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Final Project Proposal - Group K

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Introduction

Shared bicycle systems (for example, Taipei's YouBike) offer convenient transportation in metropolitan areas. YouBike has expanded to over $1,000$ stations across Taipei City. However, one of the biggest challenges in operating such systems is the imbalance between supply and demand during peak times, leading to either "no bikes available" or "no docking spaces available." High-demand stations frequently experience bike shortages, whereas popular return stations often run out of docking capacity. This imbalance negatively impacts user experience and reduces overall operational efficiency. Up to now, the system has recorded more than 6 hundred million rides, and even a small proportion of stations being imbalanced can affect thousands of users' commutes.

To address this issue, operators typically dispatch trucks to redistribute bicycles among stations, moving bikes from stations with surplus (where extra docking spaces are available) to those with deficits (where docking spaces are limited). However, determining which stations need bikes, which stations have surplus bikes, and devising efficient truck routing is a highly complex optimization problem. Poor dispatch decisions can lead to wasted resources or missed service opportunities.

Although real-time monitoring technologies have been adopted to better track station usage and accelerate dispatch responses, there remain significant challenges. For instance, the New Taipei City government requires the YouBike operator to establish a daily dispatch plan based on big data analysis and often mobilizes up to 250 dispatch personnel and 40 dispatch trucks to maintain supply-demand balance during peak hours. Still, designing an optimal, or near-optimal, plan for so many stations is extremely difficult when done manually or using heuristic rules.

Given the above background, we propose a mathematical programming approach to address the YouBike redistribution problem. Our objective is to maximize the overall service level by balancing the bicycle counts across stations under limited resources (trucks and labor). The sections that follow describe the problem context and data assumptions, the mathematical formulation in detail, and the expected benefits of the proposed solution.

Problem Description

The essence of the YouBike bicycle dispatching problem is finding a balance between **supply** and **demand**. Users may rent bikes at station i and return them at station j, causing continual changes in bike distribution. Taipei City has around $1,000$ YouBike stations, each accommodating approximately 10–30 bicycles. Without timely adjustments, there is a high risk of station overload or shortage.

Figure 1 shows part of Taipei City's YouBike 2.0 stations on 2025/03/07 13:00. Green dots indicate stations with reasonable availability, yellow dots indicate no bikes to rent, and red dots indicate no available docks for returns.

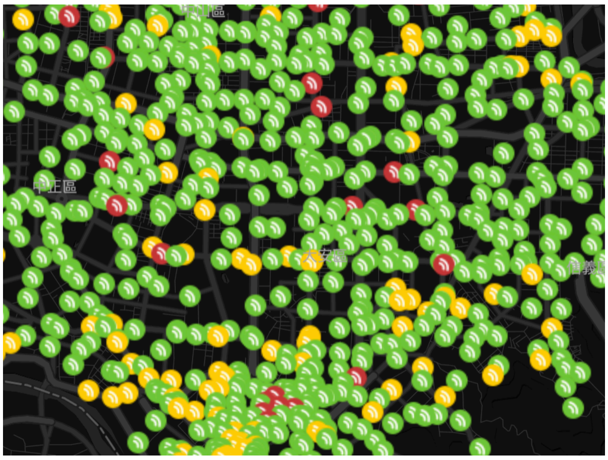

Figure 1: Distribution of some YouBike 2.0 stations.

In this study, we use the open YouBike 2.0 station data in Taipei (see [1]). The relevant API provides snapshots every five minutes, reporting each station's current number of bikes and empty docks, allowing the calculation of the "empty dock ratio."

The operator has a certain number of dispatch trucks and personnel to balance bike availability. Each truck has a maximum capacity and a certain loading/unloading rate. We also have a matrix of travel times between stations (approximated by direct-line distance at 30 km/h for simplicity).

Our goal is to plan the dispatch of these trucks within a specified time window, aiming to maximize the number of stations whose "empty dock ratio" remains in a desirable range 30%–

70% to mitigate the "no bikes" and "no docks" problems. Concretely, the model will schedule trucks to move bicycles among stations during a time window (e.g., 30 minutes) so that, upon completion of the dispatch, as many stations as possible have their bike count within the ideal threshold range. Key steps include:

1. **Determining the adjustment need (how many bikes to move in or out) at each station:** Using the snapshot data, we estimate the required number of bikes to be removed or added to each station to fall within our defined threshold.

2. **Finding optimal truck routing and distribution plan:** Each truck starts from a depot (a designated station or warehouse), collects surplus bikes from some stations, and delivers bikes to stations with deficits.

Constraints include:

- No truck may exceed its capacity.
- The bikes collected from a station cannot exceed that station's available bikes, and bikes delivered cannot exceed the station's spare docks.
- All dispatching must be completed within the designated time window.
- Each truck has a maximum travel distance/time.

Since demand can shift rapidly, we focus on short-term (e.g., 30-minute) dispatch intervals, ensuring that each dispatch cycle reflects the current situation and mitigates bike shortages or full stations.

Mathematical Model

Parameters and Sets

Station set:	$i \in N$
	C_i: total capacity of station i
	B_i: current number of bikes at station i
	$E_i = C_i - B_i$: current empty docks
Truck set:	$k \in K$
	Q: maximum capacity per truck
Time parameters:	d_{ij}: travel time between stations i and j
	L: loading/unloading time per bike
	T: total dispatch time window

Decision Variables

1. **Routing Variables**

   $x_{i,j,k} \in \{0,1\} \quad i,j \in N,\ i \neq j,\ k \in K$: If truck k travels directly from station i to station j, then $x_{i,j,k} = 1$; otherwise, 0. We let node 0 represent the depot.

