UNIT - I

Basics of Probability Theory & Distribution :-

Basic Probability Theory :-

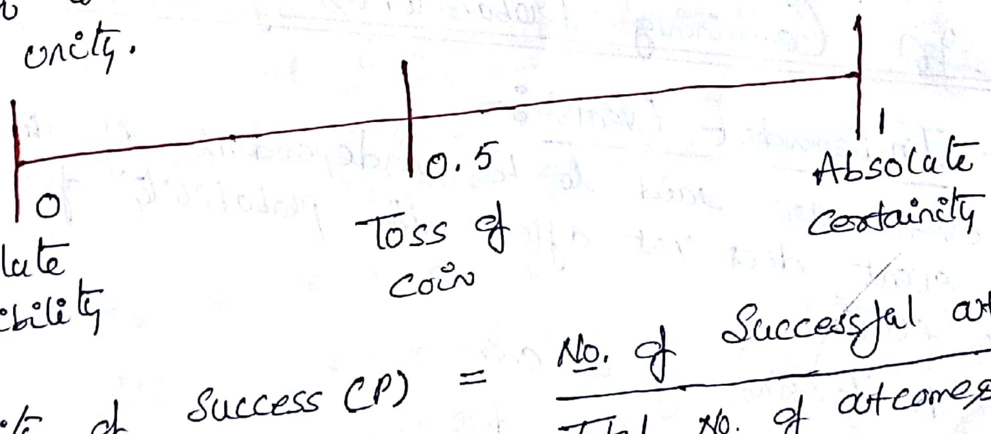
Definition of Reliability :-

Reliability is the probability of a device performing its purpose adequately for the period intended under the given operating conditions.

Probability Concepts :-

The ward probability is used frequently in a loose sense implying that a certain event has a good chance of occurring. In this sense it is a qualitative or subjective measure. ie in scientific "measure of chance". Mathematically it is a numerical index that can vary between zero which defines an absolute impossibility to unity which defines an absolute certainty. The scale of probability is shown in fig.

For example, the probability that a man will live for ever is zero, and the probability that one day he will die is unity.



$$\text{probability of Success (P)} = \frac{\text{No. of Successful outcomes}}{\text{Total No. of outcomes}}$$

$$p = \frac{s}{s+f}$$

where s = No. of successful outcomes
f = No. of failure outcomes

Probability of failure (q) = $\frac{\text{No. of failure outcomes}}{\text{Total No. of outcomes}}$

$= \frac{f}{s+f}$

$\therefore \boxed{p+q=1}$

Ex :-

1. Tossing a coin

Getting head is a success

Getting tail is a failure

$p = \frac{1}{1+1} = \frac{1}{2}$, $q = \frac{1}{1+1} = \frac{1}{2}$

2. Throwing a die

It has six faces, probability of getting 1 (say) is a success.

$p = \frac{1}{6}$ $q = 1 - \frac{1}{6} = \frac{5}{6}$

Rules for Combining probabilities of Events :-

Rule 1 :- Independent Events :-

Two events are said to be independent if the occurence of one event does not affect the probability of occurance of other event.

Ex : 1. Tossing of a coin
2. Throwing of a die.

Rule 2 : Mutually Exclusive Events :-

Two events are said to be mutually exclusive if one event A occurs other event B does not occur.

Ex : 1. Tossing of a coin
2. All six possible outcomes when a die is thrown

Rule 3 : Complementary Events :-

Two events are said to be Complementary such that the sum of those probabilities of the events is unity.

(or)

Two outcomes of an event are said to be Complementary if, when one outcome does not occur, the other must.

From fig.1, if the two outcomes A and B have probabilities P(A) and P(B), then

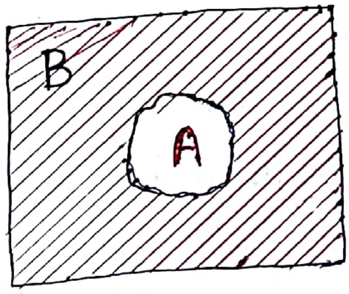


Fig.1. Complementary events

$$P(A) + P(B) = 1 \text{ (or)}$$

$$P(B) = P(\bar{A})$$

where $P(\bar{A})$ is the probability of A NOT occurring.

Ex:- 1. When tossing a coin, the outcomes head and tail are Complementary

$P(head) + P(tail) = 1$ (or) $P(head) = P(\overline{tail})$

2. A device can either be in a success state or in its failed state, there success and failed states are Complementary.

Rule 4 : Conditional Events (or) Dependent Events :-

Conditional events are those which occur conditionally depending on the occurrence of other events.

Consider two events A and B

The probability of event A occurring under the condition that event B has occurred.

$$P(A|B) = \frac{P[A \cap B]}{P[B]}$$

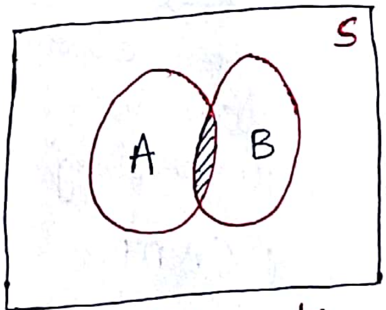


Fig: Intersection

$$P[A \cap B] = P(A|B)\,P(B)$$

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Suppose there is no Conditional occurrence

$$P[A|B] = P[A] \text{ itself.}$$

$$P[A \cap B] = P[A].P[B]$$

Rule 5 : Simultaneous Occurrence of Events

Two events A, B are said to occur simultaneously. If A and B occur together as whatever be the sequence.

Mathematically it is known as intersection of the two events and is represented as :

$$A \cap B \;,\; A \text{ AND } B \text{ or } AB$$

In this rule, there are two cases :

(a) Events are independent :-

If two events are independent, then the probability of occurrence of each event is not influenced by the probability of occurrence of the other. In this case

$$P(A|B) = P(A) \leftrightarrow P(A \cap B) = P(A|B)P(B)$$

$$P(B|A) = P(B) \leftrightarrow P(B \cap A) = P(B|A)P(A)$$

The probability that they both occurs is

$$P(A \cap B) = P(A).P(B)$$

$$P(B \cap A) = P(B).P(A)$$

This is known as product rule of independent events.

(b) Events are dependent :-

If two events are not independent, then the probability of occurrence of one event is influenced by the probability of occurrence of the other. Thus

$$P(A \cap B) = P(B|A).P(A)$$

$$= P(A|B).P(B)$$

Rule 6: Occurrence of at least one of two events:

Consider two events A, B. Occurrence of atleast one event of A, B means occurrence of either A OR B OR AB both. In terms of a Venn diagram, shown in fig. Mathematically it is known as the union of the two events and is expressed as :

$(A \cup B)$, $(A \text{ OR } B)$ (or) $(A + B)$

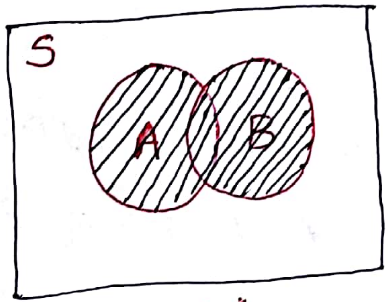

Fig: Union

In this rule there are three cases :

(a) Events are independent but not mutually exclusive :

$$
\begin{aligned}
P(A \cup B) &= P[A \text{ OR } B \text{ OR BOTH } A \text{ AND } B] \\
&= 1 - P[\text{NOT } A \text{ AND NOT } B] \\
&= 1 - P[\bar{A} \cap \bar{B}] \\
&= 1 - P[\bar{A}] \cdot P[\bar{B}] \\
&= 1 - (1 - P(A))(1 - P(B)) \\
&= P(A) + P(B) - P(A) \cdot P(B) \quad \text{- for independent events} \longrightarrow (1)
\end{aligned}
$$

$$
\begin{aligned}
P(A \cup B) &= P(A) + P(B) - P(A \cap B) \quad \text{- Union rule.} \\
&= P(A) + P(B) - P[A|B] \cdot P[B] \\
&= P(A) + P(B) - P[B|A] \cdot P[A] \quad \text{- for dependent conditional events.} \\
&= P(A) + P(B) - P(A) \cdot P(B) \quad \text{- for independent events.}
\end{aligned}
$$

(b) Events are independent and mutually exclusive.

In the case of events A and B being mutually exclusive, then the probability of their simultaneous occurrence $P(A) \cdot P(B)$ must be zero by definition. Therefore from eq (1)

$$P(A \cup B) = P(A) + P(B)$$

If there are 'n' independent and mutually exclusive events

$$P[A_1 \cup A_2 \cup \ldots \cup A_i \cup \ldots \cup A_n] = \sum_{i=1}^{n} P(A_i)$$

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(c) Events are not independent :

If the two events A and B are not independent then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= P(A) + P(B) - P(B|A) \cdot P(A)$$
$$= P(A) + P(B) - P(A|B) \cdot P(B)$$

~~Rule 7~~ : Suppose there are three events A, B, C

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$ - Union rule.

$$P(A \cup B \cup C) = P(A) + P(B) + P(C)$$ - If events A, B, C are independent and mutually exclusive (or) Summation rule.

$$P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$$

$$P(A_1 \cap A_2 \cap \ldots\ldots \cap A_n) = \prod_{i=1}^{n} A_i$$ → Product Rule.

