

Problem Set 0

Please write your solutions in the \LaTeX and Python templates provided. Aim for concise solutions; convoluted and obtuse descriptions might receive low marks, even when they are correct.

This assignment is meant to be an evaluation of your **individual** understanding coming into the course and should be completed **without collaboration** or outside help. You **may** ask for logistical help concerning \LaTeX formatting and/or code submission.

Problem 0-1. Let $A = \{i + \binom{5}{i} \mid i \in \mathbb{Z} \text{ and } 0 \leq i \leq 4\}$ and $B = \{3i \mid i \in \{1, 2, 4, 5\}\}$.

Evaluate: (a) $A \cap B$ (b) $|A \cup B|$ (c) $|A - B|$

Problem 0-2. Let X be the random variable representing the number of heads seen after flipping a fair coin three times. Let Y be the random variable representing the outcome of rolling two fair six-sided dice and multiplying their values. Please compute the following expected values.

Evaluate: (a) $E[X]$ (b) $E[Y]$ (c) $E[X + Y]$

Problem 0-3. Let $A = 600/6$ and $B = 60 \bmod 42$. Are these statements True or False?

Evaluate: (a) $A \equiv B \pmod{2}$ (b) $A \equiv B \pmod{3}$ (c) $A \equiv B \pmod{4}$

Problem 0-4. Prove by induction that $\sum_{i=1}^n i^3 = \left[\frac{n(n+1)}{2} \right]^2$, for any integer $n \geq 1$.

Problem 0-5. Prove by induction that every connected undirected graph $G = (V, E)$ for which $|E| = |V| - 1$ is acyclic.

Problem 0-6. An **increasing subarray** of an integer array is any consecutive sequence of array integers whose values strictly increase. Write Python function `count_long_subarrays(A)` which accepts Python Tuple $A = (a_0, a_1, \dots, a_{n-1})$ of $n > 0$ positive integers, and returns the number of longest increasing subarrays of A , i.e., the number of increasing subarrays with length at least as large as every other increasing subarray. For example, if $A = (1, 3, 4, 2, 7, 5, 6, 9, 8)$, your program should return 2 since the maximum length of any increasing subarray of A is three and there are two increasing subarrays with that length: specifically, subarrays $(1, 3, 4)$ and $(5, 6, 9)$. You can download a code template containing some test cases from the website.

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