \frac{K}{s\tau + 1} U(s) \Longrightarrow \tau\dot{\omega} + \omega = Ku \Longrightarrow \dot{\omega} = -\frac{1}{\tau}\omega + \frac{K}{\tau}u$$

- K_θ and K_ω are calibrated as follows. Notice that the value of K_θ and K_ω may differ for different machines. Therefore, the calibration has to be carried out individually for different equipment.

- The calibration of K_θ:
  - Adjust the motor shaft dial manually so that the potentiometer output $X_1 = 0$. Define the current angular position as $\theta = 0$.
  - Rotate the rotor and stops it at a different angular position. Record the value of θ and X_1. Repeat the procedure for several times until you get enough sampling points in a circle.
  - Estimate K_θ using the recorded values of θ and X_1 in a circle.

CONTROL SYSTEM DESIGN
- STATE-SPACE REALIZATION OF THE PLANT

- The calibration of K_θ:
  - The value of X_1 can be read from LabVIEW front panel.



Read X_1 from here

CONTROL SYSTEM DESIGN
- STATE-SPACE REALIZATION OF THE PLANT

- The calibration of K_θ:
  - Write down the value of θ and X_1 in a table that looks like the following.

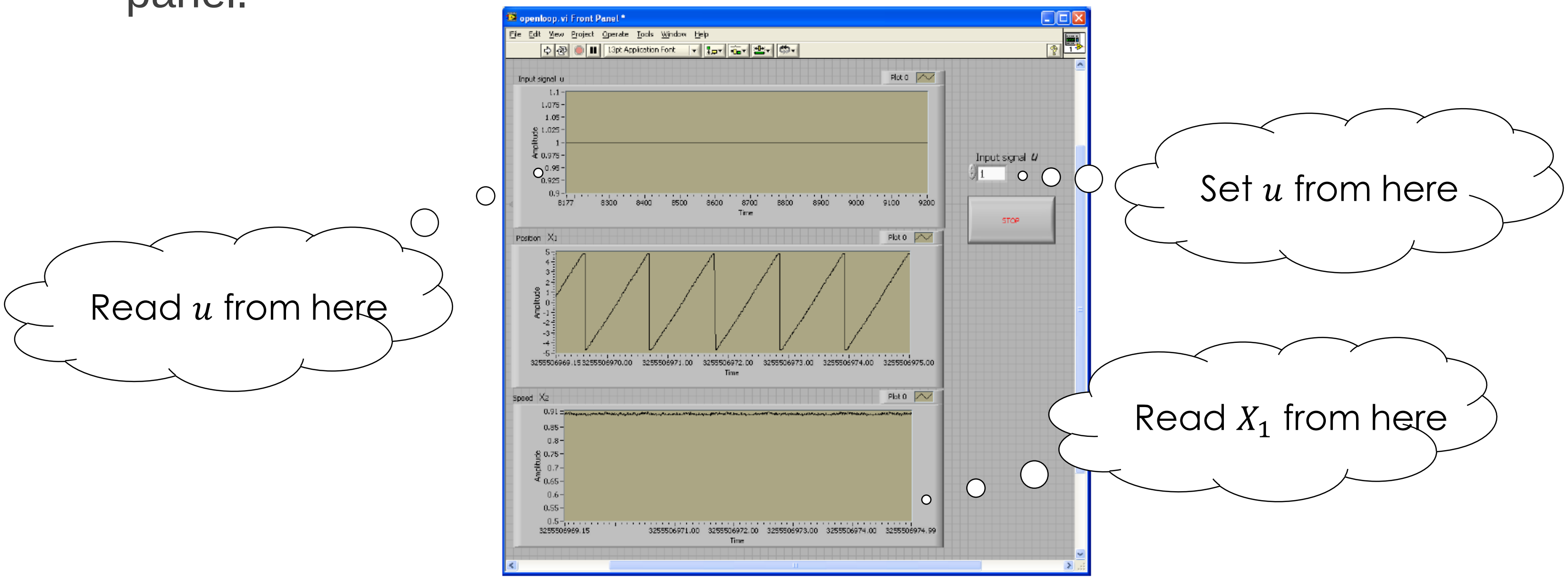
θ (in degrees)	θ (in rad)	X_1 (in volts)
$\vdots$	$\vdots$	$\vdots$
30	$\pi/6$	0.87
0	0	0
-30	$-\pi/6$	-0.86
$\vdots$	$\vdots$	$\vdots$

  - Since $X_1 = K_\theta \theta$, the value of K_θ can be estimated statistically, for instance, using Linear Regression (a build in function in MATLAB and Microsoft Excel).

- The calibration of K_ω:
  - In LabVIEW, set $u = 0$. The rotor of the motor should stay still, giving $\omega = 0$ and $X_2 = 0$.
  - Set different values for u between $\pm 5V$. Record the value of ω and X_2. Repeat the procedure for several times until you get enough sampling points.
  - Estimate K_ω using the recorded values of ω and X_2.

CONTROL SYSTEM DESIGN
- STATE-SPACE REALIZATION OF THE PLANT

- The calibration of K_{θ}:
  - The value of u and X_2 can be set (u only) and read from LabVIEW front panel.



- The calibration of K_θ:
  - Write down the value of ω and X_2 in a table that looks like the following.

u (in volts)	ω (in rpm)	ω (in rad/s)	X_2 (in volts)
$\vdots$	$\vdots$	$\vdots$	$\vdots$
1	48	5.03	0.62
0	0	0	0
-1	-45	-4.71	-0.6
$\vdots$	$\vdots$	$\vdots$	$\vdots$

$1 \text{ rpm} = \frac{2\pi}{60\text{s}} \approx 0.1047 \text{ rad/s}$

  - Since $X_2 = K_\omega \omega$, the value of K_ω can be estimated statistically, for instance, using Linear Regression (a build in function in MATLAB and Microsoft Excel).

EXTRA

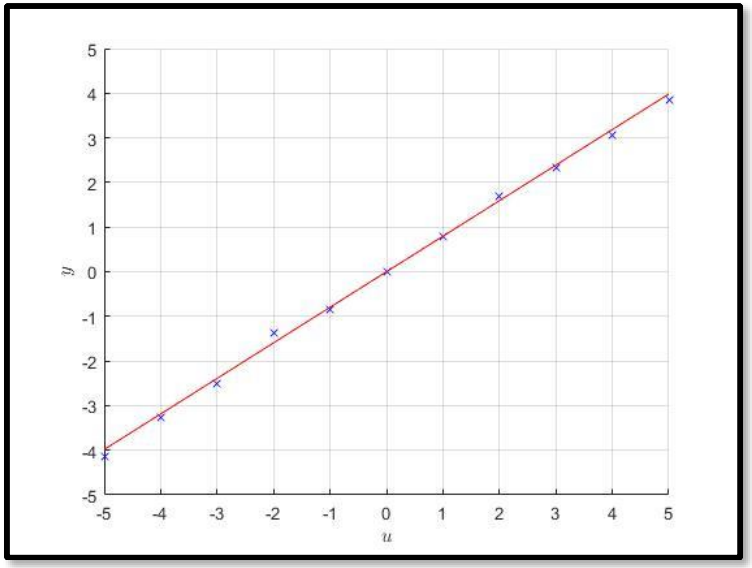
CONTROL SYSTEM DESIGN
- STATE-SPACE REALIZATION OF THE PLANT

- An example of applying Linear Regression on parameter estimation using MATLAB. (*Statistics and Machine Learning Toolbox* is required for MATLAB function 'fitlm')

```
% Linear Regression Using MATLAB: An Example
% The linear model is $y=ku$, where k is the parameter to be estimated.
u = [-5,-4,-3,-2,-1,0,1,2,3,4,5]';
y = [-4.15,-3.27,-2.51,-1.37,-0.86,0.01,0.78,1.69,2.32,3.06,3.86]';
mdl = fitlm(u,y,'intercept',false);
fprintf('The estimation of the parameter is: %d.\n',mdl.Coefficients{1,1})

figure
hold on
plot(u,y,'bx')
plot(u,mdl.Fitted,'r-')
grid on
xlabel('$u$','interpreter','latex')
ylabel('$y$','interpreter','latex')
axis([-5,5,-5,5])
hold off
```

Result:
The estimation of the parameter is: 7.965455e-01.



