

EE5132 / EE5023 – Wireless & Sensor Networks
Part 1: Tutorial 1 – Answers

1. Given: $v = 80 \text{ kph} = 80 \times 10^3 / 3600 = 22.2 \text{ m/s}$
 $f_c = 850 \text{ MHz}$; $A^2 / 2\sigma^2 = 0.01$; $A / 2\sigma = 0.1$; $c = 3 \times 10^8 \text{ m/s}$

$$(a) \quad N_A = \sqrt{2\pi} f_d \frac{A}{\sqrt{2}\sigma} \exp\left(-\frac{A^2}{2\sigma^2}\right)$$

Now $f_d = \frac{v}{\lambda} = v \frac{f_c}{c} = 22.2 \frac{850 \times 10^6}{3 \times 10^8} = 62.96$ and substituting this into N_A , we have:

$$N_A = \sqrt{2\pi} (62.96)(0.1) \exp(-0.01) = 15.63 \text{ fades/s}$$

$$(b) \quad \bar{t}_F = \frac{1}{\sqrt{2\pi} f_d} \frac{\sqrt{2}\sigma}{A} \left[\exp\left(\frac{A^2}{2\sigma^2}\right) - 1 \right] = \frac{1}{\sqrt{2\pi} (62.96)} (0.1) [\exp(0.01) - 1] = 0.000637 \text{ seconds}$$

$$(c) \quad \bar{t}_{IF} = \frac{1}{\sqrt{2\pi} f_d} \frac{\sqrt{2}\sigma}{A} = \frac{1}{\sqrt{2\pi} (62.96)} \frac{1}{0.1} = 0.06336 \text{ seconds}$$

- (d) Received carrier frequency = $f_c + f_d = 850 \text{ MHz} + 62.96 \text{ Hz} = \underline{850,000,062.96 \text{ Hz}}$

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- (e) Bit rate = 200 kbps \Rightarrow Symbol rate = 100 ksymbols/s
 \therefore Symbol duration = $1 / (100 \times 10^3) = 10^{-5}$ seconds

Since $N_A = 15.63$, $\therefore 1/N_A = 1/15.63 = 0.064$ seconds

\therefore On average, each fade occurs over many symbols and we have slow fading.

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2. Given: $N_A = 100$ fades/s ; $A^2/2\sigma^2 = 0.01$; $c = 3 \times 10^8$ m/s ; $f_c = 1$ GHz

(a) Using $N_A = \sqrt{2\pi}f_d \frac{A}{\sqrt{2}\sigma} \exp\left(-\frac{A^2}{2\sigma^2}\right)$, we have:

$$100 = \sqrt{2\pi}f_d(0.1) \exp(-0.01)$$

$$f_d = \frac{100}{\sqrt{2\pi}(0.1) \exp(-0.01)} = 402.95$$

$$\text{Since } f_d = \frac{v}{\lambda} = v \frac{f_c}{c}$$

$$\therefore v = \frac{f_d c}{f_c} = \frac{402.95 \times 3 \times 10^8}{10^9} = 120.89 \text{ m/s}$$

(b) $\bar{t}_F = \frac{1}{\sqrt{2\pi}f_d} \frac{\sqrt{2}\sigma}{A} \left[\exp\left(\frac{A^2}{2\sigma^2}\right) - 1 \right]$

$$0.01 = \frac{1}{\sqrt{2\pi}f_d} \left(\frac{1}{0.1} \right) [\exp(0.01) - 1]$$

$$\therefore f_d = \left(\frac{1}{0.01} \right) \frac{1}{\sqrt{2\pi}} \left(\frac{1}{0.1} \right) [\exp(0.01) - 1] = 4.0094$$

$$\therefore v = f_d \frac{c}{f_c} = (4.0094) \frac{3 \times 10^8}{10^9} = 1.20283 \text{ m/s}$$

3. Given: $v = 15$ m/s ; $f_c = 2$ GHz ; $c = 3 \times 10^8$ m/s ; $A^2/2\sigma^2 = 0.01$

(a) When the vehicle is at the boundary of coverage and is 30° from the direction of motion that we have the maximum relative velocity at $v \cdot \cos(30^\circ)$.

(b) Maximum relative velocity = $v \cdot \cos(30^\circ) = 15 \cos(30^\circ) = (15\sqrt{3})/2$ m/s

$$f_d = v \frac{f_c}{c} = \frac{15\sqrt{3}}{2} \frac{2 \times 10^9}{3 \times 10^8} = 50\sqrt{3} \text{ Hz}$$

$$N_A = \sqrt{2\pi}f_d \frac{A}{\sqrt{2}\sigma} \exp\left(-\frac{A^2}{2\sigma^2}\right) = \sqrt{2\pi}(50\sqrt{3})(0.1) \exp(-0.01) = 21.495 \text{ fades/s}$$

4. Given: $\bar{\tau} = 1.5 \mu\text{s}$

$$\therefore \text{Max. symbol rate} = \frac{1}{\bar{\tau}} = \frac{1}{1.5 \times 10^{-6}} = 666.67 \text{ ksymbols/s}$$

5. Given: $v = 20 \text{ m/s}$; $f_c = 2 \text{ GHz}$; $c = 3 \times 10^8 \text{ m/s}$; $A^2/2\sigma^2 = 0.01$

(a) $f_d = v \frac{f_c}{c} = 20 \frac{2 \times 10^9}{3 \times 10^8} = \frac{400}{3} \text{ Hz}$

$$N_A = \sqrt{2\pi} f_d \frac{A}{\sqrt{2}\sigma} \exp\left(-\frac{A^2}{2\sigma^2}\right) = \sqrt{2\pi} \left(\frac{400}{3}\right) (0.1) \exp(-0.01) = 33.089 \text{ fades/s}$$

$$\bar{t}_F = \frac{1}{\sqrt{2\pi} f_d} \frac{\sqrt{2}\sigma}{A} \left[\exp\left(\frac{A^2}{2\sigma^2}\right) - 1 \right] = \frac{1}{\sqrt{2\pi}} \left(\frac{3}{400}\right) \left(\frac{1}{0.1}\right) [\exp(0.01) - 1] = 0.00030071$$

$$\therefore \text{Prob. of being in a fade} = \bar{t}_F N_A = (0.00030071)(33.09) = 0.0099502$$

(b) Note that the prob. of being in a fade is

$$\bar{t}_F N_A = \left[\frac{1}{\sqrt{2\pi} f_d} \frac{\sqrt{2}\sigma}{A} \left\{ \exp\left(\frac{A^2}{2\sigma^2}\right) - 1 \right\} \right] \left[\sqrt{2\pi} f_d \frac{A}{\sqrt{2}\sigma} \exp\left(-\frac{A^2}{2\sigma^2}\right) \right] = \left[\left\{ \exp\left(\frac{A^2}{2\sigma^2}\right) - 1 \right\} \right] \left[\exp\left(-\frac{A^2}{2\sigma^2}\right) \right]$$

This is independent of the vehicular speed.

Hence the prob. of being in a fade will not change with speed.