

# High temperature levels for the 21st century.

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- [Ipcc, 2021]'s assessment report released: **1.1°C Global warming level in 2015** (Stronger over land at **1.59 °C** ).
- Due to **human activities** (greenhouse gas emissions, land-use change, etc.)
- Increasing knowledge of the warming phenomena, using both **observations and climate models**.
- Scenarios used in climate models to **simulate future conditions**.
- Combine **socioeconomic narratives** and **radiative forcing targets** for 2100 (1.9 to 8.5 W/m<sup>2</sup>)[Ipcc, 2021]

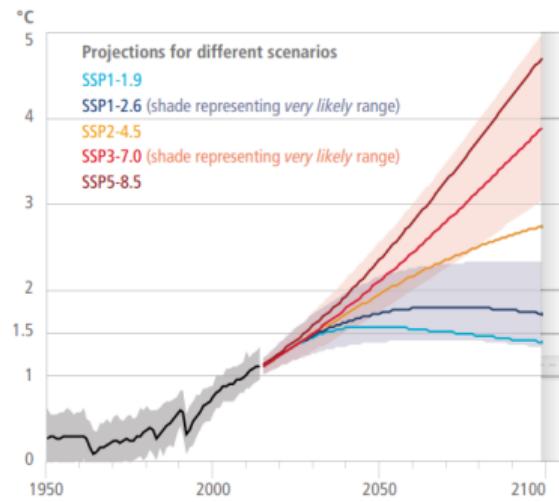


Figure: Global surface temperature changes for various emission scenarios.[Ipcc, 2021]

### Local observations

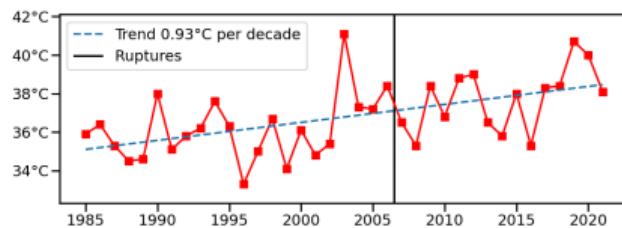


Figure: Annual maxima in Pierrelatte

- **On site** measurement.
- Good quality with few breaks.
- No knowledge on **future evolution**.

### Climate model simulations

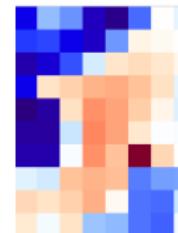
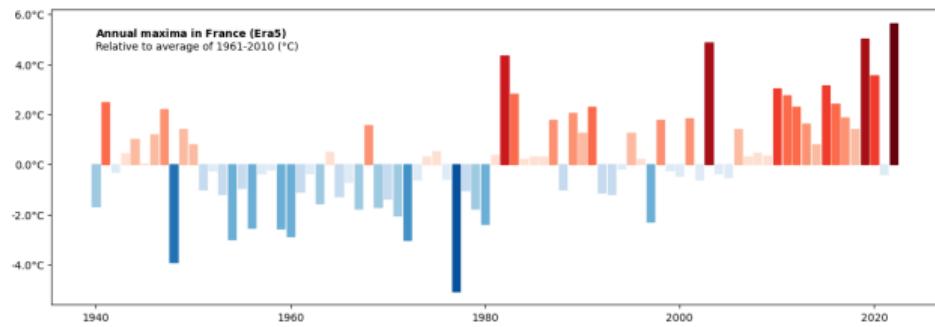


Figure: UKESM1-o-LL - France

- Various models from CMIP6.
- **Historical and scenario runs.**
- **Large grids** from 70 to 300 km.

⇒ How to combine observations and climate model to describe future temperatures?

- Extremes have a **disproportionate impact** on health, ecosystems and infrastructures.
- Increase in **frequency and intensity** of extremely hot events [Ipcc, 2021]
- In France, new records have been attained recently, for eg. in **intensity** (2019).



**Figure:** Anomaly of the annual maxima of temperature over France between 1940 and 2023.

## Safety concerns :

- Reliability of **safety-significant equipment**.
- Building codes using stationary return levels which may **vary during the building's life**.
- Danger for **human's health** during heatwaves.

⇒ How to summarize the non-stationary distribution of extremes into a meaningful and decision-relevant risk index?

⇒ What temperature levels for various spatial scales by the end of the century, and how do the data influence the projections?



Figure: Map of french nuclear sites.

**Question:** How can we define design levels for extreme temperature by the end of the century in a changing climate?

How ?

- Adapting the stationary return level to a **non-stationary context**, considering the lifetime of the building.
- Estimating extreme temperature levels, integrating **information from climate models and local observations**, using tools based on Extreme Value Theory and a Bayesian framework.
- Providing a usable estimate taking **uncertainty** into account.
- Adapting the method to **various places of interest**, taking into account the data limitations of each zone.





## Issue with return levels - Non-stationarity

In a stationary context,  $z_p = F^{-1}(1 - p)$ .

**Issue:** with non-stationarity,  $Z_{2023} \neq Z_{2050}$ :

- Annual probability of excess **changes every year.**
- Unique return level over a period is undefined:

$$z_p(t) = F_t^{-1}(1 - p)$$

⇒ Use **another risk index**, but keep continuity with existing methodology

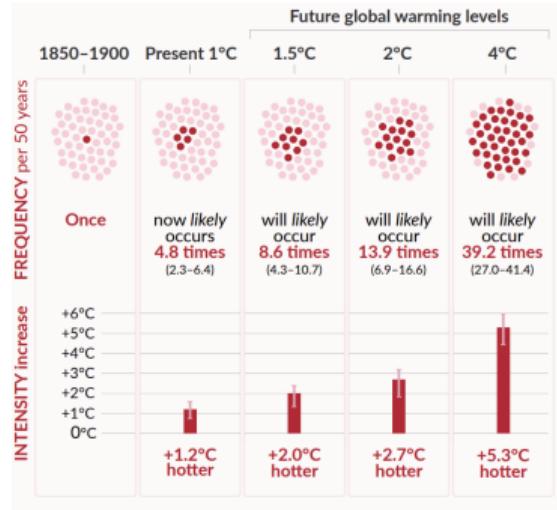


Figure: IPCC, 2021: Summary for Policymakers by Masson Delmotte, V et al.[Ipcc, 2021]

**Issue:** The **risk of excess over the period** is higher than the annual risk.

Stationary: Using  $z$ , return level of period  $T = 100$  years.

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Stationary: Using  $\mathbf{z}$ , return level of period  $T = 100$  years.

For year 1:  $P[Z_1 > \mathbf{z}] = 1 - (1 - \frac{1}{T})$

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Stationary: Using  $\mathbf{z}$ , return level of period  $T = 100$  years.

For year 1:  $P[Z_1 > \mathbf{z}] = 1 - (1 - \frac{1}{T}) = 0.01$

**Issue:** The risk of excess over the period is higher than the annual risk.

Stationary: Using  $\mathbf{z}$ , return level of period  $T = 100$  years.

$$\text{For year 1: } P[Z_1 > \mathbf{z}] = 1 - (1 - \frac{1}{T}) = 0.01$$

For year 2 :

**Issue:** The risk of excess over the period is higher than the annual risk.

Stationary: Using  $\mathbf{z}$ , return level of period  $T = 100$  years.

$$\text{For year 1 : } P[Z_1 > \mathbf{z}] = 1 - (1 - \frac{1}{T}) = 0.01$$

$$\text{For year 2 : } P[\max(Z_1, Z_2) > \mathbf{z}] = 1 - (1 - \frac{1}{T})^2$$

**Issue:** The risk of excess over the period is higher than the annual risk.

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For year 50 :

**Issue:** The risk of excess over the period is higher than the annual risk.

Stationary: Using  $\mathbf{z}$ , return level of period  $T = 100$  years.

$$\text{For year 1 : } P[Z_1 > \mathbf{z}] = 1 - (1 - \frac{1}{T}) = 0.01$$

$$\text{For year 2 : } P[\max(Z_1, Z_2) > \mathbf{z}] = 1 - (1 - \frac{1}{T})^2 = 0.02$$

$$\text{For year 50 : } P[\max(Z_1, \dots, Z_{50}) > \mathbf{z}] = 1 - (1 - \frac{1}{T})^{50}$$

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$$\text{For year 50 : } P[\max(Z_1, \dots, Z_{50}) > \mathbf{z}] = 1 - (1 - \frac{1}{T})^{50} = 0.400$$

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For period  $T_1 - T_2$ :

**Issue:** The **risk of excess over the period** is higher than the annual risk.

Stationary: Using  $\mathbf{z}$ , return level of period  $T = 100$  years.

$$\text{For year 1: } P[Z_1 > \mathbf{z}] = 1 - (1 - \frac{1}{T}) = 0.01$$

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$$\text{For period } T_1 - T_2: P[\max(Z_{T_1}, \dots, Z_{T_2}) > \mathbf{z}] = 1 - (1 - \frac{1}{T})^{T_2 - T_1 + 1}$$

⇒ **Reliability concept.**

⇒ Define separately the **period of interest** from the **return period** (annual probability).

**Various alternatives:** Expected Waiting Time,  
Average Design Life Level, Expected Number of  
Events, Design Life levels  
[Rootzén and Katz, 2013]

**Equivalent Reliability [Liang et al., 2016]:**  
For period  $t_1 : t_2$  and annual probability  $p$ ,  $\mathbf{z}_p$  is  
solution of :

$$R_{t_1:t_2}(z_p^{\text{ER}}) = (1 - p)^{t_2 - t_1 + 1}$$

**Advantages:**

- Assess risk over the **full period of interest**  $t_1 : t_2$  using reliability.
- Separate the **period of interest** from the **return period** (annual probability  $p = \frac{1}{T}$ ).
- Applied similarly **with or without stationarity**.



Constraints:

- **Extreme values** analysis: Annual maxima, use of GEV distribution.

$$Z_t \sim GEV(\mu(t), \sigma(t), \xi)$$

$$\begin{cases} \mu(t) = \mu_0 + \mu_1 X_t \\ \sigma(t) = \exp(\sigma_0 + \sigma_1 X_t) \\ \xi(t) = \xi_0 \end{cases}$$

- **Non-stationarity:** Mean European temperature as covariate for relationship with time and **scenario integration**.

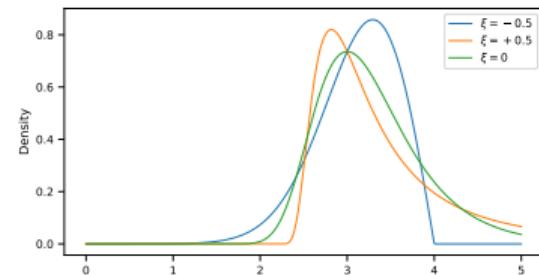


Figure: GEV probability density function

A nuclear site :

- In the Rhône Valley (Topography)
- Active since 1980
- Elevation: 54m (Google Earth)

Available data :

- **Local meteorological measurements** (37 years, expert assessment).
- Time series for **26 global climate models**.

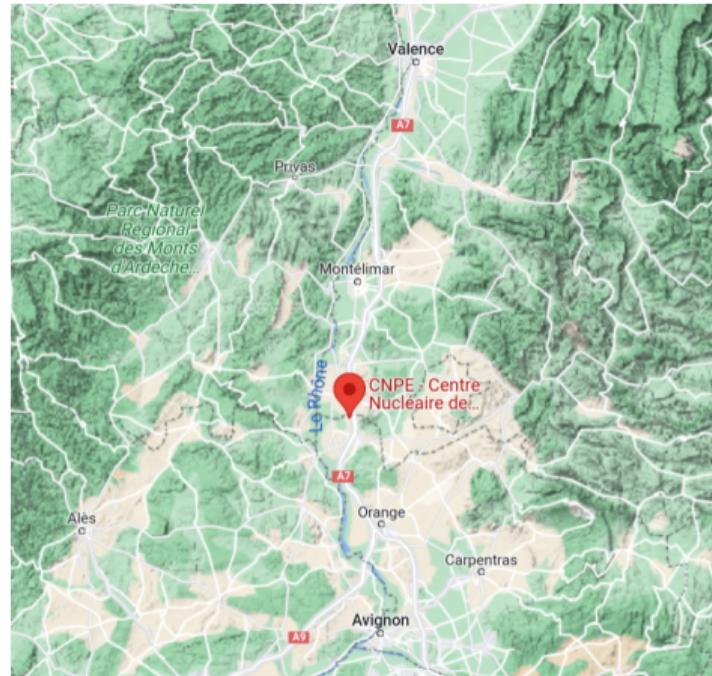
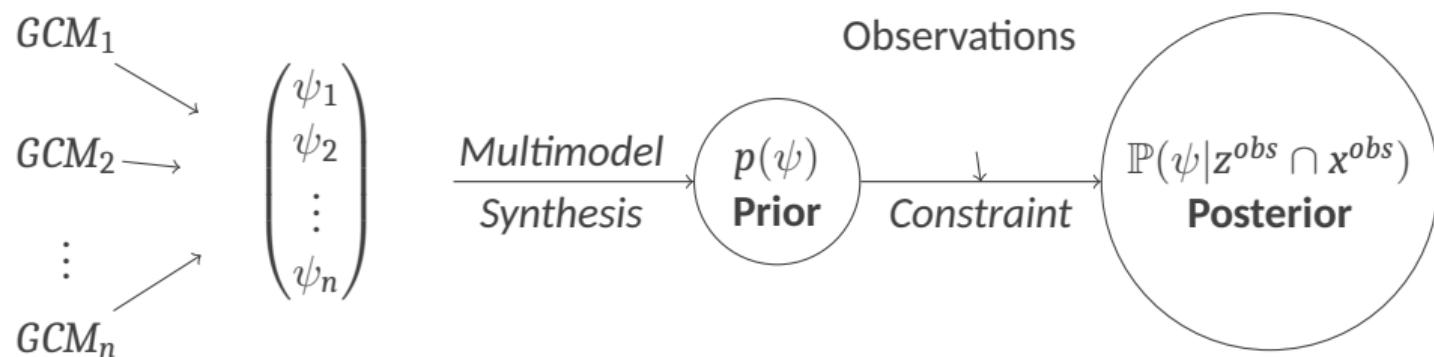


Figure: Situation of a nuclear powerplant

Usually, either observations or climate model simulations are used to produce projections.

[Robin and Ribes, 2020]'s framework **combines both**.

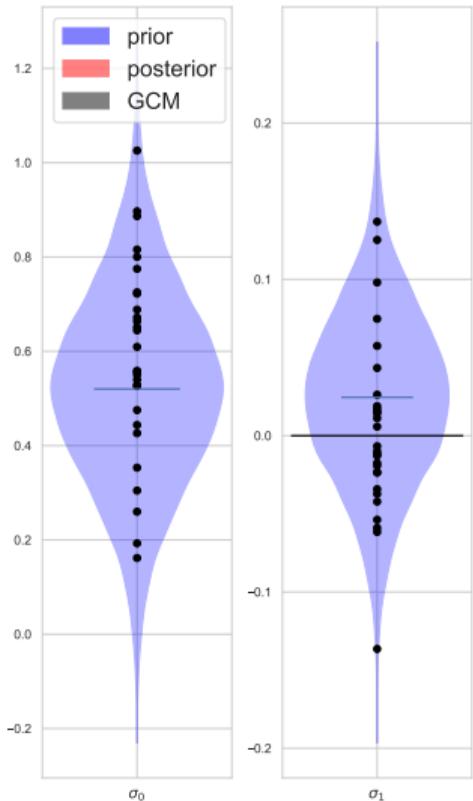
Using  $\psi = \{X_{1850} - X_{2100}, \mu_0, \mu_1, \sigma_0, \sigma_1, \xi\}$ ,  
 $X_t$  included because **the warming level is uncertain**.



## Bayesian framework [Robin and Ribes, 2020]

### A-priori knowledge

- Maximum likelihood GEV fit for each GCM.
- Multi-gaussian prior includes only information from **climate models**. (historical and scenario).