CONTROL SYSTEM DESIGN
- REFERENCE MODEL AND PERFECT MODEL-FOLLOWING SYSTEM

- To apply Model Reference Adaptive Control on the DC motor, a reference model of 2nd order is designed.
- Consider using the reference model with the following transfer function.

$$H_m(s) = \frac{\omega_m^2}{s^2 + 2\zeta_m \omega_m s + \omega_m^2}$$

- The following state-space realization of the reference model can be obtained.

$$\dot{x}_m = A_m x_m + g_m b r \quad (2)$$

$$A_m = \begin{bmatrix} 0 & 1 \\ -\omega_m^2 & -2\zeta_m \omega_m \end{bmatrix}, g_m = \omega_m^2 \quad (3)$$

CONTROL SYSTEM DESIGN

- REFERENCE MODEL AND PERFECT MODEL-FOLLOWING SYSTEM

- Assume that A_p and g in 1 are known. Substitute the control signal described by 4 into 1 to obtain the closed-loop system described by 5 .

$$u = \theta_x^T x_p + \theta_r r \qquad 4$$

$$\dot{x}_p = A_p x_p + gb\left(\theta_p^T x_p + \theta_r r\right)$$

$$= \left(A_p + gb\theta_x^T\right) x_p + gb\theta_r r \qquad 5$$

- If in 4 we choose $\theta_x = \theta_x^*$, $\theta_r = \theta_r^*$, where θ_x^* and θ_r^* are such constant that

$$\left(A_p + gb\theta_x^{*T}\right) = A_m,\ gb\theta_r^* = g_m b$$

the closed-loop system in 5 would be identical to the reference model in 2 . A perfect model-following system is obtained.

CONTROL SYSTEM DESIGN
- ADAPTIVE CONTROL SYSTEM DESIGN

- In practice, however, θ_x^* and θ_r^* cannot be calculated since A_p and g are unknown.

- Instead of letting θ_x and θ_r in the control signal 4 be any constant value, consider making them time-varying gain that follow particular adaptive law.

$$\dot{\theta}_x = -\mathrm{sgn}(g)\Gamma e^T P b x_p$$
$$\dot{\theta}_r = -\mathrm{sgn}(g)\gamma e^T P b r$$

- The adaptive law is given in Note02[post-class]. Let us see whether we can derive it by ourselves in the following slides.

- Define the tracking error of the state $e = x_p - x_m$ in the closed-loop system.

- It needs to be proved that by applying certain adaptive control that we designs, $e \to 0$ as $t \to \infty$. For the proof, V, a function that involves e, is introduced. We will prove that V satisfies particular criteria, which would further result in $e \to 0$.

EXTRA

- To be more precise in mathematical expression, the proof mainly includes the following steps:
  - First, find a Lyapunov function V that involves e. (V is a function of e, ψ_x and ψ_r, where $\psi_x = \theta_x - \theta_x^*$ and $\psi_r = \theta_r - \theta_r^*$)
  - Second, prove that V satisfies *Lyapunov's stability theorem*, thus e, ψ_x and ψ_r are Lyapunov stable.
  - Third, use the properties of Lyapunov stability to further prove that $e \to 0$ as $t \to \infty$.

- A review: Lyapunov stability and asymptotic stability.

Aleksandr Mikhailovich Lyapunov
(1857-1918)

Russian Mathematician

Let the system state vector denoted by x, following

$$\dot{x} = f\big(x(t)\big)$$

and it has a equilibrium point at x_e, where $f(x_e) = 0$.

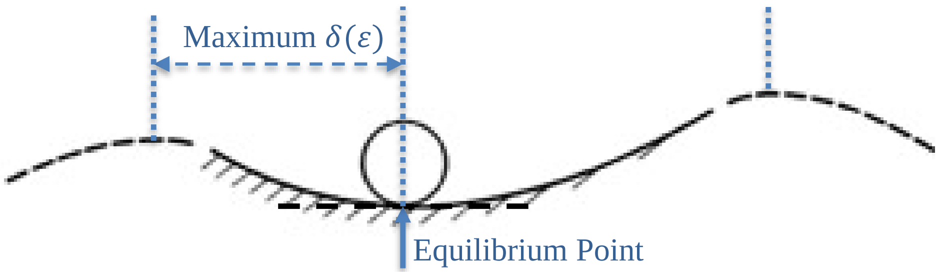
The solution $x(t) = x_e$ is Lyapunov stable (or stable) if for a given $\varepsilon > 0$, there exists a number $\delta(\varepsilon)$ such that for any initial condition $\|x(0) - x_e\| < \delta(\varepsilon)$,

$$\|x(t) - x_e\| < \varepsilon \quad \text{for} \quad 0 \leq t < \infty$$

Further more, the solution $x(t) = x_e$ is asymptotically stable if it is stable and

$$\lim_{t \to \infty} \|x(t) - x_e\| = 0$$

- Some Remarks on Lyapunov Stability:
  - If regardless of the value of $x(0)$ the solution $x(t) = x_e$ is always (asymptotically) stable, then it is said to be globally (asymptotically) stable.
  - Lyapunov stability refers to stability of a particular equilibrium point x_e and not to the differential equation $\dot{x} = f\big(x(t)\big)$.



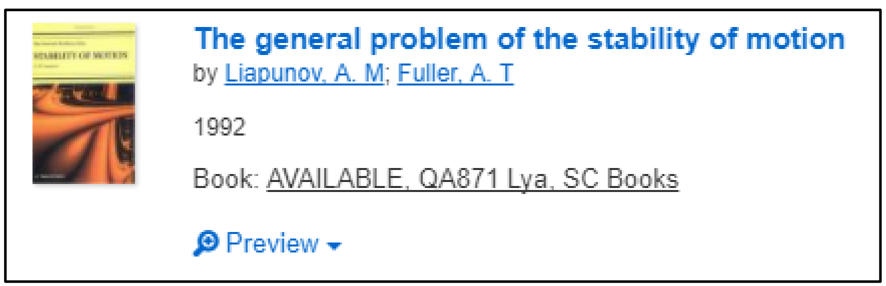
An example of local Lyapunov stability.

EXTRA

CONTROL SYSTEM DESIGN
- ADAPTIVE CONTROL SYSTEM DESIGN

- The proof of Lyapunov stability is often achieved using Lyapunov's stability theorem (or Lyapunov stability criterion).

- Lyapunov's stability theorem is proposed by Lyapunov in 1892 in his doctoral dissertation (originally in Russian). Later on, it is translated into French then into English* and eventually becomes a book published in 1992.



*Lyapunov, Aleksandr Mikhailovich. "The general problem of the stability of motion." International Journal of Control 55.3 (1992): 531-534.

- Lyapunov's Stability Theorem

For a system of n-th order

$$\dot{x} = f\big(x(t)\big)$$

having an equilibrium at $x(t) = 0$, if there exists a function $V(x): R^n \to R$ such that:

- $V(x)$ is positive definite
  - $V(x) = 0$ iff $x = 0$
  - $V(x) > 0$ iff $x \neq 0$
- $\dot{V}(x)$ is negative semidefinite

Then the solution to $x(t) = 0$ is Lyapunov stable.