Rule 7 : Conditional probability of several events :-

Consider event A is dependent upon the occurrences of number of mutually exclusive events B_i.

$$P(A \cap B) = P(A|B) \cdot P(B) \quad \text{------ (1)}$$

The following set of equations can be deduced for each B_i.

$$P(A \cap B_1) = P(A|B_1) \cdot P(B_1)$$
$$P(A \cap B_2) = P(A|B_2) \cdot P(B_2)$$
$$\vdots$$
$$P(A \cap B_i) = P(A|B_i) \cdot P(B_i)$$
$$\vdots$$
$$P(A \cap B_n) = P(A|B_n) \cdot P(B_n)$$

(or) when combined

$$\sum_{i=1}^{n} P(A \cap B_i) = \sum_{i=1}^{n} P(A|B_i) \cdot P(B_i) \longrightarrow (2)$$

If the events are independent

$$P(A|B_1) = P(A)\,P(B_1)$$

$$P(A|B_2) = P(A)\,P(B_2)$$

$$P(A|B_1, B_2) = P(A|B_1) + P(A|B_2) \rightarrow \text{Baye's Theorem.}$$

If eq (2) is summed over the exhaustive list of event B_i then, as illustrated by the consideration of four such events in fig.

$$\sum_{i=1}^{n} P(A \cap B_i) = P(A)$$

which reduces eq (2) to the following conditional probability equation.

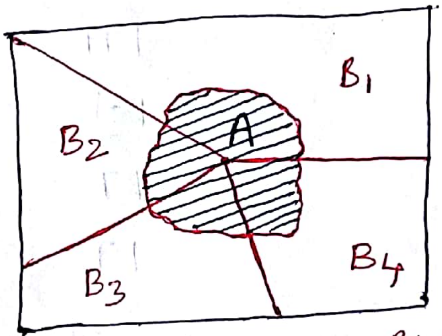

Fig: Conditional Probability

$$P(A) = \sum_{i=1}^{n} P(A|B_i) \cdot P(B_i) \longrightarrow (3)$$

If the occurrence of an event A is dependent upon only two mutually exclusive events for component B, i.e success and failure, which can be designated as B_s and B_f respectively. then eq (3) becomes as

$$P(A) = P(A|B_s)\,P(B_s) + P(A|B_f)\,P(B_f)$$

Let event A be system success

P(System Success) = P[System Success / X is good] P[X is good]
+ P[System success / X is bad] P[X is bad]

P(System Failure) = P[System failure / X is good] P[X is good]
+ P[System failure / X is bad] P[X is bad]

$\longrightarrow$ Principle of Decomposition

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Bernoulli's Trials :-

1. Consider tossing of a coin only once the possible events are getting H or T.

Getting Head is taken as success say p

Getting Tail is taken as failure say q

$$\therefore \boxed{p + q = 1}$$

2. Consider tossing of a coin two times, the possible events are. $p_A = p_B = p = 1/2$ $q_A = q_B = 1/2 = q$ (say)

A (success)	B (failure)	Probability density
1 H	1 H	$p_A \cdot p_B = p^2 = 1/4$
1 H	1 T	$p_A \cdot q_B = pq = 1/4$
1 T	1 H	$q_A \cdot p_B = qp = 1/4$
1 T	1 T	$q_A \cdot q_B = q^2 = 1/4$

Adding all the probabilities

$$= p^2 + pq + qp + q^2$$

If the ordering of the events is ignored

$$\therefore \text{Sum} = p^2 + 2pq + q^2 = (p+q)^2$$

3. Consider the coin is tossed thrice, the possible events are

$$p_A = p_B = p_C = \frac{1}{2} = p$$

$$q_A = q_B = q_C = \frac{1}{2} = q$$

A	B	C	Probability Density
H	H	H	$p_A \cdot p_B \cdot p_C = p^3 = 1/8$
H	H	T	$p_A\, p_B\, q_C = p^2 q = 1/8$
H	T	H	$p_A\, q_B\, p_C = p^2 q = 1/8$

A	B	C	Probability Density
T	H	H	$q_A p_B p_C = p^2 q = 1/8$
T	T	H	$q_A q_B p_C = q^2 p = 1/8$
T	H	T	$q_A p_B q_C = q^2 p = 1/8$
H	T	T	$p_A q_B q_C = q^2 p = 1/8$
T	T	T	$q_A q_B q_C = q^3 = 1/8$

$$\therefore \text{Sum} = p_A p_B p_C + p_A p_B q_C + p_A p_C q_B + q_A p_B p_C + q_A q_B p_C + q_A p_B q_C + p_A q_B q_C + q_A q_B q_C$$

$$= \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$$

$$= 1$$

If the ordering of the events are not considered

$$= p^3 + 3p^2 q + 3pq^2 + q^3$$

$$= (p+q)^3$$

Similarly extending for 'n' trials

$$\text{Sum} = (p+q)^n = 1$$

The above expression can be obtained from Binomial ~~Distribution~~ expansion where the probability density of r failures out of n trials

$$= {}^nC_r \, p^{n-r} q^r$$

where $${}^nC_r = \frac{n!}{r!(n-r)!}$$

The evaluation of these binomial coefficients

(a) when 'n' is small, direct hand evaluation of ${}^nC_r$ is practical.

(b) Also, when 'n' is relatively small, the coefficients can be found from Pascal's Triangle as follows.

Pascal's Triangle

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

etc.

(c) For large factorials, the coefficients can be approximated using Stirling's formula

$$n! \simeq e^{-n} n^{n} \sqrt{2\pi n}$$

Random Variables :-

Consider an experiment which is repeated many times ie a set of outcomes is generated, all the possible outcomes make up the sample space. The values of a random variables thus depends on the chance outcome of an experiment.

For example : 1. If the experiment consists of a sampling of shipment of resistors, the random variable could be the resistance value of each sample (outcomes).

2. When tossing two dice, the random variable could be the sum of the faces.

Random Variables can be continuous (such as resistances) (or) discrete (such as the sum of the faces)

* A discrete random variable is one that can have only a discrete number of states or countable number of values. For example, the toss of a coin is a discrete variable since there are only two discrete states that can occur, head and tail.

* A continuous random variable is one that can have an infinite number of values i.e. the range must extend from $-\infty$ to $+\infty$ only that there are an infinite number of possibilities of the value. For example, an electric current can have ~~only~~ any value between 5A and 10A but no other, it is a continuous random variable.

Probability Distribution Functions :-

If $P(X < x)$ is used to designate the probability that X has a value less than x. $P(a < x < b)$ is the probability that X has a value between a and b.

$$\therefore F(X) = P(X \leq x)$$

probability that X has a value less than (or) equal to x is referred to as cumulative distribution function (CDF).

The probability that X lies between x and $x + \Delta x$ as Δx becomes infinitely small is denoted by

$$f(x)\,\Delta x = P\{x \leq X \leq x + \Delta x\} \longrightarrow (1)$$

where $f(x)$ is the probability density function (PDF).

Since both $f(x)$ and $F(x)$ are probabilities they must be greater than or equal to zero for all values of x.

From eq (1) it is also evident that

$$\int_{-\infty}^{+\infty} f(x)\,dx = 1$$

The connection between the PDF and CDF of a random variable is given by

$$F(x) = \int_{-\infty}^{x} f(x)\,dx \quad (or)$$

$$f(x) = \frac{dF(x)}{dx}$$

The standard discrete distributions are the binomial and poisson's distributions.

The typical standard continuous distributions are the normal, exponential & weibull distributions

Mathematical Expectation :-

The expected value $E(x)$ of a discrete random variable x having n outcomes x_i each with a probability of occurrence P_i

$$E(x) = \sum_{i=1}^{n} x_i P_i$$

Where $\sum_{i=1}^{n} P_i = 1$

In the case of continuous random variable

$$E(x) = \int_{-\infty}^{+\infty} x\, g(x)\,dx$$

Where $\int_{-\infty}^{+\infty} f(x)\,dx = 1$

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Binomial Distribution :-

- The binomial distribution can be represented by the general expression

$$(p+q)^n$$

For this expression to be applicable, the following are the conditions

1. There must be a fixed number of trials ie n is known.
2. Each trial must result in either a success (or) a failure ie only two outcomes are possible and $p+q=1$.
3. All trials must have identical probabilities of success and therefore failure ie the value of p and q remain constant.

$$(p+q)^n = p^n + np^{n-1}q + \frac{n(n-1)}{2!}p^{n-2}q^2 + \cdots\cdots$$

$$\cdots + \frac{n(n-1)\cdots\cdots(n-r+1)}{r!}p^{n-r}q^r + \cdots$$

$$\cdots + q^n \quad \text{———} \quad (1)$$

From the definition of ^nC_r the probability of exactly r successes (or) $(n-r)$ failures in n trials can be evaluated from

$$P_r = \frac{n!}{r!(n-r)!}p^r q^{n-r}$$

$$= {}^nC_r\, p^r q^{n-r} \quad \text{———} \quad (2)$$

Substituting eq (2) in eq (1) we have

$$(p+q)^n = \sum_{r=0}^{n} {}^nC_r\, p^r q^{n-r} = 1$$

Expected Value :-

Let there are 'n' trials of which 'x' successes are there

probability density of x successes of n trials

$= {}^{n}C_{x}\, p^{x} q^{n-x}$

Expected Value $E(x) = \sum_{i=0}^{n} x_i P_i$

$= \sum_{i=0}^{n} x\, {}^{n}C_{x}\, p^{x} q^{n-x}$

$= \sum_{x=0}^{n} x . \dfrac{n!}{x!(n-x)!} p^{x} q^{n-x}$

As the contribution to this summation made by $x=0$ then

$E(x) = \sum_{x=1}^{n} x \dfrac{n(n-1)!}{x(x-1)!(n-x)!} p . p^{x-1} q^{n-x}$

$E(x) = np \sum_{x=1}^{n} \dfrac{(n-1)!}{(x-1)!(n-x)!} p^{x-1} q^{n-x}$

Let $n-1 = m$ when $x=1,\ y=0$

$x-1 = y$

$m-y = n-x$

$E(x) = np \sum_{y=0}^{m} \dfrac{m!}{y!(m-y)!} p^{y} q^{m-y}$

$= np \sum_{y=0}^{m} {}^{m}C_{y}\, p^{y} q^{m-y}$

$\therefore \boxed{E(x) = np}$ $\left[\because \sum P_i = 1\right]$

The expected value of x successes after the n trials of a binomial distribution is equal to number of trials multiplied by the probability of occurrence of success of each trial.