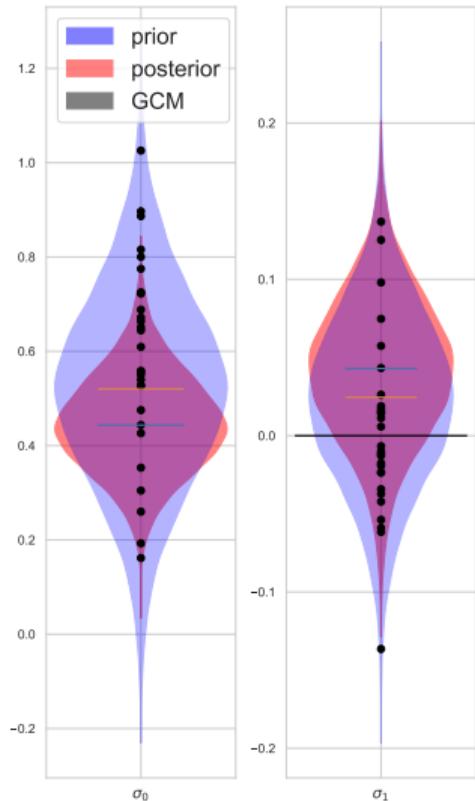
## Bayesian framework [Robin and Ribes, 2020]

### A-priori knowledge

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### Updated using observations

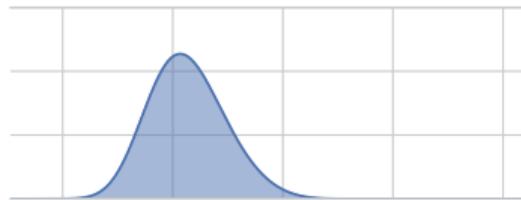
- Covariate constraint using a **conjugate**.
- Maxima constraint using **Markov chain Monte Carlo** (NUTS) with past local observations.



- Cannot directly express the posterior distribution, so **sample using MCMC**.
- Original implementation of Metropolis-Hastings had **issues of chains quality**.
- Alternative relying on **Stan's implementation** of the No-U-Turn Sampler (NUTS) [Homan and Gelman, 2014].
- Allow for a **large improvement** of speed, number and quality of samples.
- Added to the next iteration of Y. Robin's framework (Currently being reviewed in Geoscientific Model Development).
- **Allow a larger range of applications**, such as S. Qasmi 's real-time event attribution (accepted in Bulletin of the American Meteorological Society) .



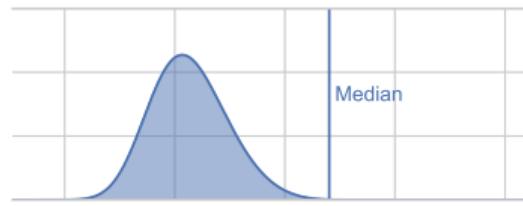
## Predictive distribution - Illustration



- Using all draws: for return levels  
**median**



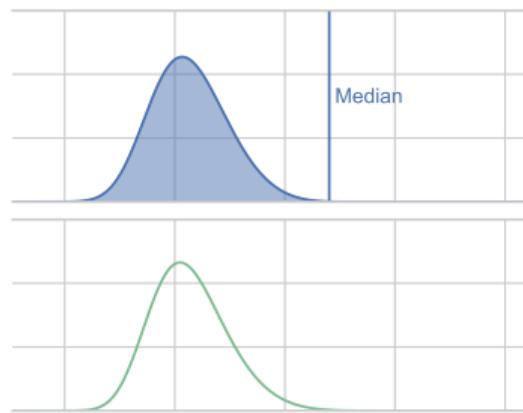
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Median distribution      Predictive distribution

## Predictive distribution - Illustration

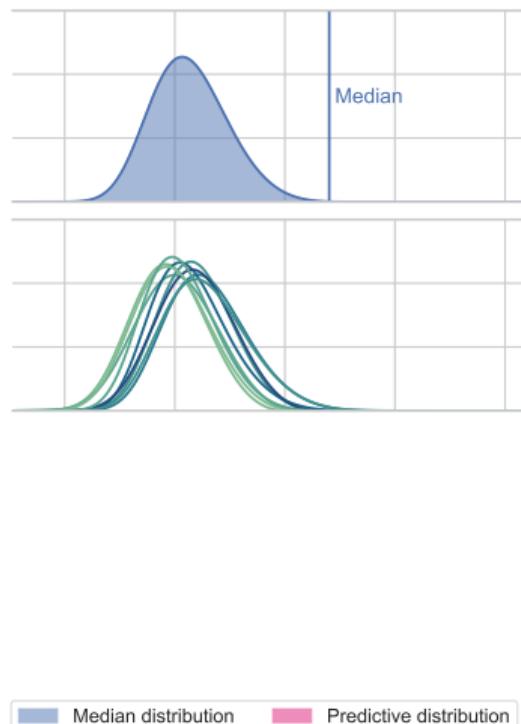


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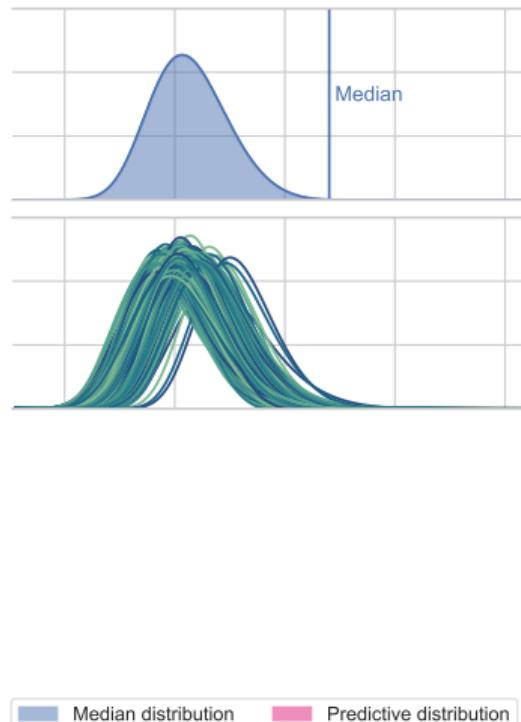
Median distribution

Predictive distribution

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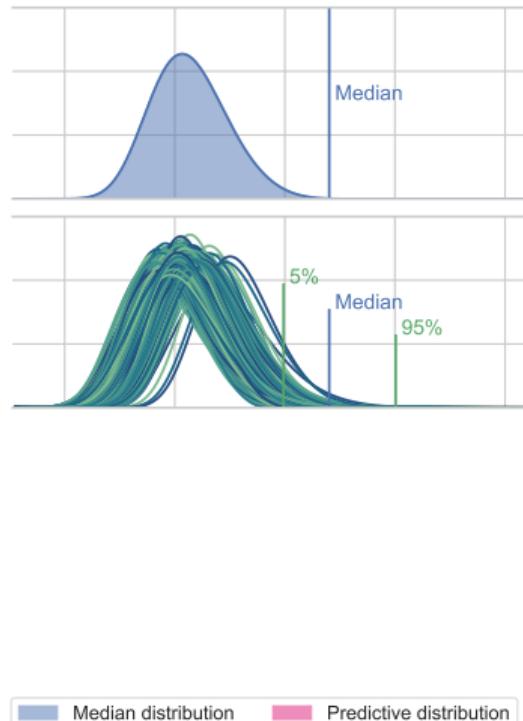


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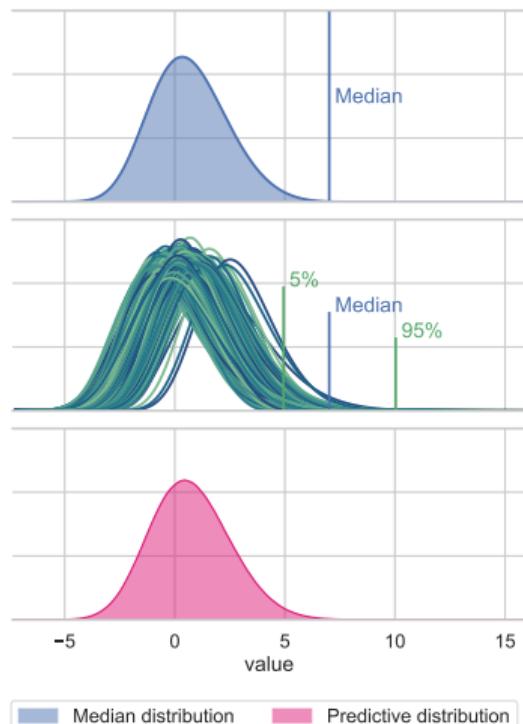


- Using all draws: for return levels  
**median and credibility intervals**

## Predictive distribution - Illustration

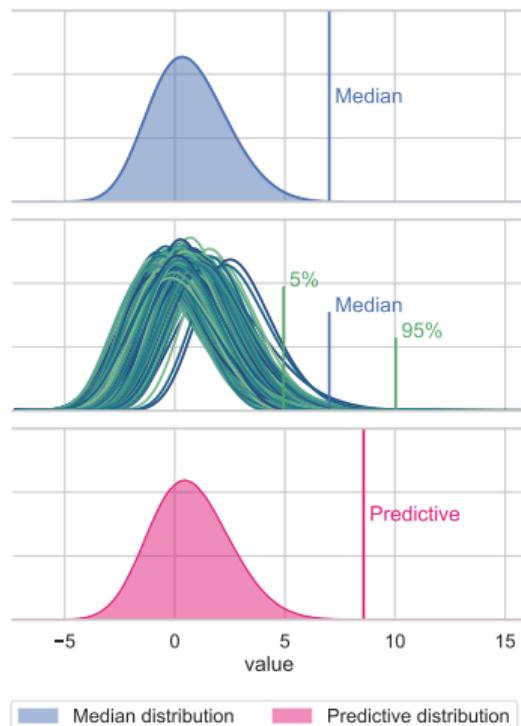


- Using all draws: for return levels **median** and **credibility intervals**
- **Issue :** Confidence level is **another parameter** to choose.



- Using all draws: for return levels **median** and **credibility intervals**
- Issue :** Confidence level is **another parameter** to choose.
- Predictive distribution**  
**[Coles and Tawn, 2005]:** One distribution **averaged** over the distribution of the model parameters.

$$P(Z \leq z | z_{obs}) = \int_{\Psi} P(Z \leq z | \psi) \pi(\psi | z_{obs}) d\psi$$



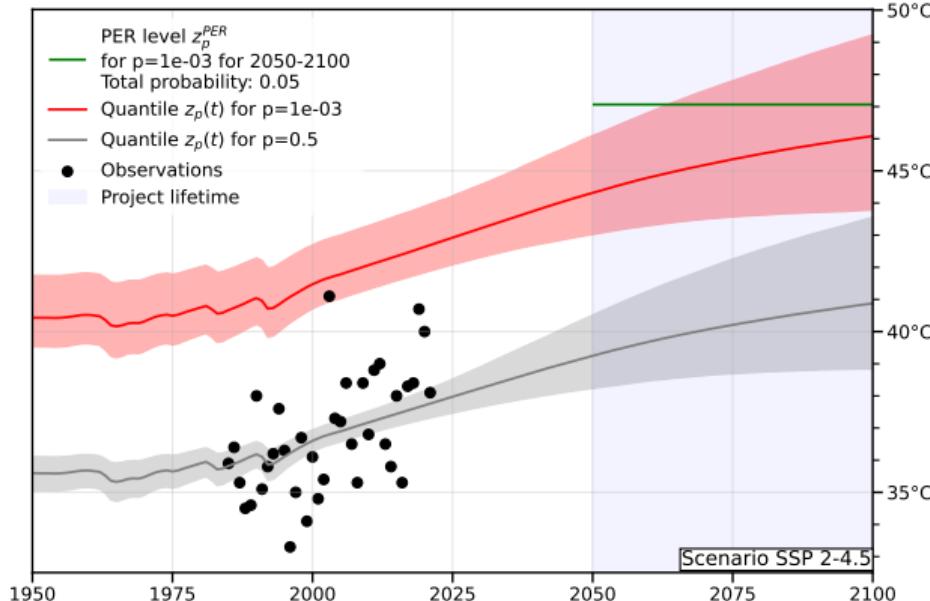
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- One distribution **blending all draws**: Account for **estimation error** and **stochastic error**.



## Results



**Figure:** Return levels and PER levels in Tricastin for emission scenario SSP2-4.5.

Period of interest: **2050-2100**.  
Annual probability of excess:  $\frac{1}{1000}$

### Interpretation

47°C has a **5% probability of excess over 2050-2100**.

## Applications to various scenarios

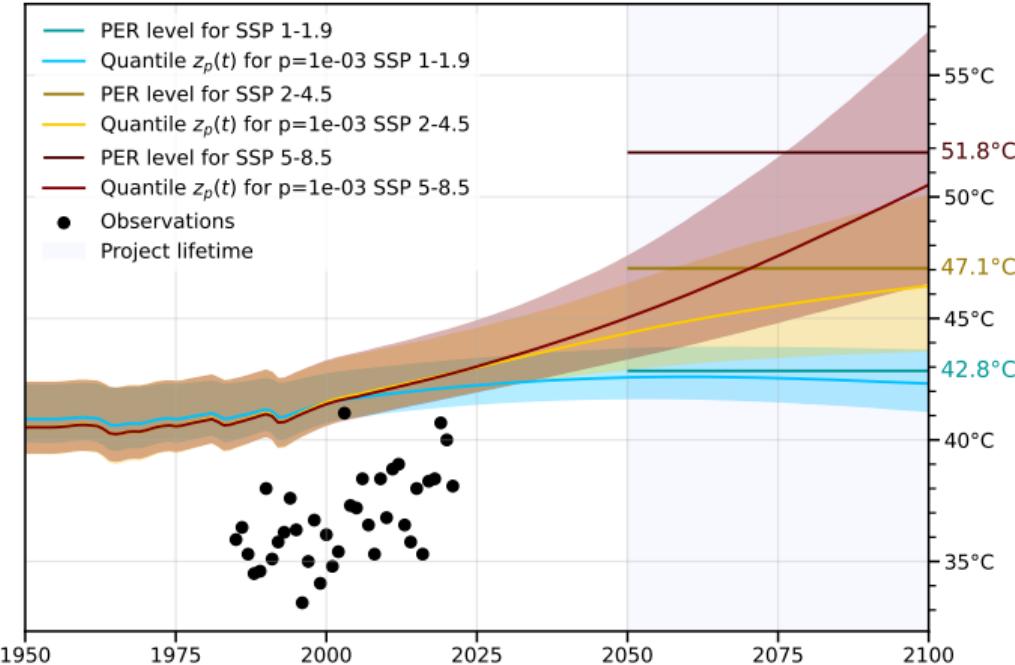


Figure: Return levels and PER levels in Tricastin under three emission scenarios.



## Integrating non-stationarity and uncertainty in design life levels based on climatological time series

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### ARTICLE INFO

Dataset link: <https://github.com/Occitane-Barbaux/Integrating-non-stationarity-and-uncertainty-in-design-life-levels>

**Keywords:**

Uncertainty  
Non-stationarity  
Risk  
Climate change

### ABSTRACT

This work focuses on inferring design life levels for extreme events under non-stationary conditions. Its objectives are twofold. The first one is to provide a single indicator that summarizes relevant and interpretable information about large values in time series, even when stationarity cannot be assumed. Classical risk indicators such as the 100-year return level become difficult to interpret in a non-stationary framework. To address this, we leverage the existing concept of the equivalent reliability (ER) level. Under stationarity, the ER level coincides with the classical return level, but it differs otherwise. More precisely, the ER level ensures that the probability of having all observations below the ER level during a specified design period is controlled. This definition ensures interpretability in terms of safety or failure risk. A second objective is to capture stochastic and estimation uncertainty, a key aspect in any risk analysis, as uncertainties due to inference schemes can grow with extreme intensities. We incorporate both by using the Bayesian predictive distribution. Although well known in Bayesian statistics, the predictive distribution has rarely been applied to climatological time series risk analysis.

Our approach is demonstrated on simulated data and on a case study of annual maxima of temperatures at a site in Southern France. To do so, a non-stationary Bayesian hierarchical extreme value model is used to combine data from 26 CMIP6 general circulation model simulations (SSP2-4.5, 1850-2100) with observations. The resulting predictive ER levels clearly indicate that non-stationarity over a design period of interest, as well as sampling and estimation uncertainty, have to be taken into account for risk assessment. For example, the 1000-year posterior predictive ER level for 2050-2100 is higher than any non-stationary 1000-year return level median estimate over the same period, reflecting the increasing risk due to the non-stationarity of the SSP 2-4.5 pathway.