Moreover, if

- $V(x) \to \infty$ as $\|x\| \to \infty$

Then the solution is globally Lyapunov stable.

If $\dot{V}(x)$ is negative definite, the solution is asymptotically stable. Globally asymptotically stable follows similarly.

Proof of the theorem can be found in *.

* Åström, Karl J., and Björn Wittenmark. Adaptive control. Courier Corporation, 2013.

CONTROL SYSTEM DESIGN
- ADAPTIVE CONTROL SYSTEM DESIGN

- The closed-loop system dynamics is given in 5 . It is equivalent to

$$\begin{aligned}\dot{x}_p &= \left(A_p + gb\theta_x^T\right)x_p + gb\theta_r r + gb\theta_x^{*T}x_p - gb\theta_x^{*T}x_p + gb\theta_r^* r - gb\theta_r^* r \\ &= \left(A_p + gb\theta_x^{*T}\right)x_p + gb\theta_r^* r + gb\left[\left(\theta_x^T - \theta_x^{*T}\right)x_p + (\theta_r - \theta_r^*)r\right] \\ &= A_m x_p + g_m br + gb\left[\psi_x^T x_p + \psi_r r\right] \qquad 6\end{aligned}$$

where $\psi_x = \theta_x - \theta_x^*$, $\psi_r = \theta_r - \theta_r^*$.

- We want to prove that by applying some adaptive law to the control gain, we can make $e = x_p - x_m$ converges to 0.
- Instead of directly proving it, consider the intermediate step: apply some adaptive law to the control gain and we can make e, ψ_x and ψ_r Lyapunov stable at the equilibrium point 0.

CONTROL SYSTEM DESIGN
- ADAPTIVE CONTROL SYSTEM DESIGN

- As the first step, we need to prove that 0 is the equilibrium point of the system composed of e, ψ_x and ψ_r.
- In other words: given $e = 0$, $\psi_x = 0$ and $\psi_r = 0$, it must be guaranteed that $\dot{e} = 0$, $\dot{\psi}_x = 0$ and $\dot{\psi}_r = 0$.

Subtract 2 from 6 to obtain $\dot{e}$ as follows. It is obvious that $\dot{e} = 0$ when $e = 0$, $\psi_x = 0$ and $\psi_r = 0$.

$$\dot{e} = A_m e + gb\left[\psi_x^T x_p + \psi_r r\right] \tag{7}$$

From $\psi_x = \theta_x - \theta_x^*$, $\psi_r = \theta_r - \theta_r^*$,

$$\dot{\psi}_x = \dot{\theta}_x = -\mathrm{sgn}(g)\Gamma e^T P b x_p$$
$$\dot{\psi}_r = \dot{\theta}_r = -\mathrm{sgn}(g)\gamma e^T P b r$$

Later when specifying the adaptive law, we must make sure that $\dot{\psi}_x$ and $\dot{\psi}_r$ are 0 when $e = 0$, $\psi_x = 0$ and $\psi_r = 0$.

- To prove the Lyapunov stability of e, ψ_x and ψ_r using Lyapunov's Stability Theorem, we need to find a Lyapunov Function V. Notice that V must have the following properties:
  - V is a function of e, ψ_x and ψ_r, i.e. $V(e, \psi_x, \psi_r)$
  - V is positive definite
    - $V = 0$ when $e = 0$, $\psi_x = 0$ and $\psi_r = 0$
    - $V > 0$ otherwise
  - $\dot{V}$ is negative semidefinite
  - For global stability, $V \to \infty$ when either e, ψ_x or ψ_r is unbounded. (So that no matter how large e, ψ_x and ψ_r initially are, their Lyapunov stability about the equilibrium point 0 is always guaranteed.)

- The Lyapunov function is chosen as follows. (Notice that in our example e, ψ_x are vectors and ψ_r is a scalar.)

$$V(e,\psi_x,\psi_r) = \frac{1}{2}e^T P e + \frac{1}{2}|g|\psi_x^T \Gamma^{-1}\psi_x + \frac{1}{2}|g|\frac{1}{\gamma}\psi_r^2 \quad (8)$$

where P is the solution to the Lyapunov equation $A_m^T P + P A_m = -Q$ with Q any arbitrarily chosen symmetric positive definite matrix (A_m must be asymptotically stable); Γ is a positive definite diagonal matrix; γ is a positive scalar.

EXTRA

Theorem: Assume that a linear system $\dot{x} = Ax$ is asymptotically stable. For each symmetric positive definite matrix Q, there exists a unique symmetric positive definite matrix P such that $A^T P + PA = -Q$. Furthermore, $V(x) = x^T P x$ is a Lyapunov function of the system.

- If V satisfies all the criteria in Lyapunov's Stability Theorem, e, ψ_x and ψ_r are Lyapunov stable. Let us check the criteria one by one.

CONTROL SYSTEM DESIGN
- ADAPTIVE CONTROL SYSTEM DESIGN

- V is a positive definite function
  - Obviously, $V = 0$ iff $e = 0$, $\psi_x = 0$ and $\psi_r = 0$; otherwise $V > 0$.
- $\dot{V}$ is negative semidefinite

$\dot{V}$ can be calculated as follows. From 8 ,

$$\dot{V} = e^T P \dot{e} + |g| \psi_x^T \Gamma^{-1} \dot{\psi}_x + |g| \frac{1}{\gamma} \psi_r \dot{\psi}_r \qquad 9$$

Substitute 7 into 9 ,

$$\begin{aligned} \dot{V} &= e^T P \left(A_m e + gb\left(\psi_x^T x_p + \psi_r r\right)\right) + |g| \psi_x^T \Gamma^{-1} \dot{\psi}_x + |g| \frac{1}{\gamma} \psi_r \dot{\psi}_r \\ &= e^T P A_m e + e^T P g b \psi_x^T x_p + e^T P g b \psi_r r + |g| \psi_x^T \Gamma^{-1} \dot{\psi}_x + |g| \frac{1}{\gamma} \psi_r \dot{\psi}_r \qquad 10 \end{aligned}$$

Notice that

$$e^T P A_m e = \frac{1}{2} e^T (P A_m + A_m^T P) e = -\frac{1}{2} e^T Q e$$

Therefore, 10 becomes

$$\dot{V} = -\frac{1}{2} e^T Q e + e^T P g b \psi_x^T x_p + e^T P g b \psi_r r + |g| \psi_x^T \Gamma^{-1} \dot{\psi}_x + |g| \frac{1}{\gamma} \psi_r \dot{\psi}_r \quad 11$$

Negative semidefinite since Q is positive definite and it is a quadratic form of e.

Cancel Out

Cancel Out

Try to cancel out by designing appropriate adaptive law.

CONTROL SYSTEM DESIGN
- ADAPTIVE CONTROL SYSTEM DESIGN

Design the adaptive law as follows (in order to cancel out some elements in 11 to make sure that $\dot{V}$ is negative semidefinite).

$$\dot{\psi}_x = \dot{\theta}_x = -\mathrm{sgn}(g)\Gamma e^T P b x_p \quad (12)$$

$$\dot{\psi}_r = \dot{\theta}_r = -\mathrm{sgn}(g)\gamma e^T P b r \quad (13)$$

From 12 and 13 , we can see that $\dot{\psi}_x = 0$, $\dot{\psi}_r = 0$ when $e = 0$. Together with 7 , it is clear that 0 is the equilibrium point of the system composed of e, ψ_x and ψ_r.