2. **Bike Collection and Delivery**

   $a_{i,k} \in \mathbb{Z}_{\geq 0} \quad i \in N,\ k \in K$: The number of bikes truck k picks up from station i.

   $b_{i,k} \in \mathbb{Z}_{\geq 0} \quad i \in N,\ k \in K$: The number of bikes truck k drops off at station i.

3. **Station Balance Indicator**

   $y_i \in \{0,1\} \quad i \in N$: If station i is within 30%–70% bike occupancy after dispatch, then $y_i = 1$; otherwise, $y_i = 0$.

4. **Truck Loading State**

   $W_{i,k} \in \mathbb{Z}_{\geq 0} \quad i \in N \cup \{0\},\ k \in K$: Represents the number of bikes carried by truck k upon leaving station i. For the depot (node 0), $W_{0,k} = 0$ initially.

Objective Function

We aim to maximize the number of stations that achieve the balanced state after dispatch:

$$\max \sum_{i \in N} y_i.$$

Constraints

Station Constraints

(a) **Balanced Station Range:**

If station i is counted as balanced, the final number of bikes must lie between 30% and 70% of its capacity:

$$0.3\,C_i - M(1 - y_i) \ \leq\ B_i + \sum_{k \in K} \big(b_{i,k} - a_{i,k}\big) \quad \forall i \in N,$$

$$B_i + \sum_{k \in K} \big(b_{i,k} - a_{i,k}\big) \ \leq\ 0.7\,C_i + M\big(1 - y_i\big) \quad \forall i \in N.$$

Here, M is a sufficiently large constant. We set

$$M = \max_{i \in N} C_i.$$

(b) **Station Capacity:**

$$0 \leq B_i + \sum_{k \in K} (b_{i,k} - a_{i,k}) \leq C_i, \quad \forall i \in N.$$

(c) **Visitation-Operation Consistency:**

$$a_{i,k} \leq Q \sum_{h \in N \cup \{0\}} x_{h,i,k}, \quad b_{i,k} \leq Q \sum_{h \in N \cup \{0\}} x_{h,i,k}, \quad \forall i \in N, \ k \in K.$$

If truck k does not enter station i, then no $a_{i,k}$ or $b_{i,k}$ operations occur at that station. Conversely, if truck k does visit station i, those operations are allowed. Here, Q is the maximum capacity of a single truck, treated as a sufficiently large constant.

Truck Constraints

(a) **Route Continuity**

- *Start and End at Depot*:

$$\sum_{i \in N} x_{0,i,k} = 1, \quad \sum_{i \in N} x_{i,0,k} = 1, \quad \forall k \in K.$$

- *Visitation*:

$$\sum_{h \in N \cup \{0\}} x_{h,i,k} \leq 1, \quad \sum_{j \in N \cup \{0\}} x_{i,j,k} \leq 1, \quad \forall i \in N, \ k \in K.$$

$$\sum_{h \in N \cup \{0\}} x_{h,i,k} = \sum_{j \in N \cup \{0\}} x_{i,j,k}, \quad \forall i \in N, \ k \in K.$$

- *Preventing Subtours*:

$$\sum_{i \in S, j \in S, i \neq j} x_{i,j,k} \leq |S| - 1, \quad \forall S \subseteq N, \ |S| \geq 2, \ \forall k \in K.$$

(b) **Total Time Window**:

$$\sum_{i \in N} \sum_{j \in N} d_{i,j} \, x_{i,j,k} + L \sum_{i \in N} (a_{i,k} + b_{i,k}) \leq T, \quad \forall k \in K.$$

Truck Loading and Capacity Constraints

(a) **Load Flow**:

$$W_{j,k} \geq W_{i,k} + a_{j,k} - b_{j,k} - Q(1 - x_{i,j,k}), \quad \forall i, j \in N, \ i \neq j, \ k \in K,$$
$$W_{j,k} \leq W_{i,k} + a_{j,k} - b_{j,k} + Q(1 - x_{i,j,k}), \quad \forall i, j \in N, \ i \neq j, \ k \in K.$$

If $x_{i,j,k} = 1$, then $W_{j,k}$ is updated accordingly. We initialize $W_{0,k} = 0$ for all $k \in K$.

(b) **Truck Capacity**:

$$0 \le W_{i,k} \le Q, \quad \forall i \in N,\ \forall k \in K.$$

Flow Conservation

$$\sum_{i \in N} a_{i,k} = \sum_{j \in N} b_{j,k}, \quad \forall k \in K.$$

Variable Domains

$$x_{i,j,k} \in \{0,1\}, \quad i \neq j,\ i,j \in N,$$

$$y_i \in \{0,1\},$$

$$a_{i,k},\ b_{i,k} \in \mathbb{Z}_{\geq 0},$$

$$W_{i,k} \in \mathbb{Z}_{\geq 0}.$$

Expected Results

From this model, we anticipate the following outcomes:

- **Dispatch Plan**: The model solution indicates how many bikes need to be relocated from or to each station so that the station's bike count is restored to a stable range.
- **Truck Routing and Schedules**: For each truck, the model provides a detailed route, specifying the sequence of stations visited and the number of bikes loaded or unloaded at each stop (e.g., Truck 1 collects 3 bikes at Station A, delivers 2 bikes at Station B, then proceeds to Station C).
- **Practical Insights and Future Extensions**: We discuss how these results can inform YouBike's dispatch operations. For example, identifying station pairs with frequent surpluses/deficits for periodic rebalancing, or prioritizing key stations under limited resources. We also highlight potential improvements, such as incorporating dynamic updates (real-time data), multi-period strategies, or even crowd-based bike redistribution mechanisms suggested by previous research.

References

[1] Taipei City Government Open Data, `https://citydashboard.taipei/mapview?index=youbike`.