## Plan

### ■ Methodology

- ▶ Risk index
- ▶ Describing extreme temperatures
- ▶ Summarizing uncertainty
- ▶ Combination

### ■ Applications

- ▶ Observation selection
- ▶ Issues with the prior

### ■ Conclusion

### ■ References

### ■ Supplementary



### What extreme temperature levels worldwide?

⇒ Need for observations of global maxima :

- World record of **56.7°C** in Furnace Creek (USA) in 1913.
- But observed 55 °C in Tunisia , 54 °C in Kuwait.

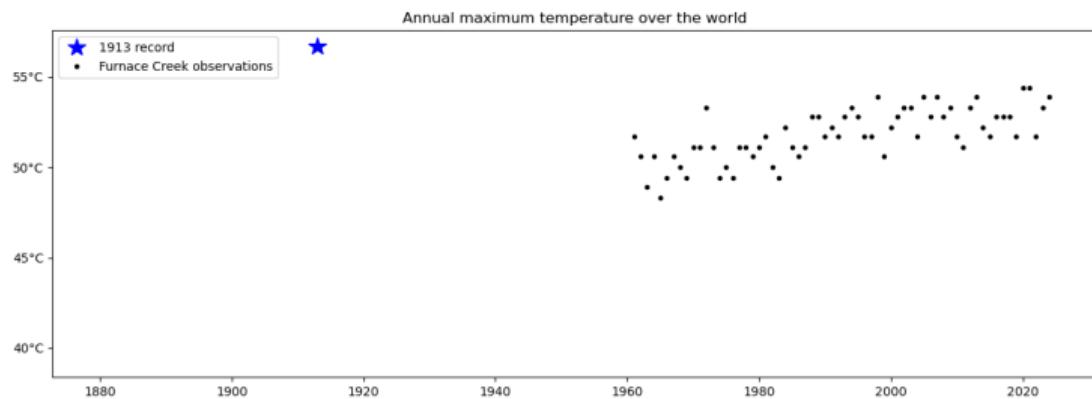


Figure: Time series of annual maxima in Furnace Creek.

## What extreme temperature levels worldwide?

⇒ Need for observations of **global maxima** :

- Raw observational datasets.

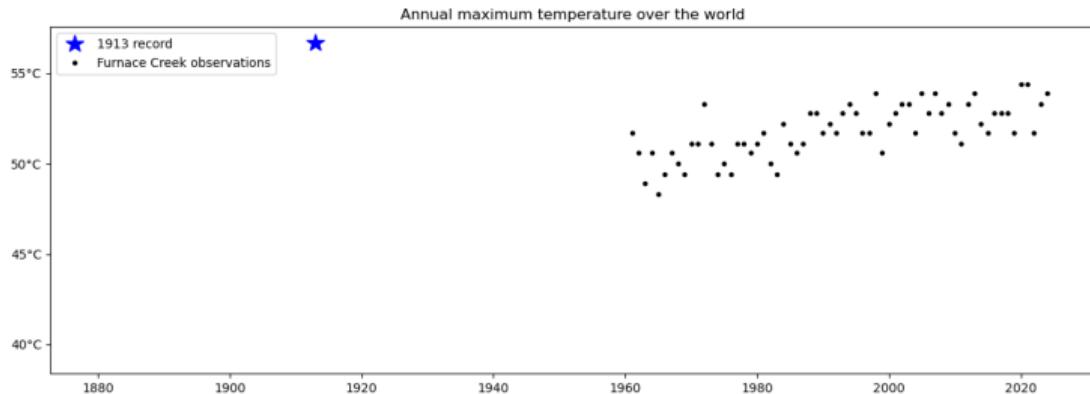


Figure: Time series of global annual maxima for three datasets.

## What extreme temperature levels worldwide?

⇒ Need for observations of **global maxima** :

- Raw observational datasets.
- Gridded observational datasets.

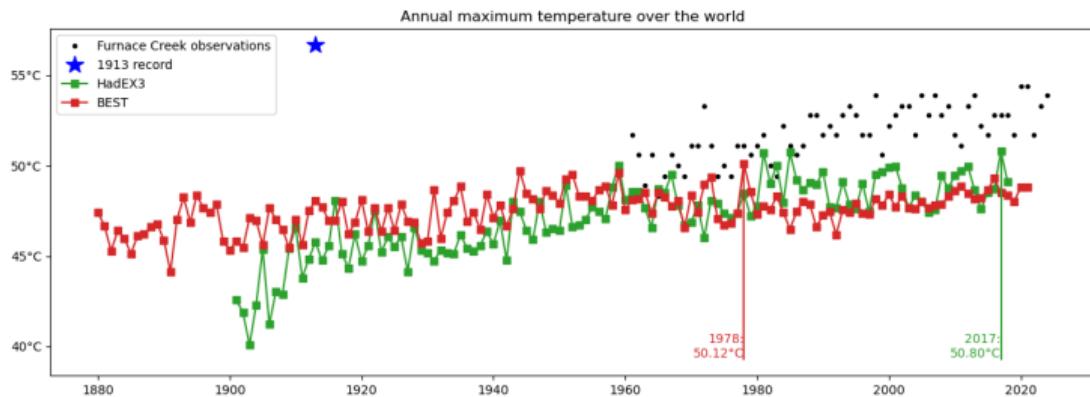


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## What extreme temperature levels worldwide?

⇒ Need for observations of **global maxima** :

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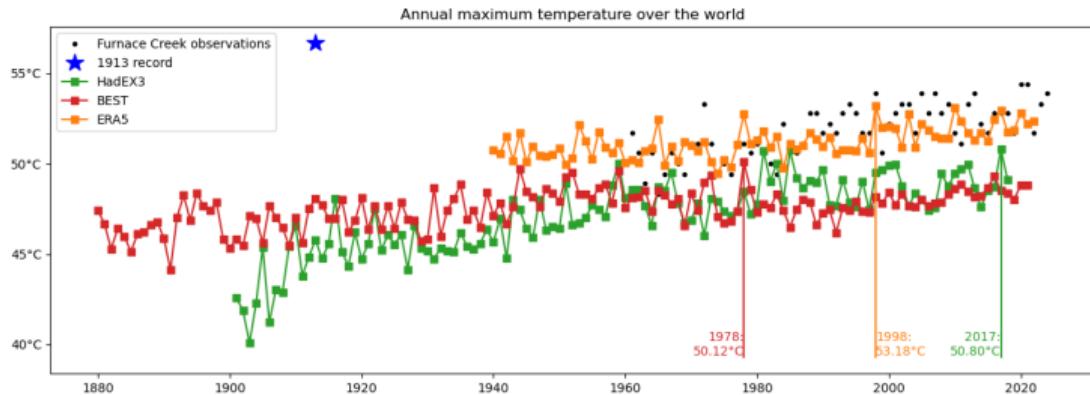


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## What extreme temperature levels worldwide?

⇒ Need for observations of **global maxima** :

- Raw observational datasets.
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⇒ Final choice: **ERA5**

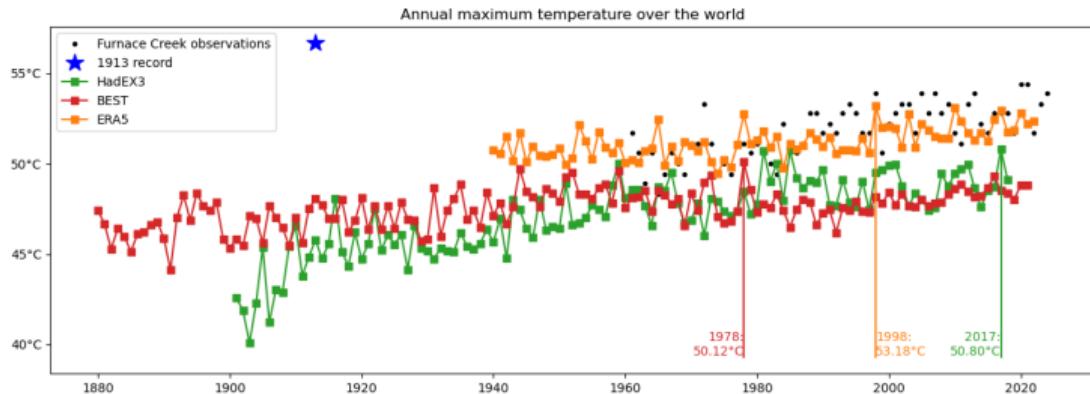


Figure: Time series of global annual maxima for three datasets.

## Temperature levels over the world

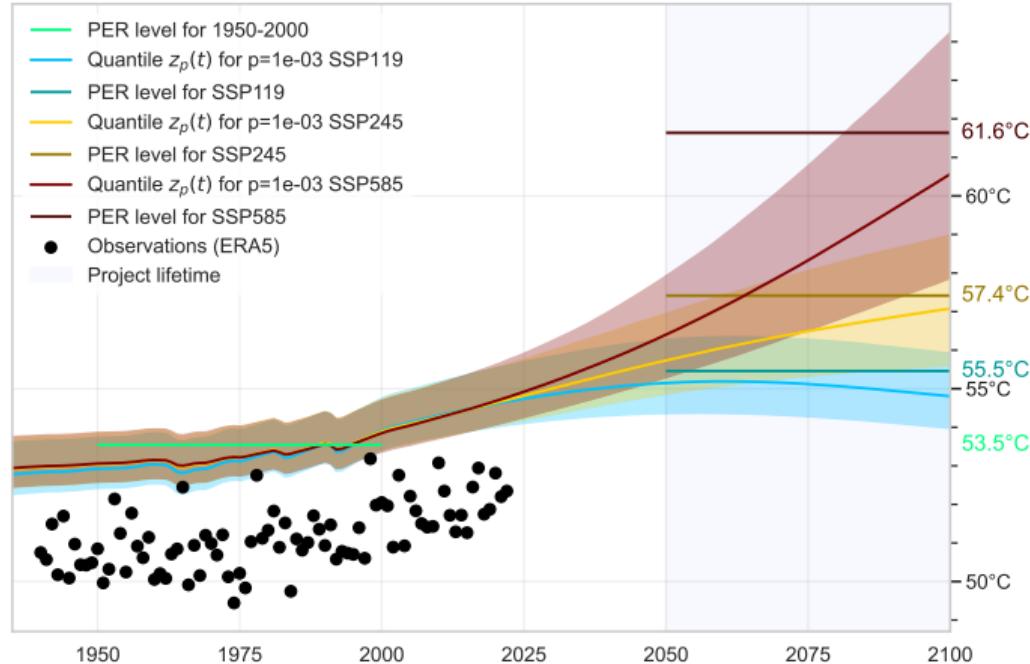


Figure: Global return levels and PER levels under three emission scenarios.

## What produces the highest temperatures ?

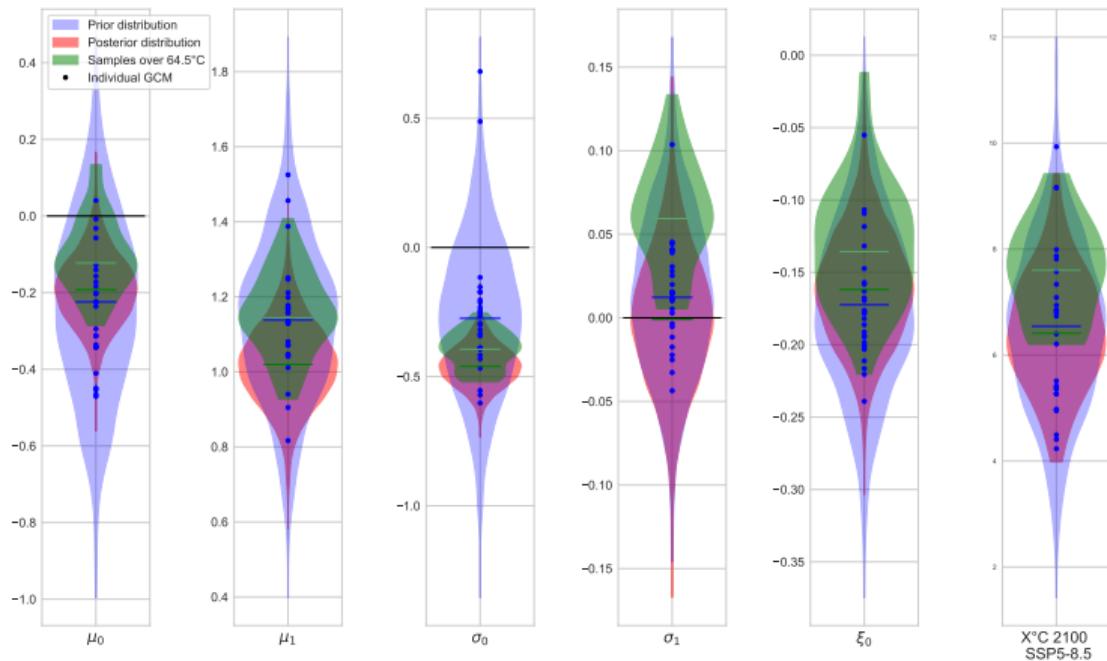
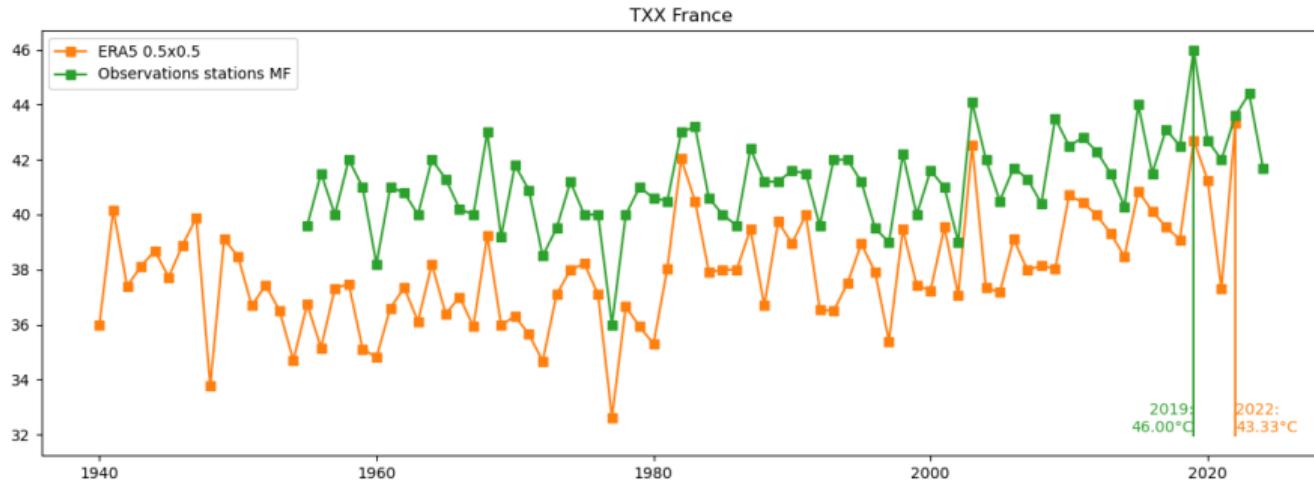


Figure: Distribution parameters producing temperatures over 64.5°C (1%).

- Obtained **global risk levels** from 55.5°C to 61.6°C depending on scenario.
- Highest values are **consistent with GCMs outputs**.
- For adaptation, necessity of **national applications**.
- Limitation: Era5 **underestimates the most extreme events** (0.5° resolution).

⇒ How sensitive is our methodology to the choice of dataset?