Substitute 12 and 13 into 11 .

$$\dot{V} = -\frac{1}{2} e^T Q e \leq 0 \quad (14)$$

Therefore, $\dot{V}$ is negative semidefinite.

- With the above discussion, we have proved that by applying the adaptive law in 12 and 13 , e, ψ_x and ψ_r are Lyapunov stable about the equilibrium point 0. Furthermore,

Let $\alpha(e, \psi_x, \psi_r)$ be given by

$$\alpha(e, \psi_x, \psi_r) = \frac{1}{2}\min\left\{\mathrm{eig}(P), \mathrm{eig}(|g|\Gamma^{-1}), |g|\frac{1}{\gamma}\right\}[e^T \quad \psi_x^T \quad \psi_r]\begin{bmatrix} e \\ \psi_x \\ \psi_r \end{bmatrix}$$

α is positive definite and is a lower bound of V, i.e. $\alpha \leq V$. It is obvious that α is unbounded when any of $\|e\|$, $\|\psi_x\|$ and $\|\psi_r\|$ is unbounded. Thus, we have

$$\|V\| \to \infty \text{ as } \|[e^T \quad \psi_x^T \quad \psi_r]^T\| \to \infty$$

- Therefore, e, ψ_x and ψ_r are globally Lyapunov stable about the equilibrium point 0.

- Now that e, ψ_x and ψ_r are globally Lyapunov stable, we are going to further prove that e is globally asymptotically stable.

- From the Lyapunov stability, we know that despite the initial values of e, ψ_x and ψ_r :
  - $\|e\|$, $\|\psi_x\|$ and $\|\psi_r\|$ are bounded for $t > t_0$ 15
  - The following integral exists and is a finite number as t tends to infinity

$$0 \le \int_{t_0}^{t} \frac{1}{2} e^T Q e \, d\tau < +\infty \qquad 16$$

This integral cannot exceed the initial value of V.

- Using 15 and 7 , we can see that $||\dot{e}||$ is bounded, which makes e a uniformly continuous function for $t \geq t_0$. This implies that e cannot perform like a impulse signal where it suddenly jumps to infinity.
- Since e is uniformly continuous and the integral in 16 is bounded, we can already intuitively comprehend that e would converge to 0.

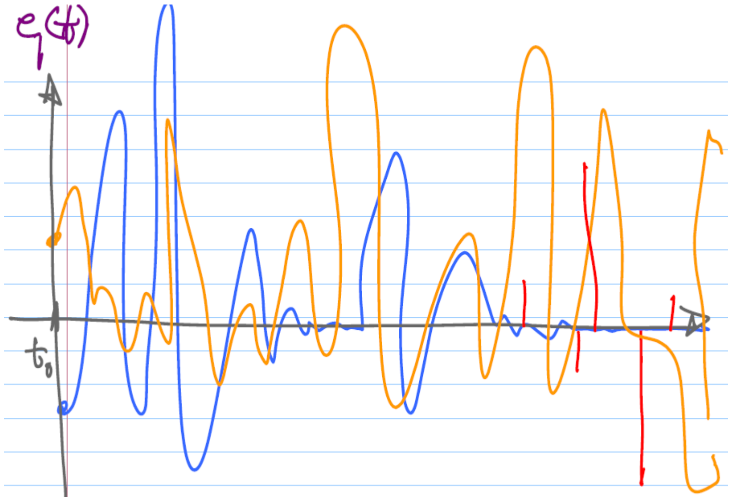
CONTROL SYSTEM DESIGN
- ADAPTIVE CONTROL SYSTEM DESIGN

- The orange kind of e is impossible since 16 is bounded.

- The red kind of e is impossible since e is uniformly continuous.

- Therefore, e must be acting like the blue kind of signal, which implies

$$\lim_{t\to\infty} e = 0$$



- This can also be illustrated using Barbalat's lemma as follows.

Barbalat's Lemma : If g is a real function of a real variable t, defined and uniformly continuous for $t \geq t_0$, and if the limit of the integral

$$\int_{t_0}^{t} g(\tau)d\tau$$

as t tends to infinity exists and is a finite number, then

$$\lim_{t\to\infty} g(t) = 0$$

Now, let $g(t) = \frac{1}{2}e^T Qe$. Using 7 ,

$$\begin{aligned}\dot{g} &= e^T Q\dot{e} \\ &= e^T Q\left[A_m e + gb\left(\psi_x^T x_p + \psi_r r\right)\right]\end{aligned}$$

From 15 , we can see that $\|\dot{g}\|$ is bounded, implying that g is uniformly continuous for $t \geq t_0$.

From 16 , we know that $\int_{t_0}^{t} g(\tau)d\tau$ is bounded as t tends to infinity.

Therefore, using Barbalat's lemma,

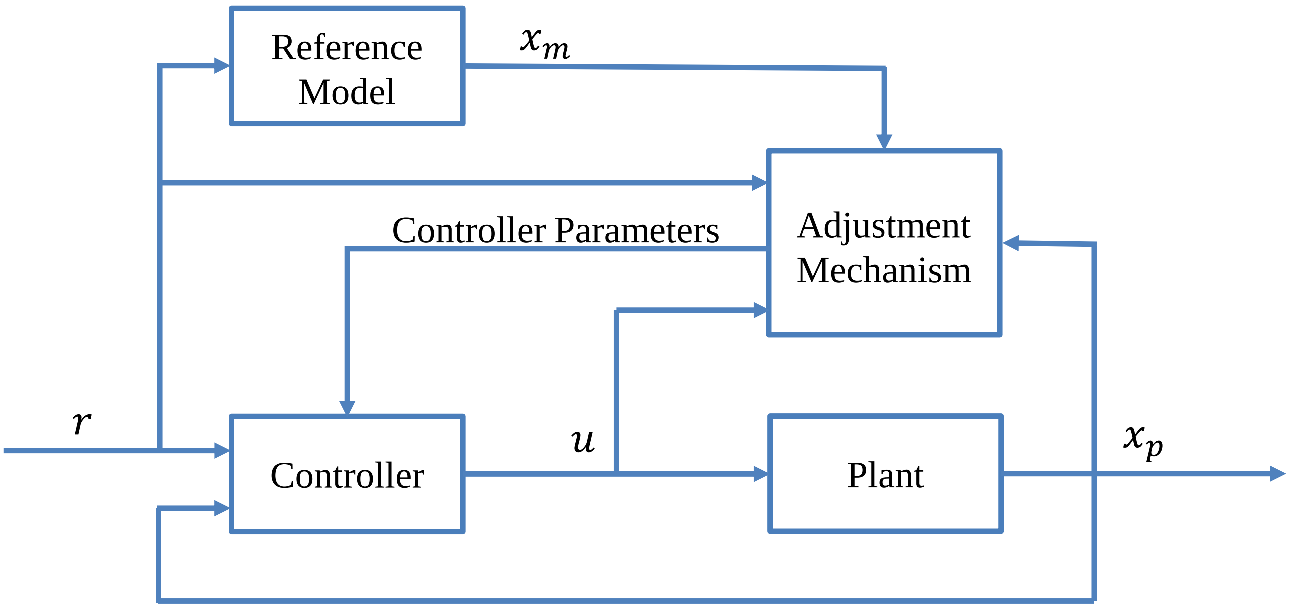
$$\lim_{t\to\infty} \frac{1}{2}e^T Qe = 0$$

Since Q is positive definite, the above equation implies

$$\lim_{t\to\infty} e = 0$$

CONTROL SYSTEM DESIGN
- ADAPTIVE CONTROL SYSTEM DESIGN

- The following is the block diagram of a model reference adaptive system with state-feedback.



CONTROL SYSTEM DESIGN
- AN EXAMPLE

- Here is an example of designing the adaptive controller for the DC motor to control its angular position.