Standard Deviation :-

Varience $V(x) = E[x^2] - [E(x)]^2$

$$E[x^2] = \sum_{x=0}^{n} x^2 \, n_{C_x} \, p^x q^{n-x}$$

$$x^2 = x(x-1) + x$$

$$= \sum_{x=0}^{n} x(x-1) \, n_{C_x} \, p^x q^{n-x} + \sum_{x=0}^{n} x \, n_{C_x} \, p^x q^{n-x}$$

$$= \sum_{x=0}^{n} x(x-1) \, n_{C_x} \, p^x q^{n-x} + np$$

Consider 1^{st} term

$$\sum_{x=0}^{n} x(x-1) \, n_{C_x} \, p^x q^{n-x}$$

$$\sum_{x=2}^{n} x(x-1) \frac{n!}{(n-x)! \, x!} p^2 p^{(x-2)} q^{n-x}$$

$$\sum_{x=2}^{n} x(x-1) \frac{n(n-1)(n-2)!}{x(x-1)(x-2)! \, (n-x)!} p^2 p^{x-2} q^{n-x}$$

$$n(n-1)p^2 \sum_{x=2}^{n} \frac{(n-2)! \, p^{x-2} q^{n-x}}{(x-2)! \, (n-x)!}$$

Let $n-2 = m$ $\qquad n-x = m-y$

$x-2 = y$

$$= n(n-1)p^2 \sum_{y=0}^{m} \frac{m!}{y!\,(m-y)!} p^y q^{m-y}$$

$$= n(n-1)p^2 \qquad \left[\because \sum p_i = 1\right]$$

$$E(x^2) = n(n-1)p^2 + np$$

Valience $V(x) = n(n-1)p^2 + np - (np)^2$

$$= n^2p^2 - np^2 + np - n^2p^2$$

$$= np(1-p) = npq$$

Standard deviation $\sigma = \sqrt{V(x)}$

$$\therefore \boxed{\sigma = \sqrt{npq}}$$

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Problems :-

1. There are four boxes A, B, C and D each containing a mixture of good and defected fuses as shown.

Box	No. of fuses	% of defective fuses	No. of good fuses
A	2000	5%	5% of 2000 – 1900
B	500	10%	450
C	1000	8%	920
D	1000	9%	910

A box is chosen and fuse is picked up from it at random. what is the probability that the picked up fuse is good.

Sol Total fuses $= 2000 + 500 + 1000 + 1000$
$= 4500$

No of good fuses
$= 1900 + 450 + 920 + 910$
$= 4180.$

$\therefore$ Probability $= \dfrac{4180}{4500} = \underline{0.9288}.$ (or)

Overall probability of picking good one
$=$ probability of picking from box A × prob. of selecting good one from A + ... B × ...... B + ...C × ..... + ... D × ..... D.

Prob: $\dfrac{1900}{2000} = 0.95$, $\dfrac{450}{500} = 0.9$, $\dfrac{920}{1000} = 0.92$, $\dfrac{910}{1000} = 0.91$

$= \frac{1}{4} \times 0.95 + \frac{1}{4} \times 0.9 + \frac{1}{4} \times 0.92 + \frac{1}{4} \times 0.91$

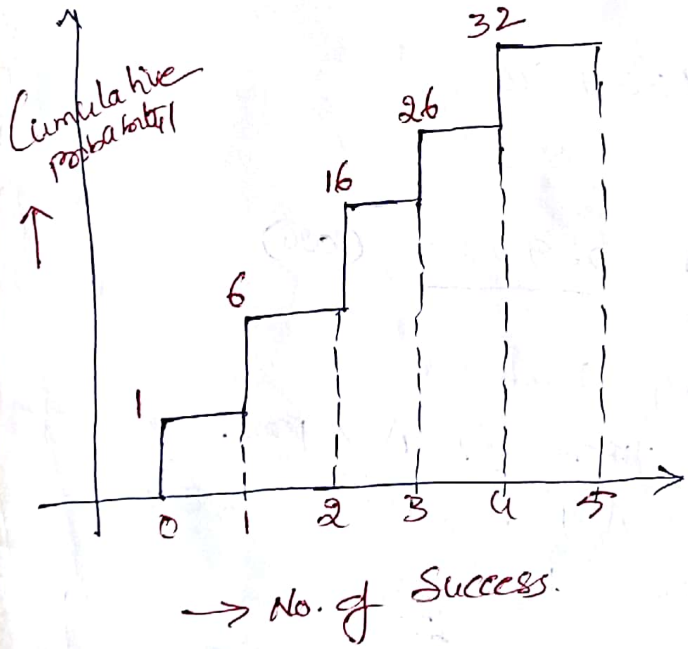
$= \underline{0.92}$

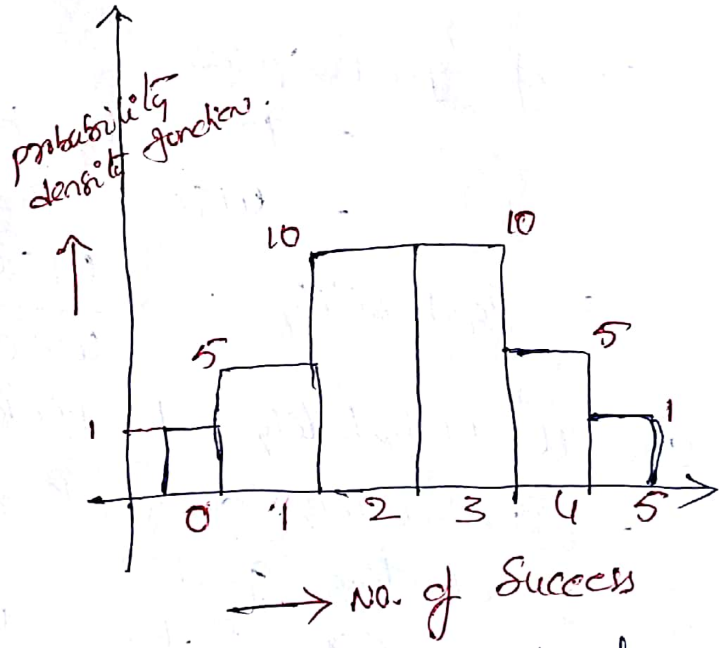
2. A coin is tossed 5 times draw the probability density and distribution functions.

Sol $n = 5, \quad p = 1/2 \quad q = 1/2 \quad \rightarrow {}^nC_r \, p^{n-r} q^r$

Probability Table :-

No. of Heads (Success)	No. of Tails (Failure)	Individual Probability (or) probability Density	Cumulative probability (or) probability Distribution
0	5	${}^5C_5 \, p^0 q^5 = 1/32$	$1/32$
1	4	${}^5C_4 \, p^1 q^4 = 5/32$	$6/32$
2	3	${}^5C_3 \, p^2 q^3 = 10/32$	$16/32$
3	2	${}^5C_2 \, p^3 q^2 = 10/32$	$26/32$
4	1	${}^5C_1 \, p^4 q^1 = 5/32$	$31/32$
5	0	${}^5C_0 \, p^5 q^0 = 1/32$	$32/32 = 1$





(a) What is the probability of getting 2 heads out of 5 trials $= 10/32$

(b) What is the probability of getting atleast 2 heads
Cumulative $= 26/32$.

Q. A generating station has 5 units each of 20 MW capacity. The probability of failure of any unit is 0.06. What is the probability of the station not been able to generate 70 MW.

Sol: Given $n = 5$, $q = 0.06$, $p = 1 - q = 1 - 0.06 = 0.94$

No. of Success	No. of Failure	Capacity out in MW.	Capacity In MW	Individual probability	Cumulative probability
5	0	0	100	${}^5C_0\, p^5 q^0 = 0.7339$	1.0
4	1	20	80	${}^5C_1\, p^4 q^1 = 0.2342$	0.26607
3	2	40	60	${}^5C_2\, p^3 q^2 = 0.0299$	0.03187
2	3	60	40	${}^5C_3\, p^2 q^3 = 0.0019086$	0.0019702
1	4	80	20	${}^5C_4\, p^1 q^4 = 0.000068$	0.000068
0	5	100	0	${}^5C_5\, p^0 q^5 = 0.776 \times 10^{-6}$	0.776×10^{-6}

$[{}^nC_r\, p^{n-r} q^r]$

Probability of the station not been able to generate 70 MW
$= 0.03187$

Probability of the station generate atleast 60 MW
$= 0.0019702$

4. A telephone exchange contains 10 lines, a line can be busy are available for calls and all lines act independently. If the probability that a line will be busy during the noon period is 0.8. What is the probability of there being atleast 3 free lines at any given time during the period.

Sol $n = 10$, probability of line busy i.e. failure

$q = 0.8$

$p = 1 - q = 0.2$

probability of getting atleast 3 lines free at any free time.

= Probability of getting 3 or more lines free

= 1 - [probability of not getting '0' lines +
" " " 1 lines +
" " " 2 lines]

$= 1 - \left[10_{C_0} p^0 q^{10} + 10_{C_1} p^1 q^9 + 10_{C_2} p^2 q^8\right]$

$= 1 - \left[(1)(0.8)^{10} + 10(0.2)^1(0.8)^9 + 45(0.2)^2(0.8)^8\right]$

$= \underline{0.322}$.