### What's the difference between ERA5 and a national database?



**Figure:** Time series of annual maxima over France for ERA5 and Météo France datasets.

## Temperature levels over France - Effect of database for SSP585

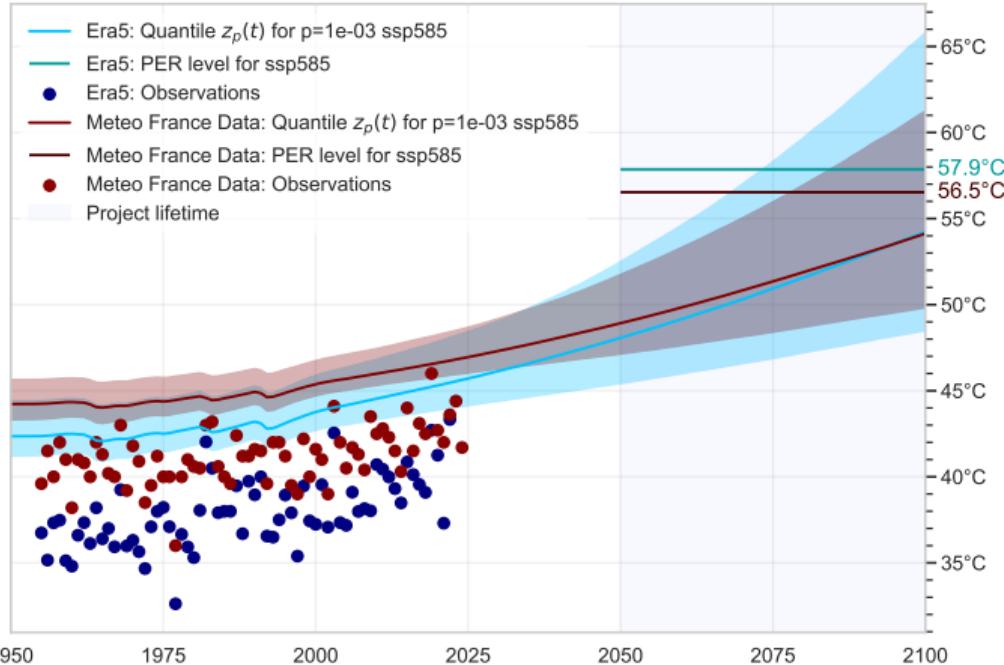


Figure: France's return levels and PER levels using either Météo France or ERA5 data (SSP5-8.5).



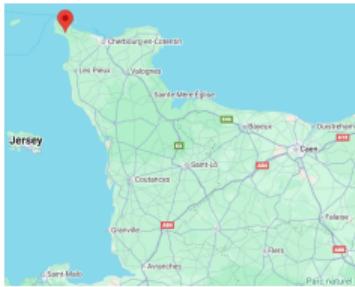


Figure: Situation of la pointe de La Hague

### La pointe de La Hague :

- On the **coastal edge** of the English Channel (Coastline)
- Strong **local climatic influences** (sea exposure).

⇒ **Biases:** systematic shifts in values, under-/overestimated uncertainty.

### Solutions:

- Statistical downscaling: **Use observations** to create a statistical link function with GCM data (Limitations for extremes and non-stationarity).
- Dynamical downscaling: **Higher resolution physical simulations** such as Regional Climate Models (RCMs).

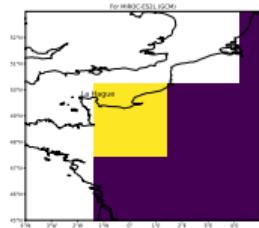


Figure: GCM MIROC-ES2L

### General circulation models:

- Large grids (from 50 to 180 km).
- May misrepresent local-scale phenomena (coastline).

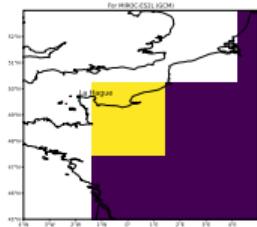


Figure: GCM MIROC-ES2L

### General circulation models:

- Large grids (from 50 to 180 km).
- May **misrepresent** local-scale phenomena (coastline).

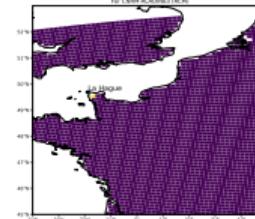


Figure: RCM ALADIN-Climat63

### Regional climate models :

- Higher resolution (10-50 km).
- Limited number of models and runs per model available (12 for Cordex).
- Use older GCMs (CMIP5) and emissions scenarios (RCP).

**Proposition:** Use the RCM to correct the prior parameter distribution.

1. Fit GEV parameter on RCM's series of maxima:  $\hat{\theta}_{HR}$
2. Fit GEV parameters on RCM's series of maxima regressed over each GCM's grid:  $\hat{\theta}_{GCM_i}$
3. Calculate difference :  $\Delta_{LR_i} = \hat{\theta}_{LR_i} - \hat{\theta}_{HR}$
4. Use it to offset the prior distribution.

## Effect of correction on parameter distributions

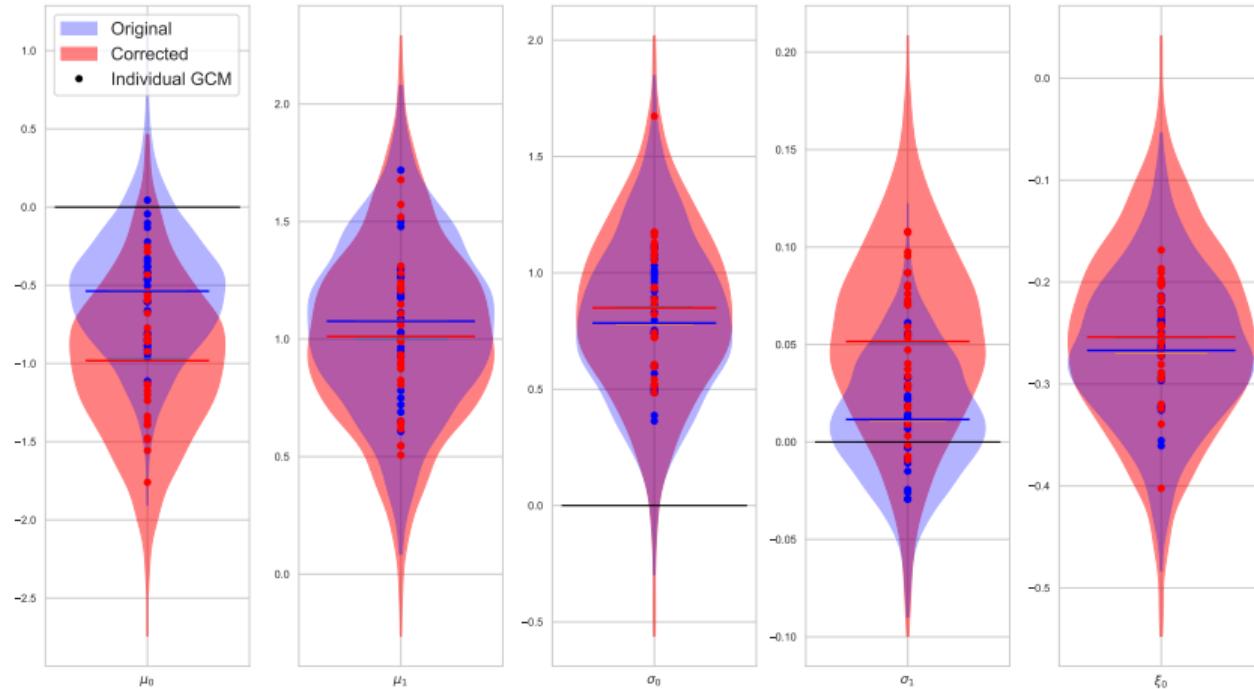


Figure: Prior parameter distributions with or without RCM-based offset.

## Effect of correction on temperature levels

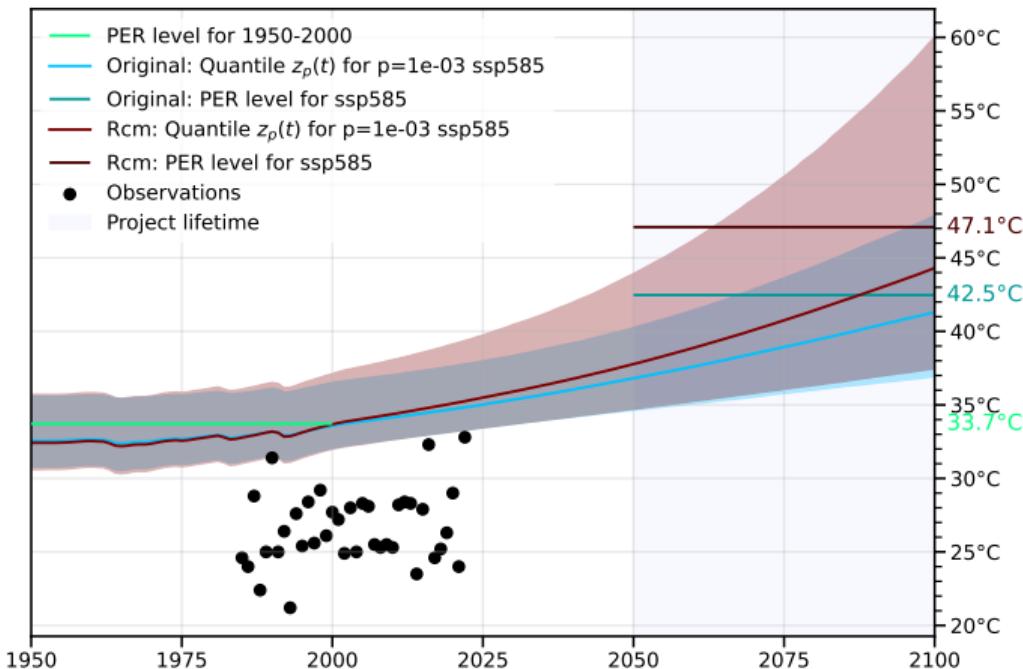


Figure: Return levels and PER levels under SSP5-8.5 at La Hague with or without RCM-based offset.



- Chose **Equivalent Reliability** as a risk index of interest.
- Adapted and improved Robin and Ribes' (2020) Bayesian estimation method.
- **Predictive** estimation taking parameter **uncertainty** into account.
- Applied the methodology to **various places of interest** (sites, countries, global temperature, etc.), accounting for limitations due to data availability.
- Global levels differ from **53.5°C to 61.6°C depending on scenario**.
- **Data used** produces differing levels design when using a different database (France) or correcting for local effects.
- Design levels produces conservatives enough values to **cover unexpected extreme events**.

- Currently testing the methodology on extreme, "**record-shattering**" events.
- Extend correction to **multiple RCMs** to assess the consistency of results and associated spread.
- Integrate the **duration** of extreme heat events in the framework.
- Incorporate a **physics-based prior distribution on the upper bound** [Noyelle et al., 2024, Noyelle et al., 2023, Zhang and Boos, 2023].
- Application to **other environmental hazards**, such as extreme precipitation.

# High temperature levels for the 21st century.

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*Thanks for listening.  
Any questions?*



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- It's possible to calculate a physical upper bound for temperature, relying on moisture and instability of the air column.
- Both Zhang and Boos (2023) and Noyelle et al. (2023) calculated a physical upper bound for temperature using ERA-5 data on western Europe, higher than a GEV upper bound estimate on similar data.
- Using IPSL GCM data, Noyelle (2024) similarly found a physical upper bound 3 to 8°C higher than a GEV upper bound over 70 years.
- Exploration of the availability of necessary data for a multi-model analysis. Necessary first step for inclusion in our Bayesian framework.

## What are records-shattering events ?

**Records-shattering events:** Break previous local records by large margins [Fischer et al., 2021].

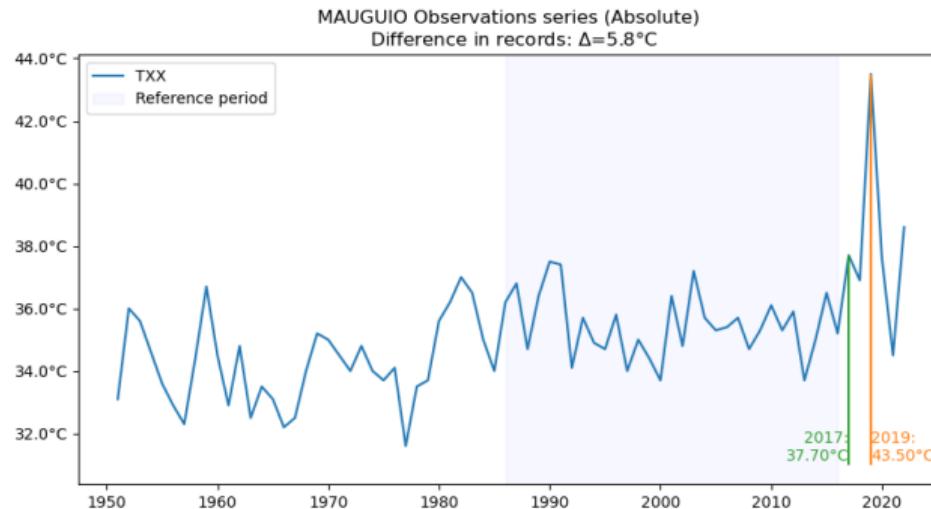


Figure: Series of annual maxima of temperature in Maugio (34).

## Coverage- fit without the record

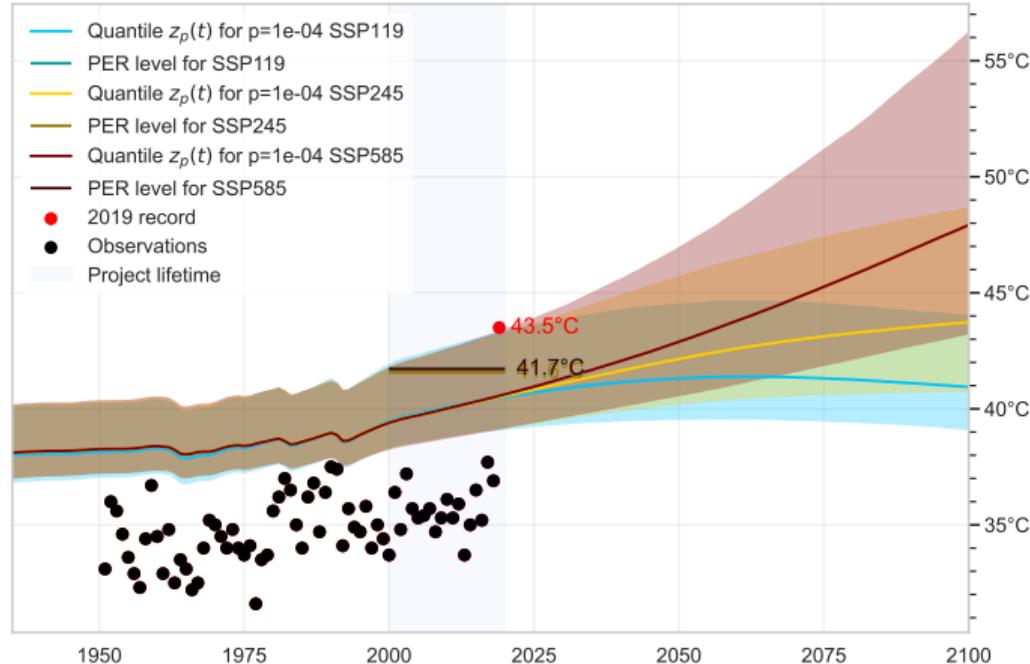


Figure: Return levels and PER levels at Mauguio.

## Distribution - fit without the record

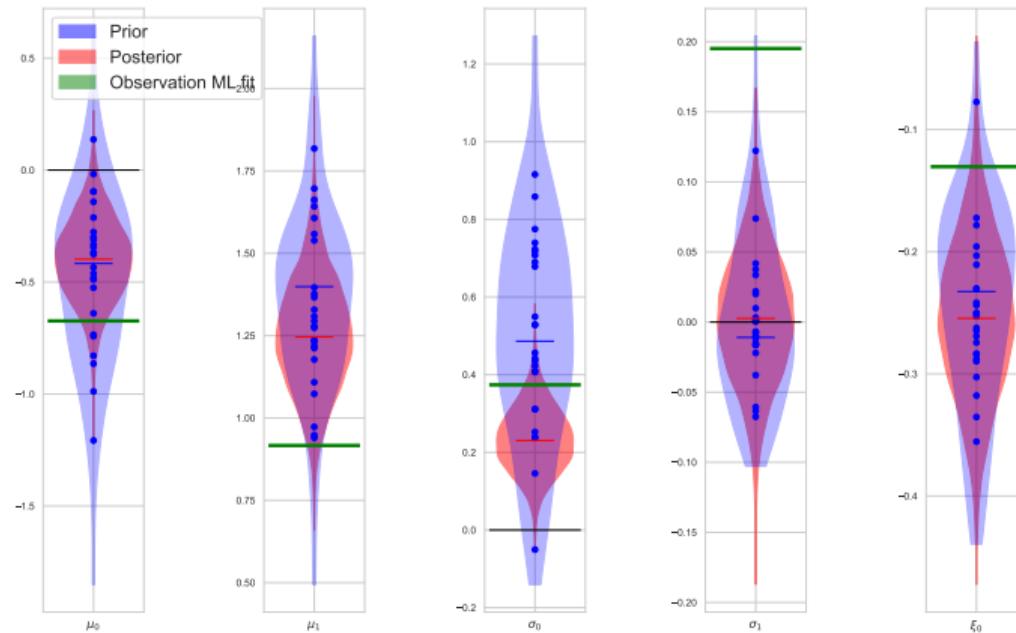


Figure: Parameter distributions.