- As a first step, based on the requirements and practical conditions, specify the natural frequency and damping ration of the reference model as follows.

$$\omega_m = 10,\ \zeta_m = 0.8$$

- The state-space realization of the reference model can be obtained as

$$\dot{x}_m = A_m x_m + g_m b r$$

$$A_m = \begin{bmatrix} 0 & 1 \\ -100 & -16 \end{bmatrix},\ g_m = 100,\ b = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

CONTROL SYSTEM DESIGN
- AN EXAMPLE

- Use the following control signal u, where θ_x, θ_r are adaptive control gains and x_p, r are the state feedback and the reference signal respectively.

$$u = \theta_x^T x_p + \theta_r r$$

- The control gain follows the following adaptive law.

$$\dot{\theta}_x = -\mathrm{sgn}(g)\Gamma e^T P b x_p \qquad \dot{\theta}_r = -\mathrm{sgn}(g)\gamma e^T P b r$$

where $e = x_p - x_m$; $\mathrm{sgn}(g)$ can be obtained by pre-test on the plant; Γ and γ are arbitrarily chosen as unity; $\theta_x(0)$ and $\theta_r(0)$ are chosen as 0. P is calculated as follows.

- Let Q to be identity matrix. Noticing that A_m is asymptotically stable, the following Lyapunov equation can be formed for the reference model.

$$A_m^T P + P A_m = -Q$$

- Substitute A_m and Q into the equation and P is solved as

$$P = \begin{bmatrix} 3.236 & 0.005 \\ 0.005 & 0.316 \end{bmatrix}$$

EXTRA

The following MATLAB commands can be used to solve the continuous Lyapunov equation.

```
% Solving Lyapunov Equation Using MATLAB
Am = [0,1;-100,-16];
Q = eye(2);
P = lyap(Am',Q);
fprintf('The solution to the Lyapunov equation is: \n')
disp(P)
```

Result:
The solution to the Lyapunov equation is:

```
3.2363    0.0050
0.0050    0.0316
```

CONTROLLER PROGRAMED IN LABVIEW

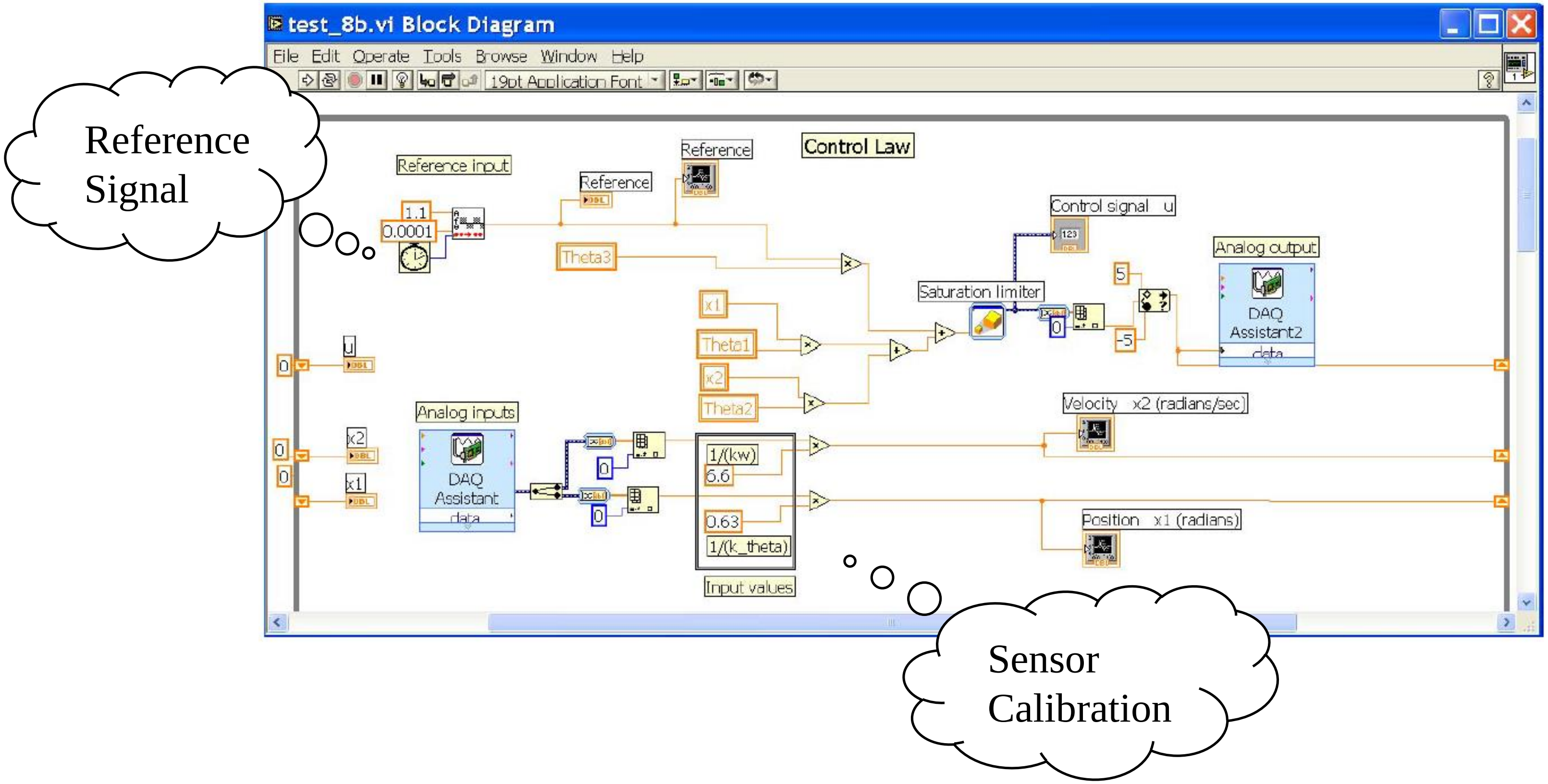
- The control signal u can be calculated in real-time with determined Γ, γ, P and x_p, r, e.

- In order to test the controller on the DC motor apparatus, the controller needs to be realized in LabVIEW as follows.