HW

5. A die is thrown six times find the probability of getting (a) two atleast ones

(b) the probability of two not getting.

6. A generating station has 2 X 20 MW units and 1 X 30 MW units. A 20 MW unit station has a probability of failure of 0.1 and 30 MW unit has a probability of failure of 0.15. Find the Combined Capacity outage probability table.

Sol $q = 0.1$, $p = 0.9$, 2 X 20 MW unit

Individual Capacity outage probability Table :-

①

Capacity out (MW)	Capacity in (MW)	Individual Probability
0	40	$p^2 = 0.81$
20	20	$2pq = 2 \times 0.9 \times 0.1 = 0.18$
40	0	$q^2 = (0.1)^2 = 0.01$
		$\Sigma = 1.0$

$p = 0.85$, $q = 0.15$, 1 X 30 MW

②

Capacity out (MW)	Capacity in (MW)	Individual probability
0	30	0.85
30	0	0.15
		$\Sigma = 1.0$

Combined Capacity outage probability Table :-

Capacity out (MW)	Capacity In (MW)	Individual Probability
0	70	$0.81 \times 0.85 = 0.6885$
20	50	$0.18 \times 0.85 = 0.153$
30	40	$0.81 \times 0.15 = 0.1215$
40	30	$0.01 \times 0.85 = 8.5 \times 10^{-3}$
50	20	$0.18 \times 0.15 = 0.027$
70	0	$0.01 \times 0.15 = 1.5 \times 10^{-3}$
		$\Sigma = 1.0$

7. If a random variable has the probability density

$$f(x) = \begin{cases} 2 \cdot e^{-2x} & \text{for } x > 0 \\ 0 & \text{for } x < 0 \end{cases}$$

Find the probabilities that it will take on a value

1. Between 1 and 3
2. Greater than 0.5

Sol. 1. B/w 1 and 3

$$F(x) = \frac{d}{dx} R(x)$$

$$R(x) = + \int_1^3 2e^{-2x} dx .$$

$$= + 2 \left[\frac{e^{-2x}}{-2} \right]_1^3$$

$$= \underline{\underline{0.1329}}$$

2. Greater than 0.5

$$R(x) = + \int_{0.5}^{\infty} 2e^{-2x}$$

$$= + 2 \left[\frac{e^{-2x}}{-2} \right]_{0.5}^{\infty}$$

$$= e^{-\infty} - e^{-1}$$

$$= \underline{\underline{0.3679}}$$

8. If the probability density of a random variable is given by

$$f(x) = Kx^3 \quad 0 < x < 1$$
$$= 0 \quad \text{elsewhere}$$

Find the value of K and the probability that the random variable takes on a value

1. Between 1/4 and 3/4
2. Greater than 2/3.

Sol (1) b/w 1/4 and 3/4

$$R(X) = +\int_{1/4}^{3/4} Kx^3\,dx = K\left[\frac{x^4}{4}\right]_{1/4}^{3/4}$$

$$= \frac{K}{4}\left[\frac{3^4}{4^4} - \frac{1^4}{4^4}\right]$$

$$= \frac{K}{4}\left[0.31640 - 0.003906\right] = 0.31249$$

$$= 0.07812K$$

$$=$$

(2) $$R(X) = +\int_{2/3}^{1} Kx^3 = +K\left[\frac{x^4}{4}\right]_{2/3}^{1}$$

$$= +\frac{K}{4}\left[\left(\frac{2}{3}\right)^4 - (1)^4\right]$$

$$0.2006K = 0.8024$$

$$=$$

we know

$$\int_{-\infty}^{+\infty} f(x)\,dx = 1.$$

$$\int_{-\infty}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx + \int_{1}^{+\infty} f(x)\,dx = 1$$

$$0 + \int_0^1 Kx^3 + 0 = 1 \Rightarrow K\left[\frac{x^4}{4}\right]_0^1 = 1 \Rightarrow \frac{K}{4} = 1$$

$$\therefore \boxed{K = 4}$$

9. A four engine air craft can operate only if atleast two engines are working. It has two generators driven by the engines, each one of which can supply the minimum load demand. write down the various system configurations possible and compare reliabilities.

HW

Reliability of engine $R_E = 0.8$

Reliability of Generator $R_G = 0.9$

Sol

$R_E = 0.8 \qquad R_G = 0.9$

$Q_E = 0.2 \qquad Q_G = 0.1$

① $R = {}^4C_2 (P)^2 (q)^{4-2} + {}^4C_3 (P)^3 (q)^{4-3} + {}^4C_4 (P)^4 (q)^{4-4}$

$= 6(0.8)^2 (0.2)^2 + 4(0.8)^3 (0.2)^1 + 1(0.8)^4$

$= 0.1536 + 0.4096 + 0.4096$

$= \underline{0.9728}$

② $R = {}^2C_1 (P)^1 (q)^{2-1} + {}^2C_0 (P)^0 (q)^{2-0}$

$= 2[0.9]^1 [0.1]^1 + 1[1][0.1]^2$

$= 0.18 + 0.01 = \underline{0.19}$

10. Let X be the random variable that denotes life in hours of a certain electronic device. The probability density function is

$$f(x) = \frac{2000}{x^3}, \quad x > 100$$
$$= 0 \quad \text{elsewhere}$$

Find the expected life of this device.

Sol Expected life $E(x) = \int_{-\infty}^{+\infty} x f(x)\,dx$

$$= \int_{100}^{\infty} x f(x)\,dx$$

$$= \int_{100}^{\infty} x \cdot \frac{2000}{x^3}\,dx$$

$$= \int_{100}^{\infty} \frac{2000}{x^2}\,dx$$

$$= 2000 \left[\frac{x^{-2+1}}{-2+1}\right]_{100}^{\infty} = -2000 \left[0 - \frac{1}{100}\right]$$

Expected life = 20 years

11. The foreman of a casting section in a factory finds that on the average 1 in every 5 castings made is defective. If the section makes 8 castings a day. What is the probability that exactly 2 castings will be defective.

Sol $p = \frac{1}{5}$ $q = \frac{4}{5}$, $n = 8$, $r = 2$

$$p = {}^{n}C_{r}\, p^{r} q^{n-r}$$

$$= {}^{8}C_{2} \left[\frac{1}{5}\right]^2 \left[\frac{4}{5}\right]^{8-2}$$

$$= (28)(0.04)(0.2621)$$

$$= 0.293552$$

12. If the Probability is 0.4 that steam will condense in a thin-walled aluminium tube at 10 atm pressure, use the formulae for binomial distribution to find the probability.

Sol

$p = 0.4$

$q = 0.6$

$$Prob = {}^{n}C_{r} (p)^{r} (q)^{n-r}$$

$$= {}^{10}C_{0} (0.4)^{0} (0.6)^{10-0}$$

$$= 0.060466$$

Problems: A

1. A generating station consists of 6 units of 30 MW each. If the prob of failure of each unit is 0.1. Calculate the prob of the system (1) supplying at least 120 MW (2) Not able to supply 20 MW.

(2) A lot containing 7 components is sampled by a quality inspector, the lot contains 4 good components and 3 defective components. A sample of 3 is taken by the inspector. Find the prob value of number of good components in this sample.

(3) A product is claimed to be 90% free of defective. What is the expected value & S.D of the number of defects in a sample of 4.

(4)

UNIT - II

Network Modelling & Reliability Analysis :-

Graph (or) Networks are classified as

1. Single input Single o/p N/w or SISO N/w.
2. Single input multi o/p N/w or SIMO N/w.
3. Multi - Input Single output N/w or MISO N/w
4. MIMO or Multi Input Multi output

Single Input Single Output N/w or SISO N/w :-

(a) Series Systems

(b) parallel Systems

1. Fully Redundant System.
2. partially Redundant System.

(c) Series - parallel Systems

1. Fully Redundant System.
2. partially Redundant System.

(d) Complex Systems (Non-Series - parallel Systems)

1. Using decomposition principle or Baye's Theorem.
2. pathset based approach to determine Network Reliability
3. Cutset based approach to determine Network unreliability

(a) Series Systems :-

Let R_A, R_B be the probabilities of success ~~and~~ of components A and B respectively.

Q_A, Q_B be the probabilities of failure of components A and B respectively.

Assume the probability of success of each component is independent

System success which implies that both the components must function.

$\therefore$ using the product rule of combining events then

Let R_S = Probability of success of the system

= Reliability of the system.

$$\therefore \boxed{R_S = R_A \cdot R_B}$$

We know

$R_A + Q_A = 1 \Rightarrow R_A = 1 - Q_A$

$R_B + Q_B = 1 \Rightarrow R_B = 1 - Q_B$

$$R_S = (1 - Q_A)(1 - Q_B)$$

$$= 1 - [Q_A + Q_B - Q_A Q_B]$$

Let Q_S = Probability of failure of system

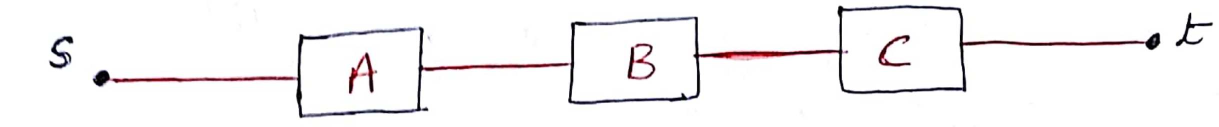
$$R_S + Q_S = 1$$

$$Q_S = 1 - R_S$$

$$= 1 - [1 - (Q_A + Q_B - Q_A Q_B)]$$

$$\therefore \boxed{Q_S = Q_A + Q_B - Q_A Q_B}$$ $\rightarrow$ Called union rule of unreliability of a series system.