## Distribution - fit with the record

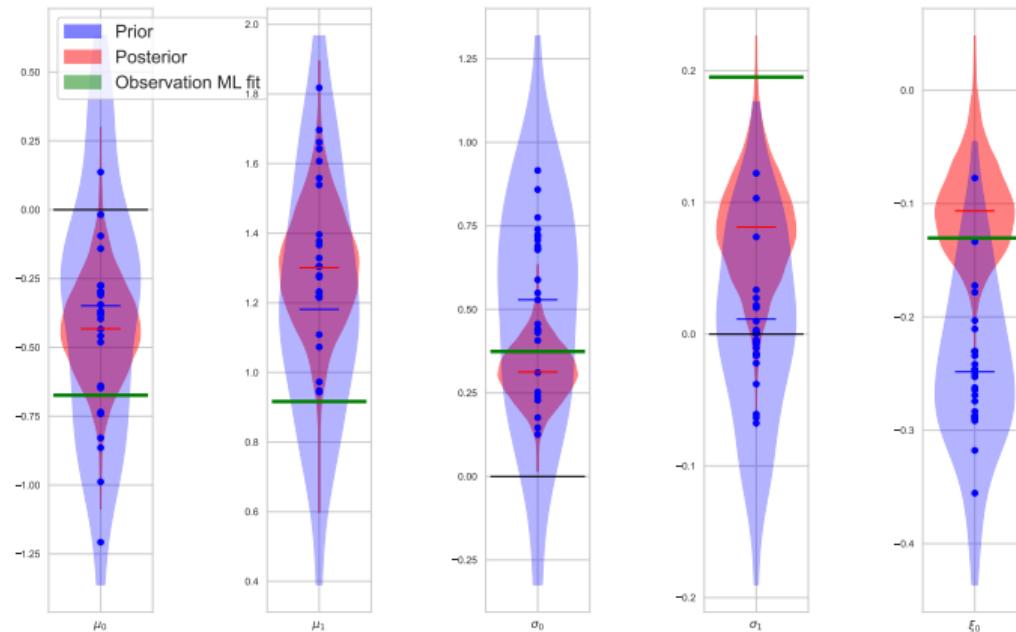


Figure: Parameter distributions.

## Coverage - fit with the record - RL 1000

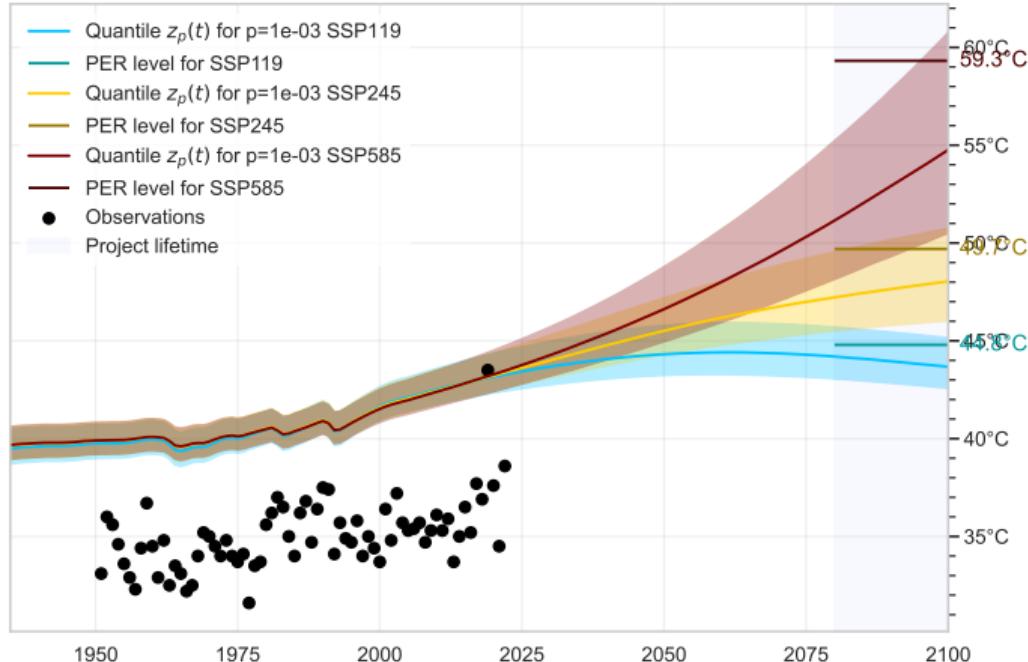


Figure: Return levels and PER levels at Mauguio.

## Coverage - fit with the record - RL 10000

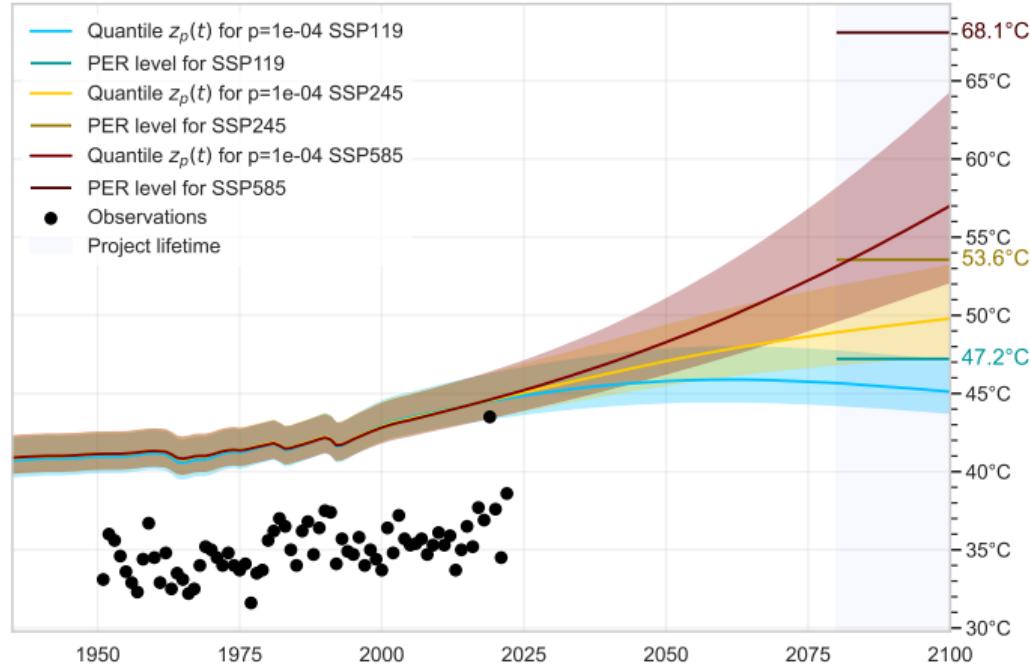


Figure: Return levels and PER levels at Mauguio.

Only need to be able to simulate from conditional distributions. (Maybe possible use of  $X_T$ )

Multivariate :  $\psi = (\psi_1, \dots, \psi_d)'$ , full conditionnals are  $\pi(\psi_i | \psi_{-i}) = \pi_i(\psi_i)$

Description of algorithm:

- Initialisation:  $k=1$ , initial state of chain  $\psi^{(0)}$
- Boucle: For new value  $\psi^{(k)}$  :
  - $\psi_1^{(k)} \sim \pi(\psi_1 | \psi_{-1}^{(k-1)})$
  - $\psi_2^{(k)} \sim \pi(\psi_2 | \psi_{-1,2}^{(k-1)}, \psi_1^{(k)})$
  - ...
  - $\psi_d^{(k)} \sim \pi(\psi_d | \psi_{-d}^{(k)})$

$\pi(\psi)$  is still the density of interest. We now have a transition kernel  $p(\psi_{i+1}, \psi_i)$ , easy to simulate from, to get successive values.

- Initialisation :  $k=1$ , initial state of chain  $\psi^{(0)}$
- Boucle: For new value  $\psi^{(k)}$  :
  - Generate new proposed value  $\psi'$  using the kernel transition function.
  - Calculate Acceptance Probability (ratio)  $A(\psi^{(k-1)}, \psi')$  of the proposed change of value:

$$A(\psi^{(k)}, \psi') = \min\left\{1, \frac{\pi(\psi')L(\psi'|\mathbf{x})p(\psi', \psi^{(k-1)})}{\pi(\psi^{(k-1)})L(\psi^{(k-1)}|\mathbf{x})p(\psi^{(k-1)}, \psi')}\right\}$$

- Accept  $\psi^{(k)} = \psi'$  with probability  $A(\psi^{(k)}, \psi')$  and keep  $\psi^{(k)} = \psi^{(k-1)}$  otherwise.

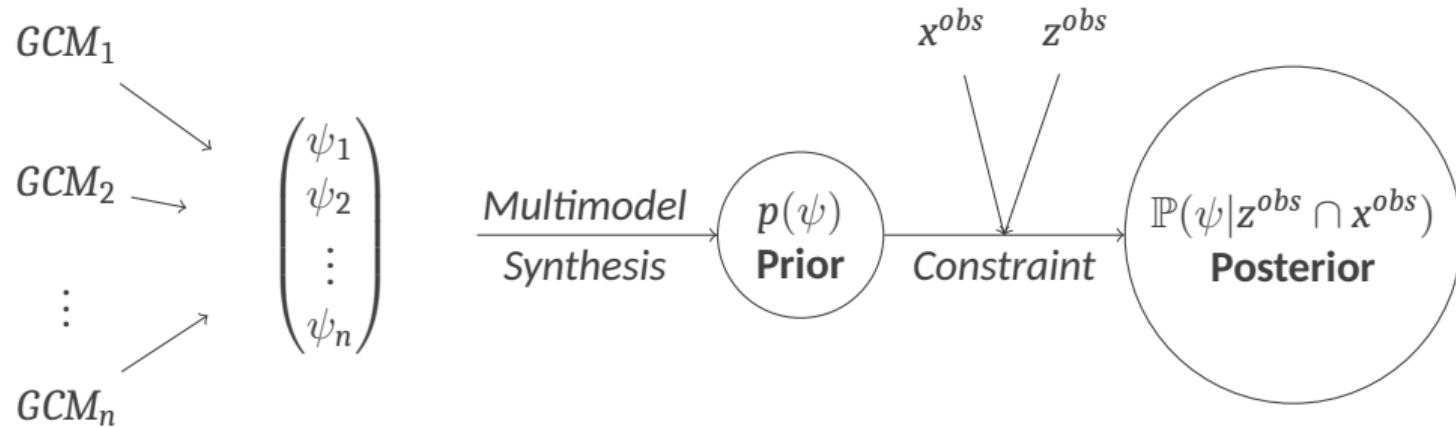
Based on Bayesian Modelling of Extreme Rainfall Data from Elizabeth Smith  
Gibbs concept (each parameter is updated in turn) and conditionals are MH (do we accept  
the new value produced by the transition function?)

Advantage: Each parameter has its own trajectory (One may not move much, and another  
a lot) + varying transition kernel proportionally. (not hard to do for simple MH too)  
→ Less dependence than normal MH?.

Description of algorithm:

- Initialisation :  $k=1$ , initial state of chain  $\theta^{(0)}$
- Boucle: For new value  $\theta^{(k)}$  :
  - In turn, for each parameter  $\theta_j^{(k)}$ 
    - $\theta_j' = \theta_j^{(k-1)} + \varepsilon_j$
    - Accept or refuse using  $A(\theta_j^{(k-1)}, \theta_j')$  with  $\theta_{-j}^{(k)}$  seen as known.

NUTS is based on Hamiltonian Monte Carlo (HMC) [Neal, 2011], which generates proposals using gradient-informed trajectories rather than random walks. This allows for more efficient exploration of high-dimensional and correlated parameter spaces[Betancourt, 2018]. NUTS further adapts key hyperparameters such as step size and trajectory length during sampling, making it more robust to correlation in the posterior distribution and scaling issues [Betancourt, 2018].



For  $X_2 100_{ssp585}$  : E3SM-1-O\_i1p1f1: 9.93 (n=5), CanESM5\_i1p1f1 9.16 (n=25),  
CanESM5\_i1p2f1 9.16 (n=25).

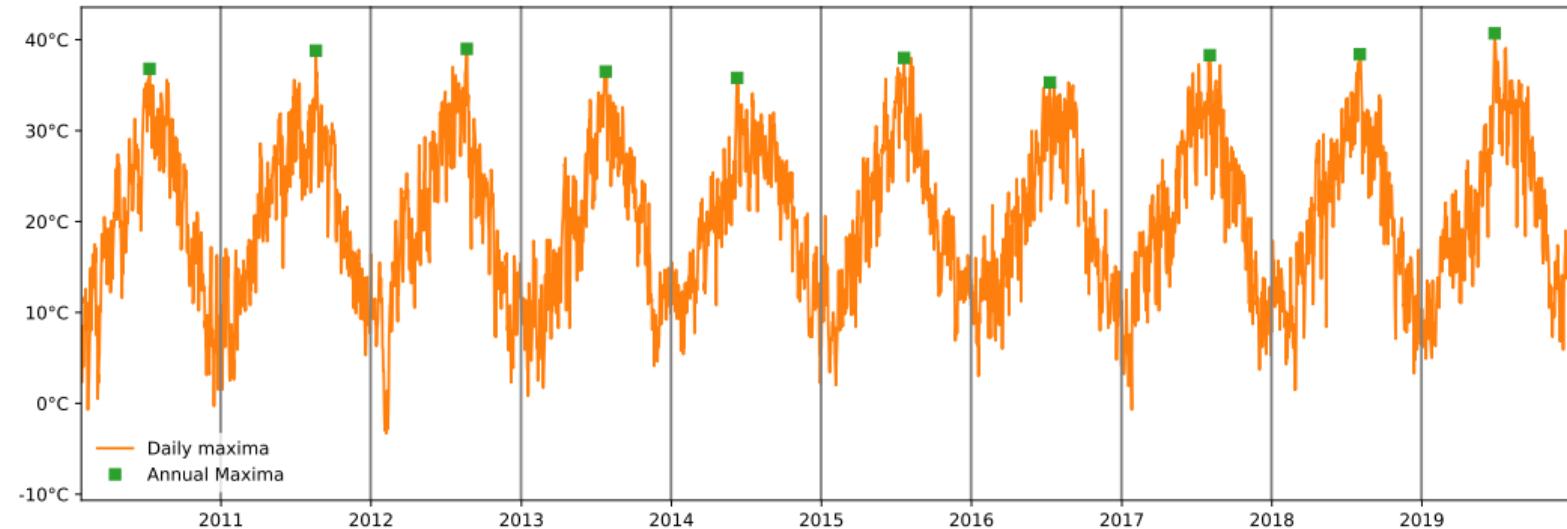
For  $\mu_0$ : ACCESS-ESM1-5\_i1p1f1: 0.356 (n=40), NorESM2-MM\_i1p1f1: 0.041 (n=1) ,  
CNRM-CM6-1-HR\_i1p1f2: -0.008 (n=1).

For  $\mu_1$ : MIROC6\_i1p1f1: 1.53 (n=50), ACCESS-ESM1-5\_i1p1f1 : 1.46 (n=40),  
CNRM-ESM2-1\_i1p1f2: 1.39 (n=5).

For  $\sigma_0$ : ACCESS-ESM1-5\_i1p1f1: 0.68 (n=40), BCC-CSM2-MR\_i1p1f1 : 0.49 (n=1),  
IPSL-CM6A-LR\_i1p1f1 : -0.116 (n=7).

For  $\sigma_1$ : ACCESS-ESM1-5\_i1p1f1: 0.104 (n=40), ACCESS-CM2\_i1p1f1 : 0.0454 (n=10),  
TaiESM1\_i1p1f1 : 0.0443 (n=1).

For  $\xi$ : MRI-ESM2-O\_i2p1f1: -0.0552 (n=1), BCC-CSM2-MR\_i1p1f1: -0.107 (n=1),  
TaiESM1\_i1p1f1: -0.109 (n=1).



