APPENDIX A

Trigonometric identities

Formulas for rotation about the principal axes by θ:

$$R_X(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}, \tag{A.1}$$

$$R_Y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}, \tag{A.2}$$

$$R_Z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{A.3}$$

Identities having to do with the periodic nature of sine and cosine:

$$\begin{aligned} \sin\theta &= -\sin(-\theta) = -\cos(\theta + 90^\circ) = \cos(\theta - 90^\circ), \\ \cos\theta &= \cos(-\theta) = \sin(\theta + 90^\circ) = -\sin(\theta - 90^\circ). \end{aligned} \tag{A.4}$$

The sine and cosine for the sum or difference of angles θ_1 and θ_2:

$$\begin{aligned} \cos(\theta_1 + \theta_2) &= c_{12} = c_1 c_2 - s_1 s_2, \\ \sin(\theta_1 + \theta_2) &= s_{12} = c_1 s_2 + s_1 c_2, \\ \cos(\theta_1 - \theta_2) &= c_1 c_2 + s_1 s_2, \\ \sin(\theta_1 - \theta_2) &= s_1 c_2 - c_1 s_2. \end{aligned} \tag{A.5}$$

The sum of the squares of the sine and cosine of the same angle is unity:

$$c^2\theta + s^2\theta = 1. \tag{A.6}$$

If a triangle's angles are labeled a, b, and c, where angle a is opposite side A, and so on, then the "law of cosines" is

$$A^2 = B^2 + C^2 - 2BC\cos a. \tag{A.7}$$

The "tangent of the half angle" substitution:

$$\begin{aligned} u &= \tan\frac{\theta}{2}, \\ \cos\theta &= \frac{1 - u^2}{1 + u^2}, \\ \sin\theta &= \frac{2u}{1 + u^2}. \end{aligned} \tag{A.8}$$