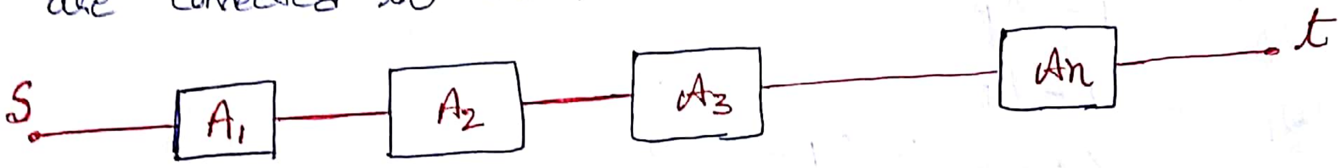
Consider three systems Components Connected in series



$$R_S = R_A R_B R_C$$

$$Q_S = Q_A + Q_B + Q_C - Q_A Q_B - Q_B Q_C - Q_C Q_A + Q_A Q_B Q_C$$

Consider there are 'n' Components say $A_1, A_2 \ldots A_n$ are Connected in series.



Let $R_S = P(A_1) P(A_2) \ldots \ldots P(A_n)$

$$R_S = \prod_{i=1}^{n} P(A_i)$$

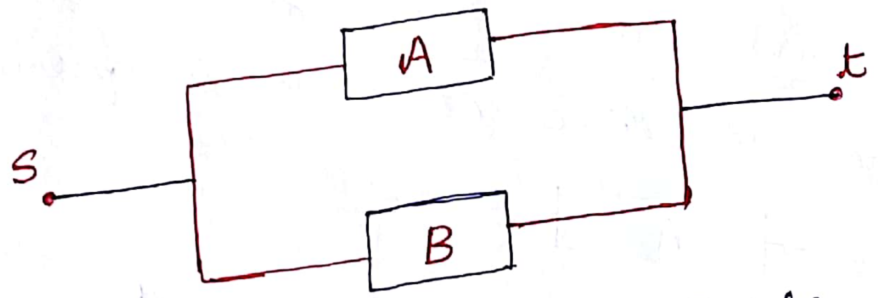
$$R_S = \prod_{i=1}^{n} R_i$$

no longer useful. or needed

Where $R_i = P(A_i)$ for $i = 1$ to n.

2. parallel Systems (or) Redundant Systems :-

Consider two Components are connected in parallel



Either A or B or Both A and B are success implies the system is success.

or

If both A and B fails then the system will fail.

Let Q_P = probability of failure of parallel connected system.

R_P = probability of success of parallel connected system.

$$\therefore \boxed{Q_P = Q_A Q_B}$$

i.e. product rule of unreliability for parallel connected system.

$$Q_A = 1 - R_A$$

$$Q_B = 1 - R_B$$

$$Q_P = (1-R_A)(1-R_B)$$

$$= 1 - \{R_A + R_B - R_A R_B\}$$

$$R_P + Q_P = 1$$

$$R_P = 1 - Q_P$$

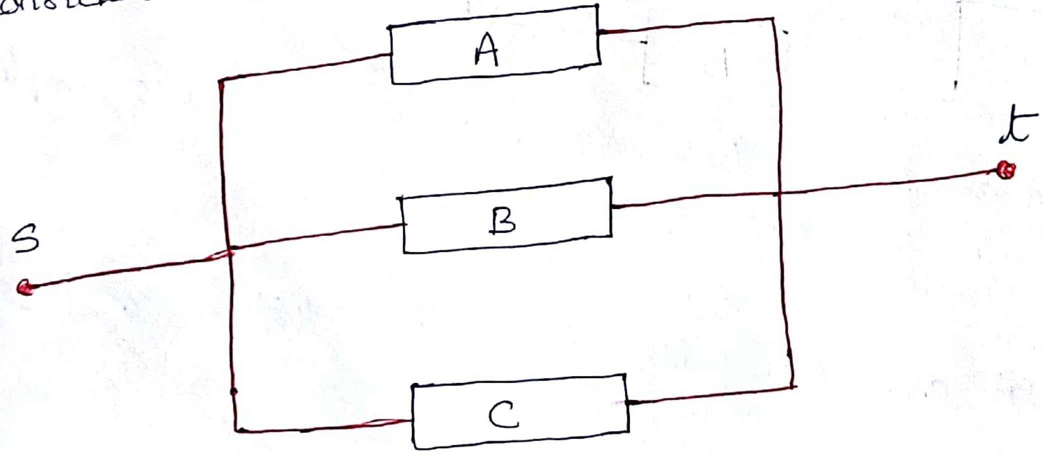
$$= 1 - \{1 - (R_A + R_B - R_A R_B)\}$$

$$\therefore \boxed{R_P = R_A + R_B - R_A R_B}$$

union rule of reliabilities for parallel connected systems

(a) Fully Redundant Configuration :-

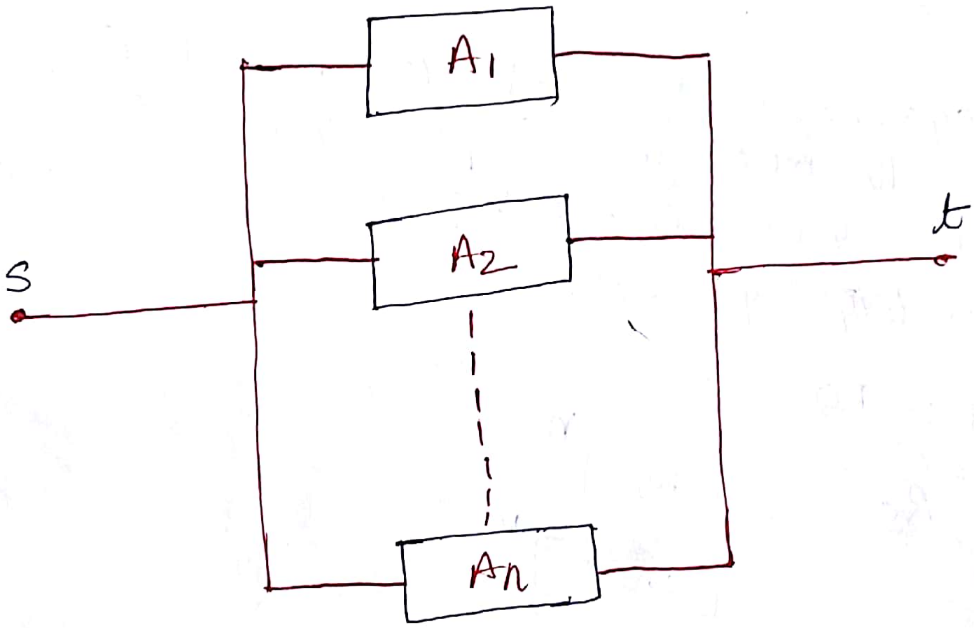
Consider there are three components in parallel



$$Q_P = Q_A Q_B Q_C$$

$$R_P = R_A + R_B + R_C - R_A R_B - R_B R_C - R_C R_A + R_A R_B R_C$$

If there are 'n' components connected in parallel



$$Q_P = \prod_{i=1}^{n} Q(A_i)$$

$$Q_P = \prod_{i=1}^{n} Q_i$$

(b) <u>Partially Redundant Configuration</u> :-

Suppose atleast two of the three must function for the system to success (R_P)

1. Fully redundant and all components have equal probabilities for 3 components

$$R_P = 1 - Q^3 = R^3 + 3R^2Q + 3Q^2R + Q^3$$

2. For partially redundant system where atleast 2 out of 3 must function and components have equal probabilities

$$R_P = 3R^2Q + R^3$$

3. Suppose each Components have unequal probabilities then

$$R_P = R_A R_B Q_C + R_A R_C Q_B + R_B R_C Q_A + R_A R_B R_C$$

Problems :-

1. A system consists of 10 identical Components all of which must work for system success. What is the system reliability if each Component have reliability of 0.95.

Sol.
$$n = 10$$
$$R_S = [R_i]^n$$
$$= [0.95]^{10}$$
$$= 0.5987$$

2. A system consists of 10 identical Components in series if the overall system reliability must not be less than 0.8. What is the minimum reliability of each Component.

Sol
$$R_S = [R_i]^{10}$$
$$0.8 = [R_i]^{10}$$
$$R_i = [0.8]^{1/10}$$
$$= 0.9779$$

3. A System consists of 4 Components in parallel each having reliability of 0.9. What is the reliability and unreliability of the system?

Sol
$$Q_A = Q_B = Q_C = Q_D = 1 - R = Q$$
$$Q = 1 - 0.9 = 0.1$$

$$Q_P = Q_A Q_B Q_C Q_D$$

$$Q_P = Q^4 = [0.1]^4 = 0.0001$$

$$R_P = 1 - Q_P = 0.9999$$

4. A system is to be designed with an overall reliability of 0.999 using components having reliabilities of 0.7. What is the individual minimum number of components that must be connected in parallel?

Sol. Given $R_P = 0.999$

$Q_P = 1 - R_P = 0.001$

We know that when 'n' components connected in parallel, the overall unreliability is

$$Q_P = Q_i^N$$

$$Q_i = 1 - 0.7 = 0.3$$

$$0.001 = (0.3)^N$$

$\ln 0.001 = N \ln 0.3$

$-6.9077 = -1.2N$

$N =$

$$N = 5.74 \simeq 6$$

5. A system consists of 4 components in parallel. If system success requires that atleast 3 out of 4 must function ① what is the system reliability if the reliability component is 0.9.