*Figure: Illustration of the annual block method selection on time series of daily and annual maxima for the period 2010-2020. This figure represents the time series of daily maxima for the period 2010-2020 (orange) and the annual maxima (green) as selected using the block-maxima method.*

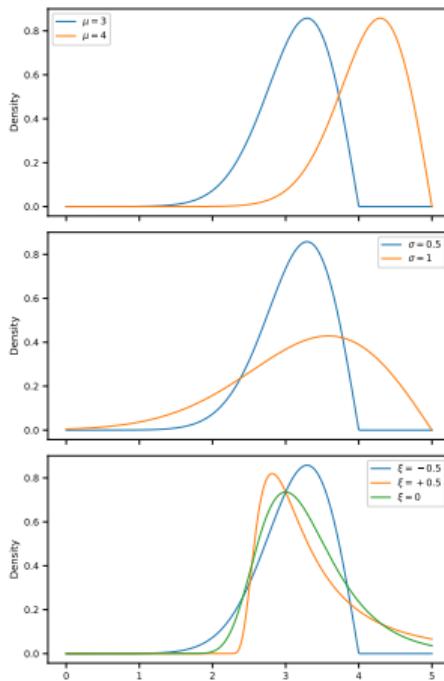
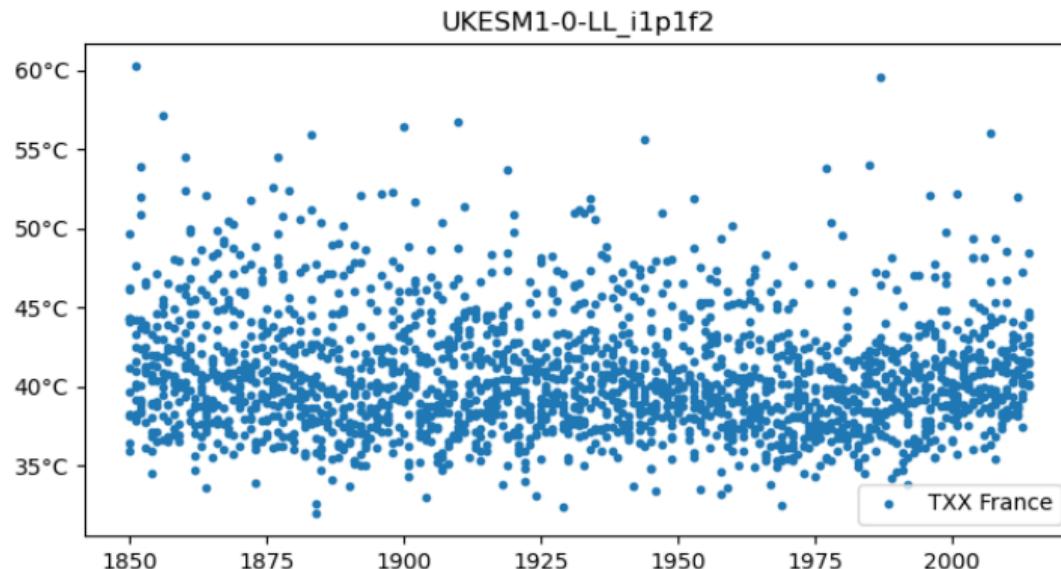
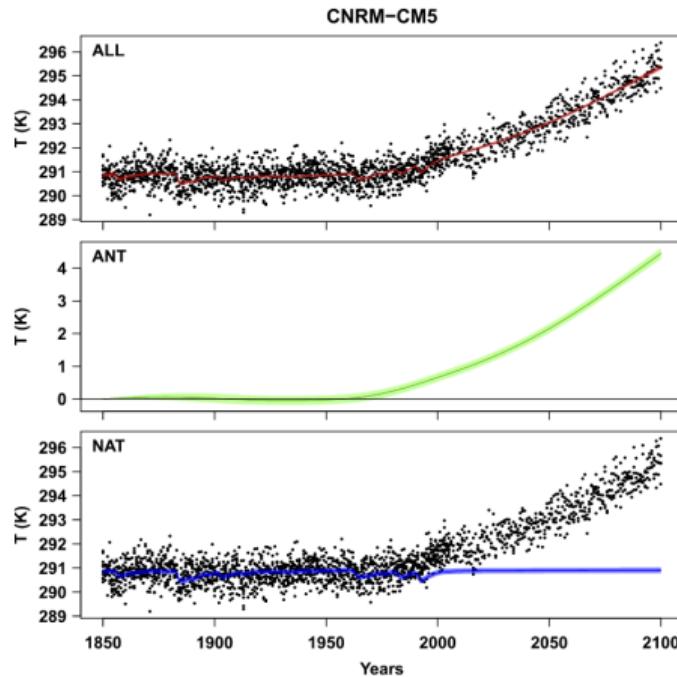


Figure: Various GEV distributions with differing  $\mu$ ,  $\sigma$  or  $\xi$ .



*Figure: Annual maxima of daily maxima of temperature for the historical experiment of model UKESM1-0-LL.*



**Figure:** Results of the decomposition of  $X_t$  into  $X_t^{\text{ANT}}$  and  $X_t^{\text{NAT}}$ .

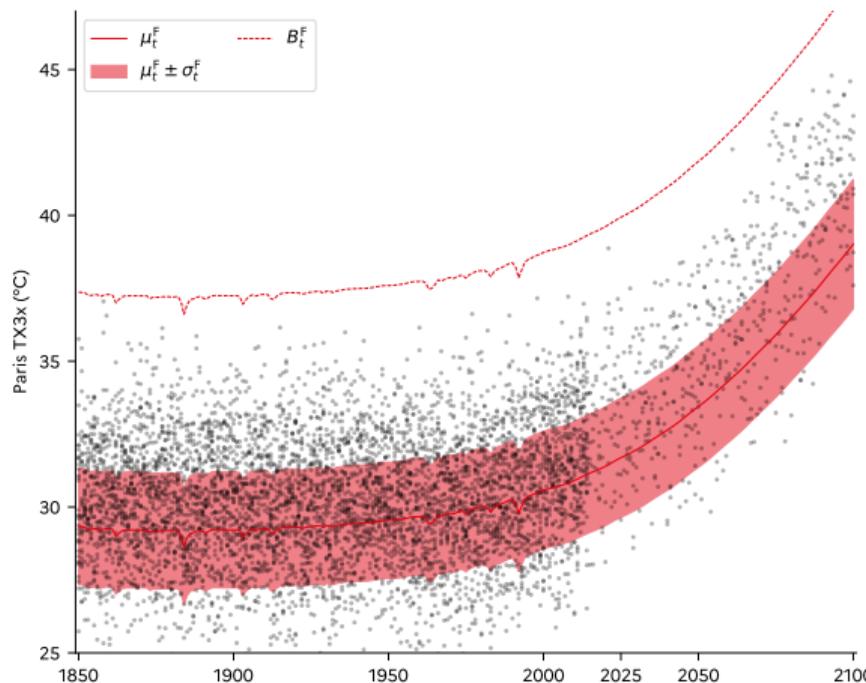


Figure: Results of the fit of a non-stationary GEV distribution. This figure is courtesy of Yoann Robin.

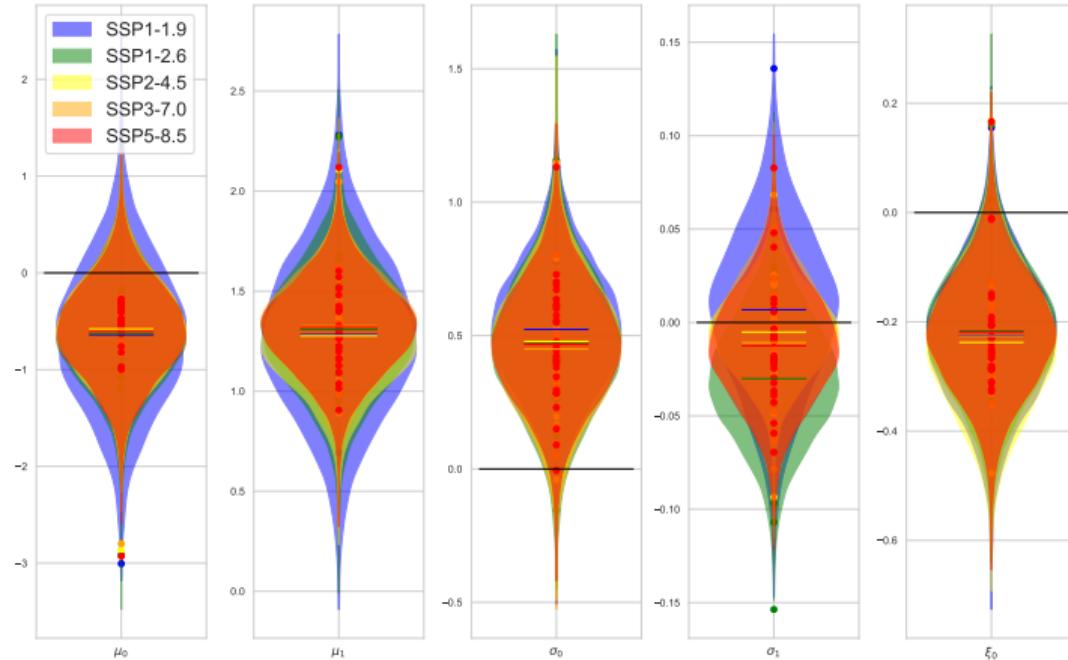


Figure: Various prior distributions obtained using time series for each single scenario.

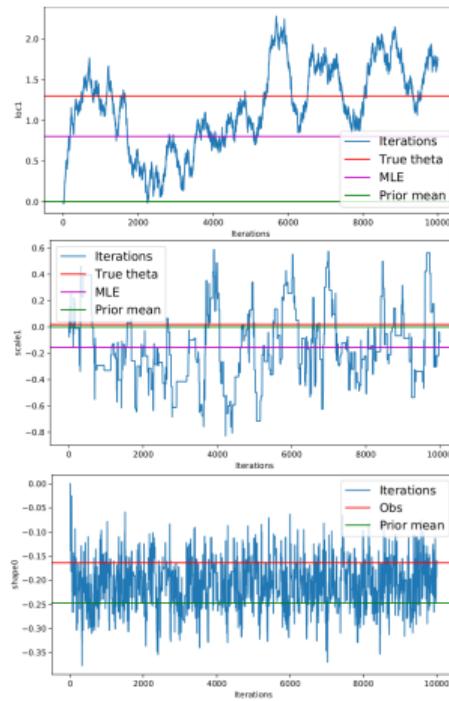
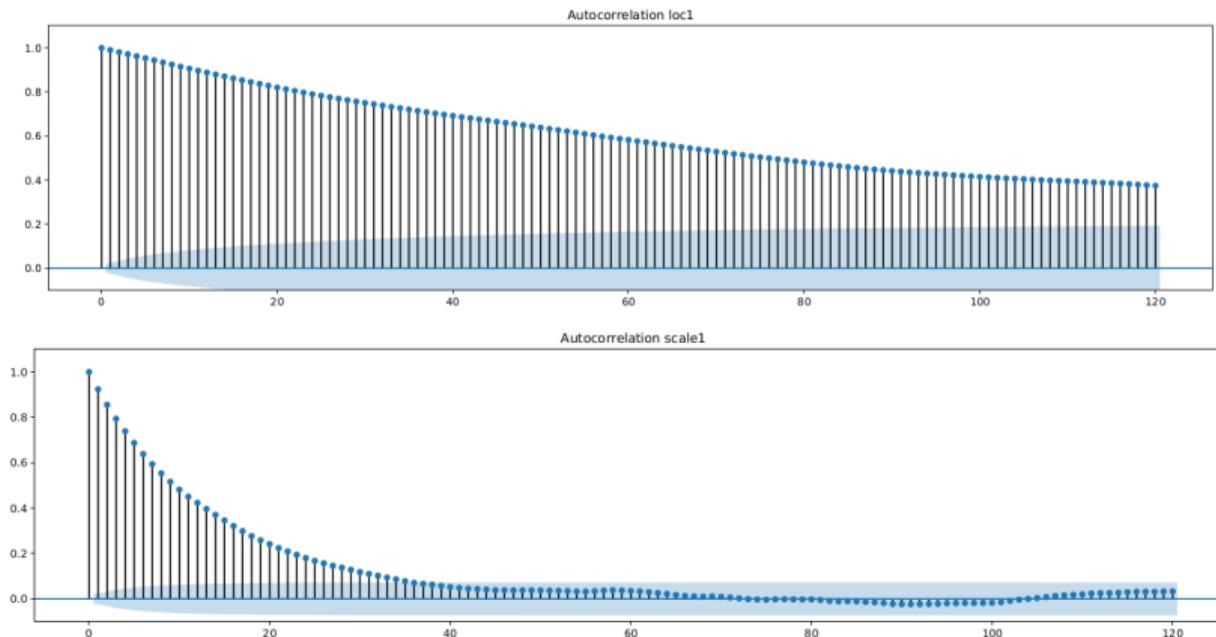


Figure: 3 different MCMC traceplots, two pathological ones and a healthy one



**Figure:** Two different autocorrelation plots, one pathological and a healthy one

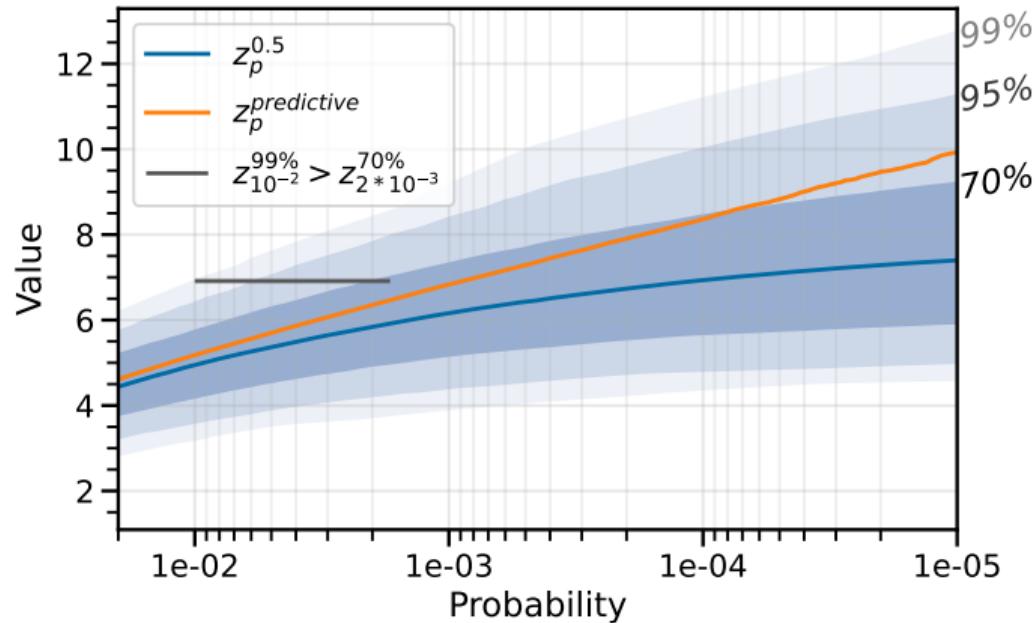
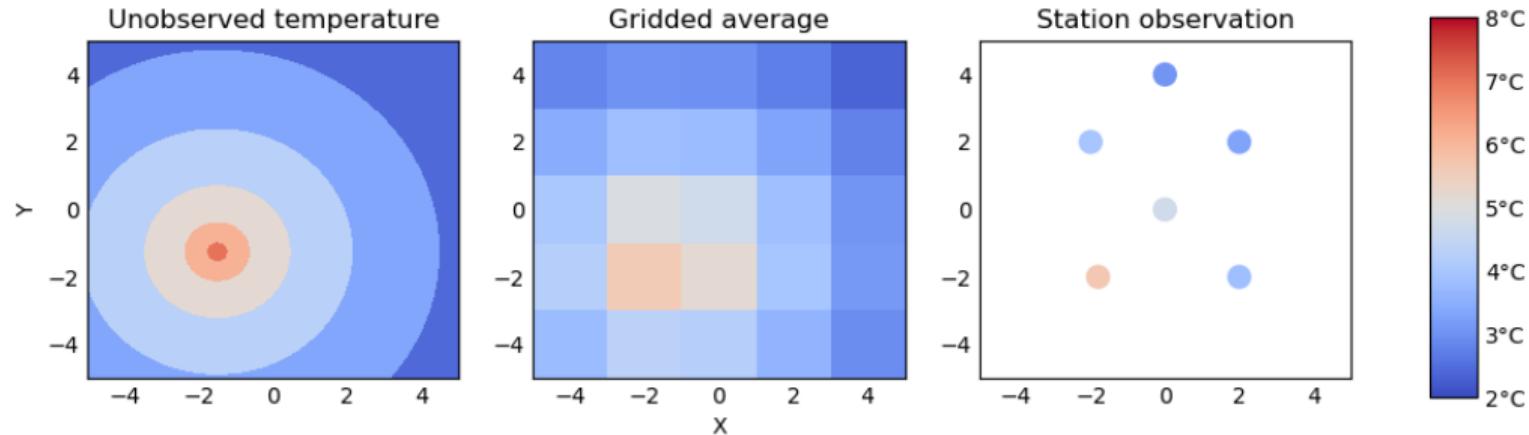


Figure: Comparison between predictive, median, and upper-bound estimates of return levels for exceedance probabilities  $p$ .