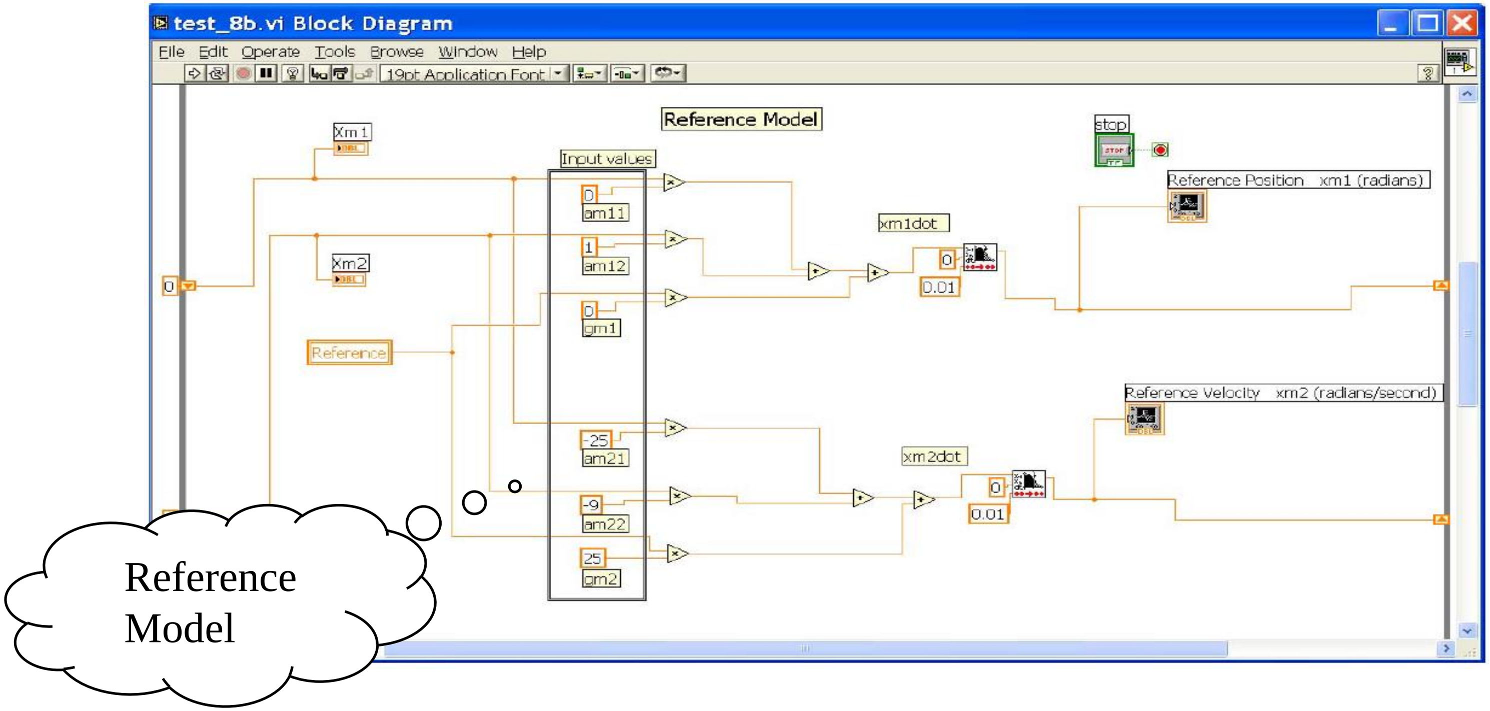
- Notice that in our example since $b = [0 \quad 1]^T$, the adaptive law is irrelevant of the value of p_{11} in P.

$$Pb = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} p_{12} \\ p_{22} \end{bmatrix}$$

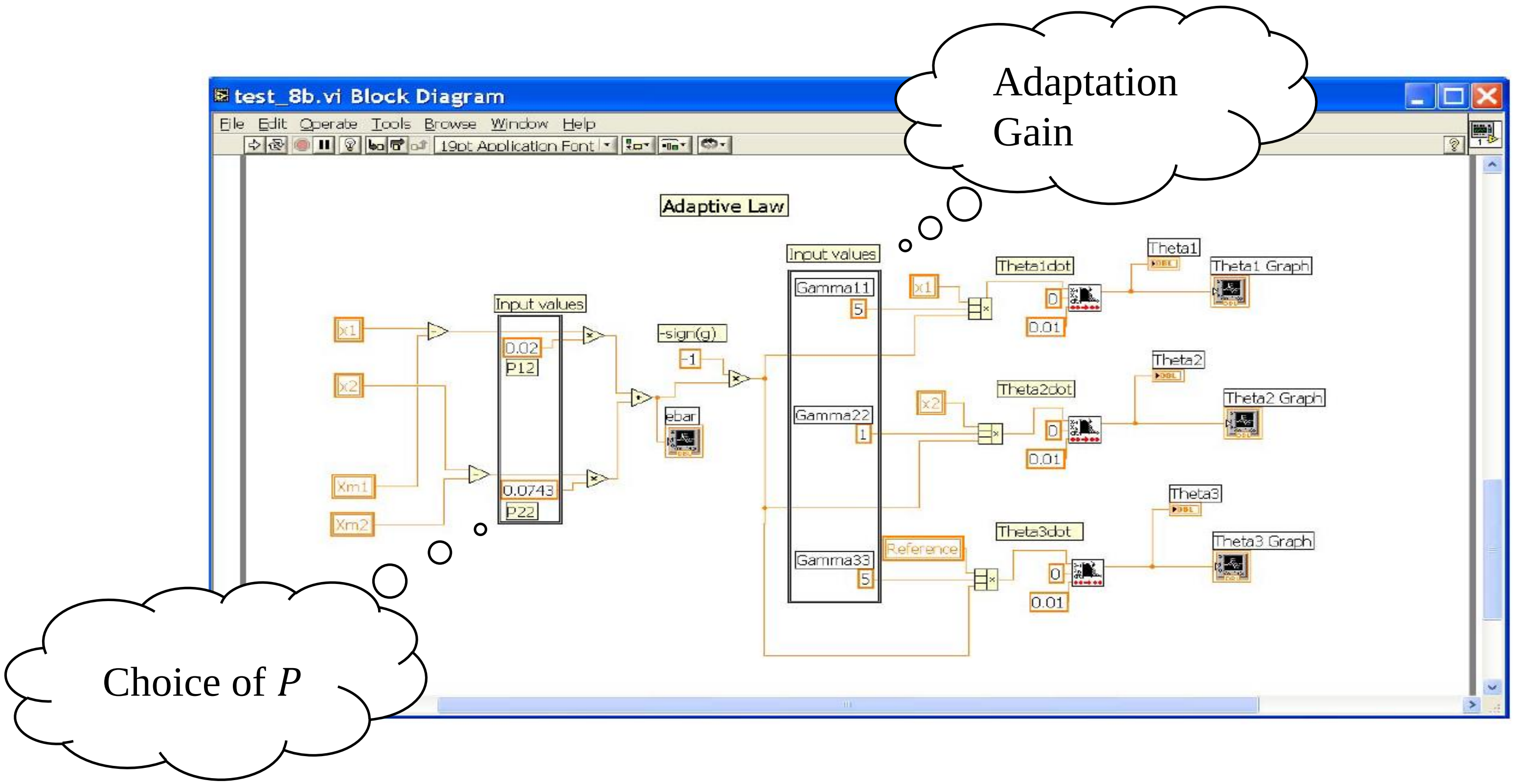
CONTROLLER PROGRAMED IN LABVIEW



CONTROLLER PROGRAMED IN LABVIEW



CONTROLLER PROGRAMED IN LABVIEW



QUESTIONS FOR EE5104/EE6104

- Required:
  - Design and implement an adaptive controller with full state measurable for angular position control.
  - For the controller, try different choice of the adaptation gain Γ and γ, and also different choice of Q in the Lyapunov equation.
  - Use LabVIEW to investigate / explore various design choice of your adaptive controller for a suitably chosen position reference signal.
  - Further investigate all signals/variables of suitable interest and discuss.

QUESTIONS FOR EE5104/EE6104

- In addition, try to discuss the following questions:
  - Will the adaptive control gain θ_x and θ_r converge to θ_x^* and θ_r^* under the following particular choice of reference signal r:
    - r is a step signal;
    - r is a random signal (for instance, white noise);
  - How does the choice of $\theta_x(0)$ and $\theta_r(0)$, i.e. the initial values of the control gain, affect the system performance, in particular
    - $\theta_x(0)$ and $\theta_r(0)$ are close to θ_x^* and θ_r^*;
    - $\theta_x(0)$ and $\theta_r(0)$ are far away from θ_x^* and θ_r^*;

QUESTIONS FOR EE6104

- Required:
  - Design and implement an adaptive controller with full state measurable for angular velocity control. (You may think of the plant as a first-order system whose state contains the angular velocity only.)
  - For the controller, try different choice of the adaptation gain Γ and γ, and also different choice of Q in the Lyapunov equation.
  - Use LabVIEW to investigate / explore various design choice of your adaptive controller for a suitably chosen position reference signal.
  - Further investigate all signals/variables of suitable interest and discuss.