② what is the system reliability if 5 components are placed in parallel to perform the same function?

Sol $R_i = 0.9 \quad \forall \; i = 1$ to 4

$Q_i = 1 - 0.9 = 0.1 \quad \forall$ for $i = 1$ to 4

4 Components Present :

$$R_P = R^4 + 4R^3Q$$

$$R_P = R_A R_B R_C R_D + R_A R_B R_C Q_D + R_A R_B Q_C R_D + R_A Q_B R_C R_D + Q_A R_B R_C R_D$$

$$R_P = (0.9)^4 + 4(0.9)^3 \times 0.1$$

$$= 0.9477$$

5 Components Present :

$$R_P = {}^5C_3 R^3 Q^2 + {}^5C_4 R^4 Q + {}^5C_5 R^5$$

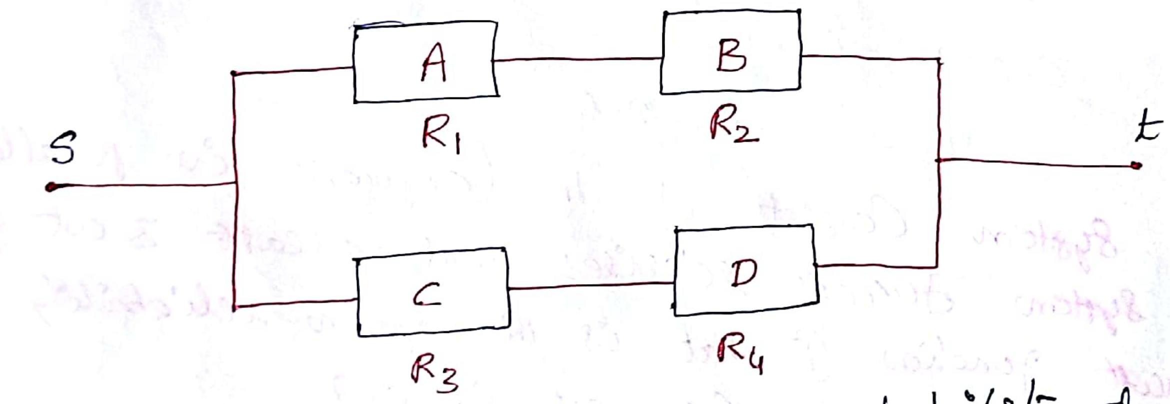
$$= {}^5C_3 (0.9)^3 (0.1)^2 + {}^5C_4 (0.9)^4 (0.1) + {}^5C_5 (0.9)^5$$

10, 5, 1

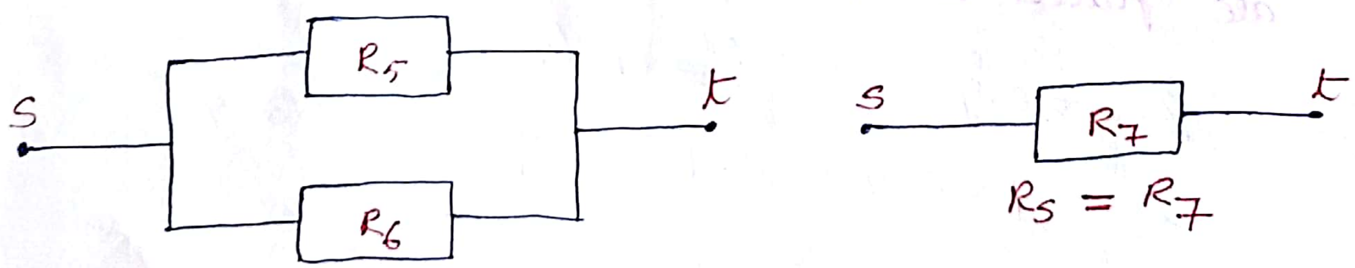
$$= 0.99144.$$

Series - Parallel Networks :-

1.



Let R_1, R_2, R_3 and R_4 be the probability of success of Components A, B, C, D respectively.



$R_5 = R_7$

$R_5 = R_1 R_2$

$R_6 = R_3 R_4$

$Q_7 = Q_5 \cdot Q_6$

where $Q_5 = 1 - R_5 = 1 - R_1 R_2$

$Q_6 = 1 - R_6 = 1 - R_3 R_4$

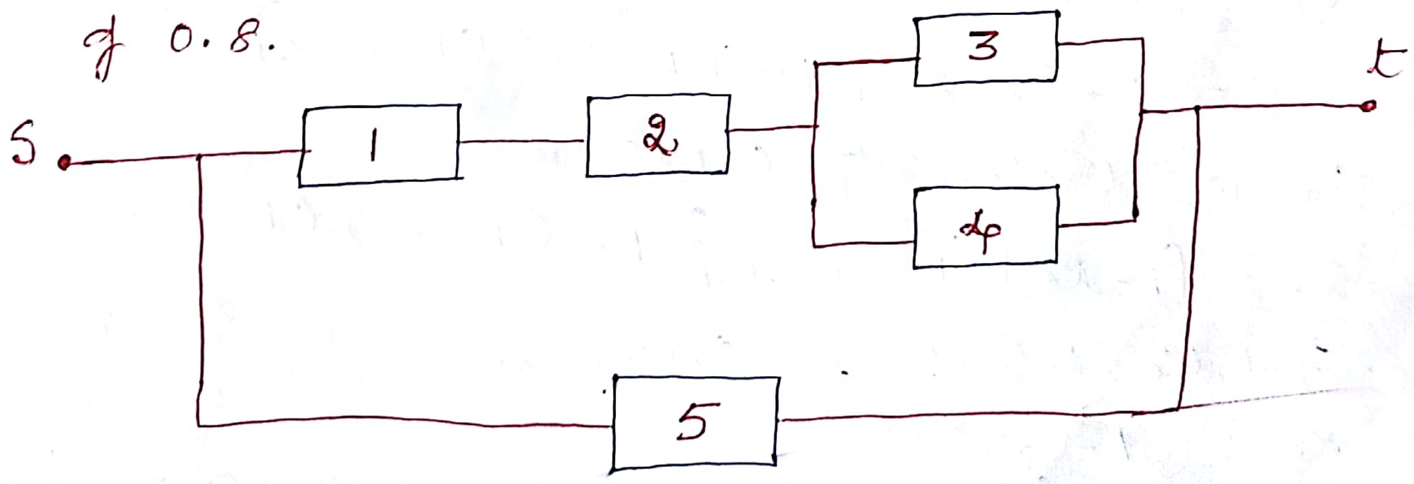
$$R_7 = 1 - Q_7 = 1 - Q_5 Q_6$$
$$= 1 - [1 - R_5][1 - R_6]$$
$$= 1 - [1 - R_1 R_2][1 - R_3 R_4]$$

This is known as symbolic reliability expression.

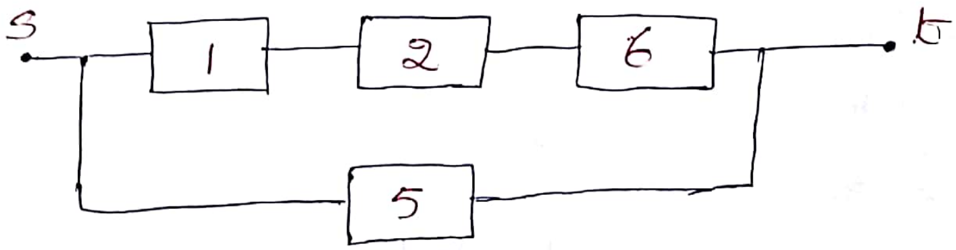
if $R_1 = R_2 = R_3 = R_4 = R$ and given say 0.9

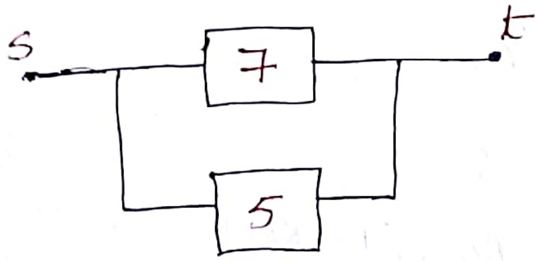
$$R_7 = 1 - (1 - R^2)(1 - R^2)$$
$$= 1 - \{1 + R^4 - 2R^2\}$$
$$R_7 = 2R^2 - R^4$$
$$= 2(0.9)^2 - (0.9)^4$$
$$R_7 = 0.9639$$

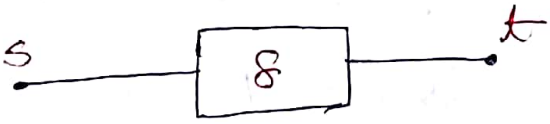
2. Derive a general expression for reliability and unreliability of the following system and hence evaluate if each Component have reliability of 0.8.



Sol







$R_s = R_8$

$Q_8 = Q_7 \cdot Q_5$

where $Q_7 = 1 - R_7 = 1 - R_1 R_2 R_6$

$Q_5 = 1 - R_5 = 1 - R_5$

$Q_6 = Q_3 Q_4$

$R_6 = 1 - Q_6 = 1 - Q_3 Q_4$

$R_7 = R_1 R_2 R_6$

$= R_1 R_2 [1 - Q_3 Q_4]$

$R_1 = R_2 = R_3 = R_4 = R_5 = R_6$ are the probability of success.

$Q_1 = Q_2 = Q_3 = Q_4 = Q_5 = Q_6$ are the probability of failure.

