author: Ocean Wong
(Hoi Yeung Wong)
supervisor: Chantal Nobs
Robin Smith
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Organization: Culham Centre for Fusion Energy

**Abstract**

A short document to explain how to choose the most effective set of activation foils for neutron spectrum unfolding. (This document will be expanded upon later, i.e. in the coming month) *Note that in this document, $\sigma$ is used for both microscopic cross-section and standard deviation. When it is appended by subscript(s), (e.g. $\sigma_i$) it is used to denote microscopic cross-section; otherwise it must be followed by a bracketed quantity (e.g. $\sigma(N)$) which denotes the standard deviation of the bracketed quantity (e.g. $N$).*

$$(\phi_{true}) \quad \overset{R}{\Rrightarrow} \tag{1}$$

# 1 Response Matrix For Effectiveness Comparison

The effective response matrix (abbreviation of Response Matrix For Effectiveness Comparison) is simply the microscopic cross-section matrix, $\underline{\underline{\sigma}}$, where each row is scaled up or down by a certain factor.

$$\sigma_{ji} = \text{microscopic cross-section at the} i^{th} \text{bin (column) , for the} j^{th} \text{reaction}$$

When the final number of daughter nuclei $(N_0)_k$ (at measurement time) and its associated error $\sigma((N_0)_k)$ are known, then the response matrix for effectiveness comparison is

$$\underline{\underline{\mathbf{R}}}'_{ki} = [\frac{1}{V_k(N_d)_k}\frac{1}{\Delta t_k}e^{\overline{(t_{dec})_j}^{\,t}}\sigma((N_0)_k)]^{-1}\sigma_{ki} \tag{2}$$

- $V_k$ = the volume of the k-th reaction's parent foil,
- $(N_d)_k$ = the number density of the k-th reaction's parent foil,

$V_k \cdot (N_d)_k$ together refers to the number of parent nuclide present in the foil,

- $\Delta t_k$ = an irradiation factor
- $exp\{-\frac{t}{(t_{dec})_j}\}$ is the decay correction factor
  * $(t_{dec})_j = 1/\ln(2)$ * half-life of the daughter nuclei.
  * $t_0$ = how much time elapsed between the irradiation time and measurement time.

$\frac{1}{\Delta t_k}e^{\overline{(t_{dec})_j}^{\,t}}$ together are (the basic form of) reaction rate correction factors. (More compliated one taking into account the irradiation profile (i.e. the temporal variation in flux) will require FISPACT calculations) At this stage I would advise assuming $\Delta t = 1$, and choosing elements whose half-lives are at least 4 times longer than the irradiation period, such that both can be set to unity.

- $\sigma((N_0)_k)$ = the standard deviation on the error (See),
- $\sigma_{ki}$ = the $k^{th}$ reaction's microscopic cross-section at the $i^{th}$ energy bin.

Therefore the equation above can be reduced to

$$R'_{ki} = [V_k(N_d)_k]\frac{\sigma_{ki}}{\sigma((N_0)_k)} \tag{3}$$

This effective response matrix can then be plotted as a heatmap:

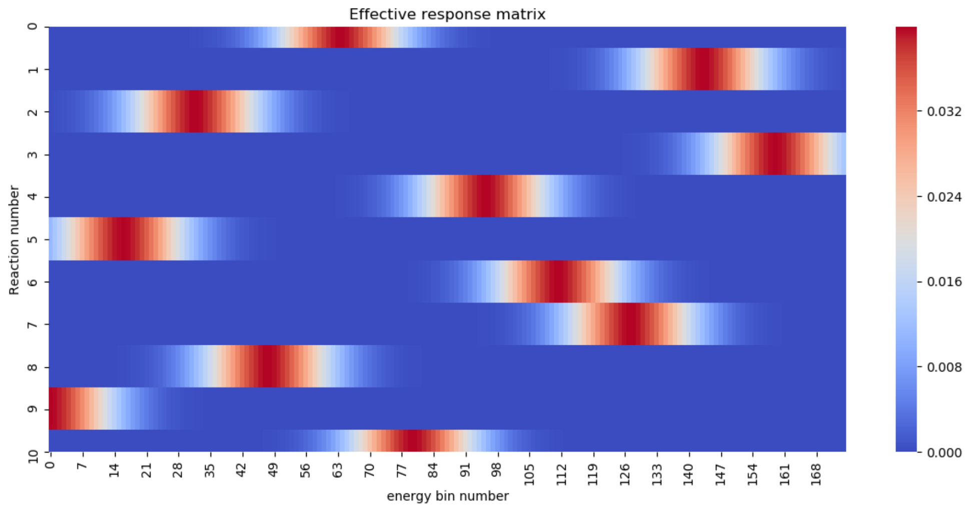


Figure 1: An example effective response matrix plotted as using `sns.heatmap(R)`.

The larger the value of $\sum\limits_k R'_k i$ is, the more intensely the system "samples" that part of the spectrum.

## 1.1 Criteria for a good effective response matrix

A good response matrix should not leave large parts of the spectrum "unsampled" (i.e. have a "cool" column).

Also, if one wishes to compare the difference in fluxes of two regions (e.g.), then one must "sample" these parts of the spectrum using two different reaction. For example, if one wishes to know how many neutrons gets upscatterd/blue-shifted to above bin 169, relative to how many neutrons are present below 169, then there must be at least two separate reactions, e.g. $k = 1$ and $k = 2$, such that $(R'_{1i} - R'_{2i})$ must experience a drastic change in value as $i$ increases from $i < 169$ to $i > 169$.

# 2 Final number of daughter nuclide and its associated error

The final number of daughter nuclide for each neutron induced reaction k = $(N_0)_k$.

This is calculated from the

# 3 Crude foil selection process

When selecting foils, the most obvious parameter to control are the volume of the foils and they

- Reaction with the same parent foil will be scaled up/down together (when the daughter's decay is not accounted for) if one were to increase/decrease its volume. This means these rows, as seen in figure 1, will get hotter/cooler together.

- Reaction rates with the same foil

# 4 Assumption

The FoilSelector currently does not take into account self-shielding effects, i.e. it assumes that the neutron flux $\phi$ is spatially independent, i.e. $\phi(E, V) = \phi(E)$. So that the reaction rate per parent nuclide $Z_k$ is linearly proportional to the average flux and is spatially independent,

$$(N_\infty)_k = \Delta t \iint \sigma_k(E)\phi(E, V)dEdV \tag{4}$$

$$(N_\infty)_k = \Delta t \iint \sigma_k(E)\phi(E)dEdV \tag{5}$$

$$(N_\infty)_k = \Delta t V_k \int \sigma k(E)\phi(E)dE \tag{6}$$

$$(N_\infty)_k = \Delta t V_k Z_k \tag{7}$$

Otherwise, if self-shielding is properly accounted for, 7 is no longer a spatially independent equation.