KEY REFERENCES

- Lecture Notes of EE5104/6104 Advanced/Adaptive Control Systems
- CA3 Experiment Requirement Sheet for EE5104 Y2015/2016 S1
- CA3 Experiment Requirement Sheet for EE6104 Y2015/2016 S1
- Dr. K. Z. Tang, *et al*, EE6104/EE5104 Advanced/Adaptive Control Systems: Briefing Notes for CA3 Mini Project, Sep. 2008

EXTRA: CALIBRATION OF THE SENSORS USING LEAST SQUARES (LS) ESTIMATION

- The estimation of K_θ and K_ω can be obtained using the Least Squares (LS) algorithm as follows. LS estimation is widely used in parameter estimation.

Consider the following linear system model

$$y_i = ku_i + \epsilon_i$$

where $i = 1, \dots, N$ is the sampling index; y_i and u_i are the known output and input of the system; ϵ_i is the i.i.d noise that is unknown (often assumed to be Gaussian distributed).

The LS estimation of the parameter $\hat{k}$ is such that the following cost function is minimized.

$$J = \frac{1}{2}\sum_{i=1}^{N}\left(y_i - \hat{k}u_i\right)^2$$

The estimation is

$$\hat{k} = \frac{\sum_{i=1}^{N} u_i y_i}{\sum_{i=1}^{N} u_i^2}$$

LS estimation is covered in the second part of the lecture.

EXTRA

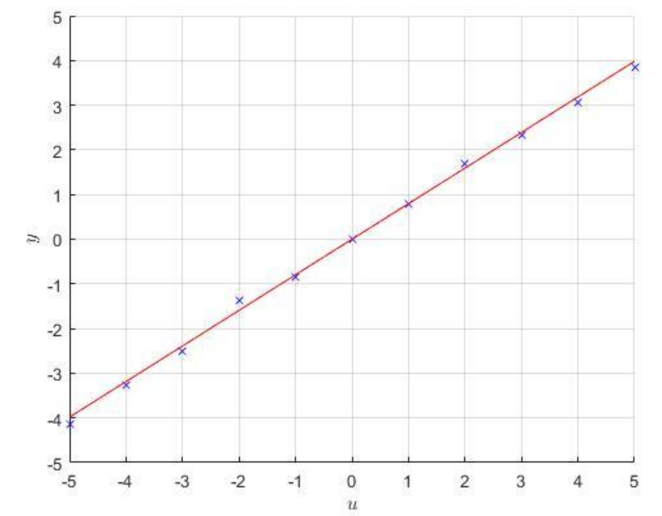
...RA: CALIBRATION OF THE SENSORS USING LEAST SQUARES (LS) ESTIMATION

- An example of applying LS algorithm on parameter estimation using MATLAB. The result is identical to that of the Linear Regression.

```
% Parameter Estimation Using LS Algorithm: An Example
% The linear model is $y=ku+\epsilon$; k is to be estimated.
u = [-5,-4,-3,-2,-1,0,1,2,3,4,5]';
y = [-4.15,-3.27,-2.51,-1.37,-0.86,0.01,0.78,1.69,2.32,3.06,3.86]';
P = (u'*u)^(-1);
khat = P*u'*y;
fprintf('The estimation of the parameter is: %d.\n',khat)

figure
hold on
plot(u,y,'bx')
plot(u,khat*u,'r-')
grid on
xlabel('$u$','interpreter','latex')
ylabel('$y$','interpreter','latex')
axis([-5,5,-5,5])
hold off
```

Result:
The estimation of the parameter is: 7.965455e-01.



EXTRA: SIMULATION OF THE CLOSED-LOOP SYSTEM USING SIMULINK

- Practical issues may occur during the experiment, including:
  - Time-varying system dynamics. (The process matrix A_p may change with time since the DC motor heats up due to the continuous operation.)
  - Process noise. (For instance, by accidentally touching the rotor of the DC motor during the experiment, the angular velocity of the motor may experience a sudden drop. This can be regarded as process noise.)
  - Measurement noise. (The measurements contain unavoidable noise due to the accuracy limits of the sensors.)

- In case the closed-loop system performance fails the expectation, we need to make it clear whether the failure is caused by the design of the controller or by the practical issues.

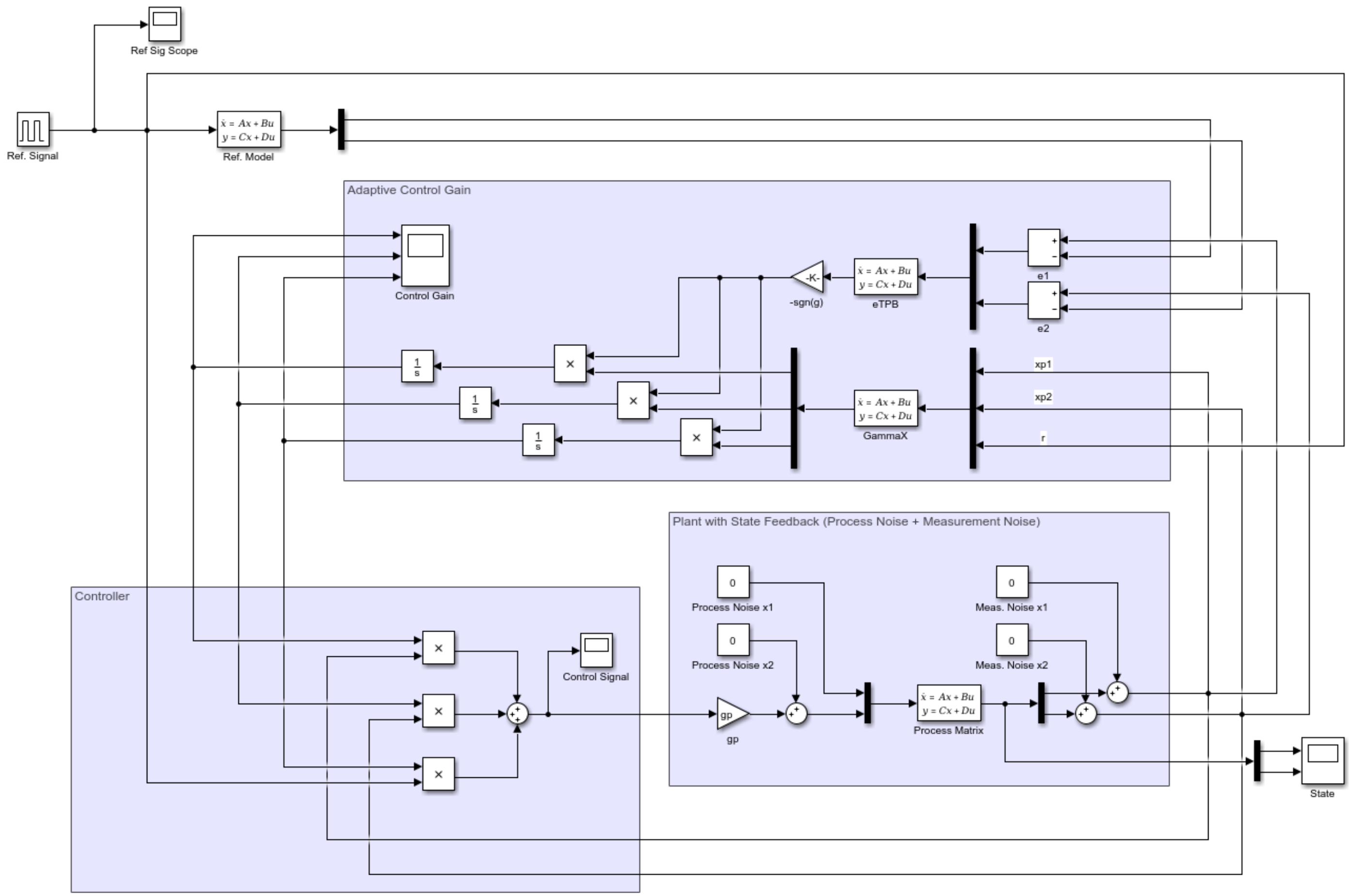
EXTRA: SIMULATION OF THE CLOSED-LOOP SYSTEM USING SIMULINK

- By simulating the closed-loop system with varieties of practical issues and comparing the system performance in reality with the simulation results, it might be possible to identify the cause of the failure.