Table 3.1

Period	50%	25%	10%	5%	1%	0.1%
1940-2020 for historical	40.60	41.25	41.94	42.44	43.51	45.22
2020-2100 for SSP1-1.9	42.61	43.38	44.21	44.78	46.11	48.31
2020-2100 for SSP1-2.6	43.39	44.25	45.15	45.78	47.24	49.49
2020-2100 for SSP2-4.5	44.61	45.61	46.67	47.38	49.02	51.66
2020-2100 for SSP3-7.0	46.54	47.80	49.13	50.07	52.26	55.85
2020-2100 for SSP5-8.5	48.48	50.30	52.18	53.47	56.29	61.08

Table: PER values in Tricastin over the periods 1940-2020 and 2020-2100 for various emission scenarios and probabilities of excess.



**Figure:** Illustration of representativity between gridded data and local observations.

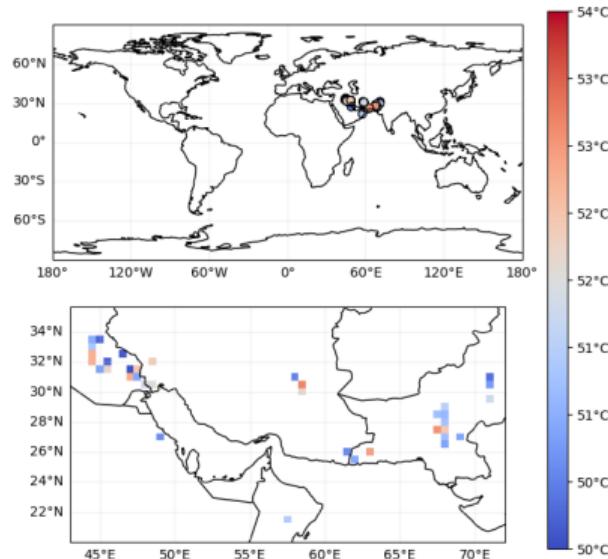


Figure: Spatial repartition of global annual maxima of daily maxima of temperature for ERA5 over 1940- 2022.

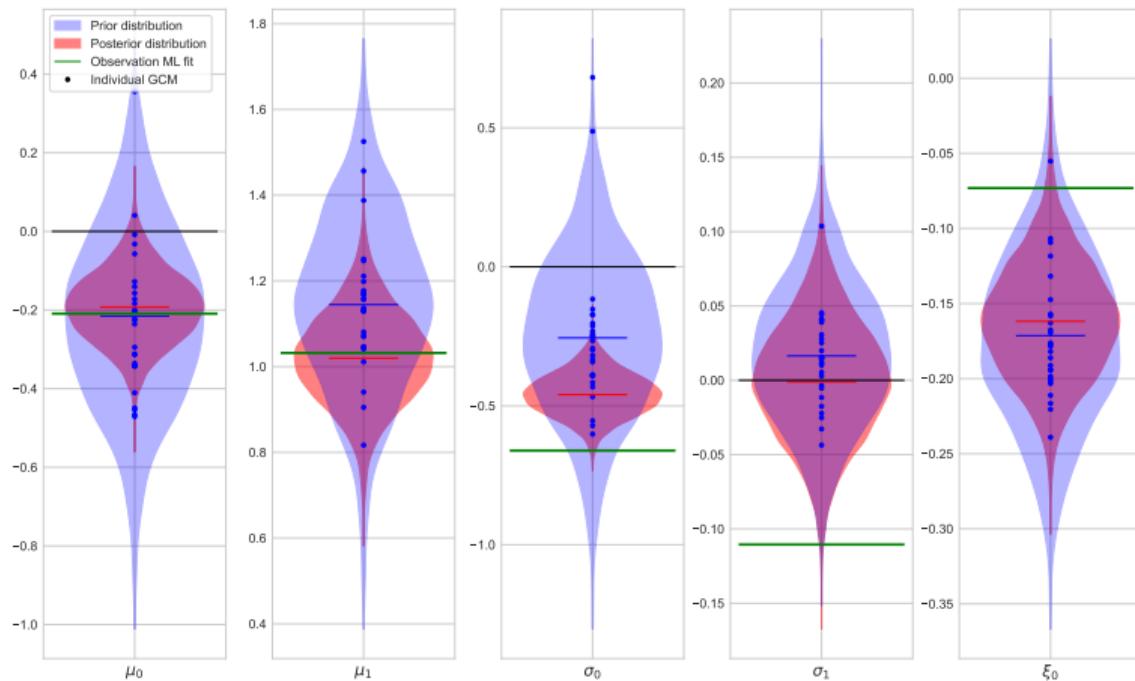


Figure: Prior and posterior parameter distributions for a fit using TX series.

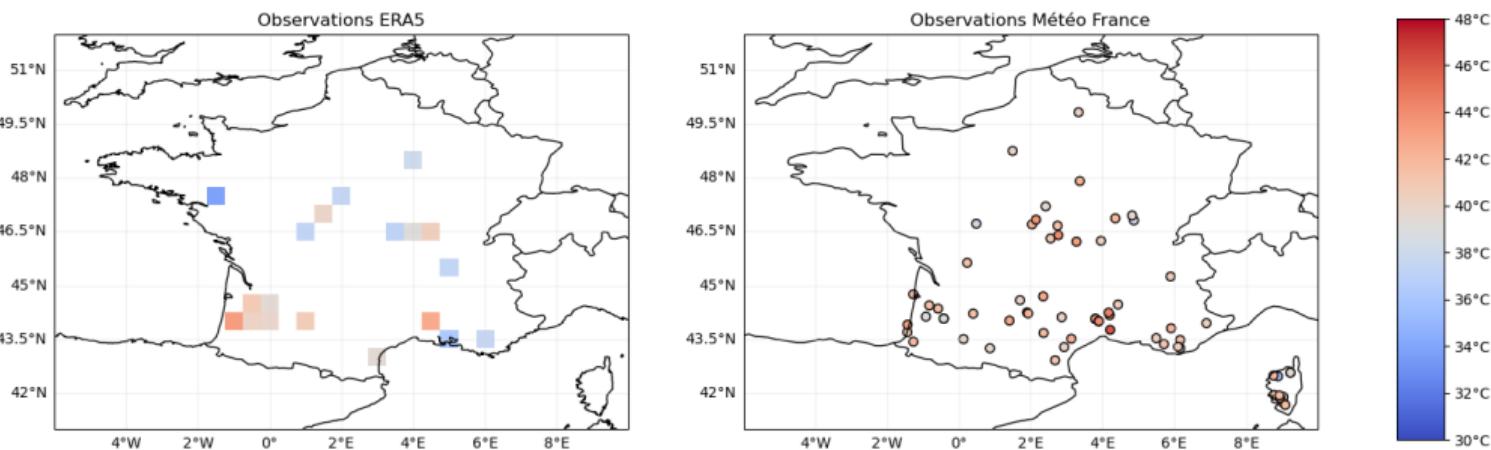


Figure: Spatial distribution of annual maxima of daily maxima of temperature over France for ERA5 and Météo France datasets.

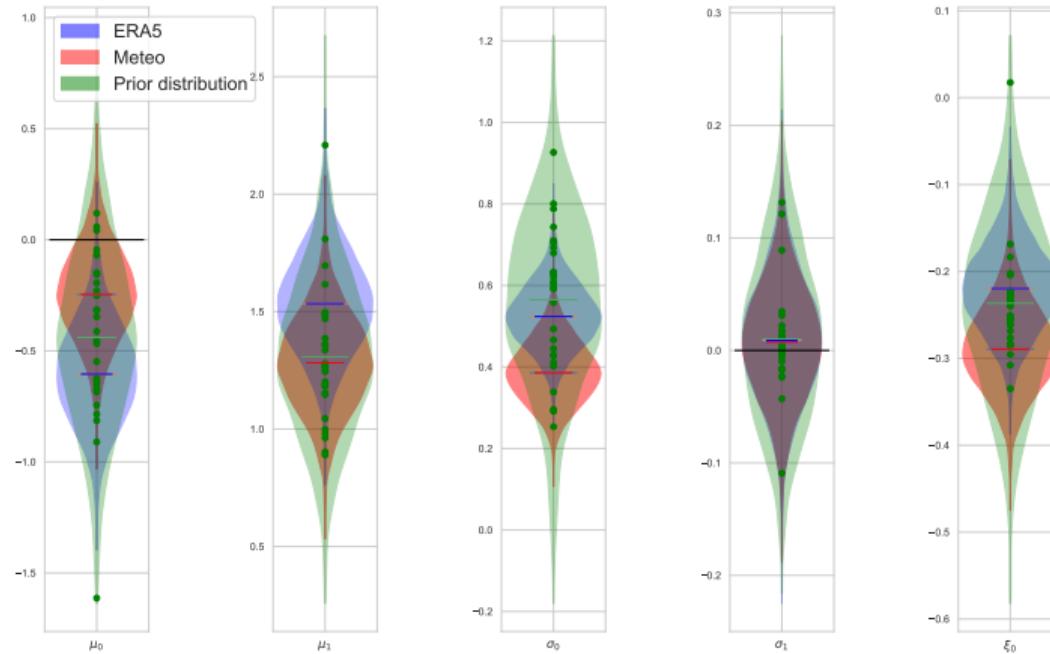


Figure: Posterior parameter distribution using either Météo France or ERA5 time series for constraint.

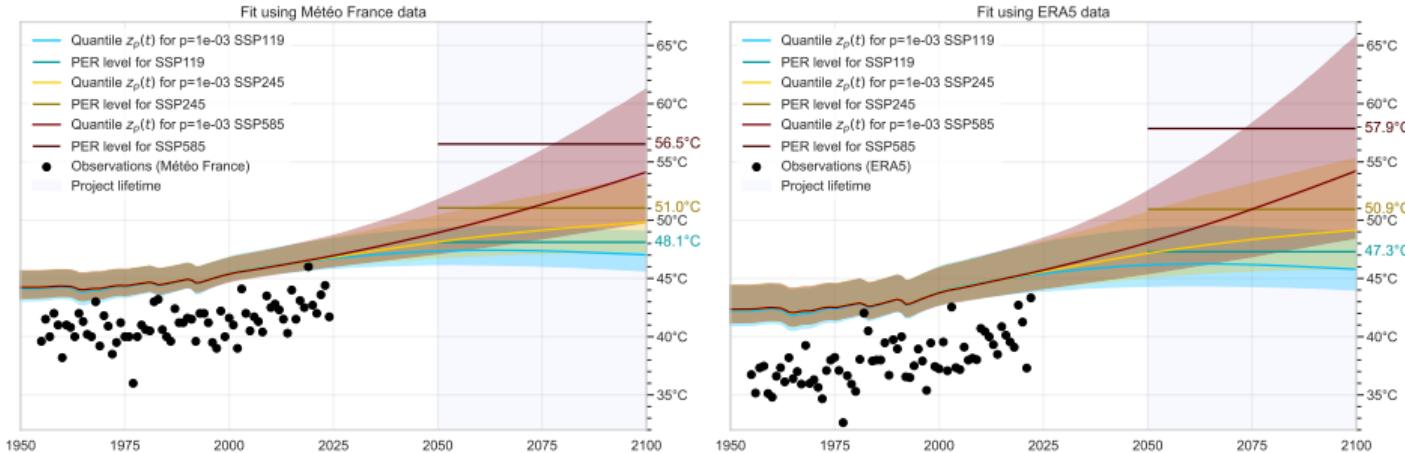
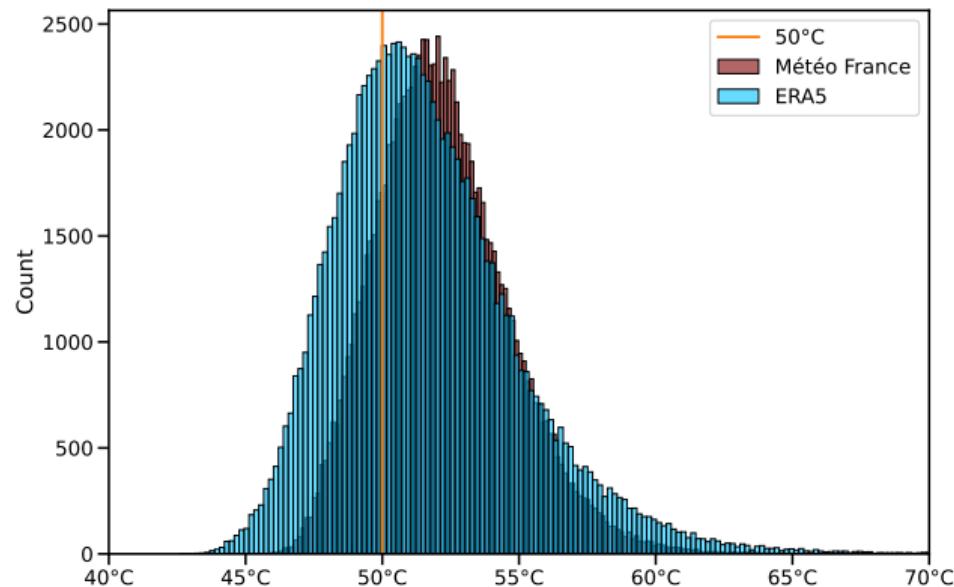


Figure: Time-dependent return levels and predictive equivalent reliability (PER) levels under three emission scenarios in France using either Météo France or ERA5 time series for constraint.

Table 4.1

	Météo France	ERA5
SSP1-1.9	0.002	0.003
SSP1-2.6	0.016	0.014
SSP2-4.5	0.146	0.095
SSP3-7.0	0.598	0.385
SSP5-8.5	0.820	0.660

**Table:** Probabilities of exceeding 50°C over the period 2050-2100 for various scenarios using either Météo France or ERA5 time series for constraint. This table presents estimated probabilities of exceeding 50°C integrated over the period 2050-2100 for scenarios SSP1.1-9, SSP1-2.6, SSP2-4.5, SSP3-7.0, and SSP5-8.5. The applications are done using Météo France time series for the left column, and ERA5 time series for the right.



*Figure: Histogram of the predictive distribution of maxima over the period 2050-2100 for scenario SSP5-8.5, using either Météo France or ERA5 time series for constraint.*

Table 4.2

Country	Past 1950-2000	SSP1-1.9	SSP1-2.6	SSP2-4.5	SSP3-7.0	SSP5-8.5
Sweden	36.98	39.27	39.85	41.18	42.96	44.67
UK	38.21	42.35	43.50	45.88	49.24	52.89
Germany	40.57	44.37	45.44	47.60	50.66	53.90
Brazil	44.80	47.03	47.57	49.20	51.47	53.63
Italy	45.14	48.10	48.95	50.50	52.66	55.23
France	43.31	47.29	48.58	50.94	54.16	57.86
China	48.55	50.12	50.42	51.52	53.11	54.92
Spain	45.37	48.77	49.79	51.77	54.17	57.25
Australia	50.91	52.36	52.68	53.78	55.36	56.98
USA	51.54	53.48	53.92	55.35	57.44	59.44
World	53.54	55.46	56.01	57.42	59.47	61.64

Table: PER values over the periods 1950-2000 and 2050-2100 for various scenarios and countries.