$Q_7 = 1 - R_1 R_2 [1 - Q_3 Q_4]$

$= 1 - R_1 R_2 [1 - (1 - R_3)(1 - R_4)]$

Symbolic Unreliability Expression

$Q_8 = [1 - R_5][1 - R_1 R_2] \{1 - (1 - R_3)(1 - R_4)\}$

Symbolic Reliability Expression

$R_8 = 1 - Q_8$

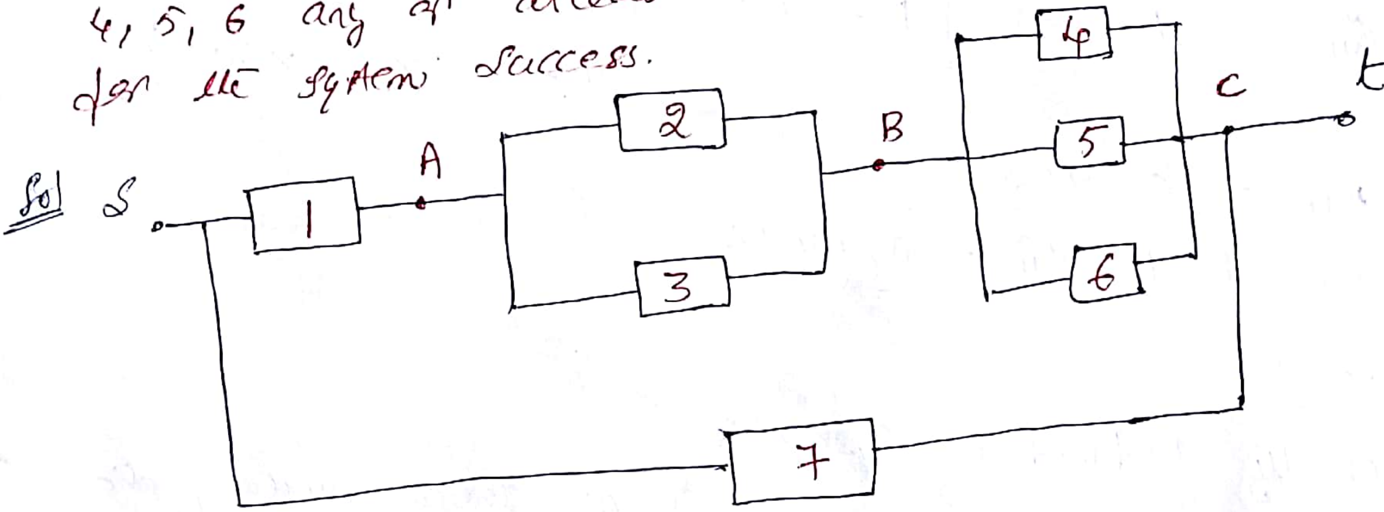
$= 1 - \{[1 - R_5][1 - R_1 R_2][1 - (1 - R_3)(1 - R_4)]\}$

$$R_S = 1 - \{(1-0.8)(1-0.8\times0.8) - [1-(1-0.8)(1-0.8)]\}$$

$$R_S = 0.92288$$

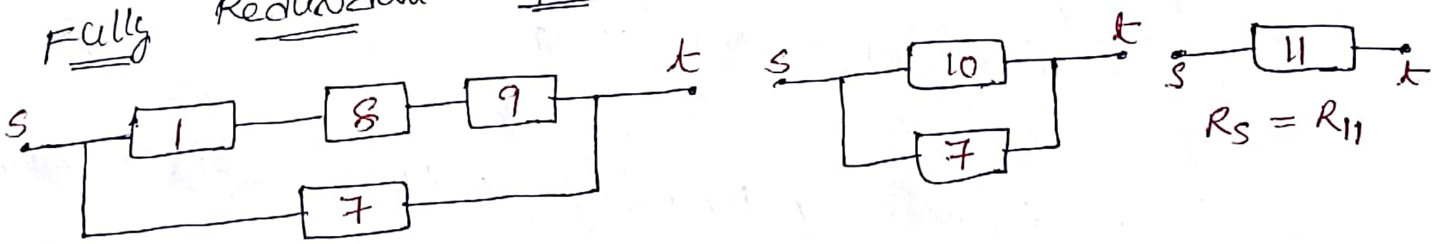
3. Derive a general expression for the unreliability and reliability of the system whose reliability logic diagram (RLD) is shown in fig. Consider the case in all the parallel branches of this system are fully redundant except that out of the components 4, 5, 6 any of atleast two components are required for the system success.

Sol



Case 1:

Fully Redundant System:



$R_S = R_{11}$

$Q_{11} = Q_{10} \cdot Q_7$

$Q_8 = Q_2 \cdot Q_3$

$R_8 = 1 - Q_2 Q_3$

$Q_9 = Q_4 Q_5 Q_6$

$R_9 = 1 - Q_4 Q_5 Q_6 = 1 - Q_9$

$R_{10} = R_1 R_8 R_9 = R_1 [1 - Q_2 Q_3][1 - Q_4 Q_5 Q_6]$

$Q_{10} = 1 - R_{10}$

$Q_{11} = 1 - \{ R_1 (1 - Q_2 Q_3)(1 - Q_4 Q_5 Q_6) \} Q_7$

$R_{11} = 1 - Q_{11}$

$= 1 - Q_7 \left[1 - \{ R_1 (1 - Q_2 Q_3)(1 - Q_4 Q_5 Q_6) \} \right]$

If $R = R_i = 0.8 \ \forall \ i = 1$ to 7

$Q_i = 0.2 \ \forall \ i = 1$ to 7

$R_{11} = 1 - 0.2 \left[1 - \{ 0.8 (1 - 0.2 \times 0.2) (1 - 0.2 \times 0.2 \times 0.2) \} \right]$

$R_{11} = 0.9523$

$Q_{11} = 1 - R_{11} = \underline{0.0476}$

Case : 2

Partially Redundant System :-

out of 4, 5, 6 atleast two of the three must function

$R_9 = R^3 + 3R^2 Q$ using principle of Bernoulli's Trials

$= (0.8)^3 + 3(0.8)^2 \times 0.2$

$= 0.896$

$= R_4 R_5 R_6 + R_4 R_5 Q_6 + R_4 Q_5 R_6 + R_5 R_6 Q_4$

$R_{10} = R_1 R_8 [R_4 R_5 R_6 + R_4 R_5 Q_6 + R_4 Q_5 R_6 + Q_4 R_5 R_6]$

$Q_{11} = Q_7 \cdot Q_{10}$

$= Q_7 [1 - R_{10}]$

$= Q_7 [1 - \{ R_1 R_8 [R_4 R_5 R_6 + R_4 R_5 Q_6 + R_4 Q_5 R_6 + R_5 R_6 Q_4] \}$

$$R_{11} = 1 - Q_{11}$$

$$= 1 - Q_7 \{ 1 - R_1(1 - Q_2 Q_3) [R_4 R_5 R_6 + R_4 R_5 Q_6 + R_5 Q_6 R_6 + Q_4 R_5 R_6] \}$$

If each $R_i = 0.8 \ \forall \ i = 1$ to 7

$Q_i = 0.2 \ \forall \ i = 1$ to 7

$R_{11} = 0.93763$

$Q_{11} = 1 - R_{11} = 0.06237$

Conditional Probability Approach :-

[For non-series - parallel configuration]

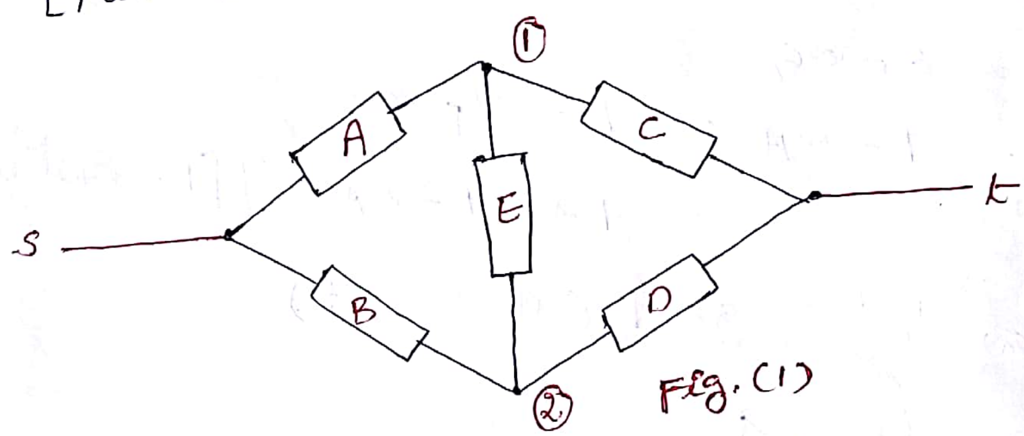


Fig. (1)

Using Conditional probability approach

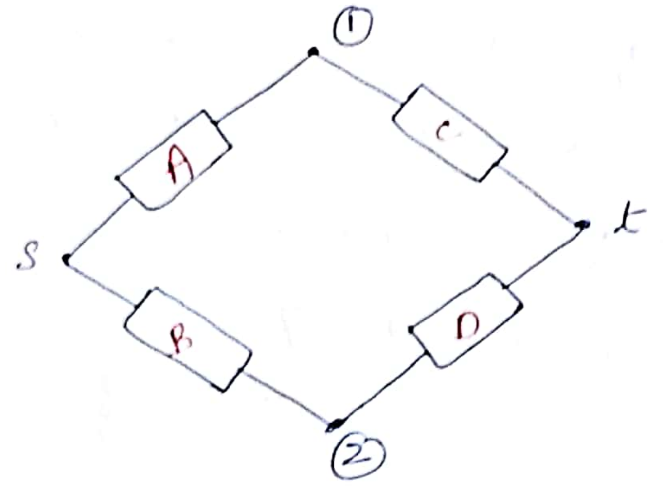
R_s = P [System Success] = P [System Success / X is good] P[X is good] + P [System success / X is bad]. P[X is bad) $\rightarrow$ (1)

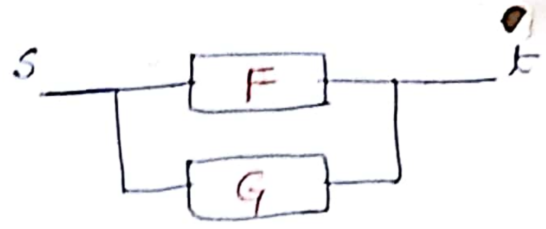
Q_s = P [System Failure] = P [System Failure / X is good] P [X is good] + P [System Failure / X is bad] P [X is bad] $\rightarrow$ (2)

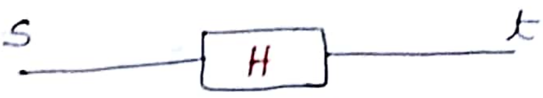
Evaluation of Network Reliability using eq (1).