- The simulation is also helpful to study the limitations of model reference adaptive controller. By deliberately adding noise to the system, it might be observed that the closed-loop system fails the expectation. After all, there is no absolute ‘perfect’ in engineering (which makes it charming).

EXTRA

RA: SIMULATION OF THE CLOSED-LOOP SYSTEM USING SIMULINK



EXTRA: SIMULATION OF THE CLOSED-LOOP SYSTEM USING SIMULINK

EXTRA

- Here is an example of using the Simulink program to analyze the system performance.
- Let the plant be

$$\dot{x}_p = A_p x_p + gbu$$

$$A_p = \begin{bmatrix} 0 & 1 \\ 0 & -4.2 \end{bmatrix}, \; g = 35.3, \; b = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

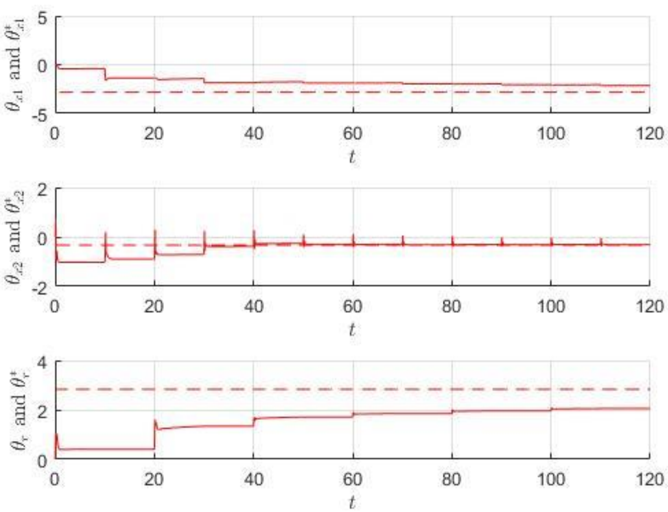
- Design the adaptive controller using

$$Q = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}, \Gamma = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}, \gamma = 10$$

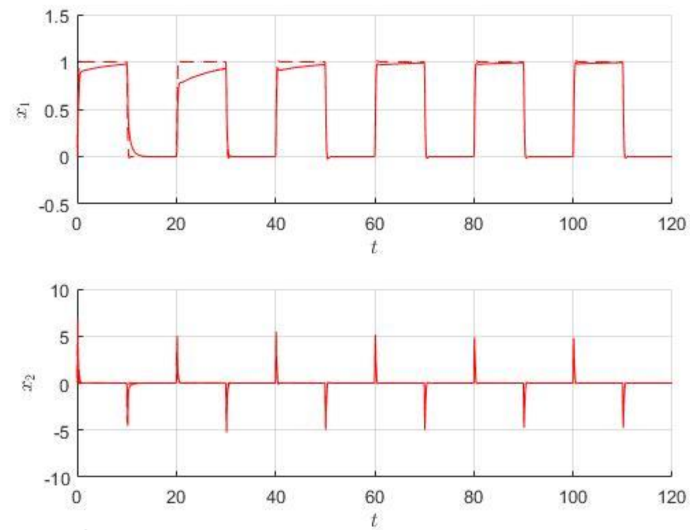
- The process noise and measurement noise are set to 0. The initial control gain is set to 0.

EXTRA: SIMULATION OF THE CLOSED-LOOP SYSTEM USING SIMULINK

- The control gain and the closed-system state trajectory in a 120s-simulation are given as follows. (Ref Signal: Unit Pulse with $T = 20s$)



Control gain:
Solid-line: θ_x and θ_r
Dashed-line: θ_x^* and θ_r^*

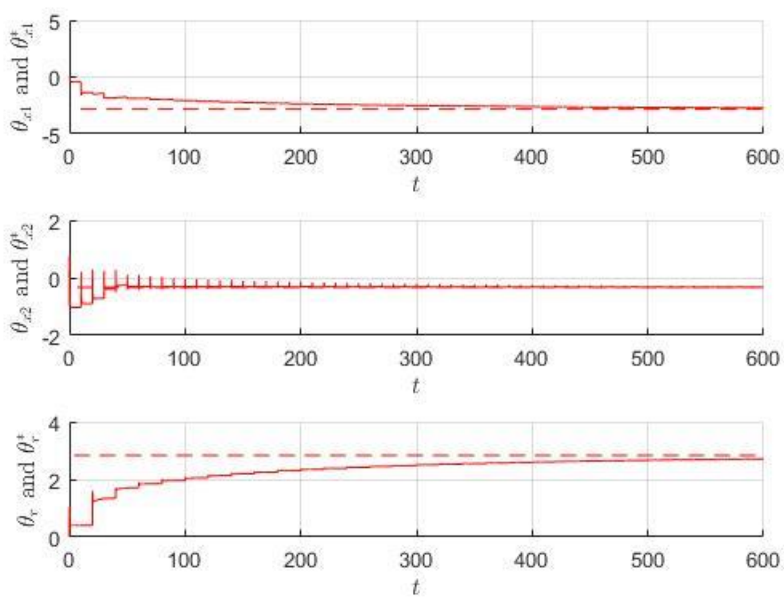


State:
Solid-line: x_p
Dashed-line: x_m

EXTRA

RA: SIMULATION OF THE CLOSED-LOOP SYSTEM USING SIMULINK

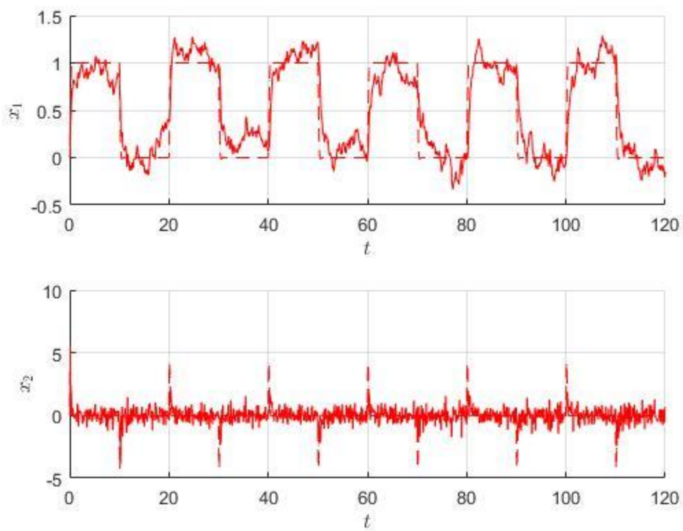
- The control gain in the first 600s of the simulation is given as follows.



EXTRA

EXTRA: SIMULATION OF THE CLOSED-LOOP SYSTEM USING SIMULINK

- Now consider applying white noise with $\text{PSD} = 0.01$ on both of the measurement of x_1 and x_2. (PSD stands for Power Spectral Density.) The following state trajectory shows how the noise affects the closed-loop system.



THANK YOU