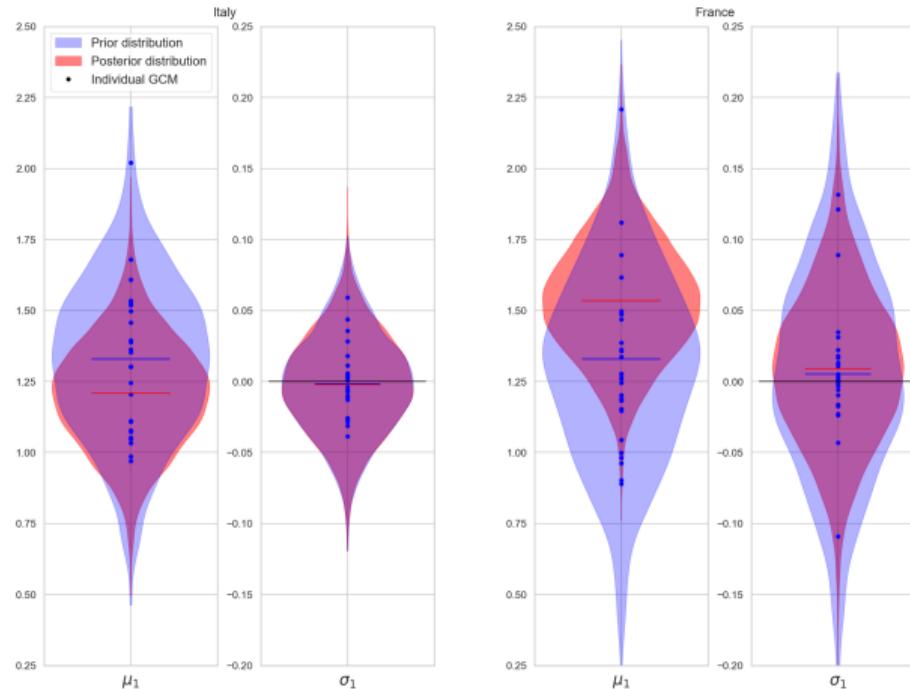


Figure: Prior and posterior parameter distributions for Italy and France.

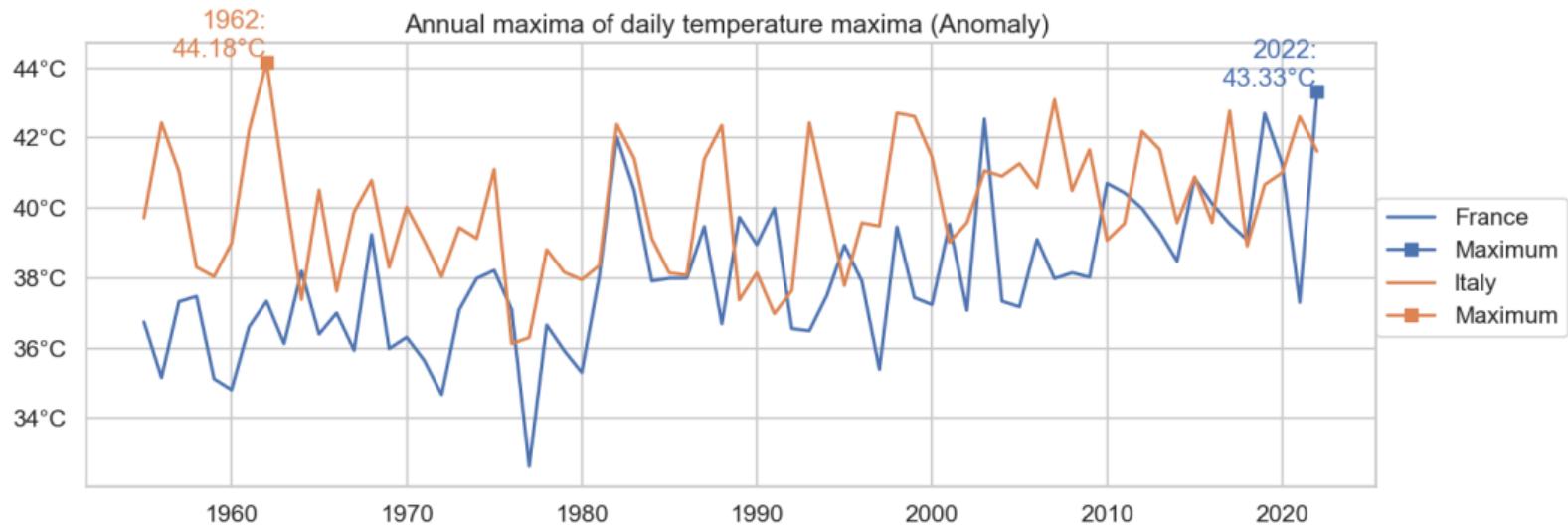
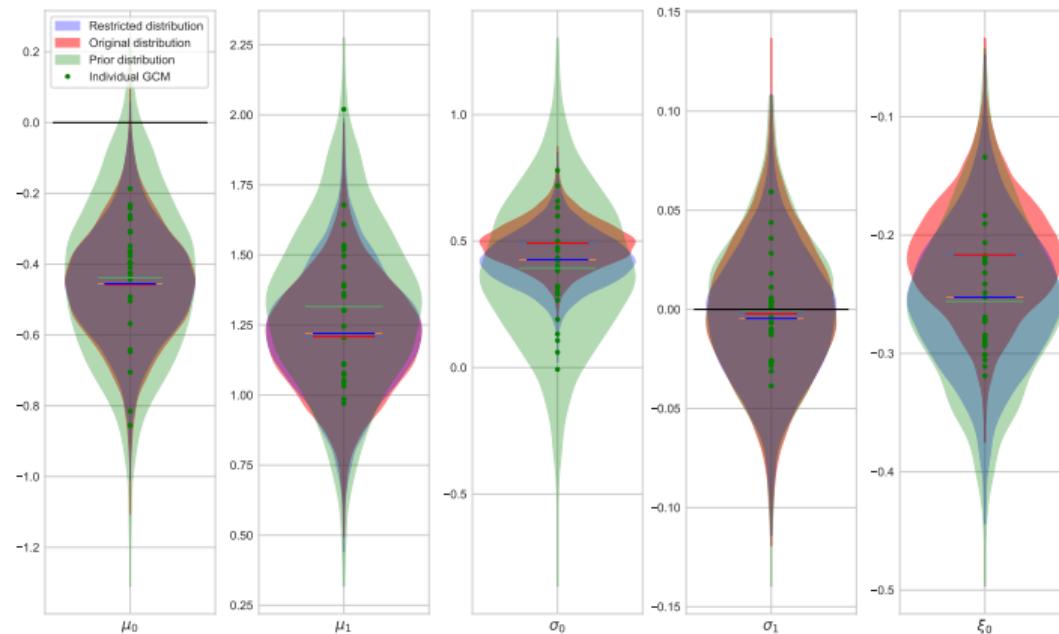


Figure: Time series of ERA5's annual maxima for France and Italy.



*Figure: Prior parameter distribution and posterior parameter distributions obtained using either 1980-2022 or 1955-2022 time series over Italy.*

Table 4.15

	Period					
	Past 1950-2000	SSP1-1.9	SSP1-2.6	SSP2-4.5	SSP3-7.0	SSP5-8.5
Time series						
1980-2022	44.53	47.66	48.58	50.27	52.52	55.13
1955-2022	45.14	48.10	48.95	50.50	52.66	55.23

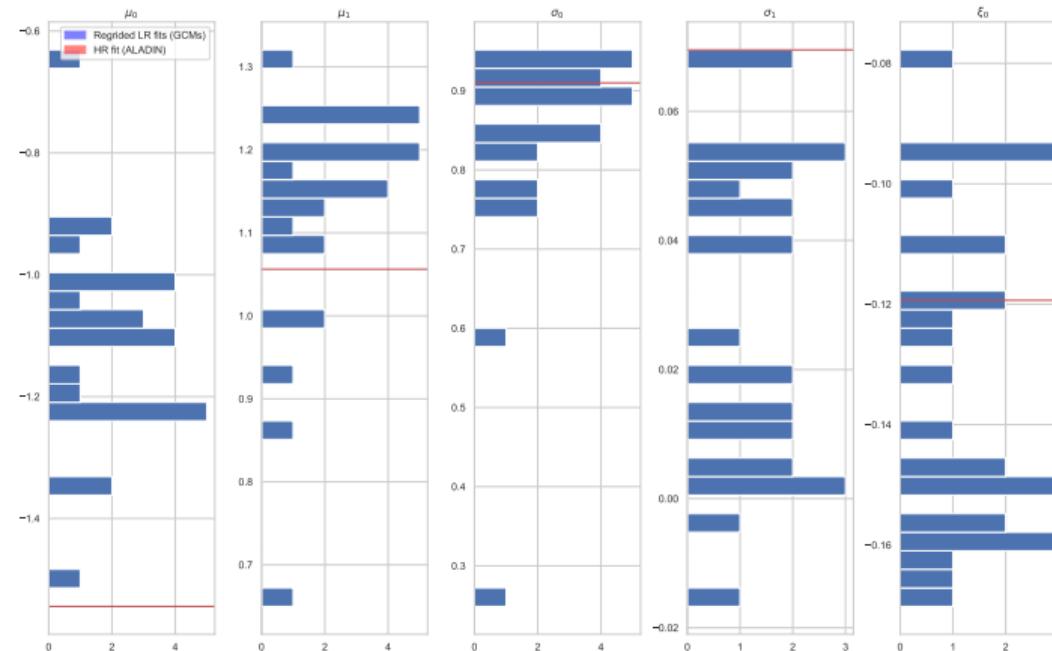
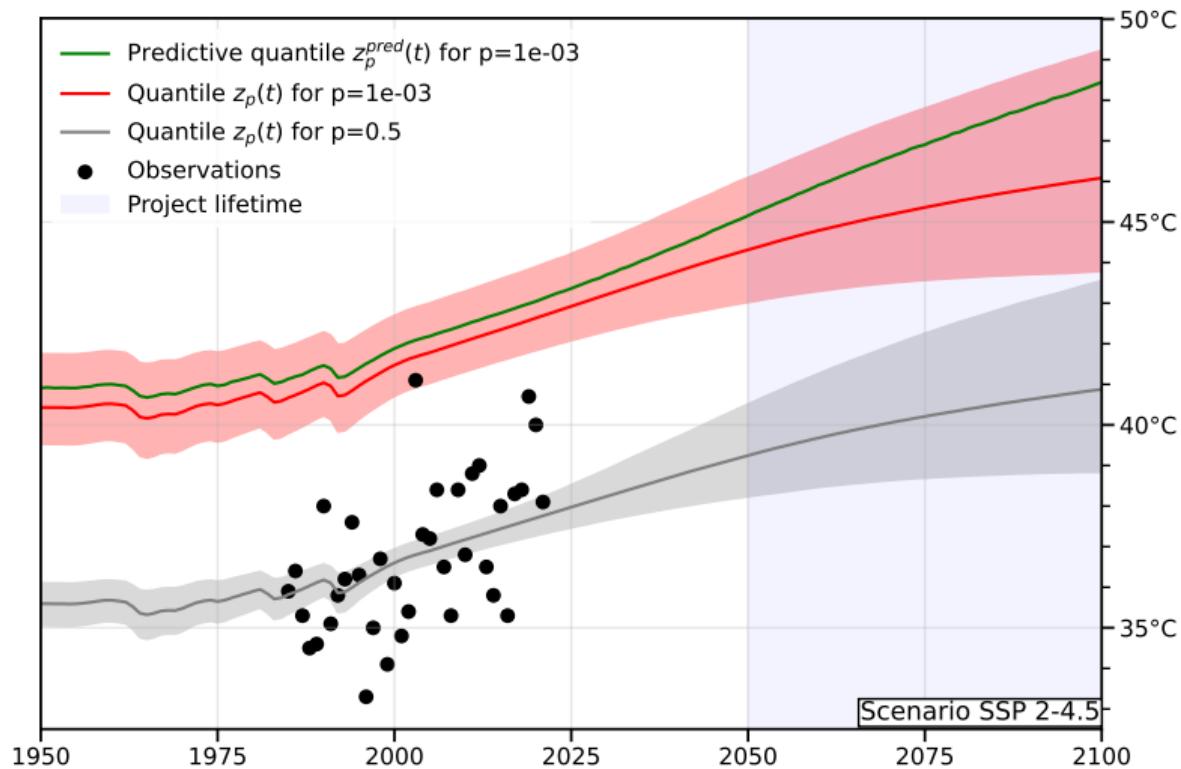
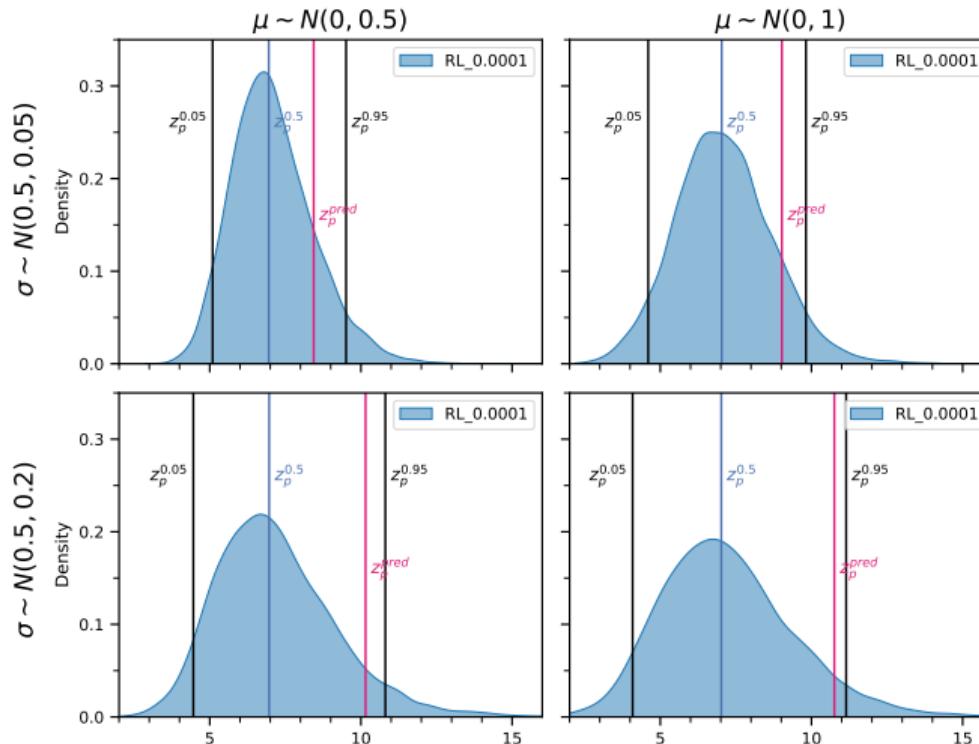


Figure: Histogram of values of  $\hat{\theta}_{HR}$ , parameter sets fitted on regridded time series, compared to  $\hat{\theta}_{HR}$ , the parameter set fitted on ALADIN63's series.





# An automatic procedure for the attribution of extreme events at the global scale: a proof of concept for heatwaves



Saïd Qasmi,<sup>a</sup> Aurélien Ribes,<sup>a</sup> Julien Cattiaux,<sup>a</sup> Occitane Barbaux,<sup>a</sup> Yoann Robin,<sup>b</sup> William Dulac,<sup>a</sup>

<sup>a</sup> Météo-France, CNRS, Univ. Toulouse, CNRM, Toulouse, France.

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The screenshot shows a preprint page from the Egusphere platform. At the top, there's a logo for 'EGUsphere' with a blue circular icon. To the right is a menu icon. Below the header is a banner featuring a blue flag with the text 'European Geosciences Union' and a yellow flag. The word 'Preprint' is overlaid on the banner. The main content area has a light gray background. At the top left of this area, it says 'Preprints / Preprint egusphere-2025-1121'. To the right is a search bar with a magnifying glass icon. Below the search bar are three buttons: 'Abstract', 'Discussion', and 'Metrics'. A DOI link 'https://doi.org/10.5194/egusphere-2025-1121' is shown, followed by a note about the Creative Commons Attribution 4.0 License. The date '08 May 2025' is indicated with a small icon.

08 May 2025

## A Bayesian statistical method to estimate the climatology of extreme temperature under multiple scenarios: the ANKIALE package

Yoann Robin, Mathieu Vrac, Aurélien Ribes, Occitane Barbaux,  
and Philippe Naveau