Consider 'E' to be taken for Conditional Probability

1. Where 'E' is bad (open ckt)







$R_{SEB} = R_H$

$R_F = R_A R_C$

$R_G = R_B R_D$

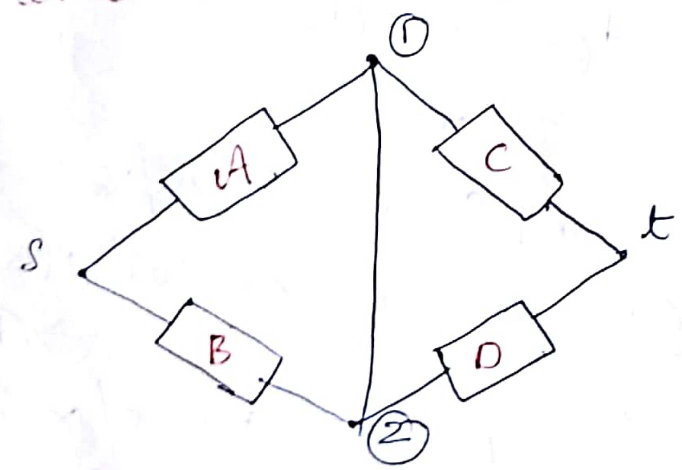
$Q_F = 1 - R_F = 1 - R_A R_C$

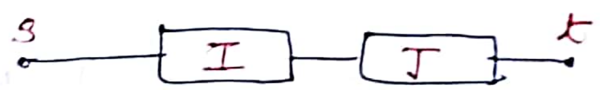
$Q_H = Q_F Q_G$

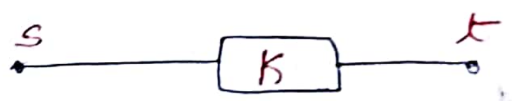
$R_H = 1 - Q_H = 1 - Q_F Q_G$

$= 1 - [1 - R_A R_C][1 - R_B R_D]$

2. when 'E' is good (short-ckt)







$R_{SEG} = R_K$

$R_K = 1 - Q_K$

$R_K = R_I R_J$

$Q_I = Q_A Q_B$

$Q_J = Q_C Q_D$

$R_K = (1 - Q_A Q_B)(1 - Q_C Q_D)$

Using eq (1), we can write as

$$R_S = R_{SEG} \cdot R_E + R_{SEB} \cdot Q_E$$

$$R_S = [(1 - Q_A Q_B)(1 - Q_C Q_D)] R_E + [1 - (1 - R_A R_C)(1 - R_B R_D)] Q_E$$

If $R_i = 0.9$ $\forall$ for all $i = A$ to E

$Q_i = 0.1$ $\forall$ for all $i = A$ to E

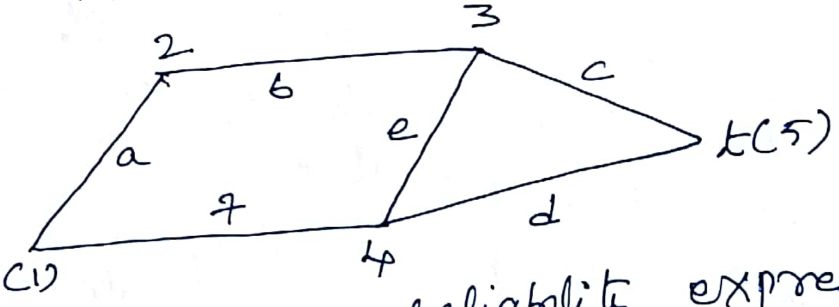
$$R_S = \{[1 - (0.1)(0.1)][1 - (0.1)(0.1)]\} 0.9 + [1 - (1 - 0.9 \times 0.9)(1 - 0.9 \times 0.9)] 0.1$$

$$R_S = 0.97848$$

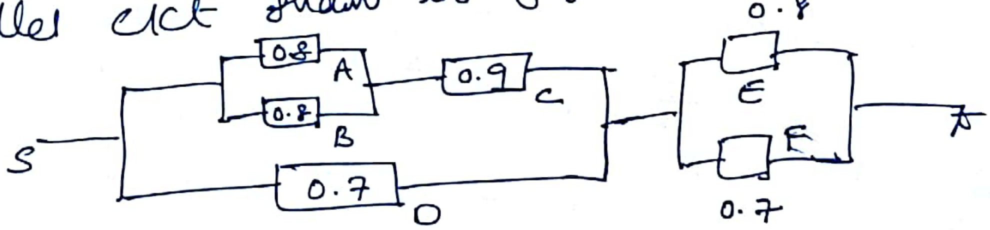
$$Q_S = 1 - R_S = 0.02152$$

Assignment Problems :-

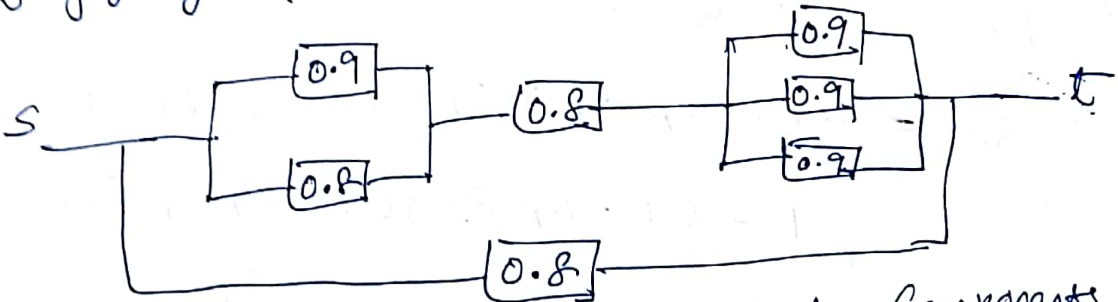
1. Develop the symbolic reliability expression for the following N/w using decomposition method. Hence, if each component has a prob of failure of 0.12, evaluate the reliability index of the system.



2. Develop the symbolic reliability expression and calculate the reliability of the system for a series-parallel ckt shown in fig. by N/w Reduction Technique.

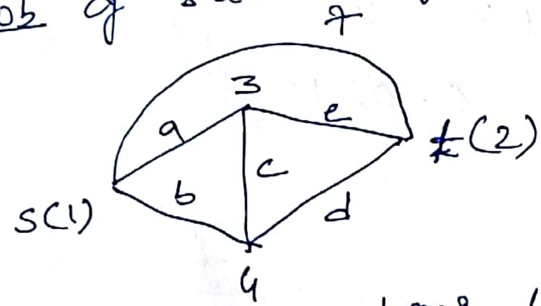


3. Calculate the reliability of the system shown in fig by N/w reduction technique

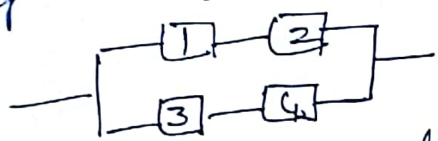


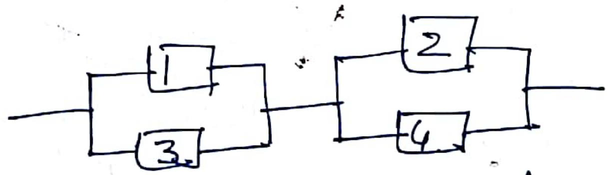
4. A series system has identical components each having a reliability of 0.998. What is the max number of components that can be allowed if the minimum system reliability is to be 0.9.

5. Evaluate reliability of the system shown in fig. using conditional prob approach, if each component has a prob of success of 0.8.



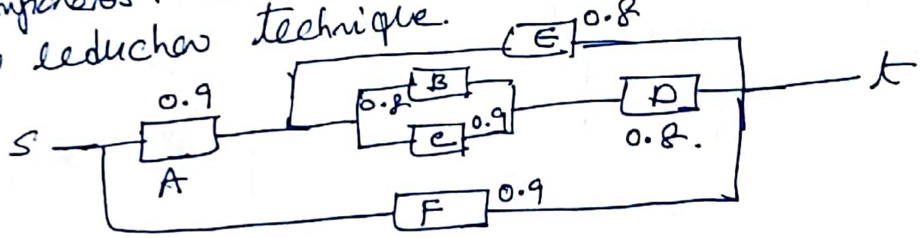
6. The following configurations represent a RLD's of a system.





Develop the symbolic reliability expressions for each & obtain the reliability of these configurations if each has a prob of failure of 0.15.

7. The reliability N/w of the system is shown in fig. The figures marked indicate the reliabilities of the components. Calculate the reliability of the system by N/w reduction technique.



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